Light quark mass dependence of the D and D_s decay constants

L.S. Geng,[1,2] M. Altenbuchinger[2] and W. Weise[2]

[1]*School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China*

[2]*Physik Department, Technische Universität München, D-85747 Garching, Germany*

(■Dated: November 7, 2018)

Abstract

We study the light quark mass dependence of the D and D_s meson decay constants, f_D and f_{D_s}, using a covariant formulation of chiral perturbation theory (χPT) at next-to-next-to-leading order (NNLO). Using the HPQCD lattice results for the $D(D_s)$ decay constants as a benchmark we show that covariant χPT can describe the HPQCD results better than heavy meson χPT (HMχPT) at both NLO and NNLO. Within the same framework, taking into account sub-leading ($1/m_Q$, with m_Q the heavy quark mass) corrections to the values of the low-energy constants and employing the lattice QCD results for $g_{BB^*\pi}$, we estimate the ratio of f_{B_s}/f_B to be $1.22^{+0.05}_{-0.04}$, which agrees well with the HPQCD result $1.226(26)$.

PACS numbers: 12.39.Fe, 13.20.Fc, 14.40.Lb, 12.38.Gc

I. INTRODUCTION

The decay constants of charged pseudoscalar mesons $\pi^\pm$, $K^\pm$, $D^\pm$, $D_s^\pm$ and $B^\pm$ play an important role in our understanding of strong interaction physics, e.g., in measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and in the search for signals of physics beyond the standard model (SM). At lowest order, the decay width of a charged pseudoscalar $P^\pm$ with valence quark content $q_1\bar{q}_2$ decaying into a charged lepton pair ($\ell^\pm\nu_\ell$) via a virtual $W^\pm$ meson is given by

$$\Gamma(P^\pm \to \ell^\pm\nu_\ell) = \frac{G_F^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right) |V_{q_1q_2}|^2, \tag{1}$$

where m_ℓ is the $\ell^\pm$ mass, $|V_{q_1q_2}|$ is the CKM matrix element between the constituent quarks $q_1\bar{q}_2$ in $P^\pm$, and G_F is the Fermi constant. The parameter f_P is the decay constant, related to the wave function overlap of the $q_1\bar{q}_2$ pair. Measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1q_2} f_P|$. A good knowledge of the value of either $|V_{q_1q_2}|$ or f_P can then be used to determine the value of the other.

These decay constants can be accessed both experimentally and through lattice Quantum Chromodynamics (lQCD) simulations. While for f_π, f_K, f_D, experimental measurements agree well with lattice QCD calculations, a discrepancy is seen for the value of f_{D_s}: The 2008 PDG average for f_{D_s} is 273 ± 10 MeV [1], about 3σ larger than the most precise $N_f = 2+1$ lQCD result from the HPQCD/UKQCD collaboration [2], 241 ± 3 MeV. On the other hand, experiments and lQCD calculations agree very well with each other on the value of f_D, $f_D(\mathrm{expt}) = 205.8 \pm 8.9$ MeV and $f_D(\mathrm{lQCD}) = 207 \pm 4$ MeV. The discrepancy concerning f_{D_s} is quite puzzling because whatever systematic errors have affected the lQCD calculation of f_D, they should also be expected for the calculation of f_{D_s}. In this context, constraints imposed by this discrepancy on new physics were seriously discussed (see, e.g., Ref. [3]).

However, the situation has changed recently. With the new (updated) data from CLEO [4–6] and Babar [7], together with the Belle measurement [8], the latest PDG average is $f_{D_s} = 257.5 \pm 6.1$ MeV [9][1]. The discrepancy is reduced to 2.4σ. Lately the HPQCD collaboration has also updated its study of the D_s decay constant [11]. By including additional results at smaller lattice spacing along with improved determinations of the lattice spacing and improved tuning of the charm and strange quark masses, a new value for the D_s decay constant has been reported[2]:

[1] The October 2010 average from the Heavy Flavor Averaging Group (HFAG) is similar: $f_{D_s} = 257.3 \pm 5.3$ MeV [10].

[2] A slightly different but less precise value of $f_{D_s} = 250.2 \pm 3.6$ MeV was obtained in Ref. [12] as a byproduct from the study of the $D \to K, \ell\nu$ semileptonic decay scalar form factor by the same collaboration.

$f_{D_s} = 248.0 \pm 2.5$ MeV. With the updated results from both the experimental side and the HPQCD collaboration, the window for possible new physics in this quantity is significantly reduced [11].

An important part of the uncertainties in heavy quark lQCD simulations comes from chiral extrapolations that are needed in order to extrapolate lQCD simulations, performed with larger-than-physical light quark masses, down to the physical point. Recent lQCD studies of the D (D_s) decay constants, both for $N_f = 2 + 1$ [2, 13] and $N_f = 2$ [14], have adopted the one-loop heavy-meson chiral perturbation theory (HMχPT) (including its partially-quenched and staggered counterparts) to perform chiral extrapolations. In particular, the HPQCD collaboration has used the standard continuum chiral expansions through first order but augmented by second- and third-order polynomial terms in $x_q = B_0 m_q / 8(\pi f_\pi)^2$ where $B_0 \equiv m_\pi^2/(m_u + m_d)$ to leading order in χPT, arguing that the polynomial terms are required by the precision of the data. It is clear that the NLO HMχPT alone fails to describe its data.

HMχPT [15–17] has been widely employed not only in extrapolating lQCD simulations but also in phenomenology studies and has been remarkably successful over the decades (see Ref. [18] for a partial review of early applications). In Ref. [19], we have argued that a covariant formulation of χPT may be a better choice for studying heavy-meson phenomenology and lQCD simulations. This was based on the observation that the counterpart in the SU(3) baryon sector, heavy baryon χPT, converges very slowly and often fails to describe both phenomenology and lattice data (particularly the latter), e.g., in the description of the lattice data for the masses of the lowest-lying baryons [20, 21]. On the other hand, covariant baryon χPT was shown to provide a much improved description of the same data [22]. Indeed, in Ref. [19] we have shown that for the scattering lengths of light pseudoscalar mesons interacting with D mesons, recoil corrections are non-negligible. Given the important role played by f_D (f_{D_s}) in our understanding of strong-interaction physics and the importance of chiral extrapolations in lQCD simulations, it is timely to examine how covariant χPT works in conjunction with the HPQCD f_D (f_{D_s}) data.

In this letter we study the light quark mass dependence of the HPQCD f_D and f_{D_s} results [2][3] using a covariant formulation of χPT. It is not our purpose to reanalyze the raw lQCD data because the HPQCD collaboration has performed a comprehensive study. Repeating such a process using a different formulation of χPT will not likely yield any significantly different results. Instead, we will focus on their final results in the continuum limit as a function of m_q/m_s, with m_q the

[3] Although the HPQCD collaboration has updated its study of the f_{D_s} decay constant, it has not done the same for the f_D decay constant, and therefore its f_{D_s}/f_D ratio remains the same but with a slightly larger uncertainty. For our purposes, it is enough to study the HPQCD 2007 data [2].

average of up and down quark masses and m_s the strange quark mass. These results can be treated as quasi-original lattice data because, for chiral extrapolations, the HPQCD collaboration has used the NLO HMχPT result plus two polynomials of higher chiral order. Therefore, any inadequacy of the NLO HMχPT should have been remedied by fine-tuning the two polynomials. Accordingly the extrapolations should be reliable, apart from the fact that the connection with an order-by-order χPT analysis is lost. Our present work tries to close this gap. Using the HPQCD continuum limits as a benchmark instead of the raw data not only greatly simplifies our analysis but also highlights the most important point we wish to make, namely that the covariant formulation of χPT is more suitable for chiral extrapolations of lQCD data than the HMχPT, at least in the present case.

This paper is organized as follows. In Section II, we introduce the relevant effective chiral Lagrangians and calculate the Feynman diagrams contributing to the $D(D_s)$ decay constants up to NNLO. In Section III, we show the numerical results and compare them with those of the HMχPT. We also estimate the ratio of f_{B_s}/f_B using the values of the low-energy constants (LECs) fixed in the present study and employing the lattice results for $g_{BB^*\pi}$. A short summary follows in Section IV.

II. THEORETICAL FRAMEWORK

The decay constants of heavy-light pseudoscalar and vector mesons with quark content $\bar{q}Q$, with q one of the u, d, and s quarks and Q either the c or b quark, are defined by

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 Q(0)|P_q(p)\rangle = -if_{P_q}p^\mu, \tag{2}$$

$$\langle 0|\bar{q}\gamma^\mu Q(0)|P_q^*(p,\epsilon)\rangle = f_{P_q^*}\epsilon^\mu, \tag{3}$$

where P_q denotes a pseudoscalar meson and P_q^* a vector meson. In this convention, f_{P_q} has mass dimension one and $f_{P_q^*}$ has mass dimension two [23]. From now on, we concentrate on the charm sector, D, D_s, D^*, and D_s^*. The formalism can easily be extended to the bottom sector.

The coupling of the D (D_s) mesons to the vacuum or to Nambu-Goldstone bosons through the left-handed current is described by the following leading chiral order Lagrangian:

$$\mathcal{L}^{(1)}_{\rm source} = a\langle (c' P^*_\mu - \frac{\partial_\mu P}{m_P})u^\dagger\rangle, \tag{4}$$

where a is a normalization constant with mass dimension two, $P = (D^0, D^+, D_s^+)$, $P^*_\mu = (D^{*0}, D^{*+}, D_s^*)$, m_P is the characteristic mass of the P triplet introduced to conserve heavy

quark spin symmetry in the $m_Q \to \infty$ limit, i.e., $\mathring{m}_D$ at NLO and m_D at NNLO (see Table 1), and $u^2 = U = \exp[\frac{i\Phi}{F_0}]$ with Φ the pseudoscalar octet matrix and F_0 their decay constant in the chiral limit. We have introduced a dimensionless coefficient c' to distinguish the vector and pseudoscalar fields, which is 1 if heavy quark spin symmetry is exact. We need to stress that in our covariant formulation of χPT we do not keep track of explicit $1/m_Q$ corrections that break heavy quark spin and flavor symmetry, instead we focus on SU(3) breaking. This implies that different couplings have to be used for $D(D^*)$ and $B(B^*)$ mesons. In the present work we only need to make such a differentiation in calculating diagram Fig. (1d). In Eq. (4) we have therefore explicitly pointed out that c' may be different from 1. In all the other places, we will simply set c' equal to 1.

The leading-order (LO) SU(3) breaking of the D meson decay constants is described by the following next-to-leading order (NLO) chiral Lagrangians

$$\mathcal{L}^{(3)} = -\frac{a}{16\pi^2 F_0^2}\left[b_D\langle(P_\mu^* - \frac{\partial_\mu P}{m_P})\mathcal{M}\rangle + b_A\langle P_\mu^* - \frac{\partial_\mu P}{m_P}\rangle\langle\mathcal{M}\rangle\right], \tag{5}$$

where b_D and b_A are two LECs and $\mathcal{M} = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$. Here and in the following $\langle\cdots\rangle$ always denotes the trace in the corresponding flavor space.

To study the NLO SU(3) breaking, one has to take into account the DD^* ($D_sD_s^*$) and DD_s ($D^*D_s^*$) mass splittings. Experimentally the DD^* and $D_sD_s^*$ splittings are similar:

$$\Delta_{DD^*} = 141.4\,\mathrm{MeV} \quad \text{and} \quad \Delta_{D_sD_s^*} = 143.8\,\mathrm{MeV}. \tag{6}$$

Therefore in our calculation we will take an average of these two splittings, i.e., $\Delta = (\Delta_{DD^*} + \Delta_{D_sD_s^*})/2 = 142.6\,\mathrm{MeV}$. It should be noted that the DD^* mass splitting is of sub-leading order in the $1/m_Q$ expansion of heavy quark effective theory. The numbers above show that SU(3) breaking of this quantity is less than 2%. The mass splitting in principle can also depend on the light quark masses but we expect that the dependence of this "hyperfine" splitting should be much weaker than that of the D mass[4], m_D, which we discuss below.

At NLO, the following Lagrangian is responsible for generating SU(3) breaking between the D and D_s masses [19]:

$$\mathcal{L}^{(2)} = -2c_0\langle PP^\dagger\rangle\langle\chi_+\rangle + 2c_1\langle P\chi_+P^\dagger\rangle, \tag{7}$$

which yields

$$m_D^2 = m_0^2 + 4c_0(m_\pi^2 + 2m_K^2) - 4c_1m_\pi^2, \tag{8}$$

[4] This seems to be supported by quenched lQCD calculations, see, e.g., Refs. [24, 25].

$$m_{D_s}^2 = m_0^2 + 4c_0(m_\pi^2 + 2m_K^2) + 4c_1(m_\pi^2 - 2m_K^2). \quad (9)$$

One may implement this mass splitting in two different ways by either using the HPQCD continuum limits on the D and D_s masses [2] to fix the three LECs: m_0, c_0, and c_1, or taking into account only the DD_s mass splitting

$$-8c_1(m_K^2 - m_\pi^2) = (m_{D_s}^2 - m_D^2 + m_{D_s^*}^2 - m_{D^*}^2)/2 = \Delta_s(m_D + m_{D_s} + m_{D^*} + m_{D_s^*})/2, \quad (10)$$

where we have introduced $\Delta_s \equiv m_{D_s} - m_D \approx m_{D_s^*} - m_{D^*} \approx (m_{D_s} - m_D + m_{D_s^*} - m_{D^*})/2$. In the second approach, using the experimental data for m_D, m_{D_s}, m_{D^*}, and $m_{D_s^*}$, one obtains $c_1 = -0.225$. We found that the HPQCD continuum limits on the D and D^* masses can be described very well using Eqs. (8,9). We also found that using Eqs. (8,9) or Eq. (10) gives very similar results in our analysis of the D (D_s) decay constants. The results shown below are obtained using Eq. (10) to implement the SU(3) breaking and light quark mass evolution of the D (D_s) masses.

In order to calculate loop diagrams contributing to the decay constants one needs to know the coupling, $g_{DD^*\phi}$, with ϕ denoting a Nambu-Goldstone boson. This is provided at the leading chiral order by the following Lagrangian [19]:

$$\mathcal{L}^{(1)} = ig\mathring{m}_D\langle P_\mu^* u^\mu P - P u^\mu P_\mu^{*\dagger}\rangle \quad (11)$$

where $u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger)$ and $\mathring{m}_D = 1972.1$ MeV, the average of D, D_s, D^*, and D_s^* masses. The coupling g can be determined from the $D^{*+} \to D^0\pi^+$ decay width, which yields $g_{DD^*\pi} = 0.60 \pm 0.07$ [19]. At the chiral order we are working, one can take $g_{DD^*\phi} = g_{DD^*\pi}$. If heavy quark flavor symmetry is exact, we expect $g_{BB^*\pi} = g_{DD^*\pi}$. Otherwise deviations are expected. We will come back to this later.

TABLE I. Numerical values of (isospin-averaged) masses [9] and decay constants (in units of MeV) used in the present study. The eta meson mass is calculated using the Gell-Mann-Okubo mass relation: $m_\eta^2 = (4m_K^2 - m_\pi^2)/3$.

$\mathring{m}_D$	m_D	Δ_s	Δ	m_B	$\Delta_s(B)$	$\Delta(B)$	m_π	m_K	m_η	f_π	F_0
1972.1	1867.2	102.5	142.6	5279.3	88.7	47.5	138.0	495.6	566.7	92.4	$1.15f_\pi$

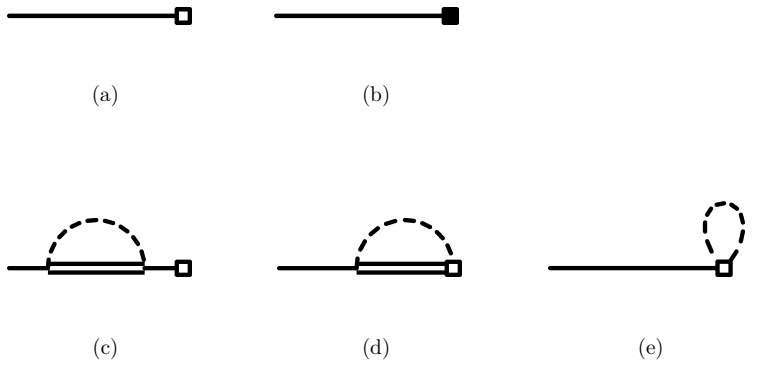

FIG. 1. Feynman diagrams contributing to the calculation of f_D and f_{D_s} up to next-to-leading order (NLO): (a) and (b) are LO and NLO tree level diagrams, loop diagrams (c), (d), and (e) contribute at NLO.

Up to NLO, the $D(D_s)$ decay constants receive contributions from the Feynman diagrams shown in Fig. 1. Studies of these decay constants within the framework of HMχPT have a long history [26–28]. Here we are going to present the first covariant χPT calculation. Insertion of the mass splittings between D, D_s, D^*, and D_s^* in the loop diagrams shown in Fig. 1 generates the NNLO contributions which are implemented in the present case by making the following replacements in the NLO results:

$$m_{D_s} \to m_D + \Delta_s, \quad m_{D^*} \to m_D + \Delta, \quad \text{and} \quad m_{D_s^*} \to m_D + \Delta + \Delta_s, \tag{12}$$

with the values of these quantities given in Table I. It should be noted that there is no new counterterm appearing at $\mathcal{O}(p^4)$.

Computation of the tree-level diagrams Figs. (1a,1b) is trivial. Fig. (1a) gives $\hat{a} = a/m_p$ with mass dimension one for both D and D_s . Fig. (1b) yields

$$\delta_1 = \hat{a}\left[-\frac{1}{16\pi^2 F_0^2}\left(b_A(2m_K^2 + m_\pi^2) + b_D m_\pi^2\right)\right], \tag{13}$$

$$\delta_2 = \hat{a}\left[-\frac{1}{16\pi^2 F_0^2}\left(b_A(2m_K^2 + m_\pi^2) + b_D(2m_K^2 - m_\pi^2)\right)\right], \tag{14}$$

where δ_1 is for D and δ_2 for D_s.

Diagram Fig. (1c) is the wave function renormalization, from which one can calculate the wave function renormalization constants, which can be written as

$$Z_i = \sum_{j,k} \xi_{i,j,k} \frac{d\,\phi_w(p_i^2, m_j^2, m_k^2)}{d\,p_i^2}|_{p_i^2 = m_i^2}, \tag{15}$$

where p_i denotes the four-momentum of D (D_s), m_i the mass of D (D_s), m_j the mass of D^* (D_s^*), and m_k the mass of π, η, and K. The coefficients $\xi_{i,j,k}$ are given in Table II. The function ϕ_w is

defined as[5]

$$\phi_w(p_i^2, m_V^2, m_M^2) = \frac{(g\mathring{m}_D)^2}{4F_0^2 m_V^2}\Big[\left(-2m_M^2\left(p_i^2+m_V^2\right)+\left(m_V^2-p_i^2\right)^2+m_M^4\right)B_0\left(p_i^2,m_M^2,m_V^2\right)$$
$$+A_0\left(m_V^2\right)\left(-p_i^2+m_M^2-m_V^2\right)+A_0\left(m_M^2\right)\left(-p_i^2+3m_M^2+m_V^2\right)\Big], \quad (16)$$

where the functions A_0 and B_0 are defined in the Appendix.

Diagram Fig. (1d) provides current renormalization, which has the following form

$$C_i = \hat{a}c' \sum_{j,k} \xi_{i,j,k}\phi_c(m_i^2, m_j^2, m_k^2), \quad (17)$$

where $\xi_{i,j,k}$ are given in Table II with i running over D and D_s, j over D^* and D_s^*, and k over π, η, K. The function ϕ_c is defined as

$$\phi_c(m_i^2, m_V^2, m_M^2) = -\frac{(g\mathring{m}_D)m_P}{8F_0^2 m_i^2 m_V^2}\Big[\left(m_M^2-m_V^2\right)\left(m_i^2-m_M^2+m_V^2\right)B_0\left(0,m_M^2,m_V^2\right)-2m_i^2A_0\left(m_M^2\right)$$
$$+\left(-2m_i^2\left(m_M^2+m_V^2\right)+m_i^4+\left(m_M^2-m_V^2\right)^2\right)B_0\left(m_i^2,m_M^2,m_V^2\right)\Big]. \quad (18)$$

It should be noted that C_i vanishes in NLO HMχPT but plays an important role in covariant χPT.

Diagram Fig. (1e) also provides current correction

$$T_i = \hat{a}\sum_{j=\pi,\eta,K}\zeta_{i,j}A_0(m_j^2)/F_0^2, \quad (19)$$

with $\zeta_{i,j}$ given in Table III.

TABLE II. Coefficients, $\xi_{i,j,k}$, appearing in Eqs. (15,17).

	D^*			D_s^*		
	π	η	K	π	η	K
D	3	$\frac{1}{3}$	0	0	0	2
D_s	0	0	4	0	$\frac{4}{3}$	0

[5] To be consistent, the product $g\mathring{m}_D$ is only appropriate for NLO. At NNLO, it has to be replaced by $g'm_D$ with $g' = g\mathring{m}_D/m_D \approx 0.63$ before performing expansions in terms of $1/m_D$ either to obtain the HMχPT results or to remove the power-counting-breaking pieces. The same applies to the calculation of C_i [see Eq. (17)].

TABLE III. Coefficients, $\zeta_{i,j}$, appearing in Eq. (19).

	π	η	K
D	$-\frac{3}{8}$	$-\frac{1}{24}$	$-\frac{1}{4}$
D_s	0	$-\frac{1}{6}$	$-\frac{1}{2}$

The total results are then

$$f_i = \hat{a}(1 + Z_i/2) + \delta_i + T_i + C_i. \tag{20}$$

Because of the large D meson masses, C_i and Z_i contain so-called power-counting-breaking (PCB) terms. As explained in detail in Ref. [19] one can simply expand these functions in terms of $1/\mathring{m}_D$ at NLO or $1/m_D$ at NNLO and then remove the PCB pieces. This procedure is in fact the same as the extended-on-mass-shell (EOMS) scheme. This scheme was first developed for baryon chiral perturbation theory [29, 30] and has been shown to be superior to heavy baryon χPT in a number of cases, see, e.g., Refs. [22, 31, 32]. With the full results for C_i and Z_i given above the expansion can easily be performed and then one obtains $\tilde{C}_i$ and $\tilde{Z}_i$, which have a proper power-counting as prescribed in Ref. [19]. At the end one finds

$$\tilde{f}_i = \hat{a}(1 + \tilde{Z}_i/2) + \delta_i + T_i + \tilde{C}_i, \tag{21}$$

the expression that is used in the actual calculations. By expanding Z_i and C_i in terms of $1/\mathring{m}_D$ at NLO or $1/m_D$ at NNLO and keeping the lowest order in $1/\mathring{m}_D$ ($1/m_D$) one can easily obtain the corresponding HMχPT results.

III. RESULTS AND DISCUSSION

Before presenting the numerical results, we should make it clear that in our present formulation of χPT we have focused on SU(3) breaking in the context of the chiral expansions but we have not utilized explicitly heavy quark symmetry that relates the couplings of the D mesons with those of the D^*, B, and B^* mesons.

In the present case, we encounter three LECs: a, b_D, and b_A. At this point, light quark mass dependent lQCD results are extremely useful. By a least-squares fit to the HPQCD results , one can fix those three LECs appearing in our calculation.

First we treat the D, D_s, D^*, and D_s^* mesons as degenerate, i.e., we work up to NLO. The corresponding results are shown in Fig. 2, where the HMχPT results are obtained by expanding our covariant results in terms of $1/\mathring{m}_D$ and keeping only the lowest-order terms. It is clear that the covariant results (with χ^2=41) are in much better agreement with the HPQCD continuum limits than the HMχPT results (with $\chi^2 = 201$) [6]. This is not surprising because as we mentioned earlier the HPQCD collaboration has added second and third order polynomial terms in x_q to perform their extrapolation. Furthermore one can notice that at larger light quark masses the difference between the covariant χPT and the HMχPT results becomes larger. This highlights the importance of using a covariant formulation of χPT in order to make chiral extrapolations if lattice simulations are performed with relatively large light quark masses. Similar conclusions have been reached in studying the light quark mass dependence of the lowest-lying octet and decuplet baryon masses [22].

Taking into account the mass splittings between D, D_s, D^*, and D_s^* as prescribed by Eq. (12) one obtains the NNLO χPT results. Fitting them to the HPQCD extrapolations, one finds the results shown in Fig. 3. Compared to Fig. 2, it is clear that the agreement between the covariant χPT results with the HPQCD extrapolations becomes even better. Furthermore the covariant χPT results (with $\chi^2 = 16$) is still visibly better than the HMχPT results (with $\chi^2 = 59$), but now the difference between the covariant and the HM χPT results becomes smaller. The three LECs in the NNLO covariant χPT have the following values: $\hat{a} = 208$ MeV, $b_D = 0.318$, $b_A = 0.166$.

If we had fitted the HPQCD extrapolations by neglecting the loop contributions, we would have obtained a even better agreement ($\chi^2 = 9$). In Ref. [22] we also found that the lattice baryon mass data could be fitted better with the LO (linear in m_q) chiral extrapolation. But there we found that the NLO chiral results in fact describe the experimental data better than the LO (linear) chiral extrapolation. This just shows that the lattice baryon mass data behave more linearly as a function of light quark masses at large light quark masses and chiral logarithms play a more relevant role at smaller light quark masses, as one naively expects.

Another way of understanding the importance of chiral logarithms is to perform separate fits for lattice simulations obtained at different light quark masses. One expects that at smaller light quark masses (e.g., $m_\pi < 300$ MeV) covariant χPT and HM χPT results should perform more or less similarly. On the other hand, if the light quark masses are larger, covariant χPT should be a better

[6] It should be noted that the absolute value of χ^2 as defined here does not have a clear-cut physical meaning. It only reflects to what extent the chiral results agree with the HPQCD extrapolations.

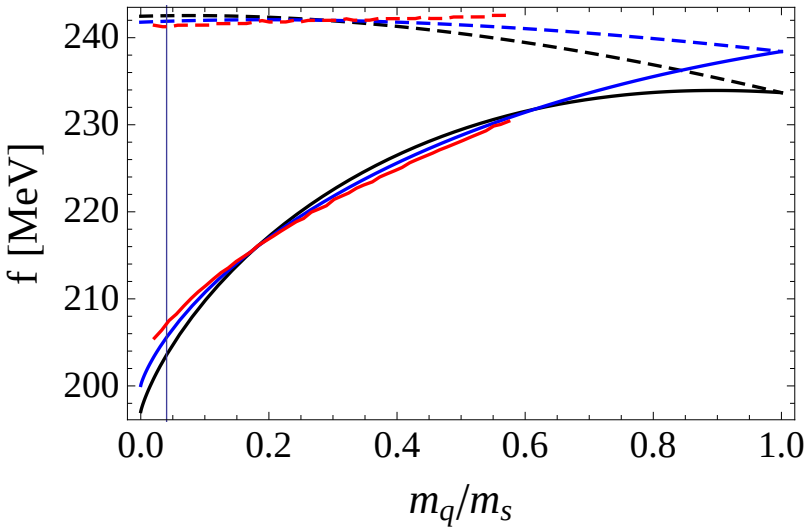

FIG. 2. (Color online) Light quark mass dependence of f_D (solid lines) and f_{D_s} (dashed lines). The black lines show the results of the NLO HMχPT and the blue lines the results of the covariant NLO ChPT. The red lines are the continuum extrapolations of the HPQCD collaboration [2]. The ratio $r = m_q/m_s$ is related to the pseudoscalar meson masses at leading chiral order through $m_\pi^2 = 2B_0 m_s r$ and $m_K^2 = B_0 m_s (r+1)$ with $B_0 = m_\pi^2/(m_u + m_d)$ and $m_q = (m_u + m_d)/2$, where m_u, m_d, and m_s are the physical up, down, and strange quark mass.

choice. In Fig. 4, we show the fitted results obtained from fitting the HPQCD extrapolations in two different regions of light quark masses, $m_q/m_s \leq 0.2$ (left panel) and $m_q/m_s > 0.2$ (right panel). It is clearly seen that fitting lattice data with large light quark masses using the HMχPT results may give unreliable extrapolations. Here we have used the NNLO HMχPT and covariant χPT results for comparison. The difference will become even larger if the NLO χPT results are used. We should also mention that even for $m_q/m_s \leq 0.2$ ($m_\pi \leq 307$ MeV) the HPQCD extrapolations are better described by covariant χPT than by HMχPT judging from the χ^2 analysis (although the difference is so small that it can hardly be appreciated by just looking at the left panel of Fig. 4).

We have checked that our covariant results are stable with respect to variations of certain input parameters within reasonable ranges, e.g., $m_\rho < \mu < 2$ GeV and $0.53 < g < 0.67$, where μ is the renormalization scale and g the $DD^*\pi$ coupling defined in Eq. (11). With our standard choice: $g = 0.6$ and $\mu = 1$ GeV, we have also noticed that for the NNLO covariant χPT to produce a smaller χ^2 than the linear chiral extrapolation, c' has to be larger than 1.23. If we use the quenched lQCD result, $c' = 1.35 \pm 0.06$ [33], the fit is even better[7]. On the other hand, our results remain

[7] Using the results from a more recent calculation by the UKQCD collaboration [34], one obtains $c' \approx 1.18 \pm 0.13$,

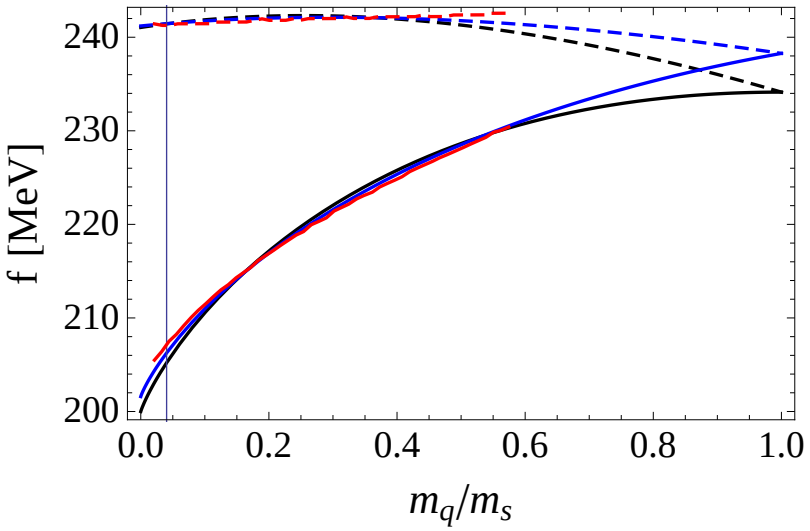

FIG. 3. (Color online) Same as Fig. 2, but the chiral expansions are calculated up to NNLO.

qualitatively the same with either $c' = 1$ or $c' = 1.35$. Therefore we have presented the results obtained with $c' = 1$.

Chiral perturbation theory not only helps extrapolating lQCD simulations to the physical light quark masses. It also benefits from this process because once the values of the relevant LECs are fixed by fitting the lQCD data, χPT predicts observables involving the same set of LECs. In the present case, assuming that the $1/m_Q$ corrections to the values of the three LECs b_D, b_A, and g are small, we can calculate the ratio of f_{B_s}/f_B by making the following replacements in our NNLO covariant χPT results:

$$m_D \to m_B, \quad \Delta \to \Delta(B), \quad \text{and} \quad \Delta_s \to \Delta_s(B). \tag{22}$$

It is found that deviations of b_D and b_A from those determined from the $D(D^*)$ mesons affect the f_{B_s}/f_B ratio only by small amounts. Changing b_D and b_A by $\sim 15\%$ changes f_{B_s}/f_B ratio by only about 1%. On the other hand, the effect of $g_{BB^*\pi}$ is much larger. If heavy quark flavor symmetry were exact, one would have $g_{BB^*\pi} = g_{DD^*\pi} = 0.6$. However, lattice QCD simulations indicate that $g_{BB^*\pi}$ is most likely smaller than $g_{DD^*\pi}$. For instance, two most recent $N_f = 2$ studies give $g_{BB^*\pi} = 0.516(5)(33)(28)(28)$ [35] and $g_{BB^*\pi} = 0.44 \pm 0.03^{+0.07}_{-0.00}$ [36]. Using 0.516 as the central value and 0.60 (0.44) as the upper(lower) bounds for $g_{BB^*\pi}$, we find:

$$f_{B_s}/f_B = 1.22^{+0.05}_{-0.04}, \tag{23}$$

which is compatible with the result of Ref. [33] but with larger uncertainties.

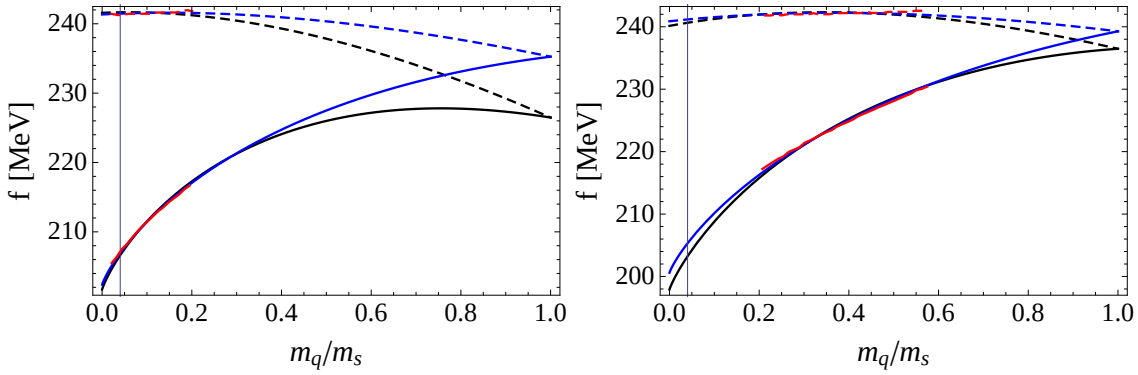

FIG. 4. (Color online) Same as Fig. 3, but on the left panel only the lattice extrapolations with $m_q/m_s \leq 0.2$ are fitted while on the right panel only those with $m_q/m_s > 0.2$ are fitted.

which agrees very well with the most precise result from the HPQCD collaboration: $f_{B_s}/f_B = 1.226(26)$ [37]. The uncertainty of ~ 0.05 does not take into account all sources of uncertainties[8], but nevertheless it represents a reasonable estimate by covering a range of $g_{BB^*\pi}$ values suggested by the two recent lQCD calculations and possible $1/m_Q$ corrections to b_D and b_A.

IV. SUMMARY AND CONCLUSIONS

We have derived a covariant formulation of χPT in order to study the D and D_s decay constants. To simplify the analysis, we have taken extrapolated lattice data (the HPQCD continuum limits) as a benchmark and focused on the light quark mass evolution of f_D and f_{D_s}, in particular on the SU(3) breaking pattern. We find that covariant χPT describes the HPQCD extrapolations considerably better than HMχPT at a given order, although both approaches show improvement when going from NLO to NNLO. Our studies show once more that if the lattice simulations are performed with relatively large light quark masses (e.g., $m_\pi > 300$ MeV), a covariant formulation of χPT is a better choice for chiral extrapolations, particularly at low chiral orders.

Lattice QCD calculations have made remarkable progress in recent years. For "gold plated" physical quantities such as the decay constants studied in this work, the overall uncertainties have been reduced to a few percent. Chiral perturbation theory plays an important role in understanding some of the systematic errors, such as those from finite volume and extrapolations of the light quark masses to their physical values. On the other hand these precise lattice data are also valuable to fix the relevant LECs appearing in χPT, which can then be used to predict physical observables

[8] For instance, the small uncertainties propagated from the lQCD results of f_D (f_{D_s}).

involving the same LECs. In the present work, we have used the HPQCD D (D_s) data in combination with two lattice determinations of $g_{BB^*\pi}$ in order to predict the ratio $f_{B_s}/f_B = 1.22^{+0.05}_{-0.04}$. This ratio turns out to be more sensitive to the value of $g_{BB^*\pi}$ than to possible $1/m_Q$ corrections to the two relevant LECs, b_D and b_A.

V. ACKNOWLEDGEMENTS

This Work is supported in part by BMBF, the A.v. Humboldt foundation, the Fundamental Research Funds for the Central Universities (China) and by the DFG Excellence Cluster "Origin and Structure of the Universe." LSG thanks Prof. Christine Davies for useful and informative communications regarding the HPQCD data.

VI. APPENDIX

The functions A_0 and B_0 appearing in the calculation of the D and D_s meson decay constants in the text are defined as, respectively:

$$A_0(m^2) = -\frac{1}{16\pi^2} m^2 \log\left(\frac{\mu^2}{m^2}\right), \tag{24}$$

$$B_0(p_i^2, m_1^2, m_2^2) = \begin{cases} -\frac{1}{16\pi^2}\left[\log\left(\frac{\mu^2}{m_1^2}\right) - 1\right] & \text{if } p_i^2 = 0 \text{ and } m_1 = m_2 \\ -\frac{1}{16\pi^2}\left[\frac{m_1^2 \log\left(\frac{\mu^2}{m_1^2}\right) - m_2^2 \log\left(\frac{\mu^2}{m_2^2}\right)}{m_1^2 - m_2^2}\right] & \text{if } p_i^2 = 0 \text{ and } m_1 \neq m_2 \\ -\frac{1}{16\pi^2}\frac{1}{2p_i^2}\left[2p_i^2\left\{\log\left(\frac{\mu^2}{m_2^2}\right) + 1\right\} + (m_2^2 - m_1^2 - p_i^2)\log\left(\frac{m_1^2}{m_2^2}\right)\right. & \\ +2\sqrt{2m_1^2(p_i^2 + m_2^2) - (m_2^2 - p_i^2)^2 - m_1^4} & \\ \left.\times\left\{\tan^{-1}\left(\frac{m_2^2 - m_1^2 - p_i^2}{\sqrt{2m_1^2(p_i^2+m_2^2) - (m_2^2 - p_i^2)^2 - m_1^4}}\right) - \tan^{-1}\left(\frac{m_2^2 - m_1^2 + p_i^2}{\sqrt{2m_1^2(p_i^2+m_2^2) - (m_2^2 - p_i^2)^2 - m_1^4}}\right)\right\}\right] & \end{cases}. \tag{25}$$

In the present work, the loop results are regularized using the modified minimal subtraction scheme and, unless otherwise specified, the regularization scale μ is set at 1 GeV.

[1] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).

[2] E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. Lett. **100**, 062002 (2008).

[3] B. A. Dobrescu and A. S. Kronfeld, Phys. Rev. Lett. **100**, 241802 (2008).

[4] J. P. Alexander *et al.* [CLEO Collaboration], Phys. Rev. D **79**, 052001 (2009).

[5] P. U. E. Onyisi *et al.* [CLEO Collaboration], Phys. Rev. D **79**, 052002 (2009).

[6] P. Naik *et al.* [CLEO Collaboration], Phys. Rev. D **80**, 112004 (2009).

[7] J. P. Lee *et al* [The BABAR Collaboration], arXiv:1003.3063 [hep-ex].

[8] L. Widhalm *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 241801 (2008).

[9] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010)

[10] The Heavy Flavor Averaging Group *et al.*, http://www.slac.stanford.edu/xorg/hfag.

[11] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na and J. Shigemitsu, arXiv:1008.4018 [hep-lat].

[12] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, arXiv:1008.4562 [hep-lat].

[13] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaborations], PoS **LAT2009**, 249 (2009).

[14] B. Blossier *et al.*, JHEP **0907**, 043 (2009).

[15] M. B. Wise, Phys. Rev. D **45**, 2188 (1992).

[16] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D **46**, 1148 (1992) [Erratum-ibid. D **55**, 5851 (1997)].

[17] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).

[18] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. **281**, 145 (1997).

[19] L. S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise, Phys. Rev. D **82**, 054022 (2010).

[20] A. Walker-Loud *et al.*, Phys. Rev. D **79**, 054502 (2009).

[21] K. I. Ishikawa *et al.* [PACS-CS Collaboration], Phys. Rev. D **80**, 054502 (2009).

[22] J. Martin Camalich, L. S. Geng and M. J. Vicente Vacas, Phys. Rev. D **82**, 074504 (2010).

[23] A. V. Manohar, M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **10**, 1-191 (2000).

[24] X. H. Guo and A. W. Thomas, Phys. Rev. D **65**, 074019 (2002).

[25] J. Hein, S. Collins, C. T. H. Davies *et al.*, Phys. Rev. **D62**, 074503 (2000).

[26] B. Grinstein, E. E. Jenkins, A. V. Manohar, M. J. Savage and M. B. Wise, Nucl. Phys. B **380**, 369 (1992).

[27] J. L. Goity, Phys. Rev. D **46**, 3929 (1992).

[28] C. G. Boyd and B. Grinstein, Nucl. Phys. B **442**, 205 (1995).

[29] J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).

[30] T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

[31] L. S. Geng, J. Martin Camalich, L. Alvarez-Ruso and M. J. V. Vacas, Phys. Rev. Lett. **101**, 222002 (2008).

[32] L. S. Geng, J. Martin Camalich and M. J. Vicente Vacas, Phys. Rev. D **79**, 094022 (2009).

[33] A. Abada *et al.*, Nucl. Phys. B **376**, 172 (1992).

[34] K. C. Bowler, L. Del Debbio, J. M. Flynn, G. N. Lacagnina, V. I. Lesk, C. M. Maynard and D. G. Richards [UKQCD Collaboration], Nucl. Phys. B **619**, 507 (2001).

[35] H. Ohki, H. Matsufuru and T. Onogi, Phys. Rev. D **77**, 094509 (2008).

[36] D. Becirevic, B. Blossier, E. Chang and B. Haas, Phys. Lett. B **679**, 231 (2009).

[37] E. Gamiz *et al.* [HPQCD Collaboration], Phys. Rev. **D80**, 014503 (2009).