

# Past and future of some universes

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## Abstract

We consider a class of toy models where a spatially flat universe is filled with a perfect fluid. The dynamics is found exactly for all these models. In one family, the perfect fluid is of the phantom type and we find that the universe is first contracting and then expanding while the dynamics is always accelerated. In a second family, the universe is first in an accelerated expansion stage, then in a decelerated expansion stage until it reaches a turning point after which it contracts in a decelerated way (increasing contraction rate) followed by another accelerated stage (decreasing contraction rate). We also consider the possibility to embed this perfect fluid in a realistic cosmology. The first family cannot be viable in a conventional big bang universe and requires a rebound in the very early universe. The second family is viable in the range  $0 < 1 + w_{DE,0} \lesssim 0.09$  for a spatially closed universe with a curvature satisfying current bounds. Though many of the models in this family cannot be distinguished today from a universe dominated by a cosmological constant, the present accelerated expansion is transient and these universes will reach a turning point in the future before entering a contraction phase.

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# 1 Introduction

Since the discovery of the universe present accelerated expansion rate [1], our view of expanding universes has been durably affected. There has been a lot of interest in models producing the accelerated expansion [2] as we see it at the present time. The simplest models introduce dark energy (DE) in the form of a comoving perfect fluid with a sufficiently negative pressure today. A very simple and appealing example is provided by a cosmological constant  $\Lambda$  which agrees well with existing data. Remarkably, this corresponds to a constant energy density and a constant pressure satisfying  $\rho_\Lambda + p_\Lambda = 0$  or  $w_\Lambda = -1$ . Leaving aside the conceptual problems raised by the existence of such a tiny cosmological constant, it remains to be seen whether more accurate observations will force us into a paradigm shift. In any case it remains an important issue to look for other viable alternatives where the energy density and pressure will have a different physical interpretation and behaviour. In particular it may well be that no DE component is needed at all if the present accelerated expansion is caused by modifications of gravity on cosmic scales. A microscopic interpretation of the DE energy density and pressure is given in models like quintessence [3], a time-dependent homogeneous scalar field minimally coupled to gravity. Through an Einsteinian interpretation of the modified Friedmann equations, a microscopic description of an effective DE energy density and pressure can be given as well for some models going beyond General Relativity, (see e.g.[4] in the context of  $f(R)$  models) where gravity is modified on cosmic scales. This radical departure from conventional expansion histories has also brought interest for DE of the phantom type ( $w_{DE} < -1$ ) [5] and for the future of our universe [6]. As we will see below, our work touches upon both issues.

Here we introduce a class of phenomenological perfect fluids and we study the corresponding universe dynamics. The crucial property is that the energy density of this perfect fluid can vanish in the course of expansion or contraction, depending on the specific model. We consider toy models where spatially flat universes contain only this perfect fluid and we solve for their dynamics. As expected this gives rise to very peculiar dynamics. In one family the perfect fluid is forever of the phantom type and it will tend to a cosmological constant in the asymptotic future after passing through a bounce. In the second family the universe expands till a turning point after which it starts contracting. We also consider the possibility to include such a component in a realistic cosmology. We will find that viable universes can exist with a radical departure from standard cosmological scenarios either in the past, for the perfect fluid of the phantom type, or in the future for the second family. The departure in the universe past in the first case comes from a conceptual problem and requires probably to go beyond General Relativity in the early universe. However for the second family a universe with a peculiar future can be obtained in the framework of General Relativity.

## 2 The model

We consider a Friedmann-Lemaître- Robertson-Walker (FLRW) spatially flat universe filled with a comoving perfect fluid of energy density  $\rho$  and pressure  $p$ . We have the following equations

$$H^2 = \frac{8\pi G}{3} \rho \quad (1)$$

$$\dot{H} = -4\pi G (\rho + p) , \quad (2)$$

The time evolution of the perfect fluid

$$\dot{\rho} = -3H(\rho + p) = -3H(1 + w)\rho , \quad (3)$$

is contained in (1),(2) and we have introduced the equation of state (EoS) parameter  $w \equiv p/\rho$ . Let us start with a perfect fluid satisfying

$$p = \text{constant} . \quad (4)$$

It is easy to derive from (1),(2)

$$d(\rho a^3) = -p da^3 = -d(p a^3) , \quad (5)$$

which leads to

$$\rho a^3 = -p a^3 - B , \quad (6)$$

or

$$\rho = -p - \frac{B}{a^3} \geq 0 . \quad (7)$$

Setting  $p = -A$ , we can write

$$\rho = A - \frac{B}{a^3} . \quad (8)$$

It is easy to understand why both terms can appear in (8): from (3), the first term leads to  $\rho$  and  $p$  being constant while the second term corresponds to  $p = 0$ . The important point here is that  $B$  is an arbitrary integration constant. It can vanish and while it does not, it can have *arbitrary* sign, the sign is not specified by (4).

The case  $B > 0$  corresponds to a phantom equation of state,  $w < -1$ . This will always be the case, there is no crossing of the phantom divide line. We have in particular from (2)

$$\dot{H} \geq 0 . \quad (9)$$

It is not possible in this case to interpret (8) as a cosmological plus some dust because the additional dustlike component would have a negative energy density due to the “wrong” sign of  $B$ . It is the sum of both terms in (8) that gives a physically sound energy density satisfying  $\rho \geq 0$ .

Solving for the scale factor  $a$  as a function of  $t$  we obtain

$$a = a_B \cosh^{\frac{2}{3}} \left[ \frac{3}{2} H_\infty (t - t_B) \right] . \quad (10)$$

Here  $a_B$  is the minimal value of the scale factor reached at time  $t_B$  where the Hubble parameter  $H$  vanishes. Equation (8) can be recast as

$$\rho = A \left[ 1 - \left( \frac{a_B}{a} \right)^3 \right] . \quad (11)$$

Though the universe we consider is spatially flat, we get a bouncing universe with contraction at times  $t < t_B$  and expansion at times  $t > t_B$ . We have defined

$$H_\infty^2 \equiv \frac{8\pi G}{3} A , \quad (12)$$

which represents the value of  $H$  in the asymptotic future. The Hubble parameter  $H$  in function of  $t$  is readily obtained from (10), viz.

$$H = H_\infty \tanh \left[ \frac{3}{2} H_\infty (t - t_B) \right] , \quad (13)$$

from which we see again that  $H$  tends to  $H_\infty$  for  $t \rightarrow \infty$ . To summarize, our universe is always in a phantom stage, it is first contracting then expanding and tends to a pure de Sitter universe with  $H = H_\infty$  for  $t \rightarrow \infty$ .

It is interesting to compare (10) and (13) with the solutions obtained in a matter dominated universe with a cosmological constant  $\Lambda$

$$a = \left( \frac{\Omega_{m,0}}{\Omega_{\Lambda,0}} \right)^{\frac{1}{3}} a_0 \sinh^{\frac{2}{3}} \left( \frac{3}{2} H_\infty t \right) \quad (14)$$

$$H = H_\infty \coth \left( \frac{3}{2} H_\infty t \right) , \quad (15)$$

with  $H_\infty^2 \equiv \frac{\Lambda}{3}$ . In this case we have of course a ‘‘Big Bang’’ singularity with  $H \rightarrow \infty$  as  $t \rightarrow 0$ . It is the past which is radically different for both universes.

We can look for more general behaviours

$$\rho = A - \frac{B}{a^n} \quad A, B > 0 . \quad (16)$$

We have in particular  $A - \rho > 0$ . It is easy to obtain the corresponding pressure  $p$

$$p = -\rho - \frac{n}{3} (A - \rho) . \quad (17)$$

We consider first the case  $n > 0$ . We check that a phantom type equation of state is obtained for  $n > 0$ , a fact already clear by inspection of (16). Hence a universe filled with a perfect fluid that evolves according to (16) undergoes only accelerated expansion. We can generalize the expressions (10), (13), viz.

$$a = a_B \cosh^{\frac{2}{n}} \left[ \frac{n}{2} H_\infty (t - t_B) \right] . \quad (18)$$

$$H = H_\infty \tanh \left[ \frac{n}{2} H_\infty (t - t_B) \right] . \quad (19)$$

Actually these expressions apply to negative  $n$  as well. It is interesting to consider also the case  $n < 0$  as well. Of course, now we have no phantom type equation of state and  $\dot{H} < 0$ . The universe is a de Sitter space in the infinite past and we can call its (constant) Hubble parameter  $H_\infty$ . It will reach some turning point in the future with expansion followed by a contraction stage. We have now (for  $n < 0$ )

$$a = a_B \cosh^{-\frac{2}{|n|}} \left[ \frac{|n|}{2} H_\infty (t - t_B) \right] . \quad (20)$$

$$H = H_\infty \tanh \left[ -\frac{|n|}{2} H_\infty (t - t_B) \right] . \quad (21)$$

It is seen that  $a_{-n} = a_n^{-1}$  from which we obtain, in accordance with eqs. (18) and (20)

$$H_{-n} = -H_n . \quad (22)$$

For  $n < 0$ ,  $\dot{H} < 0$  so that the expansion is not always accelerated. Actually the evolution is now very different. The universe is first expanding and then it starts contracting at time  $t_B$ . In the asymptotic past,  $t \rightarrow -\infty$ , the universe tends to an expanding de Sitter space while it tends to a contracting de Sitter space in the asymptotic future,  $t \rightarrow \infty$ . During some (symmetric) interval around  $t_B$  one has  $\ddot{a} < 0$  similarly to a spatially closed universe going over from expansion to contraction. At some time  $t_a < t_B$ , the evolution goes over from accelerated expansion to decelerated expansion on the interval  $t_a = t_B - \Delta t < t < t_B$  with  $\Delta t \equiv t_B - t_a$ . This is followed by decelerated contraction on the interval  $t_B < t < t_B + \Delta t$ . For  $t > t_B + \Delta t$  the universe undergoes accelerated contraction. The interval  $\Delta t$  is easily found, viz.

$$\sinh^2 \left[ \frac{|n|}{2} H_\infty \Delta t \right] = \frac{|n|}{2} . \quad (23)$$

It is seen that the interval  $\Delta t$  depends on  $|n|$ . Hence for  $n < 0$ , the universe evolution can be summarized as follows:

$$\ddot{a} > 0 \quad |t - t_B| > \Delta t \quad (24)$$

$$\ddot{a} < 0 \quad |t - t_B| < \Delta t , \quad (25)$$

with  $\Delta t$  found from (23), and further

$$\dot{a} > 0 \quad t < t_B \quad (26)$$

$$\dot{a} < 0 \quad t > t_B . \quad (27)$$

Let us turn our attention to the equation of state (EoS) parameter  $w = \frac{p}{\rho}$ . It is straightforward to derive

$$w = -1 - \frac{n}{3} \left( \frac{A}{\rho} - 1 \right) , \quad (28)$$

which shows that  $w$  diverges whenever  $H = 0$ . This has to be the case as we show now.

Assuming a contracting universe ( $n > 0$ ) starts from some regular initial conditions at time  $t_1$  and reaches a bounce at time  $t_B$ , the following equality must hold from (1),(2)

$$\int_{t_1}^{t_B} dt (1 + w) = -\infty . \quad (29)$$

Clearly, this equality cannot be satisfied for  $t_B < \infty$  if  $w$  is regular. Assuming an expanding universe with  $n < 0$  starts from some regular initial conditions at time  $t_1$  and reaches a turning point at time  $t_B$ , we get analogously

$$\int_{t_1}^{t_B} dt (1 + w) = \infty , \quad (30)$$

leading to a similar conclusion.

### 3 Embedding in a realistic cosmology

A further interesting issue concerns the embedding of our perfect fluid in a realistic cosmology. This is what will be investigated now. It is natural to assign to our perfect fluid the role of dark energy responsible for the late-time accelerated expansion of the universe. Hence our fluid will be called below dark energy with energy density  $\rho_{DE}$  and pressure  $p_{DE}$ . For our discussion it is convenient to introduce the notation

$$B = B_0 a_0^n , \quad (31)$$

so that (16) can be recast in the form

$$\rho = A - B_0 \left( \frac{a}{a_0} \right)^{-n} , \quad (32)$$

where  $B_0$  has dimensions of energy with  $B_0 = A - \rho_0$ . It is easy to show

$$\frac{a_B}{a_0} = \left( \frac{B_0}{A} \right)^{\frac{1}{n}} . \quad (33)$$

When  $n > 0$ ,  $a_B$  corresponds to a bounce with  $a_B \rightarrow 0$  for  $B_0 \rightarrow 0$ . When  $n < 0$ ,  $a_B$  corresponds to a turning point with  $a_B \rightarrow \infty$  for  $B_0 \rightarrow 0$ . The Friedmann equations yield now

$$H^2 = \frac{8\pi G}{3} \left[ \rho_m + \rho_r + A - B_0 \left( \frac{a}{a_0} \right)^{-n} \right] + \Omega_{k,0} H_0^2 \frac{a_0^2}{a^2} \quad (34)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho_m + 3\rho_r - 2\rho_{DE} - n(A - \rho_{DE})] \quad (35)$$

where we have allowed for spatially non-flat universes. We finally note that  $n = 0$  corresponds to a cosmological constant, only the difference  $A - B_0$  matters with  $8\pi G (A - B_0) \equiv \Lambda$  and this case needs no further consideration.

### 3.1 $n > 0$

When  $n > 0$ , the universe will tend in the future to a de Sitter space. The dark energy sector is of the phantom type, it tends asymptotically to a cosmological constant from below ( $w_{DE} < -1$ ) which is not possible for usual quintessence (a minimally coupled scalar field). We have curiously a “growing” rather than “decaying” effective cosmological constant.

The past of the universe must be changed compared to the standard big bang scenario. While our perfect fluid has no influence of the universe dynamics in the past – actually it becomes dynamically significant only at  $z < 1$  – the universe is not allowed to cross in the past the value  $a = a_B$  if we insist on the condition  $\rho_{DE} \geq 0$ . On the other, if we want a universe where the main features of the hot big bang universe is retained,  $B_0$  has to be small enough so that

$$\left(\frac{B_0}{A}\right)^{\frac{1}{n}} \ll 10^{-10}, \quad (36)$$

which requires a fine tuning of the model in addition to the fine tuning of  $A$  (the cosmic coincidence). Viability of such a universe requires further that the universe has a bounce at  $a > a_B$  when  $\rho_{DE}$  is still positive. This requires to go beyond General Relativity in the very early universe (e.g. [7]). In any case, due to (36), such a universe is undistinguishable observationally from a universe with a cosmological constant. We believe the main merit of this model is that it provides an interesting conceptual problem.

### 3.2 $n < 0$

When  $n < 0$ , the primordial universe can be governed by equations (34),(35) with a standard big bang cosmology. The universe future is changed with a turning point after which the universe will be contracting.

The danger with such a universe is that  $a_B$  could lie in our past which is forbidden as further expansion would force  $\rho_{DE}$  to be negative and the universe could not reach the present day expansion stage. The condition  $B_0 < A$  is obviously sufficient to guarantee that  $a_B$  lies in the future,  $a_B > a_0$ . Taking  $\frac{B_0}{A} \ll 1$  will push  $a_B$  (far) in the future. In such a model dark energy is represented by a slowly “decaying” cosmological constant. Actually, even if  $a_B \gg a_0$ , once the point  $a = a_B$  is reached, further expansion – which is unavoidable because of the presence of dustlike matter (and radiation) however tiny their energy densities – would inevitably imply  $\rho_{DE} < 0$ . This is reminiscent of the situation encountered with a Big-Brake singularity in the future [8] if one is willing to include dustlike matter.

The only way to avoid this is to have a closed universe. Let us call  $a_1$  the value of the scale factor at which the dustlike term and the curvature term cancel each other in the right hand side of the Friedmann equation. We then have

$$\frac{a_1}{a_0} = \frac{\Omega_{m,0}}{|\Omega_{k,0}|}. \quad (37)$$

Inclusion of radiation is possible but it will have negligible influence. To ensure that the turning point is reached before  $\rho_{DE}$  becomes negative, we must impose  $a_1 \leq a_B$ . This can be expressed as a lower bound on  $|\Omega_{k,0}|$  in function of the model parameters

$$|\Omega_{k,0}| \geq \Omega_{m,0} \left( \frac{B_0}{A} \right)^{\frac{1}{|n|}} . \quad (38)$$

In a realistic cosmology,  $-0.0065 \leq \Omega_{k,0} \leq 0.0012$  (95% C.L.) with central value  $\Omega_{k,0} = -0.0027$  [9]. So there is preference for a slightly spatially closed universe which is exactly what we need. Condition (38) can obviously be satisfied for a large range of parameter values satisfying

$$\frac{\Omega_{B,0}}{\Omega_{A,0}} \lesssim (0.026)^{|n|} . \quad (39)$$

where we have defined  $\Omega_{B,0} \equiv \frac{B_0}{\rho_{cr,0}}$  and  $\Omega_{A,0} \equiv \frac{A}{\rho_{cr,0}}$  and we have taken  $\Omega_{m,0} \simeq 0.25$ . Violation of (38) for some set of parameter values implies that the observational upper bound on spatial curvature is not large enough to ensure  $\rho_{DE} \geq 0$  in the future.

Inequality (39) can be recast into

$$\frac{\Omega_{B,0}}{\Omega_{DE,0}} \lesssim \frac{1}{(0.026)^{-|n|} - 1} \quad (40)$$

For example for  $|n| = 2$  we have  $\Omega_{B,0} \lesssim 0.00067 \Omega_{DE,0}$ , for  $|n| = 0.5$ , one obtains  $\Omega_{B,0} \lesssim 0.192 \Omega_{DE,0}$ . For  $|n| \lesssim 0.5$ ,  $\Omega_{B,0}$  can be substantial while it becomes negligible for  $|n| \gtrsim 2$ .

On the other hand, the expansion must be accelerated at the present epoch which yields

$$\Omega_{B,0} < \frac{2\Omega_{DE,0} - \Omega_{m,0}}{|n|} \lesssim \frac{1.25}{|n|} , \quad (41)$$

Condition (41) corresponds to  $w_{DE,0} < -\frac{1}{3}\Omega_{DE,0}^{-1}$  (neglecting here  $\Omega_{k,0}$ ). We know however from the observations that  $w_{DE,0}$  is significantly smaller. Hence a much stronger condition comes from the smallness of  $1 + w_{DE,0}$ . Indeed we have

$$\frac{\Omega_{B,0}}{\Omega_{DE,0}} = \frac{3}{|n|} (1 + w_{DE,0}) . \quad (42)$$

If we take  $1 + w_{DE,0} \lesssim 0.1$  and  $\Omega_{DE,0} \simeq 0.75$ , we obtain  $\Omega_{B,0} \lesssim \frac{0.225}{|n|}$ . As expected, this is much stronger than (41).

Combining condition (40) with (42), we get a new inequality

$$1 + w_{DE,0} \lesssim \frac{|n|}{3} \frac{1}{(0.026)^{-|n|} - 1} \quad (43)$$

It is interesting that (43) implies

$$1 + w_{DE,0} \lesssim 0.09 . \quad (44)$$



Hence this model is able to cover the range of values relevant for  $1 + w_{DE,0} > 0$ . As the observational bound on  $|\Omega_{k,0}|$  will be more stringent, the range of allowed values for  $1 + w_{DE,0} > 0$  will decrease.

Let us consider the regime  $|n| \geq 2$ . In that case we have  $\Omega_{B,0} < 6 \times 10^{-4} \Omega_{A,0}$ ,  $\Omega_{B,0} < 6 \times 10^{-4} \Omega_{DE,0}$  and  $\Omega_{A,0} \simeq \Omega_{DE,0}$ . We also get in this case from (43) that  $1 + w_{DE,0} < 4 \times 10^{-4}$ . These models cannot be distinguished from a cosmological constant model with  $\Omega_{A,0} \simeq \Omega_{\Lambda,0}$ . Actually for models with  $|n| \gtrsim 0.95$ , the quantity  $1 + w_{DE,0}$  is strongly constrained with  $1 + w_{DE,0} \lesssim 0.01$ . For large  $|n|$  values both quantities  $\frac{\Omega_{B,0}}{\Omega_{DE,0}}$  and  $\frac{\Omega_{B,0}}{\Omega_{A,0}}$  are much smaller than one. It is possible to have substantially higher values for  $1 + w_{DE,0}$  when  $|n| \rightarrow 0$ . For  $|n| \approx 0.5$ , the quantity  $1 + w_{DE,0}$  is allowed to be inside the range  $0 < 1 + w_{DE,0} \lesssim 0.03$ . For lower nonzero values  $|n| \approx 0$ ,  $1 + w_{DE,0}$  can be as large as 0.09. In this range of small  $|n|$  values,  $\frac{\Omega_{B,0}}{\Omega_{DE,0}}$  is allowed to be much larger than 1 with  $\Omega_{B,0} \simeq \Omega_{A,0}$  (and  $\Omega_{B,0} < \Omega_{A,0}$ ,  $\Omega_{A,0} - \Omega_{B,0} = \Omega_{DE,0} \approx 0.75$ ). The quantity  $\Omega_{B,0}$  can be large because  $|n|$  is very small, and therefore the time evolution of  $\rho_{DE}$  is slow enough so that one can still obtain  $a_B \gg a_0$  and  $a_B \geq a_1$ .

If observations show evidence for  $w_{DE,0}$  significantly higher than  $-1$  but still satisfying  $w_{DE,0} \lesssim 0.09$ , then only the low  $|n|$  values remain viable. In all these universes the late-time accelerated expansion is a transient phenomenon. It is followed by decelerated expansion and eventual contraction.

## 4 Summary

We have considered spatially flat toy models universes filled with a particular perfect fluid and we have solved exactly the corresponding dynamics. In one family of models ( $n > 0$ ) the perfect fluid is of the phantom type and an ever accelerating dynamics is obtained: the universe is first contracting, then passing through a bounce after which it is expanding. We have exponential contraction in the infinite past and the universe tends to a de Sitter expanding universe in the asymptotic future. In the second family of models ( $n < 0$ ) the universe is exponentially expanding in the asymptotic past and exponentially contracting in the asymptotic future. At some time ( $t_a$ ) the expansion becomes decelerated, the scale factor then reaches a maximum followed by a contracting phase, first decelerating then accelerating (decreasing contraction rate). We have also considered the possibility to embed our perfect fluid in a realistic cosmology in which case it can play the role of dark energy. When ( $n > 0$ ) we have the strange situation that while the corresponding energy density is negligible already at low redshifts, a standard big bang singularity would force the energy density of our perfect fluid to become negative. We conjecture that this could be avoided by having a rebound before this happens, which could be the case if we go beyond General Relativity in the very early universe. On the other hand, at the present time this universe cannot be distinguished from a  $\Lambda$  dominated universe and it tends to a de Sitter (expanding) universe in the future with  $1 + w_{DE} < 0$ .

As for the other family of solutions ( $n < 0$ ), a slightly spatially closed universe is required in order to avoid  $\rho_{DE}$  to become negative in our future. Interestingly, using the current observational bound on  $|\Omega_{k,0}|$  we find that all universes with  $0 <$

$1 + w_{DE,0} \lesssim 0.09$  can be accommodated by this model. The maximal values  $1 + w_{DE,0} \simeq 0.09$  are obtained for small values  $0 \neq |n| \ll 1$ . For  $|n| \geq 2$ , we have essentially a  $\Lambda$  dominated universe with  $\Omega_{B,0} \ll \Omega_{A,0} \simeq \Omega_{\Lambda,0}$  and  $1 + w_{DE,0} \ll 1$ . The strong inequality  $\Omega_{B,0} \ll \Omega_{A,0}$  can be seen as an additional fine tuning. These models certainly do not pretend to get rid of any of the conceptual problems of the  $\Lambda$  model, what is interesting is that they can provide a radically different future. Because the effective cosmological constant is slowly decaying, the accelerated expansion is a transient phenomenon and the present expansion stage will be followed by a contracting phase even though this universe cannot be distinguished today from a  $\Lambda$  dominated universe. The future of our universe may depend crucially on the precise form of the dark energy sector even if it cannot be distinguished today from a cosmological constant, and our model provides just such a example.

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