

Augmentation of nucleon-nucleus scattering by information entropy.

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Quantum information entropy is calculated from the nucleon nucleus forward scattering amplitudes. Using a representative set of nuclei, from ${}^4\text{He}$ to ${}^{208}\text{Pb}$, and energies, $T_{lab} < 1$ [GeV], we establish a linear dependence of quantum information entropy as functions of logarithm nuclear mass A and logarithm projectile energy T_{lab} .

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I. INTRODUCTION

Studies of many body systems, like the sophisticated fields of atomic, nuclear and particle physics, have seen new and versatile enrichment under the auspices of quantum information theory. On one side we are aware of the implications of a first principle treatment of complex quantum mechanical systems and on the other side we are aware of less sophisticated phenomenological solutions that simplify the many body systems significantly. Phenomenology requires only a few degrees of freedom and introduces effective intensive and extensive matter properties.

The growing awareness and hope to exploit entanglement of quantum mechanical systems, as a new resource, has fostered many fields to investigate their realm also within the terminology of information theory. This is undoubtedly the case for a wide range of molecular and atomic systems and less for nuclear and particle physics. At least currently, the latter systems are not guided by practical applications but rather epistemological virtues.

Nuclear geometries are realized with nuclear structure calculations or measurements in conjunction with direct nuclear reactions [1]. Their values are now available in comprehensive tables and the data-groups continue their program as new facilities advent. In terms of information theory, nuclear physics is still in its infancy.

Shannon information entropy, together with quantum information theory, has been the subject of many theoretical studies [2, 3]. For its realization we know many cases with a fundamental microscopic approach as well as approaches which prefer a phenomenological access to information theory [4]. In any case, the Shannon information entropy and quantum information entropy (IE) are closely related. The first uses a normalized set of probabilities, the other uses the density matrix formalism and a normalized trace. The density matrix is diagonalized and the eigenvalues are taken as probabilities to

calculate IE

$$S = - \sum_{i=1}^n p_i \ln p_i, \quad \text{with} \quad \sum_{i=1}^n p_i = 1. \quad (1)$$

The eigenvectors are generally not used to define a probability distribution, but investigations along such line have been done and produced interesting results and conclusions. To distinguish their input approach, from the full density matrix IE , it is said, Shannon information entropy is calculated, with entities in r - and k -space, for atoms [5], and more recently for nuclei, atomic clusters and boson traps [6] in particular.

In subnano physics IE is everywhere present. It reduces radically any genuine and detailed information of a many particle system and the normalization condition eliminates any aspect of relative scales which are existing within physical quantities and are the key for prevalence and subordinate. Within the set p_i , the succession is irrelevant. As such, the pure mathematical aspect of IE is impossible to escape and any set of positive definite normalized physical data are used as a potential set $\{p_i\}$. Thus, the resulting information entropy does not automatically have a physical interpretation since an abstract scheme of order or disorder is now in vogue.

With this outline in mind, we do not hesitate to augment nucleon-nucleus (NA) cross sections with the expression of information entropy. In this way we concatenate geometry, kinematics and dynamics of nuclear reactions. We expect that these entities are separable and thus imply a linear relation for measures of IE .

This conjecture is based upon a uniform probability distribution, to simplify the IE relation, for all partial wave scattering amplitudes, which form our input data $p_i = \frac{1}{n}$ and $i = 1, \dots, n$, where $n - 1 = l_{max} = kR = kx_0A^{1/3}$ is the grazing (largest relevant) angular momentum. This is determined from the grazing radius $R = x_0A^{1/3}$, for which a nuclear Fermi distribution implies $x_0 = r_0 + 3a/A^{1/3}$ with $r_0 = 1.15$ [fm] and $a = 0.55$ [fm]. The masses (projectile, target) m_1 , m_2 and $T = T_{lab}$ determine the wave number

$$(\hbar c)^2 k^2 = \frac{m_2^2(T^2 + 2m_1T)}{(m_1 + m_2)^2 + 2m_2T}$$

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$$\begin{aligned}
&= \frac{2m_1m_2^2}{(m_1+m_2)^2}T + \frac{(m_1-m_2)^2m_2^2}{(m_1+m_2)^4}T^2 + \dots \\
&\approx \frac{2m_1m_2^2}{(m_1+m_2)^2}T. \tag{2}
\end{aligned}$$

A uniform distribution, $\{p_i = 1/n\}$, implies IE to be $S = -\sum_{i=1}^n \frac{1}{n} \ln \frac{1}{n} = \ln n$. Together with the approximated k and R values, this gives a desirable linear dependence

$$S = a + \frac{1}{2} \ln T + \frac{1}{3} \ln A \tag{3}$$

with

$$a = \ln \frac{x_0 \sqrt{2m_1}}{\hbar c} + \ln \frac{m_2}{(m_1+m_2)}. \tag{4}$$

Eq. (3) is interesting in itself, as it recalls the well known physiological sensitivity of biological sense-organs which often show a logarithmic dependence on magnitudes, i.e. sound pressure or light intensity. As one knows, animals alike humans are very able to notice and detect small structural differences, even within a chaotic environment, thus we anticipate propitious support also from information entropy for the physics of many body systems and in particular for quantum scattering. It is not difficult to improve Eq. (3) and foresee departures from simplicity with nonlinear dependencies between the variables $(m_1, m_2, T, N+Z, N-Z)$. To this end, we performed accurate numerical studies with realistic input data and compiled results for many nuclei and projectile energies, ranging from low to medium energies, $T_{lab} < 1$ [GeV].

The following sections contain the relevant scattering theory, related input data, numerical results and the final summary and conclusions.

II. LINK OF NA SCATTERING WITH IE

The NA scattering amplitudes for zero spin targets

$$\begin{aligned}
f(\theta) &= \frac{i}{2k} \sum_{l=0}^{lmax} [(l+1)(1-\eta_l^+) + l(1-\eta_l^-)] e^{i\sigma_l} P_l(\cos\theta) \\
&\quad - \frac{1}{2k} \sum_{l=0}^{lmax} (\eta_l^+ - \eta_l^-) e^{i\sigma_l} \frac{d}{d\theta} P_l(\cos\theta) + f_c(\theta), \tag{5}
\end{aligned}$$

are readily available for any type of cross section and spin observable [8, 9]. In this application we suppress the Coulomb amplitude $f_c(\theta)$ and use only the sum of partial wave amplitudes. The associated angle integrated cross sections, (e) for elastic, (r) for reaction and (t) for total, are

$$\sigma^{(t)} = \sigma^{(e)} + \sigma^{(r)} = \sum_{l=0}^{lmax} \sigma_l^{(e)} + \sum_{l=0}^{lmax} \sigma_l^{(r)}, \tag{6}$$

where partial wave cross sections are

$$\sigma_l^{(e)} = \frac{\pi}{k^2} [(l+1)|1-\eta_l^+|^2 + l|1-\eta_l^-|^2], \tag{7}$$

$$\sigma_l^{(r)} = \frac{\pi}{k^2} [(l+1)(1-|\eta_l^+|^2) + l(1-|\eta_l^-|^2)]. \tag{8}$$

After normalization, by $\sigma^{(t)}$, the partial wave cross section probabilities distinguish contributions from elastic and reaction channels

$$1 = \sum_{l=0}^{lmax} [p_l^{(e)} + p_l^{(r)}], \tag{9}$$

and

$$p_l^{(e)} = \frac{\sigma_l^{(e)}}{\sigma^{(t)}}, \quad \text{and} \quad p_l^{(r)} = \frac{\sigma_l^{(r)}}{\sigma^{(t)}}. \tag{10}$$

The l -dependent probabilities $p_l^{(e)}$ and $p_l^{(r)}$ include no interference between different l -values, the density matrix is diagonal and off-diagonal elements are zero. Thus the information entropy for proton and neutron nucleus scattering integrated cross sections are a straight forward sum of contributing angular momenta $0 \leq l \leq l_{max}$

$$S = - \sum_{l=0}^{lmax} [p_l^{(e)} \ln p_l^{(e)} + p_l^{(r)} \ln p_l^{(r)}]. \tag{11}$$

An ample remark. In case of angle and complex spin dependent scattering and reactions it is necessary to use the full power of the density matrix formalism with diagonalization. This formalism is well developed and general scattering programs, for low and medium energy scattering, are available. A range of such programs has been developed by J. Raynal, CEN-Saclay, with older as well as more current versions of DWBA, with a sophisticated microscopic approach suited for NA scattering, and versions of the coupled channels code ECIS [7].

For introductory studies, we suggest to use the scattering amplitude given by Eq. (5). These amplitudes can easily be arranged in a matrix

$$\rho(l_1, j_1, l_2, j_2 | T, \theta) = \frac{|l_1, j_1, T, \theta\rangle \langle l_2, j_2, T, \theta|}{\text{Tr } \rho(l, j, l, j | T, \theta)} \tag{12}$$

and diagonalized.

III. APPLICATION

There exist many theoretical and experimental studies of NA scattering amplitudes. Most of them are calculated with a Schrödinger equation and optical model potential fitted to data. Today, the potentials are microscopic optical potentials. These optical potentials are based upon a high quality NN potential. Such a microscopic approach is our choice for this study and we are using the Argonne AV18 NN potential [10]. Hereby, we

TABLE I. Eqs. (3) and (13), linear relation best fit parameters.

Reaction Type	a	b	c	Remarks
nA and pA	-1.25±0.25	1/2	1/3	uniform model
nA	-1.0796	0.7168	0.3787	20-1000 MeV
pA	-1.1866	0.7412	0.3634	20-1000 MeV
nA	-1.1267	0.7348	0.3598	50-500 MeV
pA	-1.2638	0.7638	0.3485	50-500 MeV

generalized AV18 above pion production threshold to become a complex NN optical potential [9]. Globally, these IE studies are insensitive to the use of a t - or g -matrix microscopic optical model potential [8, 11]. Undoubtedly, phenomenological optical model potentials are bound to yield results which confirm ours.

The herein used scattering amplitudes contain data for (n, A) and (p, A) scattering on a dense grid of energies, $20 < T(n, p) < 1000$ [MeV], and targets, ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$, ${}^{40}\text{Ca}$, ${}^{58}\text{Ni}$, ${}^{90}\text{Zr}$ and ${}^{208}\text{Pb}$. The Coulomb amplitude f_c is suppressed in case of (p, A) . The results are shown in Fig. 1, with the calculated IE values (circles), for our selection of nuclei and energies. We verified, with the numerical IE results, a linear dependence, as given in Eq. (3), and determined, with a χ^2 fit, the expansion coefficients

$$\chi^2 = \text{Min}_{(a,b,c)} \|S - (a + b \ln T + c \ln A)\|. \quad (13)$$

The parameters a , b and c are given, for two energy fit ranges, in Table I and Fig. 1 shows lines of the linear approximation together with the numerical results for the representative set of nuclei and T_{lab} energies.

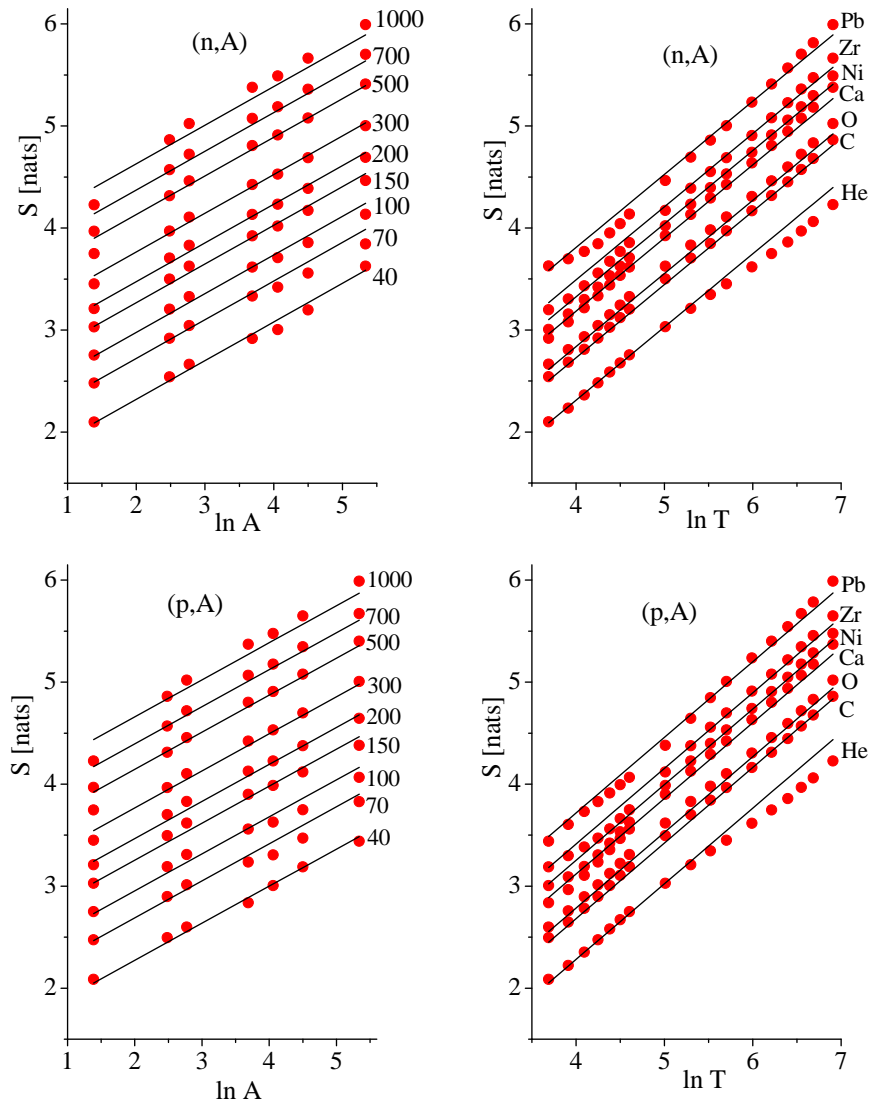


FIG. 1. Verification of a linear dependence, using parameters associated with the range 20 – 1000 [MeV] in Table I, versus $\ln T$ and $\ln A$, of numerically calculated information entropy (red dots).

It is possible to see some data scattering around the lines. This is caused by the not smooth but realistic dependence of microscopic optical model potentials on A and $N - Z$ and the spectroscopy of the target states. These particularities generate deviations from the lines of a few percent. Demanding a better agreement would be unrealistic in the first place.

In summary, the analysis confirms a linear dependence of IE in NA scattering from two entities, first the *logarithm of target A* and second the *logarithm of projectile kinetic energy T* . In particular, we suggest to use the relation for IE

$$S(A, T) = a + b \ln T + c \ln A \quad (14)$$

with best fit parameters (a, b, c) from Table I.

Finally, we present a numerical study and comparison of IE for various isotopes, $S(A, N - Z, T)$. Such study is bound to reflect the spectroscopy of a range of targets. The numerical results are shown in Fig. 2. Notice, in comparison with Fig. 1, we have increased our resolution in Fig. 2 by an order of magnitude and differentiate between scattering of neutrons and protons with full circles and squares, respectively, all having $T = 400$ [MeV]. The results are smooth and reflect shell closures for cases when the neutron number equals a magic number, *i.e.* O at $N=8$, Ca at $N=20$ and 28 , Ni at $N=28$ and 50 , Zr at $N=50$, Sn at $N=50$ and 82 , and for Pb at $N=126$.

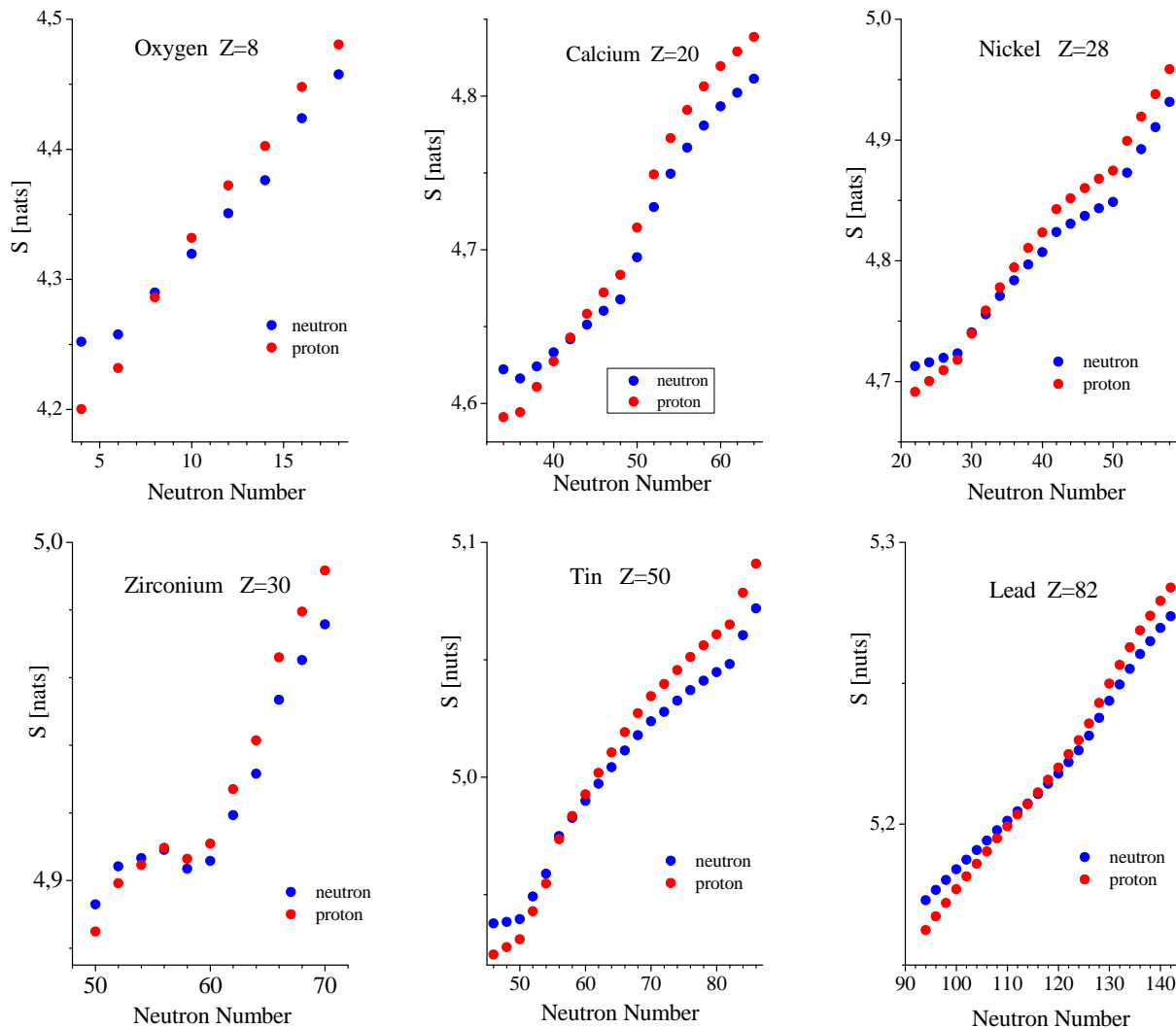


FIG. 2. Dependence of information entropy $S(A, N - Z)$ for O, Ca, Ni, Zr, Sn and Pb isotopes. Shown are numerical results for (n, A) in (blue) and (p, A) in (red), $T_{lab} = 400$ [MeV].

IV. SUMMARY AND CONCLUSIONS

Information entropy traces its roots to the work of Shannon in the first place, as a fundamental result of classical telecommunication theory, since that time it has received much attention in experiment and theory, for classical as well as quantum systems [2, 4]. IE , as a mean to quantify entanglement of quantum mechanical states, of bound or scattering states, encouraged also our study with the aim to find smooth and simple structures to emerge in the least structured quantity of nucleon-nucleus scattering, elastic and reaction cross sections at low to medium energies 20-1000 [MeV].

The results of this study predict a smooth qualitative

dependence of IE on $\ln T_{lab}$ and $\ln A$. The observed numerical scattering, around this smooth and averaged results, are first due to shell effects of the target and second due to target mass and charge dependencies, i.e. functions of $(N + Z)$ and $(N - Z)$ [11]. The mathematical nature of IE implies uncountable many applications in theoretical studies with powerful options for intuitions and conjectures beyond the known scattering analysis.

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