

Light field integration in SUGRA theories

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Abstract. We revisit the integration of fields in $\mathcal{N} = 1$ Supergravity with the requirement that the effective theory has a reliable two-derivative supersymmetric description. In particular we study, in a supersymmetric manifest way, the situation where the fields that are mapped out have masses comparable to the Supersymmetry breaking scale and masses of the remaining fields.

We find that as long as one stands in regions of the field configuration space where the analytic continuation to superspace of the F-flatness conditions be reliable equations of motion for the fields that are being mapped out, and provided their solutions are stable regardless the dynamics of the remaining fields, such a two-derivative description is a reliable truncation of the full effective theory.

The study is mainly focused to models with two chiral sectors, H and L , described by a Kähler invariant function with schematic dependencies of the form $G = G_H(H, \bar{H}) + G_L(L, \bar{L})$, which leads to a nearly decoupled theory that allows the previous requirements to be easily satisfied in a consistent way. Interestingly enough for the matters of our study this kind of models present an scenario that is as safe as the one presented in sequestered models.

It is also possible to allow gauge symmetries as long as these appear also factorized in hidden and visible sectors. Then, the integration of the hidden vector superfields is compulsory and proceeds reliably through the D-flatness condition analytically continued to superspace.

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1. Introduction

Supersymmetry (SUSY) and in general Supergravity (SUGRA) not only continues to be the preferred playground for models beyond the Standard Model, but also an ideal framework for dealing with situations where otherwise many calculations would be either impossible or unreliable. However, any constructed model has in mind only a small subset of the entire bunch of fields present in explicit realizations, and these are regarded as encoding all the important dynamics under study. Physically what one has in mind

is that the rest of the fields are either decoupled or that their dynamics are negligible. Formally the neglected fields are supposed to be integrated out in such a way that the resulting theory is, at least approximately, SUSY.

Integrating out fields in $\mathcal{N} = 1$ SUGRA theories led recently to some discussion [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] settled finally by the work of Brizi, Gomez-Reino and Scrucca [11] where, by requiring an effective two-derivative SUSY description, approximate superfield equations of motion (e.o.m.) were derived, for the fields to be integrated out, together with the estimate of the deviations from the exact effective higher order theory. These can be understood in the light of a low-energy effective theory where higher order terms appear suppressed by the mass of the fields being integrated out and, therefore, turn out to be subleading. A general result of the work by Brizi et al. is that the gravitational effects to the e.o.m. are automatically negligible once the masses of the integrated fields lie far above the characteristic energies of the effective theory, which include now the SUSY breaking scale, and therefore the leading superfield e.o.m. coincide with the ones of rigid SUSY.

There are, however, scenarios where one might like to get rid of some fields despite the fact that no hierarchy is realized. Already in ordinary field theories it is clear that in such a case higher order derivative terms are no longer suppressed, as the kinetic energies in the effective theory are comparable to the masses of the integrated fields. An obvious situation that circumvents this problem is the case where both sectors, the one to be integrated out and the one to be kept, denoted hereafter by $\{H\}$ and $\{L\}$ respectively, are completely decoupled. For rigid $\mathcal{N} = 1$ SUSY, without vectors fields, this is obtained for Kähler potential and superpotential factorized schematically as follows:

$$K = K_H(H) + K_L(L), \quad W = W_H(H) + W_L(L). \quad (1)$$

In SUGRA, instead, the theory is described by the generalized Kähler invariant function, $G = K + \ln |W|^2$, so the previous factorization does not hold in G nor in the theory. Moreover, gravitational interactions implies that even if G turns out to have a factorized form, i.e., $G = G_H(H) + G_L(L)$, the Lagrangian has not a fully decoupled structure as can be seen already in the scalar potential,

$$V = e^G \left(G^{I\bar{J}} G_I G_{\bar{J}} - 3 \right), \quad (2)$$

with $G^{I\bar{J}} \equiv (G_{I\bar{J}})^{-1}$ the inverse scalar manifold metric, the subindex I denoting derivatives respect to the superfield ϕ^I , and everything is evaluated in the lowest component of the superfields. In fact, in this context the most ideal scenario would be the sequestered models [12, 13, 14] where only gravitational interactions enter in the interplay between the sectors.

A factorizable G function leads still to some decoupling, as was first discussed in [15] and later on studied in [4, 5, 8]. A more detailed study was done by the author in [16], where also vector fields in simple setups were included, but still following a component approach where SUSY is not manifest in the prescription and, therefore, it is not completely clear how to understand the results in a fully SUSY framework.

Moreover, it is not clear the extend of validity of the constrains found there as the e.o.m. are not actually fully scanned.

The present letter follows closely the analysis in [11], now for nearly decoupled theories and no hierarchies in the game, looking for the conditions for a reliable two-derivative SUSY description in the effective theory. We find that such a description exist and is reliable as long as the analytic continuation of the F-flatness condition to the superspace is a reliable e.o.m. for the fields that are being integrated out and has stable solutions, regardless the dynamics of the field sector kept in the theory.

The superfield approach allows a neat analysis in the case of presence of gauge symmetries. In this case, in order the H sector not to be sourced back, the fields to be integrated out can be charged only under some hidden gauge group whose gauge kinetic function dependency on the L sector should be suppressed. In the same way the gauge kinetic function of the visible sector depends on the H fields in a mild way. Then, the leading superfield e.o.m.'s are the analytic continuation of the F-flatness conditions plus the D-flatness ones, leading to a two-derivative SUSY theory.

Our results support and generalize the findings of [16]. Working directly in the superspace, however, allows us to spot directly the fact that it is not necessary to restrict artificially to slow varying solutions being in fact a requirement coming from the e.o.m., as can be also understood from SUSY transformations. These findings are particularly relevant in the context of SUSY breaking scenarios, and the related issue of moduli stabilization in Superstring/M-theory, where most of the fields are regarded as SUSY preserving and a detailed description of the SUSY breaking and moduli stabilization is performed only on a tiny subset of fields. In particular the seminal work of Kachru, Kallosh, Linde and Trivedi [17] falls in the kind of scenarios where a hierarchy, dictated by the ratio between the flux and non-perturbative dynamics, is present and therefore the results of Brizi et al. apply. On the other hand, for natural Vacuum Expectation Value (VEV) of the superpotential, i.e., $\langle W \rangle \sim 1$ in Planck units, all fields get important, and of the same order, gravity contributions to the masses and therefore no hierarchy is realized. Low energy SUSY is still possible if the VEV for the G function is negatively large thanks to the universal factor e^G in the potential. This is what precisely happens in the so called Large volume scenarios (LVS) for type-IIB Superstring compactifications [18, 19] where, moreover, the coupling between the Khähler moduli, T , and the SUSY preserving dilaton and complex structure moduli, denoted by U , is described by

$$G_{mix} \sim \frac{\xi(U, \bar{U})}{\mathcal{V}(T, \bar{T})} + \frac{W_{np}(T, U)}{W_{flux}(U)} + h.c., \quad (3)$$

with ξ a function of the dilaton resulting from α' corrections [20] and $W_{flux} \gg W_{np}$ the flux induced and non-perturbative parts in the superpotential. Thus, for large values of the compact manifold volume, \mathcal{V} , the G function realizes an approximate factorizable form and our results apply (for details and numerical examples see [16]).

We should mention that, although with some broad applicability in moduli stabilization models, our analysis should be repeated for scenarios where higher order operators are relevant and the two-derivative level leads to poor descriptions, like the case of

cosmological models of inflation where the background dynamics should be taken into account [21, 22]. Then, it is necessary to keep full track of the higher order operators to get insights of the effective SUGRA theory [23]. Nevertheless, some analyses are valid at this level [24, 25, 26, 27, 28, 29, 30].

The letter is organized as follows: section two is dedicated to review the general procedure of integrating out fields defining what we call the effective description, where the degrees of freedom that are mapped out are not heavier than the ones kept in the theory, in contrast with the usual low energy effective theories. In section three the arguments in [11] are reviewed introducing SUSY as a global symmetry. Here a first instance of models with a reliable two-derivative SUSY effective description is shown. In section four gravity enters in the game regarding only chiral superfields. In here, after recovering results in [11], we show how the factorizable models can have such an effective description and the superfield e.o.m. to use in this case. Section five is dedicated to study the case where gauge symmetries are present, exploring also the possibility of having a charged hidden sector. The last section discusses the gravitational terms and the gauge fixing of the superconformal symmetry, an issue not regarded in previous studies. We close with some summary and discussion of the results.

2. Integrating out fields and effective descriptions

Let us consider a field theory with two kind of modes, H and L , described by an action $S[H, L]$. Suppose we are in a situation where only the L modes can be realized in the initial and final states. Then, the dynamics of the L fields can be described by a theory that does not depend on the H ones. This theory is the result of summing up over the H intermediate states, in a procedure that in the path integral formalism goes precisely as integration over the H modes, defining the effective action, S_{Eff} , for the remaining fields,

$$e^{-S_{Eff}[L]} = \int [dH] e^{-S[H, L]}, \quad (4)$$

where we use the Euclidean space notation. For practical purposes we expand around the classical solution for the H fields so the path integral is now over quantum fluctuations,

$$e^{-S_{Eff}[L]} = e^{-S[H_0, L]} \int [d\delta H] \exp \left\{ -\frac{\delta^2 S[H, L]}{\delta H \delta H} \delta H \delta H + \dots \right\} \Big|_{H=H_0}, \quad (5)$$

with H_0 the solutions to the classical e.o.m. $\frac{\delta S}{\delta H} = 0$, and the ellipses containing higher order terms in the quantum fluctuations. Notice, however, that the solutions to this classical e.o.m. are expected to depend on the L fields for which quantum fluctuations are still on. Therefore, in general H_0 is not the classical solution for H .[‡] At this level, then, the effective action is given by

$$S_{Eff}[L] = S[H_0, L]. \quad (6)$$

[‡] The classical e.o.m. is in fact more general once the action that is originally taken contains quantum corrections like non-perturbative effects.

So far we have not specified what we mean by the two kinds of modes. In the usual low energy effective theories the L are low energy modes, and the H high energy ones, where the distinction is made by some energy scale Λ . In this case then the path integral defining the effective action is restricted to modes with momentum higher to Λ . Formally, however, the procedure can be applied to get rid of any kind of mode, as long as one ensures that in the asymptotic states only L fields appear. For example if the action turns out to be the sum of actions for each sector, i.e., the two sector are decoupled, and in the initial states only one kind of fields appear. The situation we will deal with is of this kind, where despite the fact the decoupling is not complete any mixing term in the action will be parametrically small. This is what we will call an effective description to be distinguished from the low energy effective one.

In general the theory that one obtains by the procedure above is a higher derivative theory, and so suffers the pathologies of such [31]. Thus one might like to keep, consistently, only up to two derivative operators though higher order operator in the fields can be admitted. This reduces to consider in the e.o.m. only the contribution from the potential disregarding the kinetic terms. The truncation can be stated more precisely using the general form the effective Lagrangian of the resulting theory [32],

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}_i, \quad (7)$$

where c_i are dimensionless couplings known as Wilson coefficients, Λ is the cut-off scale and \mathcal{O}_i are operators with dimension d_i . We see, therefore, that for low energy effective theories higher order operators are naturally suppressed by powers of the energy scale Λ and the requirement above is automatic. § For the effective descriptions, instead, the consistency of the truncation procedure relies on the smallness of the couplings between the two sectors, which translates in small Wilson coefficients for operators not present in the original theory.

Another consideration behind the philosophy of effective theories is to ensure that the fields that have been mapped out be not sourced back by any process involving the L fields. For the low energy effective theories this is guaranteed kinematically since the effective theory cannot be used for describing processes with energies comparable or larger than the masses of the H fields. In the case of effective description the constraint is dynamical, coming from the decoupling. So the L fields cannot excite H modes.

3. Global SUSY effective theories

The introduction of SUSY, and the requirement of a SUSY effective description introduces a further issue, as was noticed in [11]: the usual two-derivative truncation for an effective description of a field theory is not enough when SUSY is implied, as higher order terms in the spinor bilinears and auxiliary fields are mapped, by SUSY transformations, to higher order derivative terms. Therefore, a further truncation in

§ Part of this analysis clearly relies on the assumption that the Wilson coefficients have no anomalous small or large values.

spinor bilinears and auxiliary fields should be imposed which will be reliable only if the missing terms are negligible. At the superfield level this means neglecting SUSY covariant derivatives in the Kähler potential and superpotential in the effective description, as should be clear from the fact that these derivatives have as components, both, space-time derivatives as well the spinor and auxiliary components of the fields, these last ones encoding the SUSY breaking energy scale. In other words, the solutions to the superfield e.o.m., for the fields that are being mapped out, should correspond either to field configurations where all the SUSY covariant derivatives are negligible, or such that are independent of any non-negligible one.

Let us explore better the situation for a global SUSY theory and consider models with two sectors of chiral fields $\{H^i\}$ and $\{L^\alpha\}$ (notice the distinction in the indices). The exact e.o.m. for the superfields H are obtained from the generic two-derivative Lagrangian

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K + \int d^2\theta W + h.c.. \quad (8)$$

The application of the variational principle in this action should take into account the constraint $\bar{\mathcal{D}}\Psi = 0$, with $\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\beta}^\mu \bar{\theta}^\beta \partial_\mu$ the supercovariant derivative, on the chiral superfields.¶ To make this explicit we write the D-term part of the action as a F-term one using the identity

$$\int d^2\theta d^2\bar{\theta} G(\Psi, \bar{\Psi}) = -\frac{1}{4} \int d^2\theta \bar{\mathcal{D}}^2 G(\Psi, \bar{\Psi}), \quad (9)$$

where $\mathcal{D}^2 = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}}$, the two expressions differing by total derivatives. With this consideration the superfield e.o.m. for the H^i reads:¶

$$W_i - \frac{1}{4} \bar{\mathcal{D}}^2 K_i = 0. \quad (10)$$

It is usually considered (see [34], also [11, 35]) that, around the solutions to this e.o.m., the energy scale associated to the second non-mixed holomorphic derivatives of the superpotential, i.e., W_{ij} , dominates over all others, say the ones associated to the superpotential itself, pure L -sector and mixed derivatives, e.g., $W_{\alpha\beta}$ and $W_{i\alpha}$, the space-time derivatives on the fields and the auxiliary fields VEV's. With a regular behavior in the Kähler potential this implies that the leading term in (10) is the first one so the approximate e.o.m. reads:

$$W_i = 0, \quad (11)$$

which leads to a two-derivative SUSY description for the L fields as no SUSY covariant derivative is present. In particular the solutions to this e.o.m. are vanishing H auxiliary fields implying no contribution to the SUSY breaking from the H sector at leading order. Physically the fact that the holomorphic W_{ij} derivatives dominate means that the masses of the H fields, $M_H \sim W_{ij}$, are larger than the remaining energy scales, namely, the

¶ Following conventions and notations like in [33].

¶ Along the paper we use the Latin subindex notation to denote derivatives with respect superfields, e.g., $W_i \equiv \frac{\partial W}{\partial H^i}$.

masses and kinetic energy of the L -sector fields as well the SUSY breaking scale. Then, the theory obtained from this leading e.o.m. coincides with the full higher order operator effective theory at first order in an expansion in derivatives, spinor bilinears and auxiliary fields, with the missing terms suppressed by M_H , precisely like in any standard low-energy effective description.

A second possibility is one where the dynamics ruling both sectors are of the same order and therefore no significant hierarchies appear. Still one might like to get rid of one sector, in which case one should consider the situation of factorization of the action, where the Kähler potential and superpotential have the form in (1), so the e.o.m. reads

$$W_{H,i}(H) - \frac{1}{4}\overline{\mathcal{D}}^2 K_{H,i}(H, \overline{H}) = 0. \quad (12)$$

Then, the integration of the H sector is completely arbitrary as there is not L field dependence on the e.o.m, so the solutions would not affect the L sector dynamics, which in particular continue to be described by a two-derivative SUSY theory. Still, let us explore the possibility of neglecting the second part of the equation, such that the e.o.m. would be $W_{H,i} = 0$. The components of this equation are given in [Appendix A](#) where is also shown that there is a trivial solution, i.e., vanishing spinor and auxiliary components and null space-time derivatives. Nicely enough these are precisely the components of the covariant derivatives appearing in the second part of the e.o.m. and, therefore, this kind of solutions are also solutions of (12). Thus, the integration is well described by an e.o.m where the supercovariant derivatives are simply not considered.

This kind of solutions to the e.o.m are SUSY preserving, a requirement that in presence of gravity, as we will see, is a compulsory one. Another motivation to stick to the SUSY preserving solution is that we can generalize our analysis by including a small mixing, i.e., $W_{mix}(H, L)$ or $K_{mix}(L, \overline{L}, H, \overline{H})$, that makes the decoupling not exact. In this case, if we parametrize the magnitude of the mixing terms by $\epsilon \ll 1$ we expect deviations to the previous solutions of this order. Then, the approximated e.o.m. is reliable up to terms of $\mathcal{O}(\epsilon)$ and the exact solutions are given by $H = H_0 + \delta H$, with H_0 the solution to $W_{H,i} = 0$ and

$$\delta H^i = \epsilon O^{ij} \left[\frac{1}{4}\overline{\mathcal{D}}^2 K_{mix,j} - W_{mix,j} + \frac{1}{4}K_{H,j\bar{j}}\overline{W}_H^{\bar{j}\bar{i}}\overline{\mathcal{D}}^2 \left(\frac{1}{4}\mathcal{D}^2 K_{mix,\bar{i}} - \overline{W}_{mix,\bar{i}} \right) \right] \Big|_{H_0} + \mathcal{O}(\epsilon^2), \quad (13)$$

with $O^{ij} \equiv (W_{H,ij} - \frac{1}{16}K_{H,i\bar{j}}\overline{W}_H^{\bar{j}\bar{i}}K_{H,\bar{i}j}\overline{\mathcal{D}}^2\mathcal{D}^2)^{-1}$ and $\overline{W}_H^{\bar{j}\bar{i}} \equiv (\overline{W}_{H,\bar{j}\bar{i}})^{-1}$. Notice that the mixing term induces dependency of the solution on SUSY derivatives on the L superfields, which by no means we can suppress *a priori*. It is thanks to the small parameter that these are controlled and the whole δH is small. Plugging the solution back in the original theory induces, schematically,

$$W_{eff} = W(H_0) + \frac{1}{2}\epsilon^2 W_{ij}(H_0)\delta H^i\delta H^j + \mathcal{O}(\epsilon^3), \quad (14)$$

$$K_{eff} = K(H_0) + \epsilon K_i(H_0)\delta H^i + \epsilon K_{\bar{i}}(H_0)\delta \overline{H}^{\bar{i}} + \mathcal{O}(\epsilon^2), \quad (15)$$

where we have factored out the parameter ϵ characterizing the magnitude of δH . The $\mathcal{O}(\epsilon)$ corrections to the Kähler potential at first sight seem to affect drastically the theory

by inducing terms with size comparable to that of the original ones, in particular the terms in K_{mix} . However, since H_0 is a constant c-number and δH is a chiral superfield, the corrections correspond to a Kähler transformation and leads only to total derivatives in the Lagrangian. We conclude, therefore, that since the supercovariant derivatives appear from $\mathcal{O}(\epsilon^2)$, a truncation at the $\mathcal{O}(\epsilon)$ level leaves an effective description that has the structure of a two-derivative SUSY theory.

We have, then, that in both cases the superfield equation to be used in the integration of fields is the promotion of the F-flatness condition to the superspace level. We can state this in the effective theory language, where always is only a small portion of the field configuration space that is explored, by saying that as long we stand in a region where the field configuration for the H sector is approximately SUSY preserving, say with deviations parametrically of order ϵ , the theory obtained by integrating out this sector is a theory that at leading order in ϵ is a two-derivative SUSY theory.⁺ For the case of factorizable models the reliability goes up to the $\mathcal{O}(\epsilon)$.

One should expect that once the full decoupling is loosed further constrains might proceed. This since even in the case of small coupling after long periods of time, and space variations, the two sectors share enough energy to affect one to the other.* This constrain is however already encoded in the fact that the solution for the H sector has leading part with vanishing space-time derivatives, implying that for the L sector the H one is an homogeneous one from which such an energy transfer does not occur. But once the coupling is allowed to be of order ϵ , the space-time variations are also of this order and therefore the reliability of the two-derivative description holds for space-time scales of order ϵ^{-1} . In this case of SUSY theories, this will be also the space-time scale for which a SUSY solution is reliable, for the H auxiliary components start to be non negligible. The same considerations apply for the following SUGRA case.

A final remark is in order. Although a hierarchy or decoupling is a necessary condition for a small perturbation on the solution for the H sector, which we need to preserve SUSY, a further assumption is required, namely that the H sector be indeed stabilized. Understanding by stabilization points the ones where the Hessian of the scalar potential has no negative nor zero eigenvalues in these directions, so that indeed these acquire positive mass squared and so fluctuations are not dramatic for the stability of the solution.

4. Two-derivative SUGRA effective theories

We work directly with the Kähler invariant function, $G = K + \ln |W|^2$, as this Kähler gauge is usually cleaner in the results and therefore convenient for cases where the superpotential is non vanishing or, as in our case, does not introduce any important scaling by, say, a tiny VEV. It is also convenient to use the superconformal formalism and compensator technique to write down the action [36, 37, 38]. In this setup the off-

⁺ The small parameter for the case with hierarchies is given by the ratio of mass scales, i.e., $\epsilon \sim m_L/m_H$.

* We thank an anonymous referee for pointing out this important point.

shell minimal SUGRA supermultiplet is split and one of the two auxiliary fields is now contained in a compensator chiral supermultiplet Φ , required by Weyl symmetry, which later on is gauge fixed in order to recover the actual symmetries of SUGRA. Under this formalism the tensor calculi are almost the same of rigid SUSY, allowing to write down the Lagrangian as an integral over rigid supercoordinates. In our Kähler gauge, for the moment without gauge interactions, the generic two-derivative Lagrangian reads [38, 39]:

$$\mathcal{L} = -3 \int d^2\theta d^2\bar{\theta} e^{-G/3} \Phi \bar{\Phi} + \int d^2\theta \Phi^3 + h.c. + \dots, \quad (16)$$

the ellipses containing terms implying the graviton, gravitino and the remaining auxiliary field from the SUGRA multiplet, also including couplings with the matter multiplets. For the moment we neglect them in our analysis and comment about the consistency of the procedure at the end.

Again we consider models with two sectors of chiral fields $\{H^i\}$ and $\{L^\alpha\}$, then the exact superfield e.o.m. for the H^i reads:

$$-\frac{1}{4} \Phi \bar{\mathcal{D}}^2 (G_i e^{-G/3} \bar{\Phi}) = 0, \quad (17)$$

where we have used again the identity (9) and the fact that the superfield Φ is chiral. Regarding $\Phi \neq 0$ and expanding the previous expression we have,

$$e^{-G/3} \bar{\Phi} \left(G_{i\bar{I}\bar{J}} \bar{\mathcal{D}}^{\bar{I}} \bar{\mathcal{D}}^{\bar{J}} \bar{\Phi} + G_{i\bar{I}} \bar{\mathcal{D}}^2 \bar{\Phi} \right) + G_i \bar{\mathcal{D}}^2 (e^{-G/3} \bar{\Phi}) + 2G_{i\bar{I}} \bar{\mathcal{D}}^{\bar{I}} \bar{\mathcal{D}} (e^{-G/3} \bar{\Phi}) = 0, \quad (18)$$

where for simplicity in the notation we omit the spinor index in the SUSY covariant derivatives, and the I, J indices run over all superfields H^i and L^α . From previous arguments, the SUSY two-derivative description is reliable if somehow around the solution to the e.o.m. we can neglect the covariant derivatives.

Like before we can consider the case where the fields to be integrated out are heavy compared with the other energy scales (see for example [6, 7, 8, 11]) in which case the e.o.m. is dominated by the term proportional to G_i , whose leading part is W_i/W . Then at leading order in inverse powers of the heavy masses the solution satisfy the equation $W_i = 0$ which is precisely the one found for the case of rigid SUSY, and not depending on the SUSY covariant derivative leads to a reliable two-derivative SUGRA theory.

One can as well study the case of no hierarchy in the masses with decoupled sectors, though in SUGRA exact decoupling is not possible due to the gravitational interactions. We leave the most ideal case of sequestered models for a later comment, and concentrate in a second class of models, proposed first in [15] and studied at the level of the scalar potential in [4, 8, 16]. The main property of such models can be summarized in a Kähler invariant function with the following structure

$$G = G_H(H, \bar{H}) + G_L(L, \bar{L}) + \epsilon G_{mix}(H, \bar{H}, L, \bar{L}), \quad (19)$$

with G_H and G_L of the same order of magnitude and ϵ small, parameterizing the coupling between the two sectors. We emphasize, however, that the smallness of the mixing is not necessarily due to a small coupling but rather that around the solutions to the e.o.m.

all mixed terms turn out to be small. This form for the G function in the superfield e.o.m. implies that all mixed derivatives, $G_{i\alpha}$ or higher order, are suppressed so that the leading terms in (18) are proportional either to covariant derivatives of the H fields or to the G_i . Schematically this is:

$$G_i \bar{\mathcal{D}}^2 (e^{-G/3} \bar{\Phi}) + \mathcal{O}(\bar{\mathcal{D}}H, (\bar{\mathcal{D}}H)^2, \bar{\mathcal{D}}^2 \bar{H}) = \mathcal{O}(\epsilon). \quad (20)$$

For solutions with an approximate two-derivative SUSY description in the effective theory the second term should be negligible and, thus, the first one must vanish independently.‡ In general the factor accompanying the G_i is non zero since depends on the supercovariant derivatives on the L fields, which are expected to be large and supposed to be linear independent, then this term vanishes only if we require $G_i = 0$. Due to the factorization at leading order in ϵ the G_i composite superfield depends only on the H components (see Appendix A) and it vanishes trivially if the spinor and auxiliary components of the solution are null, as well the derivatives for the lowest and spinor components. The lowest component in turn should satisfy the F-flatness condition in the H directions. In fact, the e.o.m. we are finding is nothing but the analytical extension of these F-flatness conditions to superspace. Nicely enough, the supercovariant derivative acting on these solution automatically vanish. We conclude, therefore, that at leading order in ϵ the exact e.o.m. is solved by the solutions to the equation

$$G_i = 0. \quad (21)$$

In particular, for heavy fields, compared with the SUSY breaking scale, this e.o.m. reduces to the one found before, eq.(11).

Being more explicit, the exact solution for the H superfields has the following schematic form:

$$H = H_o + \epsilon \tilde{H}(L, \bar{L}, \bar{\Phi}, \bar{\mathcal{D}}\bar{L}, \bar{\mathcal{D}}\bar{\Phi}), \quad (22)$$

where H_o is the solution to $\partial_i G_H = 0$ and the remaining encodes the non-constant and non-holomorphic part, which in case of not being suppressed would spoil the two-derivative SUSY description. Plugging back the solution into the Kähler invariant function, we have that the effective theory is described by

$$G_{eff} = G_{H,o} + G_L(L, \bar{L}) + \epsilon G_{mix,o}(L, \bar{L}) + \mathcal{O}(\epsilon^2), \quad (23)$$

with the “nought” label indicating evaluation at $H = H_o$. Here it is clear that the theory is described, up to next to leading order in ϵ , by a valid G function with no supercovariant derivatives and therefore has a reliable two-derivative SUSY effective description.

We can apply the analysis to the sequestered case, for which the Lagrangian is given by [12, 13]

$$\mathcal{L} = -3 \int d^2\theta d^2\bar{\theta} e^{-G_H/3} \Phi \bar{\Phi} - 3 \int d^2\theta d^2\bar{\theta} e^{-G_L/3} \Phi \bar{\Phi} + \int d^2\theta \Phi^3 + h.c. + \dots, \quad (24)$$

‡ A SUSY description is possible even in the case the hidden sector breaks SUSY as long a superfield description is taken for the goldstino (see [7]). The symmetry, however, will be non-linearly realized.

where G_H and G_L depend only on H and L respectively. Here it is clear that due to gravitational interactions, encoded in the compensator, the two sectors cannot be completely decoupled, though the situation is better than in the previous case. The e.o.m. for the H superfield has an analogous expression like (18), replacing G by G_H and the index I, J running only over the H sector. Then, the analysis follows almost verbatim noticing that now the corrections to the exact SUSY condition depend only on the compensator, which plays the role of communicating any SUSY breaking effect from, or to, the visible sector in what is called gravitino mediation part of the full anomaly mediation going on in this kind of models [40]. We find, therefore, that for the effects of having a two-derivative SUSY effective description the factorizable models are as safe as the sequestered ones, although the last ones have the advantage of being further decoupled strengthening the dynamical constraint on the excitation of H fields. Let us close this section by drawing attention to a potential issue on equation (21), that is the fact that, contrary to the exact e.o.m (17), it is not a chiral superfield equation. Indeed the G composite superfield is real and therefore the equation has more components than a chiral one, preventing us, in general, from using it for the integration of chiral fields [11]. In the case of factorizable models, however, this is avoided as the antiholomorphic components of the equations are trivially consistent with the holomorphic ones, in the sense that both lead to vanishing spinor, spinor derivatives and auxiliary components in both H and \bar{H} , which at the same time are consistent with the lowest component of the equation that, as said before, is the F-flatness condition. Therefore, the leading part of the solution is given by the chiral set $H_o = \{h_o, 0, 0\}$.

5. Gauge interactions

The presence of gauge interactions modifies the analysis, first by the inclusion of the vector superfields V^A in a gauge invariant way in $G = G(\phi^I, \bar{\phi}^{\bar{I}}, V^A)$, the index A running over the gauge group generators, and then by their kinetic term,

$$\mathcal{L}_{gau-kin} = \frac{1}{4} \int d\theta^2 f_{AB}(\phi^I) \mathcal{W}^A \cdot \mathcal{W}^B + h.c. \quad (25)$$

with superfield strengths $\mathcal{W}_\alpha = -\frac{1}{4} \bar{\mathcal{D}}^2 (e^{-V} \mathcal{D}_\alpha e^V)$, α the spinor index, and the chiral superfield only entering through the gauge kinetic holomorphic function f_{AB} . We do not consider Fayet-Iliopoulos terms as they seem to be inconsistent with SUGRA [41]. Then, for a generic form for f_{AB} , the e.o.m. for the H^i superfield is corrected by

$$\begin{aligned} \partial_i \mathcal{L} \supset & -\frac{1}{4} \Phi \left[e^{-G/3} \bar{\Phi} \left(G_{i\bar{I}A} \bar{\mathcal{D}} \bar{\phi}^{\bar{I}} + G_{iAB} \bar{\mathcal{D}} V^B + G_{iA} \bar{\mathcal{D}} \right) + 2G_{iA} \bar{\mathcal{D}} (e^{-G/3} \bar{\Phi}) \right] \bar{\mathcal{D}} V^A \\ & + \frac{1}{4} f_{AB,i} \mathcal{W}^A \cdot \mathcal{W}^B. \end{aligned} \quad (26)$$

These are automatically subleading in case the H fields develop large masses. Indeed, among others, these are related to the SUSY breaking scale through a D-term breaking. For factorizable models the SUSY covariant derivatives acting on the L fields do not appear but (21) is no longer a solution due to the presence of the SUSY covariant

derivatives of vector superfields. Therefore, the two-derivative description is not valid either. Notice that all terms inside the brackets in (26) are null for neutral, i.e., gauge invariant, H fields as all mixed derivatives of G with the vector fields vanish.

Actually it is only in the case that the H fields are neutral that our construction is well stated. Indeed, under a gauge transformation we can mix the H and L sectors such that the factorization in G is lost. In physical grounds this is also clear as the fields that are supposed to be mapped out can be sourced back by its interaction with the gauge sector, not having any kinematical constraint forbidding this process.

This last observation warns us about the coupling the H sector can have with the gauge sector from the sigma model ruled by the gauge kinetic function. In the e.o.m. this is made explicit in the last term in (26), which, moreover, makes the $G_i = 0$ a non reliable e.o.m. for the H sector. We have, therefore, that for neutral fields that appear suppressed in the gauge kinetic function all terms in (26) are negligible and one can trust the solutions from (21), which lead to a two-derivative SUSY effective description. There might be particular situations where the D-term SUSY breaking turns out to be suppressed, for instance in the LVS studied in [16], and therefore the back-reaction in the H sector is mild enough to be subleading. Then, as far as for the scalar potential is concerned, up to the mass level, no suppression in the gauge kinetic function is needed for a leading SUSY freezing of the H sector [16]. However, this does not imply negligible contributions to the H superfields solutions coming from other components of $\mathcal{W}^{\alpha,A}$, e.g., the field strength and gauginos in the Wess-Zumino gauge, which would induce, in particular, non suppressed higher order derivative terms for the vector fields and higher order fermion bilinear for the gauginos. So contrary to the case studied by Brizi et al. (see also [10]), where these higher order terms are suppressed by the mass of the H fields and therefore negligible, an approximate two-derivative SUSY effective theory is only realized for suppressed dependencies of the H fields in the gauge kinetic function. We can allow the H fields to be charged under a hidden gauge sector \mathcal{G}_H , such that the whole gauge group is given by $\mathcal{G} = \mathcal{G}_H \otimes \mathcal{G}_L$ and the vector superfields are split as $V^a \in \mathcal{G}_L$ and $V^r \in \mathcal{G}_H$, labeled by lower case letters in the beginning and middle of the alphabet respectively. This avoids easily all possible pathologies we just mention for a charged H sector.

In this case, however, one should keep in mind the gauge invariance of the hidden sector, implying that

$$G_r = -iX_r^i G_i, \quad (27)$$

with X_r^i the Killing vectors, so the set of equations $\{G_i = 0\}$ is no longer linear independent and able to stabilize all H directions. Indeed, these flat directions are related to would-be Goldstone fields appearing after gauge symmetry breaking. We should, therefore, integrate out also the vector superfields that acquire masses in the process, a situation that can be easily studied using the superspace approach. With no loss of generality, in order to be more explicit, we show the Abelian case for which the

e.o.m. reads:††

$$G_r e^{-G/3} \bar{\Phi} \Phi + \frac{1}{8} \left[\mathcal{D}^\alpha \left(f_{rp} \bar{\mathcal{D}}^2 \mathcal{D}_\alpha V^p \right) + \bar{\mathcal{D}}^{\dot{\alpha}} \left(\bar{f}_{rp} \mathcal{D}^2 \bar{\mathcal{D}}_{\dot{\alpha}} V^p \right) \right] = 0, \quad (28)$$

where α and $\dot{\alpha}$ here stand for the spinor index and we have regarded no kinetic mixing in the gauge sector, i.e., $f_{ar} = 0$.

Then, requiring the H field dependencies of the gauge kinetic function for \mathcal{G}_L to be suppressed, only the SUSY covariant derivatives on the H and \mathcal{G}_H sectors appear in the e.o.m for the H fields. The same should be imposed for the dependency of the \mathcal{G}_H gauge kinetic function on the L fields, otherwise their covariant derivative would appear in the e.o.m. in (28). The implementation of the vector superfield integration corresponding to broken symmetries requires a gauge fixing, being the unitary gauge the one with clearest physical interpretation. However, in practice it is useful to work in a gauge where a chiral superfield, with no vanishing component in the would-be Goldstone direction, is simply fixed to its VEV. Then, as long as the SUSY covariant derivatives on the H and V^r superfields are negligible there is a reliable two-derivative SUSY description after the integration of the fields through the set of e.o.m.

$$G_{\tilde{i}} = 0, \quad G_r = 0, \quad (29)$$

where \tilde{i} runs over the chiral fields not affected by the gauge fixing, the integration of other fields being encoded in the longitudinal modes of the massive vector supermultiplets. Then, from these equations at leading order in ϵ we have: the lowest components impose simultaneous F and D-flatness conditions, obtained by arrangement of the lowest components of the chiral and vector superfields. The other components are again trivially solved for vanishing, spinor, vector and auxiliary components, plus their space-time derivatives. We can be more precise by working in the Wess-Zumino gauge. In this case the \tilde{i} index in the set of equations is not constrained but we can easily spot the effect of the real superfield components. Again the trivial null components is solution and the D-flatness condition is obtained, instead, through the lowest components of the chiral fields alone (see [Appendix A](#)).

Notice, that although no restriction on the space-time variation of the vector and gauginos are obtained, the only possibility of having vanishing solutions everywhere is because these are also null. Therefore, the whole supercovariant derivative on the vector superfield can be neglected as well and the solution to the set of equations (29) are solutions to the e.o.m., which moreover result in a reliable two-derivative SUSY effective description.

6. Gravitational sector and gauge fixing

In the previous analysis we disregarded the gravitational sector contribution to the action encoded in the ellipses in (16). On the other hand, we have shown that the

††To obtain this equation we use again (9) but now to write the F-terms as D-terms integrals over the whole superspace and using the supercovariant derivatives already present in the superfield strengths.

effective theory at next to leading order in ϵ is described by a theory with superconformal symmetry, namely, the one obtained by the G function with the H superfields frozen out. Therefore, since the gravitational terms are univocally dictated by the covariance of the symmetries these terms are also well described by the truncated theory.

On the other hand, the dilatation, axial and S transformations of the superconformal algebra, not being actual symmetries of SUGRA, should be gauge fixed requiring a canonical normalization in the gravity sector action, eliminating for example kinetic mixings with the matter sector. This proceeds by fixing the compensator in terms of the chiral superfields. However, since the form of G implies decoupling only between the H and L sector but not with the compensator, it is not automatically clear that the gauge fixing is the same in both descriptions or, in other words, that the SUGRA theory that is obtained upon the gauge fixing coincide at leading order. Writing the compensator components as $\Phi = \phi\{1, \chi_\phi, U\}$ the fixing reads [42]:

$$\phi \equiv e^{G/6}, \quad \chi_\phi \equiv \frac{1}{3}G_I\chi^I, \quad (30)$$

where the G function and its derivatives are evaluated in the lowest components of the superfields and χ^I are the spinor components of the chiral multiplets. Since around the solution to the e.o.m. for the H fields the terms not appearing in the truncated theory, namely, $G_i\chi^i$, are of order ϵ and the functions G and G_α coincide in both theories at next to leading order, the gauge fixing is the same modulo subleading terms.

One of the main targets of the present letter is to clarify the integration of the fields at the superfield level, however, the gauge fixing in (30) cannot be promoted to the superspace as the compensator is a chiral field and therefore cannot depend on the fields in the antiholomorphic sector contained in G . A variation to the fixing which can be performed directly in the superspace is the one proposed by Cheung et al. in [43] that in our Kähler gauge reads:

$$\Phi \equiv e^{Z/3}(1 + \theta^2 U), \quad (31)$$

with Z a chiral superfield given by

$$Z = \langle G \rangle + \langle G_I \rangle \phi^I, \quad (32)$$

where the $\langle \rangle$ means the VEV. Again since the VEV's in both descriptions coincide at leading order and the terms not appearing in the truncated description are suppressed, the Z superfields, and therefore the full and truncated theories, match at leading order.

7. Discussion

In this letter we have studied the possibility of having a SUSY two-derivative description for effective theories resulting from the integration of light fields in $\mathcal{N} = 1$ SUGRA. The consistency of a derivative expansion with SUSY transformations requires a parallel expansion in spinor bilinears and auxiliary terms, that at the superfield level is seen as an expansion in the supercovariant derivatives and a reliable two-derivative effective description is the one where these can be neglected.

A first point drawn in the paper is the possibility, in a generic fields theory, of integrating out light fields provided there is a decoupling between the different modes. This decoupling serves as dynamical constraint that keeps the modes that have been mapped out indeed out of the theory although not kinematical constraint is available. This leads to what we call effective descriptions where we further allow a small coupling between the sectors. A first instance of this kind of situation is worked out in a rigid SUSY example. Here we explore also the condition for the two-derivative effective description to be consistent with SUSY transformations. Interestingly enough the moral learned here can be extended to the SUGRA situation explored later on.

Whenever we speak about an effective description we have in mind a region in the field configuration space around particular solutions of the e.o.m. for the fields that have been integrated out. We find that the integration of superfields leads to a reliable two-derivative SUSY effective description if such solutions preserve SUSY, approximately, albeit the remaining fields stand at points where SUSY is spontaneously broken. One possibility is that the SUSY preserving sector is heavy enough to present a hierarchy with the SUSY breaking scale such that the back-reaction from the breaking is suppressed [8, 11, 10]. For Kähler potentials with no singular behavior such a hierarchy is realized if in particular the gravitational effects, e.g., the contribution to the masses, are suppressed, and therefore the leading superfield e.o.m. coincides with the one obtained in rigid SUSY.

On the other hand, no hierarchy is necessary if in the Lagrangian the two sectors are decoupled, in which case the best scenario in SUGRA would be sequestered sectors. Still, one can allow further interactions, beside the gravitational ones, and achieve some SUSY decoupling if the theory is described by a Kähler invariant G function of the form (19). Although this was previously realized at the level of the scalar Lagrangian [4, 8, 16], our analysis shows that the situation can be understood in a fully SUSY framework by working directly in the superspace, an approach that also allows the study of more involved situations not regarded before. We find that the decoupling leads to subleading contributions from the SUSY covariant derivatives on the L sector, despite the fact these can be large, and then field configurations solving (21) coincide at leading order, in a ϵ expansion, with solutions of the exact e.o.m., implying negligible supercovariant derivatives from the H sector and, therefore, a reliable two-derivative SUSY description. Nicely enough this can be summarized as the condition that the superfield equation obtained as analytical continuation of the F-flatness condition to the superspace be a reliable e.o.m. for the H sector, independently of the L sector dynamics.

This superfield equation, although can be seen as a natural and naive guess, was already criticized as e.o.m. for chiral fields. Indeed, the equation is a real composite superfield equation, so it overdetermines a chiral solution having more equations than unknowns. However, for the case heavy H fields, compared with the SUSY breaking scale, this extra terms turn out to be negligible and the equation takes the chiral form found in [11] given by (11). On the other hand if the theory has a factorizable nature the equation

continues to be a real and in particular the gravitational effects are important. However, it is trivially solved at leading order by null higher components of the chiral superfield, and therefore it is consistent with a chiral solution.

It is important to notice that, contrary to the sequestered case, for the factorizable models it is only around the SUSY configurations for the H sector that there is freedom on integrating and decoupling light fields. Indeed, in case the H sector leads the breaking of SUSY the decoupling with the L sector is lost, as can be seen in (20), and the e.o.m. for H starts to be L dependent. In this case, although nice decoupling features are preserved [25, 26], some constraints on the mass of the integrated fields appear for a decoupling to apply [27, 24]. Interestingly enough, under these circumstances for the effects of integrating out fields requiring a reliable two-derivative SUSY description we find that the factorizable models are as safe as the sequestered ones.

The fact that ours is not a low-energy description alerts about the fact that even in case the fields to be integrated out are neutral these can be sourced back by the vector fields from the coupling in the sigma model ruled by the gauge kinetic function. Thus, even if the D-term SUSY breaking is mild other terms in the covariant derivative of the gauge vector are not suppressed and therefore no reliable SUSY two-derivative description is available. One should, then, require a suppressed dependency of the gauge kinetic function on the H fields and, in this case, all the covariant derivative contributions to the e.o.m. are negligible at leading order such that the solutions are determined by (21). We can allow charged H fields but only under some hidden gauge group in which case the L fields should appear in a suppressed way in the corresponding gauge kinetic function. Possible flat directions resulting from symmetry breaking are handled by integrating out the vector supermultiplets, after a gauge fixing for the broken directions.

A nice fact of working directly in the superspace is that we are able to spot further considerations in the full set of e.o.m. that are cumbersome to find in working with the component Lagrangian. So, for example, we find that the condition for slow varying configurations in the H sector for a reliable two-derivative SUSY description is actually contained in the e.o.m. and there is no need of imposing it from outside, as was argued in [16]. That the e.o.m. induces vanishing space-time derivatives can be understood from the fact that it is the only way to preserve SUSY, since otherwise these would induce non-vanishing values of the spinor and auxiliary fields after SUSY transformations.

Then, in general, we can say that the theory obtained by integrating out fields is reliably described by a two-derivative SUSY theory if the analytic extension of the F-flatness and D-flatness conditions to the superspace are reliable superfield e.o.m. for the process of integration. In other words, SUSY preserving solutions is a sufficient condition for such a description to proceed. Clearly, in concluding this we have in mind that all operators that appear suppressed before the truncation have a counterpart in the truncated Lagrangian, otherwise will be misleading to neglect them in some contexts, like is the case of baryon number violating operators in effective descriptions of grand unified theories and the study of proton decay.

In order to be more precise about the validity of these results one should also look for

possible energy and momentum transfer from the background, dictated by the solutions for H , to the the L sector. These, in our case indeed happen as the space-time variation in H sector is not exactly null. However, being of order ϵ we can safely scan regions in the space-time that are within a radius of order ϵ^{-1} , out of which we cannot longer consider the H sector as homogeneous and in particular SUSY preserving.

Then, although our study is a step forward in the understanding one of the many simplification behind explicit constructions in supersymmetric theories it would be important to explore more carefully the nature of higher order operators. Another immediate question is to what extent our conclusions apply in more complicated setups, like ones that mix mild hierarchies and rather small couplings simultaneously, and/or with further sectors in the game. Those questions we hope to address in future works.

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Appendix A. Components for the equation $G_{H,i} = 0$.

Let us start with the case of global SUSY for which the leading e.o.m. is $W_{H,i} = 0$. To make explicit the components we write it as

$$W_{H,i}(H) = W_{H,i}(h) + W_{H,ij}(h)\Delta^j + \frac{1}{2}W_{H,ijk}(h)\Delta^j\Delta^k, \quad (\text{A.1})$$

where $\Delta^i = H^i - h^i = \sqrt{2}\theta\psi^i - \theta\theta F^i$, h^i , ψ^i and F^i the lowest, spinor and auxiliary components of the superfield H , and the argument in the fields is $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. Expanding to make explicit the $\bar{\theta}$ dependencies so far implicit in y^μ , we find the components:

$$\theta^0 \quad : W_{H,i} = 0, \quad (\text{A.2})$$

$$\theta \quad : W_{H,ij}\psi^j = 0, \quad (\text{A.3})$$

$$\theta\sigma^\mu\bar{\theta} \quad : W_{ij}\partial_\mu h^j = 0, \quad (\text{A.4})$$

$$\theta^2 \quad : -W_{H,ij}F^j - \frac{1}{2}W_{H,ijk}(h)\psi^i\psi^j = 0, \quad (\text{A.5})$$

$$\theta\sigma^\mu\bar{\theta}\theta_\alpha \quad : \partial_\mu(W_{ij}\psi^\alpha) = 0, \quad (\text{A.6})$$

$$\theta^2\bar{\theta}^2 \quad : W_{ijk}\partial^\mu h^j\partial_\mu h^k = 0. \quad (\text{A.7})$$

The first equation implies that the lowest components arrange so to satisfy the F-flatness condition. The others are then solved with vanishing spinor and auxiliary components. Moreover, vanishing space-time derivatives are required from the non-holomorphic components.

For $G_{H,i}$ we can do something similar but now expanding Δ making explicit the θ dependencies. Then,

$$\theta^0 : G_{H,i} = 0, \quad (\text{A.8})$$

$$\theta : G_{H,ij} \psi^j = 0, \quad (\text{A.9})$$

$$\bar{\theta} : G_{H,i\bar{j}} \bar{\psi}^{\bar{j}} = 0, \quad (\text{A.10})$$

$$\theta^2 : -G_{H,ij} F^j - \frac{1}{2} G_{H,ijk} \psi^j \psi^k = 0, \quad (\text{A.11})$$

$$\bar{\theta}^2 : -G_{H,i\bar{j}} \bar{F}^{\bar{j}} - \frac{1}{2} G_{H,i\bar{j}\bar{k}} \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{k}} = 0, \quad (\text{A.12})$$

$$\theta \sigma^\mu \bar{\theta} : i G_{H,ij} \partial_\mu h^j - i G_{H,i\bar{j}} \partial_\mu \bar{h}^{\bar{j}} + G_{H,ijk} \psi^j \sigma_\mu \bar{\psi}^{\bar{k}} = 0, \quad (\text{A.13})$$

$$\begin{aligned} \bar{\theta} \theta^2 : & -i \left[G_{H,ij} \partial_\mu \psi^j - \frac{1}{2} G_{H,ijk} (\partial_\mu h^j \psi^k + \partial_\mu h^k \psi^i) - G_{H,ijk} \partial_\mu \bar{h}^{\bar{k}} \psi^j \right] \sigma^\mu \\ & - 2 G_{H,ijk} F^i \bar{\psi}^{\bar{k}} - G_{H,ijk\bar{l}} \psi^j \psi^k \bar{\psi}^{\bar{l}} = 0, \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned} \theta \bar{\theta}^2 : & i \sigma^\mu \left[G_{H,i\bar{j}} \partial_\mu \bar{\psi}^{\bar{j}} - \frac{1}{2} G_{H,i\bar{j}\bar{k}} (\partial_\mu \bar{h}^{\bar{j}} \bar{\psi}^{\bar{k}} + \partial_\mu \bar{h}^{\bar{k}} \bar{\psi}^{\bar{i}}) - G_{H,i\bar{j}\bar{k}} \partial_\mu h^j \bar{\psi}^{\bar{k}} \right] \\ & - 2 G_{H,i\bar{j}\bar{k}} \bar{F}^{\bar{k}} \psi^j - G_{H,i\bar{j}\bar{k}\bar{l}} \psi^j \bar{\psi}^{\bar{k}} \bar{\psi}^{\bar{l}} = 0, \end{aligned} \quad (\text{A.15})$$

$$\begin{aligned} \theta^2 \bar{\theta}^2 : & G_{H,i\bar{j}\bar{j}} \left(F^j \bar{F}^{\bar{j}} + \partial_\mu h^j \partial^\mu \bar{h}^{\bar{j}} - \frac{i}{2} \psi^j \sigma^\mu \partial_\mu \bar{\psi}^{\bar{j}} + \frac{i}{2} \partial_\mu \psi^j \sigma^\mu \bar{\psi}^{\bar{j}} \right) \\ & + \frac{i}{4} G_{H,ijk\bar{k}} \left(\psi^j \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu h^k + \psi^k \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu h^j - 2i h^j \psi^k \bar{F}^{\bar{k}} \right) \\ & + \frac{i}{4} G_{H,i\bar{j}\bar{k}\bar{k}} \left(\psi^k \sigma^\mu \bar{\psi}^{\bar{j}} \partial_\mu \bar{h}^{\bar{k}} + \psi^k \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu \bar{h}^{\bar{j}} + 2i \bar{h}^{\bar{j}} \bar{\psi}^{\bar{k}} F^k \right) \\ & + \frac{1}{4} G_{H,ijk\bar{j}\bar{k}} \psi^j \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{k}} - \frac{1}{4} \partial_\mu \partial^\mu G_{H,i} = 0, \end{aligned} \quad (\text{A.16})$$

where the arguments for $G_{H,i}$ and its field derivatives is the lowest components h and \bar{h} . The first equation indicates that the lowest components are arranged such to solve the F-flatness condition. The higher order components imply vanishing spinor and auxiliary components. Vanishing space-time derivatives are also necessary for the solution.

The presence of vector superfields change the previous components by adding the following terms, in the Wess-Zumino gauge:

$$\theta \sigma^\mu \bar{\theta} : G_{H,is} v_\mu^s, \quad (\text{A.17})$$

$$\bar{\theta} \theta^2 : i G_{H,is} \bar{\lambda}_\alpha^s - \frac{1}{\sqrt{2}} G_{H,ijs} v_\mu^s (\psi^j \sigma^\mu)_\alpha, \quad (\text{A.18})$$

$$\theta \bar{\theta}^2 : -i G_{H,is} \lambda_\alpha^s - \frac{1}{\sqrt{2}} G_{H,ijs} v_\mu^s (\sigma^\mu \bar{\psi}^j)_\alpha, \quad (\text{A.19})$$

$$\theta^2 \bar{\theta}^2 : \frac{1}{2} G_{H,is} D^s + \frac{1}{4} G_{H,ist} v_\mu^s v^{\mu,t}$$

$$+ \frac{i}{2} G_{H,ij_s} (\partial_\mu h^j v_\mu^s - \sqrt{2} \psi^j \bar{\lambda}^s) - \frac{i}{2} G_{H,i\bar{j}_s} (\partial_\mu \bar{h}^{\bar{j}} v_\mu^s - \sqrt{2} \lambda^s \bar{\psi}^{\bar{j}}). \quad (\text{A.20})$$

The superfield e.o.m. $G_r = 0$ has analogous leading terms, obtained just by changing in the previous ones the index i by r .

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