

How the Hawking effect and prepared states affect Entanglement distillability of Dirac fields

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Abstract

How the Hawking effect and the prepared states influence the entanglement distillability of Dirac fields in the Schwarzschild spacetime is studied by using the Werner states which are composed of the maximum or generic entangled states. It is found that the states are entangled when the parameter of the Werner states, F , satisfies $\tau < F \leq 1$ in which τ is influenced both by the Hawking temperature of the black hole and energy of the fields. It is also shown that although the parameter of the generic entangled states, α , affects the entanglement, it does not change the range of the parameter, F , where the states are entangled for the case of generic entangled states.

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I. INTRODUCTION

Recently, many people have paid their attentions to the investigation of the effects of relativity on quantum information [1–18] because it is important to both the theoretical physics and practical application. The behaviors of entanglement in the noninertial frame were discussed [1–12], and the these studies showed that the entanglement of the states will be degraded for an accelerated observer because the Unruh effect [19] results in a loss of the information. More recently, in order to further investigate characterization of a quantum system Datta [13] and Wang *et al*[14] calculated the quantum discord between two relatively accelerated scalar and Dirac modes respectively. Shahpoor Moradi [15] studied the entanglement distillability of a bipartite mixed states as seen by two relatively accelerated parties. The quantum information in the background of black holes was investigated [16–18]. It was found that the entanglement and fidelity of the teleportation are degraded by the Hawking effect [20]. Eduardo Martín-Martínez *et al* [18] studied the behavior of quantum entanglement near a black hole, and they showed the entanglement is a function of the mass of the black hole, the frequency of the field and the physical distance from the noninertial observer to the horizon. M. Aspachs *et al* [21] analysed the general quantum-statistical grounds the problem of optimal detection of the Unruh-Hawking effect. Downes *et al* [22] investigated the entangling moving cavities in non-inertial frames. And Martin-Martinez *et al* [23] analysed the quantum entanglement produced in the formation of a black hole.

In this paper, we study the entanglement distillability of the Dirac fields in the Schwarzschild spacetime. We focus our attention on how the Hawking effect and prepared states influence the entanglement distillability and separability. In order to illustrate this problem convenient, we take the Werner states which are composed with the maximum or generically entangled states. It should point out here that there are two state parameters (F, α) and different value for these parameters represents different prepared states. We also assume that two observers, Alice and Rob, share the Werner states in flat Minkowski spacetime. Alice moves along a geodesic with a detector which only detects mode k , while Rob hovers outside the event horizon with a uniform acceleration with a detector sensitive only to mode s . With the result of Rob's detects, we want to see whether the characteristic of the entanglement distillability changes.

The outline of the paper is as follows. In Sec. II we discuss the relationship between kruskal vacuum and Schwarzschild vacuum. In Sec. III we investigate the entanglement distillability of the Werner states composed with maximum entangled states. In Sec. IV we discuss the entanglement

distillability of the Werner states composed with generically entangled states. In the last section, we summarize and discuss our conclusions.

II. RELATIONSHIP BETWEEN KRUSKAL VACUUM AND SCHWARZSCHILD VACUUM

The Dirac equation in curve spacetime can be written as [24]

$$[\gamma^a e_a^\mu (\partial_\mu + \Gamma_\mu)]\Psi = 0, \quad (2.1)$$

here γ^a is the Dirac matrix, e_a^μ represents the inverse of the tetrad e_μ^a , and $\Gamma_\mu = \frac{1}{8}[\gamma^a, \gamma^b]e_a^\nu e_{b\nu;\mu}$ is the spin connection coefficient. Hereafter we use $G = c = h = \kappa_B = 1$.

We solve Eq. (2.1) in the Schwarzschild spacetime and get the positive frequency outgoing solutions $\Psi_{k_i}^{I+} \sim e^{-i\omega_i u}$ and $\Psi_{k_i}^{II+} \sim e^{i\omega_i u}$, $\forall i$ [17], where $u = t - r - 2M \ln[(r - 2M)/(2M)]$ and ω_i is a monochromatic frequency of the Dirac field. Since $\Psi_{k_i}^{I+}$ and $\Psi_{k_i}^{II+}$ are analytic outside and inside the event horizon respectively, they form a complete orthogonal family. Thus, we can expand the Dirac field Ψ_{out} as

$$\Psi_{out} = \sum_{i,\sigma} \int dk [a_{k_i}^{(\sigma)} \Psi_{k_i}^{(\sigma)+} + b_{k_i}^{(\sigma)\dagger} \Psi_{k_i}^{(\sigma)-}], \quad (2.2)$$

where $\sigma = (I, II)$, $a_{k_i}^I$ and $b_{k_i}^{I\dagger}$ are the fermion annihilation and antifermion creation operators acting on the state of the exterior region, and $a_{k_i}^{II}$ and $b_{k_i}^{II\dagger}$ are the fermion annihilation and antifermion creation operators acting on the state of the interior region, respectively.

On the other hand, introducing Kruskal coordinates \mathcal{U} and \mathcal{V} for the Schwarzschild spacetime

$$\begin{aligned} u &= -4M \ln\left(-\frac{\mathcal{U}}{4M}\right), & v &= 4M \ln\left(\frac{\mathcal{V}}{4M}\right), & \text{if } r > r_+, \\ u &= -4M \ln\left(\frac{\mathcal{U}}{4M}\right), & v &= 4M \ln\left(\frac{\mathcal{V}}{4M}\right), & \text{if } r < r_+, \end{aligned} \quad (2.3)$$

and making an analytic continuation for $\Psi_{k_i}^{I+}$ and $\Psi_{k_i}^{II+}$, we find a complete basis for positive energy modes which analytic for all real \mathcal{U} and \mathcal{V} according to the suggestion of Damour-Ruffini

$$\begin{aligned} \mathcal{F}_{k_i}^{I+} &= e^{2\pi M \omega_i} \Psi_{k_i}^{I+} + e^{-2\pi M \omega_i} \Psi_{-k_i}^{II-}, \\ \mathcal{F}_{k_i}^{II+} &= e^{-2\pi M \omega_i} \Psi_{-k_i}^{I-} + e^{2\pi M \omega_i} \Psi_{k_i}^{II+}. \end{aligned} \quad (2.4)$$

Thus, we can also expand the Dirac fields in the Kruskal spacetime as

$$\Psi_{out} = \sum_{i,\sigma} \int dk [2 \cosh(4\pi M \omega_i)]^{-1/2} [c_{k_i}^{(\sigma)} \mathcal{F}_{k_i}^{(\sigma)+} + d_{k_i}^{(\sigma)\dagger} \mathcal{F}_{k_i}^{(\sigma)-}], \quad (2.5)$$

where $c_{k_i}^\sigma$ and $d_{k_i}^{\sigma\dagger}$ are the annihilation and creation operators acting on the Kruskal vacuum. It is worth mentioning that there exist an infinite number of combination of Kruskal annihilation operators $c_{k_j}^I = \sum_i C_{ij} c_{k_i}^I$ (where C_{ij} are coefficients) annihilates the same Kruskal vacuum state $|0\rangle_{\mathcal{K}}$ [18,19].

Eqs. (2.2) and (2.5) represent the decomposition of the Dirac fields in Schwarzschild and Kruskal modes respectively, so we can get the Bogoliubov transformations [25] between the Schwarzschild and Kruskal creation and annihilation operators. In consideration of the Bogoliubov relationships being diagonal, each annihilation operator $c_{k_i}^I$ can be expressed as a combination of Schwarzschild particle operators of only one Schwarzschild frequency ω_i [18, 26]

$$c_{k_i}^I = (e^{-8\pi M\omega_i} + 1)^{-\frac{1}{2}} a_{k_i}^I - (e^{8\pi M\omega_i} + 1)^{-\frac{1}{2}} b_{k_i}^{II\dagger}. \quad (2.6)$$

Then the Kruskal vacuum and excited states can express by Schwarzschild Fock space basis $|0\rangle_{\mathcal{K}} = \bigotimes_i |0_{k_i}\rangle_{\mathcal{K}}$ and $|1\rangle_{\mathcal{K}} = \bigotimes_i |1_{k_i}\rangle_{\mathcal{K}}$, where

$$|0_{k_i}\rangle_{\mathcal{K}} = (e^{-\omega_i/T} + 1)^{-\frac{1}{2}} |0_{k_i}\rangle_I |0_{-k_i}\rangle_{II} + (e^{\omega_i/T} + 1)^{-\frac{1}{2}} |1_{k_i}\rangle_I |1_{-k_i}\rangle_{II}, \quad (2.7)$$

$$|1_{k_i}\rangle_{\mathcal{K}} = c_{k_i}^{I\dagger} |0_{k_i}\rangle_{\mathcal{K}} = |1_{k_i}\rangle_I |0_{-k_i}\rangle_{II}, \quad (2.8)$$

where the $|n_{k_i}\rangle_I$ and $|n_{-k_i}\rangle_{II}$ are the orthogonal bases for the inside and outside region of the event horizon respectively, and $T = \frac{1}{8\pi M}$ is the Hawking temperature [27].

III. ENTANGLEMENT DISTILLABILITY FOR WERNER STATES COMPOSED WITH MAXIMUM ENTANGLED STATES

To study the entanglement distillability in the Schwarzschild spacetime, we consider a Dirac field which is, from an inertial perspective, in a special superposition of Kruskal monochromatic modes (see [18, 26] for details) such that, in the Schwarzschild frames, the observers detect the field in the single-mode states. The two modes share the Werner states which can be written as [28]

$$\rho_{AR} = F|\psi^-\rangle_{AR}\langle\psi^-| + \frac{1-F}{3}(|\phi^+\rangle_{AR}\langle\phi^+| + |\phi^-\rangle_{AR}\langle\phi^-| + |\psi^+\rangle_{AR}\langle\psi^+|), \quad (3.1)$$

where F is a parameter (runs from 0 to 1) which gives different prepared states, and $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ are usual entangled Bell states

$$\begin{aligned} |\phi^\pm\rangle_{AR} &= \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_R \pm |1\rangle_A|1_{k_R}\rangle_R), \\ |\psi^\pm\rangle_{AR} &= \frac{1}{\sqrt{2}}(|0\rangle_A|1_{k_R}\rangle_R \pm |1\rangle_A|0\rangle_R), \end{aligned} \quad (3.2)$$

where $|0\rangle_A(R)$ is vacuum states from a inertial viewpoint, $|1_{k_R}\rangle_R$ is a single particle excitation state which characterized by the frequency ω_R observed by Rob, and all other modes of the field are in vacuum state. Hereafter we will refer to the frequency ω_R simply as ω .

We can see from Eq. (3.1) that the Werner states are characterized by the parameter F . In a inertial frame, it was shown that for $F \leq \frac{1}{2}$ the Werner states are separable, while for $\frac{1}{2} \leq F \leq 1$ are entangled [28].

Because Alice is an inertial observer and Rob hovers over the event horizon with a constant acceleration, the states corresponding to modes s must be expanded by the bases for the inside and outside region of the event horizon in order to describe what Rob see. From the discussion in the section II, we know that the Kruskal states $|0\rangle_{\mathcal{K}}$ and $|1\rangle_{\mathcal{K}}$ are correspond to two mode states in the Schwarzschild frame described by Eqs. (2.7) and (2.8), then the density matrix ρ_{AR} takes the form

$$\rho_{A, I, II} = \begin{pmatrix} \frac{1-F}{3(e^{\frac{\omega}{T}}+1)} & 0 & 0 & \frac{(1-F)(e^{-\frac{\omega}{T}}+1)^{-\frac{1}{2}}}{3(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2F+1}{6} & 0 & \frac{(1-4F)}{6(e^{-\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & 0 & \frac{1-4F}{6(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} \\ \frac{(1-F)(e^{-\frac{\omega}{T}}+1)^{-\frac{1}{2}}}{3(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & 0 & \frac{1-F}{3(e^{\frac{\omega}{T}}+1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-4F}{6(e^{-\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & \frac{1+2F}{6(e^{-\frac{\omega}{T}}+1)} & 0 & 0 & \frac{(1+2F)(e^{-\frac{\omega}{T}}+1)^{-\frac{1}{2}}}{6(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1-F}{3} & 0 \\ 0 & 0 & \frac{1-4F}{6(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & \frac{(1+2F)(e^{-\frac{\omega}{T}}+1)^{-\frac{1}{2}}}{6(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & 0 & \frac{1+2F}{6(e^{\frac{\omega}{T}}+1)} \end{pmatrix}, \quad (3.3)$$

where the matrix is written on the basis of $|0\rangle_A|0\rangle_I|0\rangle_{II}$, $|0\rangle_A|0\rangle_I|1\rangle_{II}$, $|0\rangle_A|1\rangle_I|0\rangle_{II}$, $|0\rangle_A|1\rangle_I|1\rangle_{II}$, $|1\rangle_A|0\rangle_I|0\rangle_{II}$, $|1\rangle_A|0\rangle_I|1\rangle_{II}$, $|1\rangle_A|1\rangle_I|0\rangle_{II}$, $|1\rangle_A|1\rangle_I|1\rangle_{II}$. We have to trace over the states in this region because Rob is causally disconnected from the interior region of the black hole. Thus, we get

the mixed density matrix between Alice and Rob

$$\rho_{A,I} = \begin{pmatrix} \frac{1-F}{3(e^{-\frac{\omega}{T}}+1)} & 0 & 0 & 0 \\ 0 & \frac{1+2F}{6} + \frac{1-F}{3(e^{\frac{\omega}{T}}+1)} & \frac{(1-4F)}{6(e^{-\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 \\ 0 & \frac{(1-4F)}{6(e^{-\frac{\omega}{T}}+1)^{\frac{1}{2}}} & \frac{1+2F}{6(e^{\frac{\omega}{T}}+1)} & 0 \\ 0 & 0 & 0 & \frac{1-F}{3} + \frac{1+2F}{6(e^{\frac{\omega}{T}}+1)} \end{pmatrix}, \quad (3.4)$$

where the matrix is written on the basis of $|0\rangle_A|0\rangle_I$, $|0\rangle_A|1\rangle_I$, $|1\rangle_A|0\rangle_I$, $|1\rangle_A|1\rangle_I$. To determine whether the mixed states are the entanglement states or not, we use the partial transpose criterion [29] which states that the states are entanglement one if one eigenvalue of the partial transpose is negative at least. It worth to mention that the states with positive partial transpose density matrix have no distillable entanglement, but may have nondistillable entanglement if the dimension is larger than 2. We find the eigenvalues of the partial transpose of ρ_{AR} are

$$\lambda^{(1)} = \frac{1+2F}{6(e^{-\frac{\omega}{T}}+1)}, \quad (3.5)$$

$$\lambda^{(2)} = \frac{1+2F}{6} + \frac{1-F}{3(e^{\frac{\omega}{T}}+1)}, \quad (3.6)$$

$$\lambda^{(3)} = \frac{1-F}{3} + \frac{4F-1}{12} \left(\frac{1}{e^{\frac{\omega}{T}}+1} + 2\sqrt{\frac{1}{e^{-\frac{\omega}{T}}+1} + \frac{3}{2(e^{\frac{\omega}{T}}+1)(4F-1)}} \right), \quad (3.7)$$

$$\lambda^{(4)} = \frac{1-F}{3} + \frac{4F-1}{12} \left(\frac{1}{e^{\frac{\omega}{T}}+1} - 2\sqrt{\frac{1}{e^{-\frac{\omega}{T}}+1} + \frac{3}{2(e^{\frac{\omega}{T}}+1)(4F-1)}} \right). \quad (3.8)$$

It is clearly that $\lambda^{(1)}$ $\lambda^{(2)}$ and $\lambda^{(3)}$ are positive but $\lambda^{(4)}$ is negative when $F > \tau$ with $\tau = \frac{3e^{\omega/T}+5}{6e^{\omega/T}+8}$.

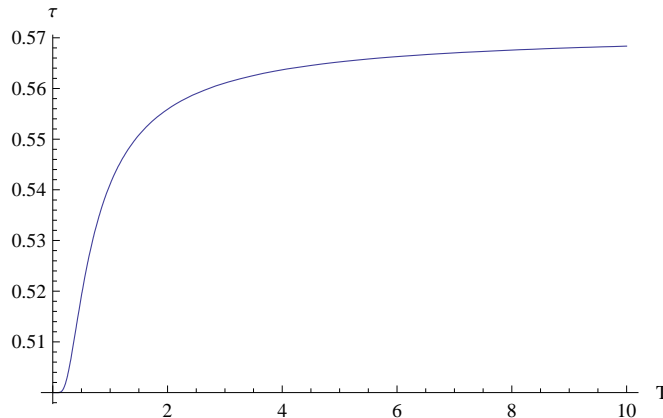


FIG. 1: Plot of $\tau = \frac{3e^{\omega/T}+5}{6e^{\omega/T}+8}$ as a function of the Hawking temperature (we take $\omega = 1$.)

From these eigenvalues we know that the states are entangled for $\tau < F \leq 1$ in the Schwarzschild spacetime, which is different from that $1/2 < F \leq 1$ in the inertial frame. It is

obvious that the Hawking temperature T and energy ω of the states play an important role in the low boundary τ .

In Fig. 1 we study how the Hawking temperature T changes τ , and see that τ increases with the increases of the temperature T . If the Hawking temperature becomes zero, $T = 0$, we have $\tau = 1/2$. Thus, for a system with zero temperature the states are entangled for $1/2 < F \leq 1$, which is the same as that in the inertial frame. But in the limit of $T \rightarrow \infty$, we get $\tau = 0.57$. That is to say, when $T \rightarrow \infty$ the states are entangled for $0.57 < F \leq 1$.

In Fig. 2 we plot τ as a function of ω . We learn from the figure that, unlike in inertial frame, the energy of the states affects the distillability of the entanglement. It is shown that τ decrease with the increase of ω . In particular, $\tau = 0.57$ as $\omega \rightarrow 0$ and $\tau = 1/2$ as $\omega \rightarrow \infty$.

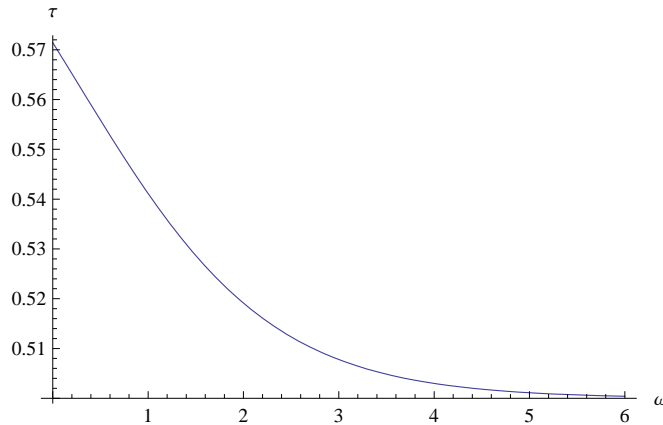


FIG. 2: Plot of $\tau = \frac{3e^{\omega/T} + 5}{6e^{\omega/T} + 8}$ as a function of the energy ω (we take $T = 1$.)

IV. ENTANGLEMENT DISTILLABILITY FOR WERNER STATES COMPOSED WITH GENERICALLY ENTANGLED STATES

We now consider the Werner states which are composed with generically entangled states

$$\begin{aligned} |\phi^\pm\rangle_{AR} &= \alpha|0\rangle_A|0\rangle_R \pm \sqrt{1-\alpha^2}|1\rangle_A|1_{k_R}\rangle_R, \\ |\psi^\pm\rangle_{AR} &= \alpha|0\rangle_A|1_{k_R}\rangle_R \pm \sqrt{1-\alpha^2}|1\rangle_A|0\rangle_R, \end{aligned} \quad (4.1)$$

where α is a real number that satisfies $0 < \alpha < 1$, and α and $\sqrt{1-\alpha^2}$ are the so-called normalized partners.

The main purpose of this section is to see whether the parameter α influences the entanglement distillability. Using Eqs. (2.7) and (2.8) we obtain the following density matrix ρ_{AR}

$$\rho_{A,II} = \begin{pmatrix} \frac{2\alpha^2(1-F)}{3(e^{\frac{-\omega}{T}}+1)} & 0 & 0 & \frac{2\alpha^2(1-F)A}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha^2(2F+1)}{3} & 0 & \frac{\alpha\sqrt{1-\alpha^2}(1-4F)}{3(e^{\frac{-\omega}{T}}+1)^{\frac{1}{2}}} & 0 & 0 & \frac{\alpha\sqrt{1-\alpha^2}(1-4F)}{3(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} \\ \frac{2\alpha^2(1-F)A}{3} & 0 & 0 & \frac{\alpha^2(2-2F)}{3(e^{\frac{\omega}{T}}+1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha\sqrt{1-\alpha^2}(1-4F)}{3(e^{\frac{-\omega}{T}}+1)^{\frac{1}{2}}} & 0 & \frac{(1-\alpha^2)(1+2F)}{3(e^{\frac{-\omega}{T}}+1)} & 0 & 0 & \frac{(1-\alpha^2)(1+2F)A}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2(1-\alpha^2)(1-F)}{3} & 0 \\ 0 & 0 & \frac{\alpha\sqrt{1-\alpha^2}(1-4F)}{3(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}} & 0 & \frac{(1-\alpha^2)(1+2F)A}{3} & 0 & 0 & \frac{(1-\alpha^2)(1+2F)}{3(e^{\frac{\omega}{T}}+1)} \end{pmatrix}, \quad (4.2)$$

where $A = \frac{(e^{\frac{-\omega}{T}}+1)^{\frac{1}{2}}}{(e^{\frac{\omega}{T}}+1)^{\frac{1}{2}}}$. Tracing over the modes in the region II we find the density matrix between Alice and Rob

$$\rho_{A,I} = \begin{pmatrix} \frac{2\alpha^2(1-F)}{3(e^{\frac{-\omega}{T}}+1)} & 0 & 0 & 0 \\ 0 & \frac{(1+2F)\alpha^2}{3} + \frac{2\alpha^2(1-F)}{3(e^{\frac{\omega}{T}}+1)} & \frac{\alpha\sqrt{1-\alpha^2}(1-4F)}{3(e^{\frac{-\omega}{T}}+1)^{\frac{1}{2}}} & 0 \\ 0 & \frac{\alpha\sqrt{1-\alpha^2}(1-4F)}{3(e^{\frac{-\omega}{T}}+1)^{\frac{1}{2}}} & \frac{(1-\alpha^2)(1+2F)}{3(e^{\frac{-\omega}{T}}+1)} & 0 \\ 0 & 0 & 0 & \frac{2(1-\alpha^2)(1-F)}{3} + \frac{(1-\alpha^2)(1+2F)}{3(e^{\frac{\omega}{T}}+1)} \end{pmatrix}. \quad (4.3)$$

To show how α affects the entanglement, we use the logarithmic negativity which is defined as $N(\rho_{A,I}) = \log_2 \|\rho_{A,I}^T\|$. $N(\rho_{A,I})$ as a function of the parameters F and α is plotted in Fig. 3.

The eigenvalues of the partial transpose of the $\rho_{A,I}$ are

$$\lambda^{(1)} = \frac{(1+2F)\alpha^2}{3} + \frac{2\alpha^2(1-F)}{3(e^{\frac{\omega}{T}}+1)}, \quad (4.4)$$

$$\lambda^{(2)} = \frac{(1-\alpha^2)(1+2F)}{3(e^{\frac{-\omega}{T}}+1)}, \quad (4.5)$$

$$\lambda^{(3)} = \frac{1}{6}[3 - 3\alpha^2 + B^2(-2F - 1 + 3\alpha^2) + \sqrt{\xi}], \quad (4.6)$$

$$\lambda^{(4)} = \frac{1}{6}[3 - 3\alpha^2 + B^2(-2F - 1 + 3\alpha^2) - \sqrt{\xi}], \quad (4.7)$$

where $\xi = 9(\alpha^2 - 1)^2 - 2B^2(\alpha^2 - 1)[(4F - 1)(8F + 1)\alpha^2 - 6F - 3] + B^4[1 + \alpha^2 + F(2 - 4\alpha^2)]^2$ and $B = (e^{\frac{-\omega}{T}} + 1)^{\frac{1}{2}}$. The eigenvalue $\lambda^{(4)}$ is negative when $F > \sigma$ with

$$\sigma = \frac{3e^{\omega/T} + 5}{6e^{\omega/T} + 8}. \quad (4.8)$$

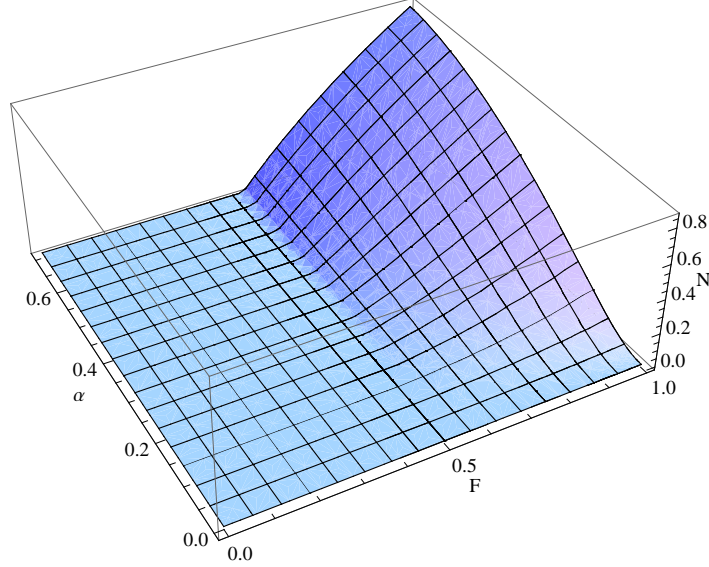


FIG. 3: Plot of logarithmic negativity N as a function of the parameter F and α with $\omega/T = 3/2$.

This implies that the states are entangled for $\sigma < F \leq 1$, which shows that the low boundary σ is independent of α and is equal to the boundary τ in the section III. Thus, we find an interesting result that although the parameter α affects the entanglement [16], it does not change the range of the parameter F where the states are entangled.

V. SUMMARY

We have investigated how the Hawking effect and the prepared states affect the entanglement distillability of the Dirac fields in the Schwarzschild spacetime using the Werner states which are composed with the maximum or generically entangled states. For the Werner states composed with the maximum entangled states, we have found that the states are entangled for $\tau < F \leq 1$ in the Schwarzschild spacetime. The relation $\tau = \frac{3e^{\omega/T} + 5}{6e^{\omega/T} + 8}$ shows that τ is influenced by the Hawking temperature T of the black hole and energy ω of the states. For a system with zero temperature the states are entangled for $1/2 < F \leq 1$, which is the same as that in the inertial frame. But the states are entangled for $0.57 < F \leq 1$ when when $T \rightarrow \infty$. On the other hand, unlike in an inertial frame, τ reduces with the increase of the energy of the states in the Schwarzschild spacetime. For the Werner states composed with the generically entangled states, we have shown that although the parameter α affects the entanglement, it does not change the range of the parameter F where the states are entangled. The results will help us to understand the information of black holes more

clearly.

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