The one-loop on-shell renormalization of some vertexes in MSSM

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Abstract

The on-shell renormalization scheme for the electroweak theory is well studied in the standard model(SM), but a consistent on-shell renormalization scheme for the minimal supersymmetric standard model(MSSM) is still unknown. In MSSM, we study the on-shell scheme for three vertexes$Z\overline{l^I}l^I$, $W^+\overline{\nu^I}l^I$ and $\tilde{L}_i^*\overline{\chi_\alpha^0}l^I$ with virtual SUSY particles (chargino, sneutrino, neutralino and slepton) at one-loop order. Instead of amplitude of a single triangle diagram, the sum of amplitude of triangle diagrams belonging to one suit can be renormalized in the on-shell scheme. One suit points out that the internal virtual particles are consistent. Zero-momentum scheme is also used for the renormalization. The two schemes can make the renormalized results decoupled. In MSSM, some special characters of the on-shell scheme are shown here. This work is propitious to complete the on-shell renormalization scheme in MSSM.

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I. INTRODUCTION

As we all know, the quantum field theory is perturbative theory. That is to say, it can not be solved exactly. To obtain finite results, renormalization is necessary and there are some typical renormalization schemes such as MS, $\overline{MS}$, MOM, zero-momentum and on-shell schemes[1–5]. In MS scheme, the counter term is just the pole term$(1/\epsilon, \epsilon \to 0)$. The counter term is proportional to $1/\epsilon + \ln 4\pi - \gamma_E$ in $\overline{MS}$ scheme. The two foregoing schemes have nothing to do with mass. For the on-shell scheme, the renormalization constants are all obtained under on-shell condition. It is the only physical scheme. For electroweak theory, the on-shell scheme is the most appropriate one.

If we can resolve the theory accurately, different renormalization schemes can give the same finite result of any physical process though the functions of the renormalized parameters are different. However, different physical predictions are produced from different renormalization schemes and different renormalized parameters, because of the curtate perturbation theory.

To obtain the counter term for the UV-divergent diagram, one can take all the external momenta of the diagram as zero, which is called zero-momentum renormalization scheme. The advantage is that in arbitrary model each divergent diagram is easy to be renormalized, and the renormalized results are decoupled[6, 7].

We focus on the on-shell renormalization scheme that is popular in electroweak theory. In the on-shell scheme, the fine structure constant α is an expansion parameter and defined in the Thomson limit. At any order of perturbation theory, the physical parameters are the same as the finite renormalized parameters. They represent clear physical meaning and can be measured directly in experiment. The renormalization procedure is summarized in the counter term approach[4].

Extending SM, physicists have developed many new models[8, 9] to explain the experimental phenomena. MSSM[10] is the most attractive one. A lot of experimentalists of high energy physics are focusing on searching for Higgs bosons in MSSM. The colliders (LHC, e^+e^- linear collider, etc.) will provide abundant information of new physics beyond SM. In MSSM, the decays $h^0(H^0, A^0) \to \tilde{\chi}^0_m \tilde{\chi}^0_n$, $\tilde{\chi}^0_m \to h^0(H^0, A^0) + \tilde{\chi}^0_n (m, n = 1, 2, 3, 4)$ and $\check{b}_a \to \chi^-_i t (a, i = 1, 2)$ are studied at one-loop order with the on-shell renormalization scheme[11, 12], but they do not give analytic results to show the elimination of UV-divergence apparently. Considering the one-loop contributions, the authors[13] completed systematic on-shell renormalization programme for gauge boson and Higgs parts. Radiative one-loop corrections to the process $e^+e^- \to l^+l^-$(hadrons) are calculated with the same scheme[14].

For supersymmetric gauge theories, a consistent regularization scheme preserving supersymmetry and gauge invariance is still not known. Two equivalent ways to solve the problem are shown here. One is to use an invariant scheme to keep the symmetries to manifest, where only those counterterms are necessary for renormalization that they themselves preserve the symmetries. The other is to use a non-invariant scheme, through using appropriate non-invariant counterterms to compensate the corresponding symmetry breaking. With appropriate non-invariant counterterms, W.Hollik[15] shows supersymmetric QED can keep the supersymmetry. Their study can be generalized to supersymmetric models with soft breakings and eventually to the supersymmetric extensions of the standard model. Although the corresponding Slavnov Taylor identities are more involved since they have to express not only the symmetries but also the spontaneous or soft breaking, their structure is the same as in SQED. Therefore, this method can also extend to full EW theory of the MSSM.

With the extension of the on-shell scheme of SM, the vertexes $(Z\overline{l^I}l^I, W^+\overline{\nu^I}l^I)$ and $\tilde{L}_i^*\overline{\chi_\alpha^0}l^I$ are studied at one-loop order in this work. We find some special characters for the on-shell scheme in MSSM. Compared with the zero-momentum scheme, it is easy to find the renormalized results in the on-shell scheme are decoupled. These selected vertexes are ordinary, and can represent the general vertexes in MSSM. The study of the on-shell scheme for these vertexes is propitious to complete the on-shell renormalization programme of MSSM. If one studies the on-shell renormalization scheme in other models, it is also helpful.

After the introduction, in Sect.2 we study both the zero-momentum scheme and the on-shell scheme of two SM vertexes in MSSM. The corresponding results of the SUSY vertex are shown in Sect.3. In Sect.4, the decoupling behaviors for the counter terms in both renormalization schemes are researched. Sect.5 is devoted to discussion and conclusion.

II. RENORMALIZATION OF SM VERTEX $(Z\overline{l^I}l^I, W^+\overline{\nu^I}l^I)$ IN MSSM

The authors[4, 5] studied the on-shell renormalization scheme of electroweak theory in SM successfully and completely. Extending the model from SM to MSSM, the condition becomes complex and faint, which needs more researches. In Feynman gauge, applying both the on-shell and zero-momentum schemes, we study the two SM vertexes $(Z\overline{l^I}l^I, W^+\overline{\nu^I}l^I)$ with virtual particles $(\tilde{L}, \tilde{\chi}^0, \tilde{\nu}, \tilde{\chi}^\pm)$ in this section. The studied one loop diagrams are shown in Fig.1. In order to obtain the counter terms, we adopt the naive dimensional regularization with the anticommuting γ_5 scheme, where there is no distinction between the first 4 dimensions and the remaining $D-4$ dimensions[16, 17].

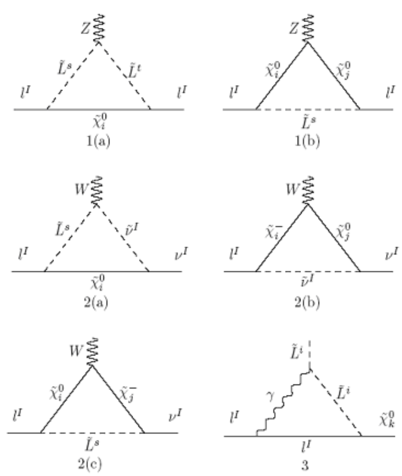

Figure 1: The studied one loop diagrams.

A. $Z\overline{l^I}l^I$ vertex with virtual SUSY particles $(\tilde{L}, \tilde{\chi}^0)$

There are two triangle diagrams for $Z\overline{l^I}l^I$ vertex with virtual SUSY particles $(\tilde{L}, \tilde{\chi}^0)$, and they are shown as diagrams 1(a) and 1(b). The two diagrams are not complete and the results do not satisfy the gauge invariant rule, but they belong to one suit and can be renormalized with some renormalization constants. In the zero-momentum scheme, each diagram has its own counter term, and the corresponding renormalized result is decoupled. Here we show the sum of the counter terms for the two triangle diagrams in the zero-momentum scheme.

$$\delta V^{(ZM),\mu}_{Z\overline{l^I}l^I} = \frac{e^3}{64\pi^2 s_W c_W}\Big\{\frac{1-2s_W^2}{2s_W^2}\Big[\Big(\frac{m_{l^I}}{c_\beta m_W}\Big)^2$$

$$
+\frac{1}{c_W^2}\Big]\gamma^\mu\omega_- - \Big[\frac{4s_W^2}{c_W^2}+\Big(\frac{m_{l^I}}{c_\beta m_W}\Big)^2\Big]\gamma^\mu\omega_+\Big\}\Delta_{UV}
$$

$$
+\Big\{\frac{e^3}{256\pi^2 s_W^3 c_W}\Big(\frac{1-2s_W^2}{c_W^2}-(1+2s_W^2)(\frac{m_{l^I}}{c_\beta m_W})^2\Big)
$$

$$
+\sum_{i,\beta=1}^{6}\sum_{j=1}^{4}\frac{e^3}{4s_W^3 c_W}(\mathcal{G})_{i\beta}(\mathcal{D}^I)_{ij}(\mathcal{D}^I)^*_{\beta j}F_1(x_{\tilde{L}_i},x_{\tilde{L}_\beta},x_{\tilde{\chi}_j^0})
$$

$$
-\frac{e^3}{8s_W^3 c_W}\sum_{s=1}^{6}\sum_{i,j=1}^{4}(\mathcal{D}^I)^*_{si}(\mathcal{R}^*)_{ji}(\mathcal{D}^I)_{sj}F_1(x_{\tilde{\chi}_i^0},x_{\tilde{L}_s},x_{\tilde{\chi}_j^0})
$$

$$
+\frac{e^3}{4s_W^3 c_W}\sum_{s=1}^{6}\sum_{i,j=1}^{4}(\mathcal{D}^I)^*_{si}(\mathcal{R})_{ji}(\mathcal{D}^I)_{sj}\sqrt{x_{\tilde{\chi}_i^0}x_{\tilde{\chi}_j^0}}F_2(x_{\tilde{\chi}_i^0},x_{\tilde{L}_s},x_{\tilde{\chi}_j^0})\Big\}\gamma^\mu\omega_-
$$

$$
+\Big\{\frac{e^3}{128\pi^2 s_W c_W}\Big(\frac{c_W^2}{s_W^2}(\frac{m_{l^I}}{c_\beta m_W})^2-\frac{4s_W^2}{c_W^2}\Big)
$$

$$
+\frac{e^3}{2s_W c_W}\sum_{i,\beta=1}^{6}\sum_{j=1}^{4}(\mathcal{G})_{i\beta}(\mathcal{C}^I)_{ij}(\mathcal{C}^I)^*_{\beta j}F_1(x_{\tilde{L}_i},x_{\tilde{L}_\beta},x_{\tilde{\chi}_j^0})
$$

$$
+\frac{e^3}{4s_W c_W}\sum_{s=1}^{6}\sum_{i,j=1}^{4}(\mathcal{C}^I)^*_{si}(\mathcal{R})_{ji}(\mathcal{C}^I)_{sj}F_1(x_{\tilde{\chi}_i^0},x_{\tilde{L}_s},x_{\tilde{\chi}_j^0})
$$

$$
-\frac{e^3}{2s_W c_W}\sum_{s=1}^{6}\sum_{i,j=1}^{4}(\mathcal{C}^I)^*_{si}(\mathcal{R})^*_{ji}(\mathcal{C}^I)_{sj}\sqrt{x_{\tilde{\chi}_i^0}x_{\tilde{\chi}_j^0}}F_2(x_{\tilde{\chi}_i^0},x_{\tilde{L}_s},x_{\tilde{\chi}_j^0})\Big\}\gamma^\mu\omega_+. \quad (1)
$$

To get Eq.(1), we use the unitary character of the mixing matrixes $\mathcal{Z}_{\tilde{L}}, \mathcal{Z}_N$ for sleptons and neutralinoes. Additionally, we adopt the abbreviation notations $c_W = \cos\theta_W, s_W = \sin\theta_W, c_\beta = \cos\beta, s_\beta = \sin\beta$, where θ_W is the Weinberg angle and $\tan\beta = v_2/v_1$ representing the ratio between the vacuum expectation values of the two Higgs doublets. $x_i = m_i^2/\Lambda_{NP}^2$ with i denoting the virtual particles in these one loop diagrams, and Λ_{NP} denotes the new physic energy scale. Here $\Delta_{UV} = 1/\epsilon + \ln(4\pi x_\mu) - \gamma_E,\ 2\epsilon = 4 - D,\ \omega_- = (1-\gamma_5)/2,\ \omega_+ = (1+\gamma_5)/2$ and the functions F_1, F_2 are shown as

$$
F_1(x,y,z) = \frac{1}{16\pi^2}\Big(1-\frac{x^2\ln x}{(y-x)(z-x)}-\frac{y^2\ln y}{(x-y)(z-y)}-\frac{z^2\ln z}{(x-z)(y-z)}\Big), \quad (2)
$$

$$
F_2(x,y,z) = \frac{1}{16\pi^2}\Big(\frac{x\ln x}{(y-x)(z-x)}+\frac{y\ln y}{(x-y)(z-y)}+\frac{z\ln z}{(x-z)(y-z)}\Big). \quad (3)
$$

The concrete forms of the vertex couplings used in Eq.(1) reads as

$$
(\mathcal{C}^I)_{tj} = \frac{-\sqrt{2}}{c_W}\mathcal{Z}_{\tilde{L}}^{(I+3)t}\mathcal{Z}_N^{1j*}-\frac{m_{l^I}\mathcal{Z}_{\tilde{L}}^{It}\mathcal{Z}_N^{3j*}}{\sqrt{2}s_W c_\beta m_W},
$$

$$
(\mathcal{D}^I)_{tj} = \frac{\mathcal{Z}_{\tilde{L}}^{It}}{c_W}(\mathcal{Z}_N^{1j}s_W+\mathcal{Z}_N^{2j}c_W)-\frac{m_{l^I}\mathcal{Z}_{\tilde{L}}^{(I+3)t}\mathcal{Z}_N^{3j}}{c_\beta m_W},
$$

$$
(\mathcal{G})_{ts} = \frac{1}{2}\mathcal{Z}_{\tilde{L}}^{It}\mathcal{Z}_{\tilde{L}}^{Is*}-s_W^2\delta^{st}, \quad (\mathcal{R})_{k\alpha} = (\mathcal{Z}_N^{4k}\mathcal{Z}_N^{4\alpha*}-\mathcal{Z}_N^{3k}\mathcal{Z}_N^{3\alpha*}). \quad (4)
$$

In the on-shell scheme, the counter term for the radiative correction to SM vertex $Z\overline{l^I}l^I$ is shown here[4].

$$\delta V^{(OS),\mu}_{Z\overline{l^I}l^I} = -\frac{e}{2}\Big[\delta Z_{AZ} - \frac{1}{2s^3_{_W}c_{_W}}\Big(\frac{\delta m^2_{_Z}}{m^2_{_Z}} - \frac{\delta m^2_{_W}}{m^2_{_W}}\Big) - \frac{(2s^2_{_W}-1)}{s_{_W}c_{_W}}\Big(\frac{\delta e}{e} + \frac{1}{2}\delta Z_{ZZ}$$
$$+\delta Z^l_L\Big)\Big]\gamma^\mu\omega_- - \frac{e}{2}\Big[\delta Z_{AZ} - \frac{s_{_W}}{c_{_W}}\Big(2\frac{\delta e}{e} + \frac{1}{s^2_{_W}}(\frac{\delta m^2_{_Z}}{m^2_{_Z}} - \frac{\delta m^2_{_W}}{m^2_{_W}}) + \delta Z_{ZZ} + 2\delta Z^l_R\Big)\Big]\gamma^\mu\omega_+. \quad (5)$$

δZ_{AZ} and δe are the renormalization constants for γZ mixing and charge respectively. Only the sum of amplitude of the two triangle diagrams can be renormalized in the on-shell scheme. That is to say, the divergent term of each diagram can not be canceled by the counter term. Another character is that just the lepton wave function renormalization constants δZ^l_L and δZ^l_R are necessary to counteract the ultra-divergent terms.

The renormalization constants for the left- and right-handed lepton wave functions are deduced from the lepton self-energy with virtual SUSY particles $(\tilde{L}, \tilde{\chi}^0)$.

$$\delta Z^l_L = -\frac{e^2}{64\pi^2 s^2_{_W}}\Big(\frac{1}{c^2_{_W}} + (\frac{m_{l^I}}{c_\beta m_{_W}})^2\Big)\Delta_{UV} - \sum_{i=1}^{6}\sum_{j=1}^{4}\Big\{\frac{1}{2s^2_{_W}}|(\mathcal{D}^I)_{ij}|^2 F_4(x_{\tilde{L}_i}, x_{\tilde{\chi}^0_j})$$
$$+x_{l^I}\Big[\frac{1}{2s^2_{_W}}|(\mathcal{D}^I)_{ij}|^2 + |(\mathcal{C}^I)_{ij}|^2 + \frac{\sqrt{2}}{s_{_W}}\mathbf{Re}[(\mathcal{C}^I)^\dagger_{ij}(\mathcal{D}^I)_{ij}]F_3(x_{\tilde{L}_i}, x_{\tilde{\chi}^0_j})\Big]\Big\},$$
$$\delta Z^l_R = -\frac{e^2}{32\pi^2}\Big(\frac{2}{c^2_{_W}} + (\frac{m_{l^I}}{\sqrt{2}s_{_W}c_\beta m_{_W}})^2\Big)\Delta_{UV} - \sum_{i=1}^{6}\sum_{j=1}^{4}\Big\{|(\mathcal{C}^I)_{ij}|^2 F_4(x_{\tilde{L}_i}, x_{\tilde{\chi}^0_j})$$
$$+x_{l^I}\Big[\frac{1}{2s^2_{_W}}|(\mathcal{D}^I)_{ij}|^2 + |(\mathcal{C}^I)_{ij}|^2 + \frac{\sqrt{2}}{s_{_W}}\mathbf{Re}[(\mathcal{C}^I)^\dagger_{ij}(\mathcal{D}^I)_{ij}]\Big]F_3(x_{\tilde{L}_i}, x_{\tilde{\chi}^0_j})\Big\}, \quad (6)$$

where the functions F_3, F_4 are shown as follows

$$F_3(x,y) = \frac{x^2 + 2xy(\ln y - \ln x) - y^2}{32\pi^2(x-y)^3},$$
$$F_4(x,y) = \frac{(2y-x)(y-x+x\ln x) - y^2\ln y}{32\pi^2(x-y)^2}. \quad (7)$$

Considering Eqs.(5)(6)(7), the needed counter terms for Diagram 1(a) and Diagram 1(b) are obtained in the on-shell scheme.

B. $W^+\overline{\nu^I}l^I$ vertex with virtual SUSY particles $(\tilde{L}, \tilde{\nu}, \tilde{\chi}^0, \tilde{\chi}^\pm)$

The condition of $W^+\overline{\nu^I}l^I$ vertex is more complex than that of $Z\overline{l^I}l^I$ vertex. The three triangle diagrams(2(a), 2(b) and 2(c)) are all necessary and belong to one suit, where the virtual SUSY particles are $\tilde{L}, \tilde{\nu}, \tilde{\chi}^0, \tilde{\chi}^{\pm}$. We collect the counter terms of these three diagrams in the zero-momentum scheme.

$$\delta V^{(ZM),\mu}_{W^+\overline{\nu^I}l^I} = \frac{e^3}{\sqrt{2}s^3_{_W}}\Big\{\frac{1}{64\pi^2}\Big[\frac{1}{c^2_{_W}} + \Big(\frac{m_{l^I}}{c_\beta m_{_W}}\Big)^2\Big]\Delta_{UV}$$
$$+\frac{1}{128\pi^2}\Big[4 - \frac{1}{c^2_{_W}} + \Big(\frac{m_{l^I}}{c_\beta m_{_W}}\Big)^2\Big]$$
$$-\frac{1}{2c_{_W}}\sum_{i=1}^{2}\sum_{J=1}^{3}\sum_{j=1}^{4}\Big((\zeta^I)^*_{Jj}(\mathcal{T})_{ji}(\mathcal{B}_i)^{IJ}F_1$$
$$+2(\zeta^I)^*_{Jj}(\mathcal{Q})_{ji}(\mathcal{B}_i)^{IJ}\sqrt{x_{\tilde{\chi}^-_i}x_{\tilde{\chi}^0_j}}F_2\Big)(x_{\tilde{\chi}^-_i}, x_{\tilde{\nu}_J}, x_{\tilde{\chi}^0_j})$$
$$+\frac{1}{4c_{_W}}\sum_{i=1}^{6}\sum_{J=1}^{3}\sum_{j=1}^{4}(\eta)^*_{iJ}(\zeta^I)^*_{Jj}(\mathcal{D}^I)_{ij}F_1(x_{\tilde{L}_i}, x_{\tilde{\nu}_J}, x_{\tilde{\chi}^0_j})$$
$$+\frac{1}{2}\sum_{i=1}^{2}\sum_{s=1}^{6}\sum_{j=1}^{4}\Big(2(\mathcal{P}^I)_{si}(\mathcal{Q})^*_{ji}(\mathcal{D}^I)_{sj}\sqrt{x_{\tilde{\chi}^-_i}x_{\tilde{\chi}^0_j}}F_2$$
$$+(\mathcal{P}^I)_{si}(\mathcal{T})^*_{ji}(\mathcal{D}^I)_{sj}F_1\Big)(x_{\tilde{\chi}^-_i}, x_{\tilde{L}_s}, x_{\tilde{\chi}^0_j})\Big\}\gamma^\mu\omega_-, \quad (8)$$

with the vertexes couplings

$$(\mathcal{A}_i)^{IJ} = \frac{-m_{l^I}\mathcal{Z}^{-*}_{2i}\mathcal{Z}^{IJ*}_{\tilde{\nu}}}{\sqrt{2}c_{_\beta}m_{_W}}, \quad (\mathcal{B}_i)^{IJ} = \mathcal{Z}^+_{1i}\mathcal{Z}^{IJ*}_{\tilde{\nu}},$$
$$(\eta)_{s_J} = \mathcal{Z}^{IJ}_{\tilde{\nu}}\mathcal{Z}^{Is}_{\tilde{L}}, \quad (\mathcal{Q})_{ji} = Z^{2j*}_N Z^+_{1i} - \frac{Z^{4j*}_N Z^+_{2i}}{\sqrt{2}},$$
$$(\mathcal{T})_{ji} = Z^{2j}_N Z^-_{1i} + \frac{1}{\sqrt{2}}Z^{3j}_N Z^{-*}_{2i},$$
$$(\mathcal{P}^I)_{si} = \frac{m_{l^I}}{\sqrt{2}c_{_\beta}M_{_W}}Z^{(I+3)s*}_{\tilde{L}}Z^{-*}_{2i} - Z^{Is*}_{\tilde{L}}Z^{-*}_{1i},$$
$$(\zeta^I)_{_{Jj}} = \mathcal{Z}^{IJ*}_{\tilde{\nu}}(\mathcal{Z}^{1j}_{_N}s_{_W} - \mathcal{Z}^{2j}_{_N}c_{_W}). \quad (9)$$

In the on-shell scheme, the counter term formula for vertex $W^+\overline{\nu^I}l^I$ can be found in Ref[4].

$$\delta V^{(OS),\mu}_{W^+\overline{\nu^I}l^I} = \frac{e}{2\sqrt{2}s_{_W}}\Big(\frac{\delta m^2_{_Z}}{m^2_{_Z}} - \frac{\delta m^2_{_Z} - \delta m^2_{_W}}{m^2_{_Z} - m^2_{_W}} + 2\delta e + \delta Z^l_L + \delta Z^\nu_L + \delta Z_{WW}\Big)\gamma^\mu\omega_-, \quad (10)$$

where δe is calculated from the virtual slepton contribution. The virtual slepton and sneutrino produce the mass renormalization constants $\delta m^2_{_Z}, \delta m^2_{_W}$ and W wavefunction renormalization constant δZ_{WW}. The wave function renormalization constants δZ^ν_L and δZ^l_L are deduced respectively from the self-energies of neutrino and lepton with virtual SUSY particles $[(\tilde{\nu}, \tilde{\chi}^0), (\tilde{L}, \tilde{\chi}^{\pm})]$ and $[(\tilde{\nu}, \tilde{\chi}^{\pm}), (\tilde{L}, \tilde{\chi}^0)]$.

To cancel the UV-divergent terms for these diagrams in the on-shell scheme, all the renormalization constants in Eq.(10) must be taken into account. Following the method in

Refs[4, 5], we obtain the needed renormalization constants.

$$
\begin{aligned}
&\frac{\delta m_Z^2}{m_Z^2} = \frac{e^2}{32\pi^2 s_W^2 c_W^2}(1-2s_W^2)^2\Delta_{UV} - \frac{e^2}{s_W^2 c_W^2}\Big\{\frac{1}{4}\sum_{j=1}^{3}F_5(x_{\tilde{\nu}_j},x_{\tilde{\nu}_j}) \\
&+\sum_{\alpha,\beta=1}^{6}|(\mathcal{G})_{\alpha\beta}|^2 F_5(x_{\tilde{L}_\alpha},x_{\tilde{L}_\beta})\Big\}, \\
&\frac{\delta m_W^2}{m_Z^2} = \frac{e^2 c_W^2}{32\pi^2 s_W^2}\Delta_{UV} - \frac{e^2 c_W^2}{2s_W^2}\sum_{i=1}^{6}\sum_{\alpha=1}^{3}|(\eta)_{i\alpha}|^2 F_5(x_{\tilde{\nu}_\alpha},x_{\tilde{L}_i}), \\
&\delta Z_{WW} = -\frac{e^2}{32\pi^2 s_W^2}\Delta_{UV} + \frac{e^2}{2s_W^2}\sum_{i=1}^{6}\sum_{\alpha=1}^{3}|(\eta)_{i\alpha}|^2 F_5(x_{\tilde{\nu}_\alpha},x_{\tilde{L}_i}), \\
&\delta e = \frac{e^2}{8\pi^2}\Delta_{UV} - e^2\sum_{i=1}^{6}F_5(x_{\tilde{L}_i},x_{\tilde{L}_i}), \\
&\delta Z_L^\nu = \frac{-e^2}{32\pi^2 s_W^2}\Big(\frac{1}{2c_W^2}+1+(\frac{m_{l^I}}{\sqrt{2}c_\beta m_W})^2\Big)\Delta_{UV} \\
&-\frac{e^2}{2s_W^2 c_W^2}\sum_{i=1}^{4}\sum_{\alpha=1}^{3}|(\zeta^I)_{\alpha i}|^2 F_4(x_{\tilde{\nu}_\alpha},x_{\chi_i^0}) - \frac{e^2}{s_W^2}\sum_{i=1}^{4}\sum_{\alpha=1}^{6}|(\mathcal{P}^I)_{\alpha i}|^2 F_4(x_{\tilde{L}_\alpha},x_{\chi_i^0}), \\
&\delta Z_L^l = -\frac{e^2}{32\pi^2 s_W^2}\Big(\frac{1}{2c_W^2}+1+(\frac{m_{l^I}}{\sqrt{2}c_\beta m_W})^2\Big)\Delta_{UV} \\
&-\frac{e^2}{s_W^2}\sum_{\alpha=1}^{3}\sum_{i=1}^{2}\Big\{|(\mathcal{B}_i)^{I\alpha}|^2 F_4 + x_{l^I}\Big[|(\mathcal{B}_i)^{I\alpha}|^2 \\
&+|(\mathcal{A}_i)^{I\alpha}|^2 + 2\mathbf{Re}[(\mathcal{A}_i^\dagger)^{I\alpha}(\mathcal{B}_i)^{I\alpha}]\Big]F_3\Big\}(x_{\tilde{\nu}_\alpha},x_{\tilde{\chi}_i^-}) \\
&-e^2\sum_{j=1}^{4}\sum_{i=1}^{6}\Big\{x_{l^I}\Big[\frac{|(\mathcal{D}^I)_{ij}|^2}{2s_W^2}+\frac{\sqrt{2}}{s_W}\mathbf{Re}[(\mathcal{C}^I)_{ij}^\dagger(\mathcal{D}^I)_{ij}] \\
&+|(\mathcal{C}^I)_{ij}|^2\Big]F_3 + \frac{1}{2s_W^2}|(\mathcal{D}^I)_{ij}|^2 F_4\Big\}(x_{\tilde{L}_i},x_{\tilde{\chi}_j^0}). \qquad (11)
\end{aligned}
$$

The function F_5 is

$$
\begin{aligned}
F_5(x,y) = &\frac{1}{288\pi^2(x-y)^3}[6(x-3y)x^2\ln x + 6(3x \\
&-y)y^2\ln y - (x-y)(5x^2-22xy+5y^2)]. \qquad (12)
\end{aligned}
$$

From Eqs.(10)(11)(12), we get the counter terms for the three diagrams(2(a),2(b) and 2(c)). The renormalization constants in Eq.(10) are all necessary at this place, which is different from the condition of $Z\overline{l^I}l^I$ vertex.

III. RENORMALIZATION OF $\tilde{L}_i^*\overline{\chi_\alpha^0}l^I$ VERTEX WITH VIRTUAL PHOTON

In order to further research the on-shell renormalization scheme in MSSM, we study the vertex $\tilde{L}_i^*\overline{\chi_\alpha^0}l^I$ at one-loop order in this section. The studied triangle diagram is Diagram 3 with virtual photon, which is the simplest instance. Diagram 3 belongs to electromagnetic interaction, and can be treated separately without considering the diagrams with virtual W and Z. The counter term for this diagram in the zero-momentum scheme is

$$\delta V^{(ZM)}_{\tilde{L}^*_i\overline{\chi^0_\alpha}l^I}(\gamma) = \frac{e^3}{16\pi^2}\Big\{\frac{(\mathcal{D}^I_{i\alpha})}{\sqrt{2}s_{_W}}\omega_- + (\mathcal{C}^I_{i\alpha})\omega_+\Big\}\Delta_{UV}$$
$$+e^3F_1(x_{\tilde{L}_i},0,x_{l^I})\Big(\frac{(\mathcal{D}^I_{i\alpha})}{\sqrt{2}s_{_W}}\omega_- + (\mathcal{C}^I_{i\alpha})\omega_+\Big). \tag{13}$$

In the on-shell scheme the counter term formula of the vertex $\tilde{L}^*_i\overline{\chi^0_\alpha}l^I$ is complicated. Following the idea of SM on-shell scheme, we show the formula here[17], where the counter term is determined by the on-shell condition.

$$\delta V^{(OS)}_{\tilde{L}^*_i\overline{\chi^0_\alpha}l^I} = \frac{e}{\sqrt{2}s_{_W}c_{_W}}\Big\{\Big[\Big(\frac{\delta e}{e}\delta_{IJ} + \frac{(\delta Z^l_L)_{JI}}{2}\Big)\delta_{ij}\delta_{\alpha\beta} + \frac{(\delta Z^\dagger_{\tilde{L}})_{ij}}{2}\delta_{IJ}\delta_{\alpha\beta}$$
$$+\frac{(\delta Z_{\chi^0})_{\beta\alpha}}{2}\delta_{IJ}\delta_{ij}\Big](Z^\dagger_{\tilde{L}})_{jJ}\Big(Z^{1\beta}_N s_{_W} + Z^{2\beta}_N c_{_W}\Big) - \frac{s_{_W}}{c_{_W}}\delta c_{_W}(Z^\dagger_{\tilde{L}})_{jJ}Z^{1\beta}_N\delta_{ij}\delta_{IJ}\delta_{\alpha\beta}$$
$$-\frac{c_{_W}}{s_{_W}}\delta s_{_W}(Z^\dagger_{\tilde{L}})_{jJ}Z^{2\beta}_N\delta_{ij}\delta_{IJ}\delta_{\alpha\beta} - \frac{m_{l^J}c_{_W}}{m_{_W}c_\beta}\Big[\Big(\frac{\delta e}{e} + \frac{\delta m_{l^J}}{m_{l^J}} + \frac{\delta m_{_W}}{m_{_W}} - \frac{\delta s_{_W}}{s_{_W}} - \frac{\delta c_\beta}{c_\beta}\Big)\delta_{IJ}\delta_{ij}\delta_{\alpha\beta}$$
$$+\frac{1}{2}(\delta Z^\dagger_{\tilde{L}})_{ij}\delta_{IJ}\delta_{\alpha\beta} + \frac{1}{2}(\delta Z_{\tilde{\chi}^0})_{\beta\alpha}\delta_{IJ}\delta_{ij} + \frac{1}{2}(\delta Z^l_L)_{JI}\delta_{\alpha\beta}\delta_{ij}\Big](Z^\dagger_{\tilde{L}})_{j(3+J)}Z^{3\beta}_N\Big\}\omega_-$$
$$+\frac{\sqrt{2}e}{c_{_W}}\Big\{-\Big[\Big(\frac{\delta e}{e} - \frac{\delta c_{_W}}{c_{_W}}\Big)\delta_{IJ}\delta_{ij}\delta_{\alpha\beta} + \frac{1}{2}(\delta Z^\dagger_{\tilde{L}})_{ij}\delta_{IJ}\delta_{\alpha\beta} + \frac{1}{2}(\delta Z^*_{\tilde{\chi}^0})_{\beta\alpha}\delta_{IJ}\delta_{ij}$$
$$+\frac{1}{2}(\delta Z^l_R)_{JI}\delta_{\alpha\beta}\delta_{ij}\Big](Z^\dagger_{\tilde{L}})_{j(3+J)}Z^{1\beta*}_N + \frac{m_{l^J}c_{_W}}{2m_{_W}s_{_W}c_\beta}\Big[\frac{\delta Z^\dagger_{\tilde{L}})_{ij}}{2}(\delta_{IJ}\delta_{\alpha\beta} + \frac{(\delta Z^*_{\tilde{\chi}^0})_{\beta\alpha}}{2}\delta_{IJ}\delta_{ij}$$
$$+\Big(\frac{\delta e}{e} + \frac{\delta m_{l^J}}{m_{l^J}} + \frac{\delta m_{_W}}{m_{_W}} - \frac{\delta s_{_W}}{s_{_W}} - \frac{\delta c_\beta}{c_\beta}\Big)\delta_{IJ}\delta_{ij}\delta_{\alpha\beta} + \frac{1}{2}(\delta Z^l_R)_{JI}\delta_{\alpha\beta}\delta_{ij}\Big](Z^\dagger_{\tilde{L}})_{jJ}Z^{3\beta*}_N\Big\}\omega_+\,. \tag{14}$$

$\delta Z^l_{L,R}, \delta Z_{\tilde{\nu}}, \delta Z_{\tilde{L}}, \delta Z_{\tilde{\chi}^-}$ and $\delta Z_{\tilde{\chi}^0}$ are the renormalization constants of wave functions for leptons and SUSY particles. The other renormalization constants come from the vertex coupling renormalization.

After tedious calculation and various compounding of renormalization constants, we find only the wave function renormalization constant of slepton $(\delta Z_{\tilde{L}})_{ij}$ is essential. That is to say, just the renormalization constant $(\delta Z_{\tilde{L}})_{ij}$ can cancel the UV-divergent term. The wave function renormalization constant $(\delta Z_{\tilde{L}})_{ij}$ is collected in the as follows.

$$F_6(x,y) = \frac{1}{32\pi^2(y-x)^3}[(y-x)(6x^2-7xy+3y^2)$$
$$+2x(2x^2-2xy+y^2)\ln x - 2y(4x^2-5xy+2y^2)\ln y],$$
$$(\delta Z^\gamma_{\tilde{L}})_{ij} = \frac{e^2}{8\pi^2}\Delta_{UV}\delta^{ij} + e^2F_6(x_{\tilde{L}_i},0)\delta^{ij}. \tag{15}$$

$(\delta Z_{\tilde{L}})^{\gamma}_{ij}$ in Eq.(15) is obtained from the self-energy of slepton with the virtual photon and slepton. In our calculation, Eq.(14) is predigested as

$$\delta V^{(OS)}_{\tilde{L}^*_i\overline{\chi^0_\alpha}l^I}(\gamma) = \frac{1}{2}(\delta Z^{\gamma}_{\tilde{L}})^{\dagger}_{ij}\Big[(\mathcal{D}^I_{j\alpha})\omega_- + (\mathcal{C}^I_{j\alpha})\omega_+\Big]. \tag{16}$$

Combining the formulas (15) and (16), Diagram 3 can be renormalized successfully in the on-shell scheme. Up to now, we have got the counter terms for the vertexes $(Z\overline{l^I}l^I, W^+\overline{\nu^I}l^I)$ and $\tilde{L}^*_s\overline{\chi^0_j}l^I$ in both the zero-momentum and on-shell schemes.

IV. THE DECOUPLING BEHAVIOR

In this section, we discuss the decoupling behavior of renormalized results in the two schemes. It is easy to prove that the renormalized results in the zero-momentum scheme are decoupled. Adopting the on-shell scheme, we must get decoupled renormalized results, if the renormalized results can not go to infinity with the incessant enlarging SUSY particle masses. To obtain the decoupling behavior of renormalized results in the on-shell scheme, we suppose all SUSY particle masses are the same and much heavier than the masses of SM particles. Compared with the decoupling character of zero-momentum counter terms, the decoupling behavior of counter terms in the on-shell scheme is obvious.

A. SM vertex $(Z\overline{l^I}l^I, W^+\overline{\nu^I}l^I)$

To obtain the decoupling behavior of the counter terms for the vertex $Z\overline{l^I}l^I$ in the zero-momentum scheme, we show the decoupling approximation of the functions $F_1$and F_2. The variables x, y, z in $F_1(x, y, z)$ are all symmetrical, and three conditions are considered here.

$$F_1(x,y,z) = \begin{cases} -\frac{\ln x}{16\pi^2} - \frac{1}{32\pi^2}, & (x=y=z) \\ -\frac{\ln x}{16\pi^2} + \dots & (x=y\gg z) \\ \frac{1-\ln x}{16\pi^2} + \dots & (x\gg y,z) \end{cases}$$
$$F_2(x,y,z) = \frac{1}{32\pi^2 x},\ (x=y=z). \tag{17}$$

With Eqs.(1) and (17), the decoupling behavior of Eq.(1) reads

$$\delta V^{(ZM),\mu}_{Z\overline{l^I}l^I} \sim \frac{e^3}{64\pi^2 s_W c_W}\Big\{\frac{1-2s^2_W}{2s^2_W}\Big[\frac{1}{c^2_W}+\Big(\frac{m_{l^I}}{c_\beta M_W}\Big)^2\Big]\gamma^\mu\omega_-$$
$$-\Big[\frac{4s^2_W}{c^2_W}+\Big(\frac{m^I_e}{c_\beta M_W}\Big)^2\Big]\gamma^\mu\omega_+\Big\}(\Delta_{UV} - \ln x_M) + \dots, \tag{18}$$

where the dots denote the terms that are finite, even when the SUSY particle masses turn to infinity. $x_{_M} = M^2/\Lambda^2_{_{NP}}$ with M representing the SUSY particle mass. In the same way, we deduce the decoupling behavior of the counter terms for the vertex $Z\overline{l^I}l^I$ in the on-shell scheme.

$$\delta V^{(OS),\mu}_{Z\overline{l^I}l^I} \sim \frac{e^3}{64\pi^2 s_{_W} c_{_W}}\Big\{\frac{1-2s^2_{_W}}{2s^2_{_W}}\Big[\frac{1}{c^2_{_W}}+\Big(\frac{m_{l^I}}{c_\beta M_W}\Big)^2\Big]\gamma^\mu\omega_-$$
$$-\Big[\frac{4s^2_{_W}}{c^2_{_W}}+\Big(\frac{m_{l^I}}{c_\beta m_{_W}}\Big)^2\Big]\gamma^\mu\omega_+\Big\}(\Delta_{UV}-\ln x_{_M})+\dots \tag{19}$$
$$F_3(x,y)=\frac{1}{96\pi^2 x},\ (x=y);\quad F_4(x,y)=-\frac{\ln x}{32\pi^2}+\frac{1}{64\pi^2},\ (x=y). \tag{20}$$

It is satisfactory that the infinite terms and undecoupled large logarithm terms in Eqs.(18) and (19) are the same. Though the finite terms represented by dots in Eqs.(18) and (19) are different, the renormalized results in both schemes are decoupled, because the zero-momentum scheme can guarantee the decoupled renormalized results.

Using the unitary character of the mixing matrixes we obtain the expectant results for the counter terms of the vertex $W^+\overline{\nu^I}l^I$ in both schemes.

$$\delta V^{(ZM),\mu}_{W^+\overline{\nu^I}l^I} \sim \Big\{\frac{e^3}{\sqrt{2}s^3_{_W}}\frac{1}{64\pi^2}\Big[\Big(\frac{m_{l^I}}{c_\beta M_W}\Big)^2+\frac{1}{c^2_{_W}}\Big](\Delta_{UV}-\ln x_{_M})\Big\}\gamma^\mu\omega_-+\dots, \tag{21}$$
$$\delta V^{(OS),\mu}_{W^+\overline{\nu^I}l^I} \sim \Big\{\frac{e^3}{\sqrt{2}s^3_{_W}}\frac{1}{64\pi^2}\Big[\Big(\frac{m_{l^I}}{c_\beta M_W}\Big)^2+\frac{1}{c^2_{_W}}\Big](\Delta_{UV}-\ln x_{_M})\Big\}\gamma^\mu\omega_-+\dots, \tag{22}$$
$$F_5(x,y)=\frac{\ln x}{48\pi^2},\ (x=y).$$

The two counter terms in Eqs.(21) and (22) can both eliminate completely Δ_{UV} and $\ln x_{_M}$ terms produced from the three triangle diagrams(2(a), 2(b) and 2(c)).

B. The MSSM vertex $\tilde{L}^*_i\overline{\chi^0_\alpha}l^I$

For the MSSM vertex $\tilde{L}^*_i\overline{\chi^0_\alpha}l^I$, the decoupling behavior of counter term is discussed here. Assuming SUSY particles are very heavy, the approximate results of Eq.(13) deduced from virtual photon are shown as

$$\delta V^{(ZM)}_{\tilde{L}^*_i\overline{\chi^0_\alpha}l^I}(\gamma) \sim \frac{e^3}{16\pi^2}\Big\{\frac{1}{\sqrt{2}s_{_W}}(\mathcal{D}^I_{i\alpha})\omega_-+(\mathcal{C}^I_{i\alpha})\omega_+\Big\}(\Delta_{UV}-\ln x_{_M})+\dots$$
$$F_6(x,y)=-\frac{\ln x}{8\pi^2}+\frac{3}{16\pi^2}+\dots,\quad (x\gg y). \tag{23}$$

The decoupling behavior of the counter term Eq.(16) in the on-shell scheme is the same as that of Eq.(23) for Δ_{UV} and $\ln x_{_M}$. In this way, we find that the renormalized results are decoupled not only in the zero-momentum scheme but also in the on-shell scheme.

V. DISCUSSION AND CONCLUSION

Up to now there have been several renormalization schemes for renormalizable theories. The on-shell renormalization scheme is approbated broadly for electroweak theory in SM, and it is well studied by theorists. For the model including new physics beyond SM, the on-shell renormalization scheme has mist to clear. MSSM is considered the most potent candidate in the new models, which has attracted much attention from many people for about twenty years. In the frame work of MSSM, some processes are calculated with the on-shell renormalization scheme. However, a consummate on-shell renormalization scheme for MSSM is still absent.

To explore the perfect on-shell renormalization scheme, at one-loop order we study two SM vertexes($Z\overline{l^I}l^I, W^+\overline{\nu^I}l^I$) and one MSSM vertex $\tilde{L}_i^*\overline{\chi_\alpha^0}l^I$ in the zero-momentum scheme and the on-shell scheme. In the zero-momentum scheme, each divergent diagram has its own counter term, and has nothing to do with other diagrams. Another important peculiarity is that the renormalized result is absolutely decoupled.

In the on-shell scheme, the counter term formulas for the SM vertexes in MSSM and in SM are similar. Almost all the renormalization constants are deduced from the one-loop self-energies of the corresponding particles. In SM, all the renormalization constants in the counter term must be taken into account. At the same time, in MSSM we can not always renormalize one triangle diagram by the counter term made up of renormalization constants. After careful study, both characters of the on-shell scheme are discovered. One character is that all the triangle diagrams belonging to one type for a vertex are essential. Only the sum of the amplitude can be renormalized completely. The other character is that not all the renormalization constants are always necessary. Which renormalization constant must be considered lies on the idiographic condition.

This work shows that for the SM vertex $Z\overline{l^I}l^I$ the lepton wave function renormalization constants $\delta Z_L^l, \delta Z_R^l$ are requisite to obtain the needed counter term. However, the condition of the vertex $W^+\overline{\nu^I}l^I$ is dissimilar. To gain the final finite results, we have to calculate all the renormalization constants in the counter term formula. For the MSSM vertex, the foregoing experience is of value of reference. The on-shell scheme for the MSSM vertex $\tilde{L}_i^* \overline{\chi_\alpha^0} l^I$ shows the property, i.e. just the wave function renormalization constant for the relevant slepton ($\tilde{L}$) is enough for completing the on-shell scheme.

In the two renormalization schemes, we study the decoupling behavior for the counter terms of these vertexes. Obviously, the counter terms obtained in the two renormalization schemes have the same characters for the infinite and large logarithm terms, when the SUSY particle masses are equal and very heavy. Because the renormalized results in the zero-momentum scheme are decoupled, the on-shell renormalization scheme can also give decoupled renormalized results.

There are a great deal of vertexes in MSSM, so it is hard to make one-loop on-shell renormalization for all of them. The studied vertexes in this work are representative, which can be helpful to upbuild a consistent on-shell renormalization scheme in MSSM. Though there are a number of questions to deal with, one can be convinced that a perfect on-shell renormalization scheme can be found in the future. This text is also propitious to study the on-shell renormalization scheme in other models, even the model is more complex than MSSM.

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