Generation of Primordial Magnetic Fields on Linear Over-density Scales

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Magnetic fields appear to be present in all galaxies and galaxy clusters. Recent measurements indicate that a weak magnetic field may be present even in the smooth low density intergalactic medium. One explanation for these observations is that a seed magnetic field was generated by some unknown mechanism early in the life of the Universe, and was later amplified by various dynamos in nonlinear objects like galaxies and clusters. We show that a primordial magnetic field is expected to be generated in the early Universe on purely linear scales through vorticity induced by scale-dependent temperature fluctuations or equivalently, a spatially varying speed of sound of the gas. Residual free electrons left over after recombination tap into this vorticity to generate magnetic field via the Biermann battery process. Although the battery operates even in the absence of any relative velocity between dark matter and gas at the time of recombination, the presence of such a relative velocity modifies the predicted spatial power spectrum of the magnetic field. At redshifts of order a few tens, we estimate a root mean square field strength of order $10^{-25} - 10^{-24}$ G on comoving scales ~ 10 kpc. This field, which is generated purely from linear perturbations, is expected to be amplified significantly after reionization, and to be further boosted by dynamo processes during nonlinear structure formation.

I. INTRODUCTION

Galaxies in the local Universe have coherent magnetic fields with strength $\sim 10^{-6}$ G [1–3]. Similar fields strengths are seen in galaxies up to redshift ~ 2 [2, 4]. In some cases, the field appears to be even stronger, e.g., a recent measurement of $30\,\mu$G in star forming galaxies [5]. One explanation is that the observed fields originated from primordial magnetic fields which were created in the very early Universe and were later amplified during the formation of the galaxies. Another possibility is that there were no primordial fields and the observed fields were generated spontaneously during the gravitational collapse of galaxies [6, 7].

There is independent evidence for a pre-galactic seed magnetic field in the inter galactic medium (IGM). This is based on the lack of detection of inverse Compton GeV radiation from charged secondaries associated with extragalactic TeV sources. A magnetic field greater than $\sim 10^{-16}$ G can deflect secondaries sufficiently to explain the observations [8, 9]; the required field strength has been reduced to 10^{-18} G in a recent study [10]. This evidence for magnetic fields in the IGM emphasizes the notion that the fields are primordial (see for further discussion Ref. [11]), although it is possible that the fields originated by baryonic outflows from already formed galaxies [6, 7]. We note that the absence of secondary radiation from TeV sources may have nothing to do with a magnetic field but be the result of beam instabilities which slow down the particles before they can produce significant inverse Compton radiation [12] (but see Ref. [13]). Other recent studies which have considered the influence of primordial magnetic fields on the cosmic microwave background (CMB) and Lyα clouds [14–16] give an upper limit on the present day large scale magnetic field in the IGM (extended up to $z \sim 5$) of $\sim 10^{-9}$ G.

In an influential study, Biermann (1950; Ref. [17]) showed that currents must flow whenever a plasma has a rotational vortex–like motion. These currents will lead to the generation of magnetic field starting from zero field. The process has been coined in the literature as the "Biermann battery", and several astrophysical applications have been discussed.These range from the generation of magnetic fields in stars [17, 18] to seed magnetic fields on galactic scales [19–23]. The latter studies typically use nonlinear gas-dynamical processes such as those that occur in shocks during structure formation.

It has been argued that magnetic fields, at the time of recombination, may be generated on large scales (> 600 kpc) through second-order couplings between photons and electrons [24]. Here we consider smaller scales, and we show that seed magnetic fields can be produced in the early Universe starting from zero field purely as a consequence of the growth of *linear* over-densities. We consider the evolution of density and temperature fluctuations of the baryonic matter after the time of recombination. We follow the approach described in Ref. [25], where the key new effect that permits the generation of magnetic fields is a spatially varying speed of sound (see below). We also consider the effect of the relative velocities between the dark matter and baryons at the time of recombination [26]. The latter effect has been shown to have a considerable effect on the evolution of over densities at high redshifts [26–33]. Here we show that it has a noticeable effect also on the growth of the magnetic field.

Throughout this paper, we adopt the following cosmological parameters: (Ω_Λ, $\Omega_{\rm M}$, Ω_b, n, σ_8, H_0)= (0.72, 0.28, 0.046, 1, 0.82, 70 km s^{-1} Mpc^{-1}) [34].

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II. LINEAR EVOLUTION OF OVER DENSITIES IN THE EARLY UNIVERSE

After cosmic recombination, the baryonic gas in the Universe decouples mechanically from the photons, but remains thermally coupled down to $z \sim 150$. This coupling is a result of CMB photons scattering off the residual free electrons, which constitute a fraction $\sim 10^{-4}$ of the bound electrons. Even at $z < 150$ the baryons still retain some memory of this heating, which induces scale-dependent temperature fluctuations or equivalently, a spatially varying speed of sound in the gas. Naoz & Barkana (2005, Ref. [25]) took this effect into account and computed the linear growth of baryonic density and temperature fluctuations separately [35]. At large wavenumbers ($k > 100$ Mpc^{-1}) the growth of baryon density fluctuations is changed significantly by the effect of the inhomogeneous sound speed, by up to 30% at $z = 100$ and 10% at $z = 20$. This has an important impact on high-z gas rich halos [36].

It was shown recently that not only the amplitudes of the dark matter and baryonic density fluctuations are different at early times, so too are their velocities [26]. After recombination, the sound speed of the baryons drops dramatically, while the dark matter velocity remains high. As a result, the relative velocity of baryons with respect to the dark matter becomes supersonic. This relative velocity, which is generally referred to as the "stream velocity" in the literature, remains coherent on scales of a few Mpc and is of the order of ~ 30 km s^{-1} at the time of recombination [26].

For completeness we write here the coupled second order differential equations that govern the evolution of the dimensionless density fluctuations of the dark matter $\delta_{\rm dm}$ and of the baryons $\delta_{\rm b}$:

$$\ddot{\delta}_{\rm dm} + 2H\dot{\delta}_{\rm dm} - f_{\rm dm}\frac{2i}{a}\mathbf{v}_{\rm bc}\cdot\mathbf{k}\dot{\delta}_{\rm dm} = \frac{3}{2}H_0^2\frac{\Omega_m}{a^3}\left(f_{\rm b}\delta_{\rm b} + f_{\rm dm}\delta_{\rm dm}\right) + \left(\frac{\mathbf{v}_{\rm bc}\cdot\mathbf{k}}{a}\right)^2\delta_{\rm dm} \quad (1)$$

$$\ddot{\delta}_{\rm b} + 2H\dot{\delta}_{\rm b} = \frac{3}{2}H_0^2\frac{\Omega_m}{a^3}\left(f_{\rm b}\delta_{\rm b} + f_{\rm dm}\delta_{\rm dm}\right) - \frac{k^2}{a^2}\frac{k_B\bar{T}}{\mu}\left(\delta_{\rm b} + \delta_T\right) \ , \quad (2)$$

where Ω_m is the present day matter density as a fraction of the critical density, k is the comoving wavenumber of the perturbation, $\mathbf{v}_{\rm bc}$ is the relative velocity between the baryons and dark matter in a local patch of the Universe, a is the scale factor of the Universe, H_0 is the present day value of the Hubble parameter, μ is the mean molecular weight of the gas, $\bar{T}$ is the mean temperature of the baryons, $f_{\rm b}$ ($f_{\rm dm}$) is the cosmic baryonic (dark matter) fraction and δ_T is the dimensionless fluctuation in the baryon temperature. Derivatives are with respect to the clock time. These equations are a compact form of equations (5) in Ref. [26], where we have used the fact that $v_{\rm bc} \propto 1/a$, and have included the pressure term appropriate to the equation of state of an ideal gas e.g., [25].

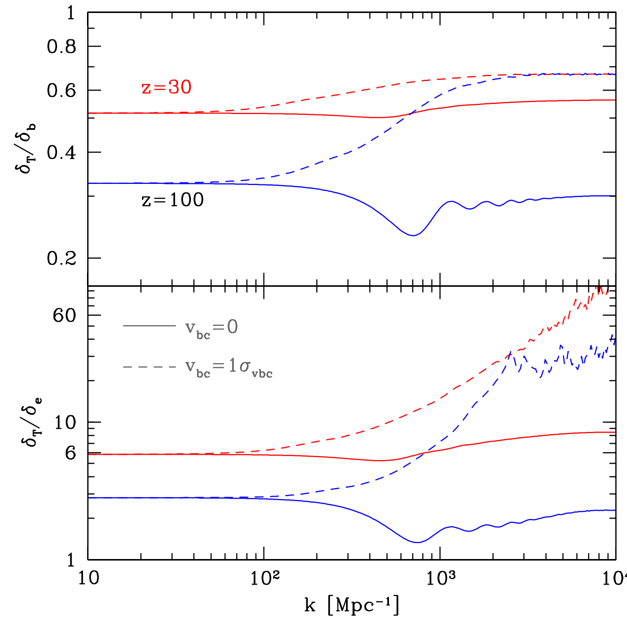

FIG. 1. Perturbation ratios δ_T/δ_b (top panel) and δ_T/δ_e (bottom panel) as a function of wavenumber k. We consider two cases: no stream velocity, $v_{\rm bc} = 0$ (solid lines), and a typical stream velocity, $v_{\rm bc} = 1\sigma_{\rm vbc}$ (dashed lines). Results are shown for two redshifts, $z = 100$ (blue lines) and z=30 (red lines).

The linear evolution of the baryon temperature fluctuations may be written down similarly [25, 37]. Including an additional term due to fluctuations of the electron over density δ_e:

$$\dot{\delta}_T = \frac{2}{3}\dot{\delta}_{\rm b} + \frac{x_e(t)}{t_\gamma}a^{-4}\left\{\frac{\bar{T}_\gamma}{\bar{T}}\left(\delta_{T_\gamma} - \delta_T\right) + \left(\delta_\gamma + \delta_e\right)\left(\frac{\bar{T}_\gamma}{\bar{T}} - 1\right)\right\} , \quad (3)$$

where δ_γ is the photon density fluctuation, $t_\gamma^{-1} = 8.55 \times 10^{-13}\,{\rm yr}^{-1}$, and $\bar{T}_\gamma$ and δ_{T_γ} are the mean photon temperature and its fluctuation, respectively.

The evolution of the mean free electron fraction x_e as a function of time is

$$\dot{x}_e = -\alpha_B(T)x_e^2 n_H(1+y) \ , \quad (4)$$

where $\alpha_B(T)$ is the case B recombination coefficient as a function of the gas temperature, n_H is the *total* hydrogen number density, and $y = n_{He}/n_H$ where n_{He} is the helium number density. Fluctuations in the electron density, $\delta_e = \Delta n_e/n_e = \Delta x_e/x_e$, evolve according as

$$\dot{\delta}_e = -\alpha_B(T)(1+y)x_e n_H(\delta_b + \delta_e) \ . \quad (5)$$

We show below that the magnetic field grows because of the presence of the residual free electrons. It is highly sensitive to the evolution of their fractional fluctuations δ_e, but not to the actual electron number density.

Equation (3) describes the evolution of the gas temperature in the post-recombination era but before the formation of the first galaxies, when the only external heating arises from Compton scattering of the remaining free electrons on the CMB photons [25]. The first term in Equation (3) describes the adiabatic cooling of the gas, while the second term is the result of Compton interactions. An important effect of this equation is that it introduces a scale dependent behavior in the fluctuations of the temperature, free electron density and baryon density. In this full thermal evolution calculation, the sound speed ($c_s^2 = dp/d\rho$, where p is the pressure of the gas), varies spatially, simply because $\delta_{\rm b}$ and δ_T have the following relation

$$1 + \frac{\delta_T}{\delta_{\rm b}} = \frac{c_s^2}{k_B \bar{T}/\mu} = \gamma_{\rm eff} \ , \tag{6}$$

where $\gamma_{\rm eff}$ is a scale dependent, effective adiabatic index.

In Figure 1 we show the ratios δ_T/δ_b (top panel) and δ_T/δ_e (bottom panel) as a function of k. At the largest scales (smallest k), the baryons follow the dark matter density, and δ_T/δ_b evolves from 1/3 (at high redshift where the baryons are tightly coupled to the relativistic CMB) to $\sim 2/3$ (lower redshift where the baryons expand adiabatically as an independent nonrelativistic fluid). Considering first the zero stream velocity case, small scales (large k) at high redshift show Jeans scale oscillations which are suppressed at lower redshift (there is only a slight minimum for $z = 30$). For $v_{\rm bc} = 1\sigma_{\rm vbc}$, the small scale baryon fluctuations drop, and are less important compared to the Compton heating [see Equation (3)] which results in a slight increase of the temperature fluctuations (compared to the zero stream velocity). These two effects result in an increase of the ratio δ_T/δ_b as a function of scale. The free electron fluctuations are further suppressed in the case of $v_{\rm bc} = 1\sigma_{\rm vbc}$ compared to the case of zero stream velocity which results in a larger increase in the ratio δ_T/δ_e.

III. BIERMANN BATTERY IN AN EXPANDING UNIVERSE

The evolution of the magnetic field via the Biermann battery process is described by a simple combination of the Maxwell–Faraday equation and the generalized Ohm's law e.g., Ref. [38]. Since we are interested in magnetic field evolution over cosmic times, we need to work with the Biermann battery equation in a flat expanding Universe. In this case, we find that the differential equation for the clock time evolution of the magnetic field $\mathbf{B}$ is given by:

$$\frac{\partial}{\partial t}\left(a^2 \mathbf{B}\right) = a \nabla \times (\mathbf{u} \times \mathbf{B}) - c \frac{\nabla n_e \times \nabla P_e}{e n_e^2} \ , \tag{7}$$

where n_e and P_e are the electron number density and pressure respectively, e is the electron charge, $\mathbf{u}$ is the peculiar velocity of the gas, and the spatial derivatives are with respect to co-moving coordinates. To relate to the literature [39, 40] this equation can be reduced to the familiar form of the Biermann battery by rescaling $\tilde{\mathbf{B}} = a^2 \mathbf{B}$, $\tilde{n}_e = n_e/a^3$, and $\tilde{P}_e = P_e/a^4$, for conformal time (η) where $\partial/\partial\eta = a\partial/\partial t$. The resulting equations are those used for example in describing a recent laboratory experiment of the Biermann battery [41]. Below we do not rescale the equations since the temperature and density fluctuations of the gas have a more complicated dependence on the scale factor [25]. The term $\nabla \times (\mathbf{u} \times \mathbf{B})$, in equation (7), describes flux freezing, i.e., the magnetic flux through any closed contour embedded in the plasma is conserved under plasma motions. The last term is the Biermann battery term. This term is proportional to the derivative with respect to time of the vorticity of the electrons; a vortex–like motion of the electrons produces an rotational electric field, and through this a magnetic field [42].

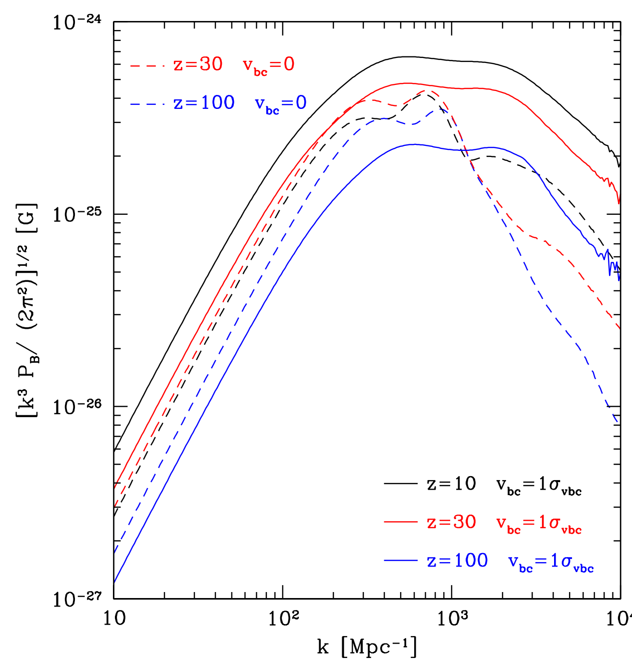

FIG. 2. Root mean square magnetic field generated by the Biermann battery as a function of wavenumber. Two cases are shown: no stream velocity, $v_{\rm bc} = 0$ (solid lines), and a typical stream velocity, $v_{\rm bc} = 1\sigma_{\rm vbc}$ (dashed lines). Three redshifts are considered: $z = 100$ (blue lines), $z = 30$ (red lines), $z = 10$ (black lines).

Consider now the Biermann term $c\nabla n_e \times \nabla P_e / e n_e^2$. The electron pressure is given by $P_e = n_e k_B T$, where, following [25], we have set $T_e = T$. Expanding the relevant quantities to linear order, i.e., $n_e = \bar{n}_e(1 + \delta_e)$ and $T = \bar{T}(1+\delta_T)$, and neglecting the flux-freezing term [43], equation (7) can be written as:

$$\frac{\partial}{\partial t}\left(a^2 \mathbf{B}\right) = -\frac{c k_B \bar{T}}{e} \nabla \delta_e \times \nabla \delta_T \ . \tag{8}$$

Note that the number density of free electrons cancels out. Thus the Biermann effect depends only on the fluctuations in the fractional electron density δ_e and not on the actual density n_e itself. Therefore, the fact that the ionization fraction of the gas is very low ($\sim 10^{-4}$) is not important.

The right hand side of equation (8) may be written in Fourier space as

$$-\nabla\delta_e \times \nabla\delta_T = \frac{1}{2}\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\left(\mathbf{k}_1 \times \mathbf{k}_2\right) e^{i\mathbf{r}(\mathbf{k}_1+\mathbf{k}_2)} \times \left[\delta_e(\mathbf{k}_1)\delta_T(\mathbf{k}_2) - \delta_e(\mathbf{k}_2)\delta_T(\mathbf{k}_1)\right] . \quad (9)$$

Fourier transforming both sides of equation (8), we then find

$$\frac{\partial}{\partial t}\left(a^2\mathbf{B_k}\right) = \frac{1}{2}\frac{ck_B\bar{T}}{e}\int \frac{d^3k_1}{(2\pi)^3}\left(\mathbf{k}_1 \times [\mathbf{k}-\mathbf{k}_1]\right) \times \left[\delta_e(\mathbf{k}_1)\delta_T(\mathbf{k}-\mathbf{k}_1) - \delta_e(\mathbf{k}-\mathbf{k}_1)\delta_T(\mathbf{k}_1)\right] , \quad (10)$$

where $\mathbf{B_k}$ has units of $\mathrm{G\,Mpc}^3$. Note that the over densities that appear in the above equation are complex, i.e., $\delta(k) = |\delta(k)|e^{i\phi_k}$, where each ϕ_k represents a random phase which is uniformly distributed over the interval 0 to 2π. The phases disappear below when we finally compute the power spectrum of the magnetic field.

Before proceeding, we note that the Biermann battery produces a magnetic field only if the gradients $\nabla\delta_e$ and $\nabla\delta_T$ in equation (8) are not parallel to each other. The equivalent condition in Fourier space is that the quantity in square brackets in equation (10) should be non-vanishing. The latter condition requires the ratio $\delta_T(\mathbf{k})/\delta_e(\mathbf{k})$ to vary with scale. This is precisely where the correct treatment of the gas thermodynamics, as described in Ref. [25], is critical. As Figure 1 shows, the ratio of temperature to density fluctuations does vary with k, and therefore we expect the cosmological Biermann battery to operate even within linear perturbation theory.

Let us define $\Delta_{e,T}(\mathbf{k},\mathbf{k}_1) = \delta_e(\mathbf{k}_1)\delta_T(|\mathbf{k}-\mathbf{k}_1|) - \delta_e(|\mathbf{k}-\mathbf{k}_1|)\delta_T(\mathbf{k}_1)$. Equation (10) then becomes

$$aH\frac{\partial\left(a^2\mathbf{B_k}\right)}{\partial a} = \frac{ck_B}{e}\int \frac{d^3k_1}{(2\pi)^3}\bar{T}(t)\left(\mathbf{k}_1 \times \mathbf{k}\right)\Delta_{e,T}(k,k_1) , \quad (11)$$

where $\partial/\partial a \equiv aH\partial/\partial t$. In this equation, only $\Delta_{e,T}$ and $\bar{T}$ depend on the time t (or equivalently the scale factor a). Thus we can write equation (11) as

$$\mathbf{B_k}(a) = \int \frac{2\pi dk_1 \sin\theta d\theta}{(2\pi)^3}\beta(a,k,k_1,\theta)\left(\mathbf{k}_1 \times \mathbf{k}\right) , \quad (12)$$

where the quantity $\beta = \beta(a,k,\sqrt{k^2+k_1^2-2kk_1\cos\theta})$ satisfies

$$aH\frac{\partial(a^2\beta(a,k,k_1))}{\partial a} = \frac{ck_B}{e}\bar{T}(a)\Delta_{e,T}(k,k_1) . \quad (13)$$

By numerically integrating equation (13), we can calculate the two dimensional array of values $\beta(k,k_1)$ as a function of scale a or redshift z. These β values still include the random phases ϕ_k. However, the phases are eliminated when we compute the power spectrum of the magnetic field P_B. The result is

$$P_B \equiv \langle\mathbf{B_k}\mathbf{B_k^\star}\rangle = \frac{1}{V}\int \frac{2\pi dk_1 \sin\theta d\theta}{(2\pi)^3}|\beta(a,k,k_1,\theta)|^2(k_1 k\sin\theta)^2 , \quad (14)$$

where V is the volume.

In Figure 2 we show the power spectrum of the magnetic field as a function of wavenumber k for different redshifts. The quantity $\sqrt{k^3P_B}$ has units of gauss. Note that the magnetic field grows most strongly on the Jeans mass scale of the baryons. This is apparent in the case of zero stream velocity, where the first peak is around $k^{-1} \sim 16$ kpc [comoving] at $z = 100$, corresponding to a mass scale $\sim 7\times10^4\,\mathrm{M}_\odot$. This mass scale is slightly above the minimum mass for which baryonic gravitational instabilities can still grow [36, 44, 45]. The second peak, where the power is maximum, is associated with smaller scales ~ 7 kpc [comoving], which correspond to where the most dramatic variation of the ratio δ_T/δ_e occurs (see Figure 1). For the case of $v_{\rm bc} = 1\sigma_{\rm vbc}$, we see the inverse behavior. Here the first peak (larger scales) has more power than the second peak (smaller scales). Note that our use of linear theory is justified, since the density perturbations are still linear for scales smaller than $\sim 1000\,\mathrm{Mpc}^{-1}$ [comoving] and become nonlinear only at $z < 10$ (see Fig. 6 in Ref. [46], see also [47]).

IV. DISCUSSION

We have shown that seed magnetic fields can be produced from zero initial magnetic field on cosmological linear over density scales through the Biermann process. The typical field strength is $\sim 10^{-25} - 10^{-24}$ G. These seeds fields may later be amplified via nonlinear dynamo processes [48, 49] and are perhaps responsible for the present day magnetic fields in galaxies. Note that baryonic outflows can still contribute to the IGM magnetic field [6]. The Biermann battery mechanism requires a vortex like motion in the plasma. We have demonstrated that the spatially varying speed of sound of gas in the early Universe produces this vorticity in the residual free electrons. The process does not depend on the fraction of free electrons in the Universe (since the electron number density cancels in the Biermann term), but only on fluctuations in this quantity.

During reionization, the temperature of the baryons as well as temperature fluctuations will increase. This will lead to even larger magnetic fields since equation (8) shows that the magnetic field growth depends linearly on $\bar{T}$, and the temperature after reionization increases to $\bar{T} \to 10^4$ K. The temperature and electron fraction fluctuations are also expected to increase substantially [50].

Thus, the magnetic field could potentially increase post-reionization by $4-6$ orders of magnitude, bringing it close to the 10^{-18} G estimated from observations [8–10]. This value is about 6 orders of magnitude smaller compared to other mechanisms in the literature that operate on the relevant scales (see Ref. [7] for review), but comparable to Ref. [51]. However, the evolution of δ_e and δ_T during and after reionization is model dependent. In contrast, we have shown in this paper that, even before reionization, magnetic field can be generated as part of the linear growth of perturbations in the Universe, and that the field strength due to this process can be estimated robustly with few uncertainties.

The effect described here (following Ref. [25]) produces a vorticity in the baryonic gas on the order of $\sim 10^{-20}$ s^{-1} at $z \sim 10$ on scales ~ 6 kpc. During reionization, as in the case of the magnetic field, the vorticity in the gas may again increase by $4-6$ orders of magnitude, bringing it close to 10^{-15} s^{-1}, which is the vorticity of the Milky Way Galaxy in the solar neighborhood.

Future measurements of the magnetic field in the IGM and in filaments (for example via Faraday rotation in the CMB, Ref. [52, 53]) would be helpful to further clarify the role of seed magnetic fields. Already, lower bounds on the magnetic field in large scale structures [8–10, 53–55] suggest that there must be a primordial seed field in the Universe. The Biermann Battery process described here, which operates through a spatially varying speed of sound, can naturally explain these seeds. Our calculation suggests that different coherent patches in the Universe with different stream velocities may have up to an order of magnitude variation in their magnetic fields. Thus, seed magnetic fields could conceivably be used in the future to study the stream velocity distribution in the Universe.

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