# Weak Interaction Neutron Production Rates in Fully Ionized Plasmas

A. Widom and J. Swain
*Physics Department, Northeastern University, Boston MA 02115*

Y.N. Srivastava
*Physics Department & INFN, University of Perugia, Perugia IT*

Employing the weak interaction reaction wherein a heavy electron is captured by a proton to produce a neutron and a neutrino, the neutron production rate for neutral hydrogen gases and for fully ionized plasmas is computed. Using the Coulomb atomic bound state wave functions of a neutral hydrogen gas, our production rate results are in agreement with recent estimates by Maiani *et al.* Using Coulomb scattering state wave functions for the fully ionized plasma, we find a substantially enhanced neutron production rate. The scattering wave function should replace the bound state wave function for estimates of the enhanced neutron production rate on water plasma drenched cathodes of chemical cells.

PACS numbers: 24.60.-k, 23.20.Nx

## I. INTRODUCTION

In years past we have been working on weak interaction inverse beta decay[1–3] including electromagnetic interactions with collective plasma modes of motion. Our considerations have been recently criticized[4]. Our neutron production rate[1, 2] is a factor of $\sim 300$ larger than that of Maiani[4]. Our purpose is to point out the source of this difference so that the physical principles may be resolved.

In Sec.II, the calculation of neutron production for a neutral plasma of Ciuchi *et al*[4] is briefly reviewed. Since the surface plasmas of hot cathodes within which neutron production is observed[3] are fully ionized, the neutral atomic gas case is not relevant. The irrelevant two body wave function[4] employed for the neutral gas case should be replaced by the two body Coulomb wave function relevant to the fully ionized plasma. This is the usual fully ionized plasma situation, for example, in the study of the weak interaction electron capture reactions

$$\text{(general)} \quad e^- + {}^A_Z X \to {}^{A}_{(Z-1)}X + \nu_e \ ,$$
$$\text{(special case)} \quad e^- + p^+ \to n + \nu_e \ , \tag{1}$$

in solar[5] physics. Scattering Coulomb wave functions also enter laboratory high energy[6] physics. The case of the fully ionized plasma is discussed in Sec.III. In Sec.IV our previous neutron production estimates[1–3] are verified employing the scattering Coulomb wave function.

In the concluding Sec.V we briefly indicate how collective many body interactions may modify the situation.

## II. NEUTRAL GAS OF ATOMS

For a gas of neutral objects which consist of a heavy electron bound to a proton, the Coulomb wave function in the zero total momentum frame

$$\psi_{e^-p^+}(\mathbf{r}) = \frac{e^{-r/a}}{\sqrt{\pi a^3}} \qquad a = \frac{\hbar^2}{me^2}, \tag{2}$$

wherein $\mathbf{r} = \mathbf{r}_{e^-} - \mathbf{r}_{p^+}$ and $m$ is the reduced mass of the heavy electron. With the lowest order Fermi cross section for a heavy electron to scatter from a proton producing a neutron and a neutrino,

$$\tilde{\nu} = v\sigma = \frac{c}{2\pi}\left(\frac{G_F m^2}{\hbar c}\right)^2 (g_V^2 + 3g_A^2) \times \left(\frac{\hbar}{mc}\right)^2 (\gamma^2 - \gamma^2_{Threshold}). \tag{3}$$

If $n$ denotes the number of bound neutral objects per unit volume, then the transition rate per unit time per unit volume to produce neutrons from the decay of the neutral objects

$$\varpi_0((e^-p^+) \to n + \nu_e) = nv\sigma|\psi_{e^-p^+}(0)|^2 \ ,$$
$$\varpi_0 = \left(\frac{n}{\pi a^3}\right) v\sigma = \left(\frac{n\tilde{\nu}}{\pi a^3}\right). \tag{4}$$

Up to this point we are in agreement with the comment of Ciuchi *et al*[4]. Our disagreement involves the more physical regime wherein the plasma is fully ionized. The particles are charged and not neutral and the wave function Eq.(2) chosen by Ciuchi *et al*[4] is thereby incorrect. The correct wave function is written below.

## III. FULLY IONIZED PLASMA MODES

For a fully ionized plasma, the constituents of the plasma are the charged heavy electron and the proton. We seek the scattering state production of neutrons

$$e^- + p^+ \to n + \nu_e. \tag{5}$$

The wave function factor $|\psi(0)|^2$ needed to include Coulomb attraction into the scattering is changed from the neutral plasma value $1/(\pi a^3)$. The positive energy $E = mv^2/2 = \hbar^2k^2/2m$ scattering Coulomb wave function[7] must replace Eq.(2) ; i.e. in terms of the

Gamma function $\Gamma(z)$ and the confluent hypergeometric function ${}_1F_1(\xi;\zeta;z)$

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\Big[e^{\pi/(2ka)}\Gamma\left(1-\frac{i}{ka}\right)\times \\ {}_1F_1\left(\frac{i}{ka};1;\frac{kr-\mathbf{k}\cdot\mathbf{r}}{ka}\right)\Big]. \quad (6)$$

If $r\to 0$, then

$$|\psi(0)|^2 = \frac{(2\pi e^2/\hbar v)}{1-exp(-(2\pi e^2/\hbar v))}\ . \quad (7)$$

The neutron production rate per unit time per unit volume is then

$$\varpi(e^- + p^+ \to n+\nu_e) = n^2 v\sigma|\psi(0)|^2 = n^2\tilde{\nu}|\psi(0)|^2\ , \\ \varpi = \frac{2\pi\alpha c n^2\sigma}{1-exp(-2\pi c\alpha/v)}\ , \quad (8)$$

wherein $\alpha = e^2/\hbar c$.

## IV. THE NEUTRON PRODUCTION RATIO

The ratio $\varpi/\varpi_0$ of the neutron production rates per unit time per unit volume can be deduced from Eqs.(4) and (8). Thermal averaging at a temperature small on the scale of the heavy electron mass $k_BT \ll mc^2$ yields[5] the transition rate per unit time per unit volume for producing neutrons

$$\eta = \frac{\varpi}{\varpi_0} = 2\pi^2\alpha n a^3\left\langle\frac{c}{v}\right\rangle,$$

$$\eta \approx 2\pi^2\alpha n a^3\sqrt{\frac{2mc^2}{\pi k_BT}}\ , \quad (9)$$

where $n$ is the number of electrons per unit volume.

Previously[2] estimated temperatures of hydride cathodes $T\sim 5\times 10^3\ {}^oK$ are in agreement with the observed hot color of their brightly light emitting surfaces[3]. The resulting neutron production as described by Eq.(9) is given by $\eta\sim 5\times 10^2$ in rough agreement with our previous estimates[1–3]. The factor of $\sim 300$ discrepancy is thereby resolved.

## V. CONCLUSION

Many body plasma effects on neutron production may be described by the correlations between the electron coordinates $(\mathbf{r}_1,\cdots,\mathbf{r}_N)$ and proton coordinates $(\mathbf{s}_1,\cdots,\mathbf{s}_N)$ as given by the correlation function

$$C = \frac{1}{N}\left\langle\sum_{i=1}^{N}\sum_{j=1}^{N}\delta\big(\mathbf{r}_i-\mathbf{s}_j\big)\right\rangle = n\xi \quad (10)$$

wherein $\xi = |\psi(0)|^2$ only if there are merely two body collisions in the plasma. Collective oscillations and many body collisions would tend to raise the value of $\xi$ but require a many body Greens function analysis to include such effects in detail. However, previous discrepancies are now understandable.

We reiterate that at the level of dilute plasma two-body correlations dealt with in previous work[4], the order of magnitude of the discrepancy has herein been resolved.


[1] A. Widom and L. Larsen, *Eur. Phys. J.* **C 46**, 107 (2006).

[2] A. Widom and L. Larsen, arXiv:0608059v2 [nucl-th] 25 Sep 2007.

[3] D. Cirillo, R. Germano, V. Tontodonato, A. Widom, Y.N. Srivastava, E. Del Giudice, and G. Vitiello *Key Engineering Materials* **495**, 104 (2012); *ibid* 124 (2012).

[4] S. Ciuchi, L. Maiani, A. D. Polosa, V. Riquer, G. Ruocco and M Vignati *Eur. Phys. J.* **C72** , 2193 (2012).

[5] J.H. Bahcall, *Phys. Rev.* **128**, 1297 (1962).

[6] G. Bardina et. al., *Nuc. Phys.* **B411**, 3 (1994).

[7] S. Flüge, "*Practical Quantum Mechanics*", pp 290-293, Springer-Verlag, Berlin (1970).