Disproof of Bell's Theorem

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We illustrate an explicit counterexample to Bell's theorem by constructing a pair of spin variables in S^3 that exactly reproduces the EPR-Bohm correlation in a manifestly local-realistic manner.

We begin by defining the detections of spin bivectors $\mathbf{L}(\mathbf{s}, \lambda^k)$ by the detector bivectors $\mathbf{D}(\mathbf{a})$ and $\mathbf{D}(\mathbf{b})$ {Ref. [1]}:

$$S^{3} \ni \mathscr{A}(\mathbf{a}, \lambda^{k}) := \lim_{\mathbf{s} \to \mathbf{a}} \left\{ -\mathbf{D}(\mathbf{a}) \mathbf{L}(\mathbf{s}, \lambda^{k}) \right\} = \begin{cases} +1 & \text{if } \lambda^{k} = +1 \\ -1 & \text{if } \lambda^{k} = -1 \end{cases}$$
 (1)

and
$$S^3 \ni \mathcal{B}(\mathbf{b}, \lambda^k) := \lim_{\mathbf{s} \to \mathbf{b}} \left\{ + \mathbf{L}(\mathbf{s}, \lambda^k) \mathbf{D}(\mathbf{b}) \right\} = \begin{cases} -1 & \text{if } \lambda^k = +1 \\ +1 & \text{if } \lambda^k = -1 \end{cases}$$
 (2)

where the orientation λ of S^3 is assumed to be a random variable with 50/50 chance of being +1 or -1 at the moment of the pair-creation, making the spinning bivector $\mathbf{L}(\mathbf{n}, \lambda)$ a random variable relative to the detector bivector $\mathbf{D}(\mathbf{n})$:

$$\mathbf{L}(\mathbf{n}, \lambda) = \lambda \mathbf{D}(\mathbf{n}) \iff \mathbf{D}(\mathbf{n}) = \lambda \mathbf{L}(\mathbf{n}, \lambda). \tag{3}$$

The expectation value of the simultaneous outcomes $\mathscr{A}(\mathbf{a}, \lambda^k) = \pm 1$ and $\mathscr{B}(\mathbf{b}, \lambda^k) = \pm 1$ is then worked out as follows:

$$\mathcal{E}(\mathbf{a}, \mathbf{b}) = \lim_{n \gg 1} \left[\frac{1}{n} \sum_{k=1}^{n} \mathscr{A}(\mathbf{a}, \lambda^{k}) \mathscr{B}(\mathbf{b}, \lambda^{k}) \right] \text{ within } S^{3} := \text{ the set of all unit (left-handed) quaternions}$$
 (4)

$$= \lim_{n \gg 1} \left[\frac{1}{n} \sum_{k=1}^{n} \left[\lim_{\mathbf{s} \to \mathbf{a}} \left\{ -\mathbf{D}(\mathbf{a}) \mathbf{L}(\mathbf{s}, \lambda^{k}) \right\} \right] \left[\lim_{\mathbf{s} \to \mathbf{b}} \left\{ +\mathbf{L}(\mathbf{s}, \lambda^{k}) \mathbf{D}(\mathbf{b}) \right\} \right] \right]$$
 (conserving total spin = 0) (5)

$$= \lim_{n \gg 1} \left[\frac{1}{n} \sum_{\substack{\mathbf{s} \to \mathbf{a} \\ \mathbf{s} \to \mathbf{b}}}^{n} \lim_{\substack{\mathbf{s} \to \mathbf{a} \\ \mathbf{s} \to \mathbf{b}}} \left\{ -\mathbf{D}(\mathbf{a}) \mathbf{L}(\mathbf{s}, \lambda^{k}) \mathbf{D}(\mathbf{b}) \equiv \mathbf{q}(\mathbf{a}, \mathbf{b}; \mathbf{s}, \lambda^{k}) \in S^{3} \right\} \right]$$
(6)

$$= \lim_{n \gg 1} \left[\frac{1}{n} \sum_{\substack{k=1 \ \mathbf{s} \to \mathbf{a} \\ \mathbf{s} \to \mathbf{b}}}^{n} \left\{ -\lambda^{k} \mathbf{L}(\mathbf{a}, \lambda^{k}) \mathbf{L}(\mathbf{s}, \lambda^{k}) \mathbf{L}(\mathbf{s}, \lambda^{k}) \lambda^{k} \mathbf{L}(\mathbf{b}, \lambda^{k}) \right\} \right]$$
(7)

$$= \lim_{n \gg 1} \left[\frac{1}{n} \sum_{\substack{k=1 \ \mathbf{s} \to \mathbf{a} \\ \mathbf{s} \to \mathbf{b}}}^{n} \left\{ -\mathbf{L}(\mathbf{a}, \lambda^{k}) \, \mathbf{L}(\mathbf{s}, \lambda^{k}) \, \mathbf{L}(\mathbf{b}, \lambda^{k}) \right\} \right]$$
(8)

$$= \lim_{n \gg 1} \left[\frac{1}{n} \sum_{k=1}^{n} \mathbf{L}(\mathbf{a}, \lambda^{k}) \mathbf{L}(\mathbf{b}, \lambda^{k}) \right] \quad \{\text{cf. Appendix B of Ref. [1]}\}.$$

Here the integrand of (6) is necessarily a unit quaternion $\mathbf{q}(\mathbf{a}, \mathbf{b}; \mathbf{s}, \lambda^k) \in S^3$ since S^3 is closed under multiplication; (7) follows upon using (3); (8) follows upon using $\lambda^2 = +1$; and (9) follows from the fact that all unit bivectors such as $\mathbf{L}(\mathbf{s}, \lambda)$ square to -1. Using $I := \mathbf{e}_x \wedge \mathbf{e}_y \wedge \mathbf{e}_z$ with $I^2 = -1$, the final sum can now be evaluated by recognizing that the spins in the right and left oriented S^3 satisfy the following geometrical relations {cf. Appendix A of Ref. [1]}:

$$\mathbf{L}(\mathbf{a}, \lambda^k = +1) \mathbf{L}(\mathbf{b}, \lambda^k = +1) = (+I \cdot \mathbf{a})(+I \cdot \mathbf{b})$$
(10)

and
$$\mathbf{L}(\mathbf{a}, \lambda^k = -1) \mathbf{L}(\mathbf{b}, \lambda^k = -1) = (+I \cdot \mathbf{b})(+I \cdot \mathbf{a}).$$
 (11)

In other words, when λ^k happens to be equal to +1, $\mathbf{L}(\mathbf{a}, \lambda^k)$ $\mathbf{L}(\mathbf{b}, \lambda^k) = (+I \cdot \mathbf{a})(+I \cdot \mathbf{b})$, and when λ^k happens to be equal to -1, $\mathbf{L}(\mathbf{a}, \lambda^k)$ $\mathbf{L}(\mathbf{b}, \lambda^k) = (+I \cdot \mathbf{b})(+I \cdot \mathbf{a})$. Consequently, the above expectation value reduces at once to

$$\mathcal{E}(\mathbf{a}, \mathbf{b}) = \frac{1}{2}(+I \cdot \mathbf{a})(+I \cdot \mathbf{b}) + \frac{1}{2}(+I \cdot \mathbf{b})(+I \cdot \mathbf{a}) = -\frac{1}{2}\{\mathbf{a}\mathbf{b} + \mathbf{b}\mathbf{a}\} = -\mathbf{a} \cdot \mathbf{b} + 0, \tag{12}$$

because the orientation λ of S^3 is a fair coin. Here the last equality follows from the definition of the inner product.

References

[1] J. Christian, Macroscopic Observability of Spinorial Sign Changes: A Reply to Gill, arXiv:1501.03393; arXiv:1211.0784.