

Minimal Fine Tuning in Supersymmetric Higgs Inflation

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Abstract

We investigate characteristic features of realistic parameter choice for primordial inflation with supersymmetric Higgs inflaton as an example of particle physics inflation model. We discuss constraints from observational results and analyze the degree of fine tuning needed to induce slow-roll inflation for wide range of soft supersymmetry breaking scale. The observed amplitude of density fluctuations implies that the minimal fine tuning for the combined electroweak scale and inflaton flatness predicts the spectral index of $n_s = 0.950 - 0.965$, which includes the central value from observational data.

1 Introduction

Recent experimental information such as the Planck satellite results [1] provides cosmological parameters with increasing accuracy, which enables us to perform detailed numerical examination of realistic model parameters in various candidate inflation models [2]. One of the motivations to investigate numerical aspects of concrete inflation models is to see how the model parameters are chosen or tuned to realize appropriate slow-roll inflation.

In this paper, among various models of inflation, we adopt a supersymmetric Higgs inflation model [3] to investigate its quantitative aspects as a simple example of particle physics model of inflation. In addition to the parameter tuning for inflation, such a particle physics model of inflation may be relevant for electroweak hierarchy tuning. We are also confronted with no discovery of superpartners so far. Hence we do not restrict ourselves to the case of weak scale supersymmetry (SUSY) in contrast to the conventional analysis.

As for the supersymmetric Higgs degrees of freedom, the quartic contributions to the scalar potential due to gauge interactions are identically zero for specific directions called D-flat directions in the field space of Higgs scalars. Along these flat directions, the scalar potential is determined by the soft SUSY breaking terms and higher order non-renormalizable operators appearing in the effective low energy theory. For suitable parameter choices, such a scalar potential can realize successful primordial inflation.

The rest of the paper is organized as follows. In the next section, we present the setup of supersymmetric Higgs inflation model, whose D-flat direction is modified to have a nearly flat inflection point. We identify the model parameters which are fine tuned to realize slow-roll inflation. In section 3, the observational constraints on temperature fluctuations of cosmic microwave background radiation are taken into account to obtain its index of spectral tilt in the supersymmetric Higgs inflation model as depicted in tables 1–3. In section 4, we consider inflationary parameter tuning to induce slow-roll inflation, the observed amplitude of density fluctuations, or eternal inflation in the present setup. The final section concludes the paper.

2 The inflaton potential

We first recapitulate the supersymmetric Higgs inflation model considered by Chatterjee and Mazumdar [3]. The Higgs fields are those of the minimal supersymmetric standard model (MSSM) with up-type and down-type ones denoted by H_u and H_d , respectively.

2.1 D-flat direction

Let us parametrize the D-flat direction of the Higgs potential as follows:

$$H_u = \left(\frac{1}{\sqrt{2}}\Phi, 0\right), \quad H_d = \left(0, \frac{1}{\sqrt{2}}\Phi\right), \quad (1)$$

where Φ is a complex scalar field. The quartic terms in the potential would vanish along this direction in the case of genuine MSSM.

The potential can be modified by non-renormalizable terms which originate from the Higgs superpotential

$$W = \mu \mathcal{H}_u \cdot \mathcal{H}_d + \sum_{k \geq 2} \frac{\lambda_k}{k} \frac{(\mathcal{H}_u \cdot \mathcal{H}_d)^k}{M_{\text{P}}^{2k-3}}, \quad (2)$$

where M_{P} denotes the reduced Planck mass $\simeq 2.4 \times 10^{18} \text{ GeV}$ and \mathcal{H}_u and \mathcal{H}_d are superfields corresponding to H_u and H_d . Hereafter, we only keep the lowest additional term with $k = 2$ in this paper.

Taking into account soft SUSY breaking terms, we have the scalar potential along the D-flat direction as

$$V(\phi, \theta) = \frac{1}{2}m^2(\theta)\phi^2 - \frac{\lambda_2\mu}{4M_{\text{P}}} \cos(2\theta)\phi^4 + \frac{\lambda_2^2}{32M_{\text{P}}^2}\phi^6, \quad (3)$$

where ϕ and θ denote the radial and angular components of the field $\Phi = \frac{1}{\sqrt{2}}\phi e^{i\theta}$ and

$$m^2(\theta) = \frac{1}{2}(2\mu^2 + m_{H_u}^2 + m_{H_d}^2 - 2b \cos 2\theta). \quad (4)$$

The soft parameters $m_{H_u}^2$, $m_{H_d}^2$, and b are coefficients of Higgs soft quadratic terms in the potential. Here, we have taken all the parameters to be positive.

The potential is minimized along the angular direction when $\theta = 0$ with the effective θ mass above the Hubble mass during inflation. Then it is given by

$$V(\phi) \equiv V(\phi, 0) = \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_2\mu}{4M_{\text{P}}}\phi^4 + \frac{\lambda_2^2}{32M_{\text{P}}^2}\phi^6, \quad (5)$$

where $m_0 \equiv m(\theta = 0)$. This m_0 may be regarded as a typical soft SUSY breaking scale so that the hierarchy between m_0 and the Z boson mass m_Z indicates the electroweak fine tuning.

2.2 Inflection point and slow-roll parameters

For the primordial Higgs inflation to occur successfully, the potential at least has to possess some region where the slow-roll conditions are satisfied. Actually, such a flat region is realized if the potential has an inflection point almost like a saddle point.

The potential (5) has a saddle point when the parameter relation $3m_0^2 = 4\mu^2$ holds. Hence we define a characteristic parameter α as

$$3m_0^2 = 4\mu^2(1 + 8\alpha^2), \quad (6)$$

which parameterizes the tuning required for slow-roll inflation, reminiscent of electroweak scale tuning in MSSM. We restrict ourselves to the regime $\alpha^2 > 0$, for which the potential is monotonic around the inflection point.

If this parameter is fine-tuned as $\alpha^2 \ll 1$, successful slow-roll inflation can occur near the inflection point $\phi = \phi_0$:

$$\phi_0 = \left(\frac{4M_P}{\sqrt{3}\lambda_2}m_0\right)^{\frac{1}{2}}(1 - \alpha^2) + \mathcal{O}(\alpha^4). \quad (7)$$

We assume $\phi_0 \ll M_P$, that is, $m_0 \ll \lambda_2 M_P$. Around the inflection point ϕ_0 , the potential can be expanded as

$$V(\phi) = V_0 + \beta_1(\phi - \phi_0) + \frac{1}{6}\beta_3(\phi - \phi_0)^3 + \dots, \quad (8)$$

where

$$V_0 = V(\phi_0) = \frac{1}{6}m_0^2\phi_0^2 + \mathcal{O}(\alpha^2), \quad (9)$$

$$\beta_1 = 8\alpha^2 m_0^2 \phi_0 + \mathcal{O}(\alpha^4), \quad (10)$$

$$\beta_3 = 8\frac{m_0^2}{\phi_0} + \mathcal{O}(\alpha^2). \quad (11)$$

The ellipsis represents higher order terms, whose effects during inflation we neglect in the following analysis.

The slow-roll parameters are given by

$$\epsilon(\phi) \equiv \frac{M_P^2}{2} \left(\frac{V'}{V}\right)^2 \simeq \frac{M_P^2}{2V_0^2} (\beta_1 + \frac{\beta_3}{2}(\phi - \phi_0)^2)^2, \quad (12)$$

$$\eta(\phi) \equiv M_P^2 \frac{V''}{V} \simeq M_P^2 \frac{\beta_3}{V_0} (\phi - \phi_0). \quad (13)$$

The absolute values of these parameters are smaller than one during inflation.

The inflation ends when the slow-roll parameter η reaches one so that the end point $\phi = \phi_{\text{end}}$ of slow-roll inflation is given by

$$\phi_0 - \phi_{\text{end}} \simeq \left(\frac{\phi_0^3}{48M_{\text{P}}^2} \right). \quad (14)$$

This should be small enough to neglect higher order terms in the potential.

3 Observational constraints

The parameters which characterize the primordial inflation model are constrained by observational data. In particular, we have restrictions on the amplitude of density fluctuations $\sqrt{A_s} \simeq 4.69 \times 10^{-5}$ and its spectral tilt parameterized by the spectral index $n_s \simeq 0.96$ with narrowing uncertainties [1].

For the supersymmetric Higgs inflation model described in the previous section, these quantities are obtained [4, 5] by means of inflaton equations of motion as

$$\sqrt{A_s} \simeq \frac{1}{72\sqrt{6}\pi} \frac{1}{X^2 \lambda_2} \sin^2[6\sqrt{6}\mathcal{N}X\lambda_2], \quad (15)$$

$$n_s \simeq 1 - 24\sqrt{6}X\lambda_2 \cot[6\sqrt{6}\mathcal{N}X\lambda_2], \quad (16)$$

where $X \equiv \alpha \left(\frac{M_P}{m_0} \right)$ and \mathcal{N} is the number of e-foldings of the present horizon after horizon exit during inflation. Note that the X indicates the combined degree of fine tuning for inflationary potential flatness and hierarchy between the soft SUSY breaking scale m_0 and the electroweak scale m_Z . For a fixed value of X , the α tuning can be compensated by the M_P/m_0 tuning or the m_Z/m_0 tuning up to the constant factor M_P/m_Z .

It turns out through numerical estimates that $X \lesssim 10^4$ and $\lambda_2 \sim 10^{-7}$ are needed to produce the right amount of density fluctuations $\sqrt{A_s} \simeq 4.69 \times 10^{-5}$. We list appropriate sample parameters sets and the corresponding theoretically obtained n_s in tables 1, 2, and 3, which show the cases of $\mathcal{N} = 40, 50$, and 60 under $\sqrt{A_s} = 4.69 \times 10^{-5}$. The uncertainty of $\sqrt{A_s}$ is negligible compared to those of \mathcal{N} and n_s in our analysis. For each value of X , there are two values of λ_2 which satisfy the constraints from the observed amplitude of density fluctuations. They are denoted by $\lambda_2^{(1)}$ and $\lambda_2^{(2)}$ and the corresponding values of n_s are obtained as $n_s^{(1)}$ and $n_s^{(2)}$. The parenthesized values in the tables are out of assumed slow-roll inflation and written just for completeness. The X takes the maximal values (see the next section) for the limiting case of $\lambda_2^{(1)} = \lambda_2^{(2)}$ and $n_s^{(1)} = n_s^{(2)}$, which is denoted in the captions to the tables.

4 Inflationary parameter tuning

Based on our numerical analysis, we now consider possible criteria for inflationary parameter tuning.

The obvious minimum requirement is that inflationary phase is at least present. The equations (12) and (13) for slow-roll parameters yield the indispensable condition for slow-roll inflation to occur as $\epsilon(\phi_0 \pm \Delta\phi_q) \lesssim 1$ and $\eta(\phi_0 \pm \Delta\phi_q) \lesssim 1$, which amount to

$$X^4 \lesssim \frac{1}{288\sqrt{3}\lambda_2} \frac{M_{\text{P}}^3}{m_0^3}, \quad \lambda_2 \lesssim \frac{\pi}{\sqrt{6}}. \quad (17)$$

Here, $\Delta\phi_q$ denotes quantum variance $\Delta\phi_q \simeq H/2\pi$ in the field ϕ during Hubble time H^{-1} . Thus, the smaller m_0/M_P is, the less the necessary fine tuning for X is.

As a realistic primordial inflation, sizable density fluctuations should be produced during inflationary phase. Let us reflect on the expressions (15) and (16) for density fluctuations and spectral index. For a fixed value of the amplitude $\sqrt{A_s}$, the allowed value of X has an upper bound, since the function $\sin^2 x/x$ in Eq.(15) has an upper bound, which is about 0.725 at $x = 1.166$. The observed amplitude of density fluctuations implies that the maximal values of X are $1.641 \times 10^4 - 2.461 \times 10^4$ for $\mathcal{N} = 40 - 60$, resulting in $n_s = 0.950 - 0.965$, which captures the central value of the observed spectral index. This maximal value of X is none other than the minimal fine tuning in the present supersymmetric Higgs inflation. Note that the advantage of small m_0/M_P in fine tuning is absent here contrary to the case of mere tuning for slow-roll inflation to occur.

As a possible further fine tuning, we finally consider the parameter tuning to realize eternal inflation. If the fine-tuned parameter α is exceedingly small, the first derivative of the potential near the inflection point ϕ_0 is extremely tiny. In such a case, when ϕ is very close to ϕ_0 , quantum effects dominate field fluctuations and keep the system in a de Sitter background effectively. This eternal inflation regime exists if quantum variance $\Delta\phi_q \simeq H/2\pi$ in the field ϕ during Hubble time H^{-1} is larger than the corresponding classical change $\Delta\phi_c \simeq |-V'/3H^2|$.

In the present model, this condition turns out to be

$$(\phi - \phi_0)^2 \lesssim \frac{1}{144\sqrt{2}\pi} \frac{m_0}{M_{\text{P}}^3} \phi_0^4 - 2\alpha^2 \phi_0^2. \quad (18)$$

For the field to be randomly kept inside the eternal inflation regime, the right-hand side of the inequality have to be larger than the inflaton quantum variance squared during the Hubble

time. This requires

$$X^2 \lesssim \frac{1}{72\pi} \left(\frac{1}{\sqrt{6}\lambda_2} - \frac{1}{2\pi} \right). \quad (19)$$

Namely, if α is small enough to satisfy this inequality, the eternal inflation regime exists near the inflection point. For $\lambda_2 \sim 10^{-7}$, this condition turns out to be $X \lesssim 10^2$, which yields $n_s = 0.900 - 0.933$ for $\mathcal{N} = 40 - 60$.

5 Conclusion

We have investigated inflationary parameter tuning in the supersymmetric Higgs inflation model as an example of particle physics model of primordial inflation. The observed tilt of the spectral index implies that the tuning is minimal to realize the sizable amplitude of density fluctuations.

That is, under the observed amplitude of density fluctuations, the maximal values of the simultaneous fine tuning parameter $X = \alpha \left(\frac{m_Z}{m_0} \right) \left(\frac{M_P}{m_Z} \right)$ for the electroweak scale and inflaton flatness is given by $X = 1.641 \times 10^4 - 2.461 \times 10^4$ for e-fold numbers $\mathcal{N} = 40 - 60$, resulting in the spectral index $n_s = 0.950 - 0.965$, which includes the central value of the observed spectral index. The advantage of small m_0/m_Z in electroweak fine tuning is compensated by the inflaton flatness tuning to realize sizable density fluctuations. Namely, the weak scale SUSY does not ameliorate the total degree of fine tuning including electroweak hierarchy in the present setup once we take into account the inflationary fine tuning.

We performed a simple case study of supersymmetric Higgs inflation model in this paper. Inflationary parameter tuning may be intriguing to explore in various models of inflation [2], by one of which we suspect the primordial inflation of our universe is well described. Future observations will further constrain realistic inflation models and enable us to make even more detailed examination on parameter choices thereof. We hope that this serves to reveal fundamental structures to determine model parameters in particle physics.

Acknowledgments

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X	$\lambda_2^{(1)}$	$n_s^{(1)}$	$\lambda_2^{(2)}$	$n_s^{(2)}$
1.6×10^4	1.054×10^{-7}	0.935	1.429×10^{-7}	0.969
1.4×10^4	0.908×10^{-7}	0.919	1.961×10^{-7}	1.007
1.2×10^4	0.847×10^{-7}	0.912	2.543×10^{-7}	1.041
1.0×10^4	0.810×10^{-7}	0.908	3.319×10^{-7}	1.078
1.0×10^3	0.751×10^{-7}	0.900	4.734×10^{-6}	1.744
1.0×10^2	0.751×10^{-7}	0.900	(5.146×10^{-5})	(3.600)
1.0×10^1	0.751×10^{-7}	0.900	(5.280×10^{-4})	(9.380)

Table 1: Sample parameters sets for the $\mathcal{N} = 40$ case. The maximal value of X to achieve $\sqrt{A_s} = 4.69 \times 10^{-5}$ is $X = 1.641 \times 10^4$ with $\lambda_2 = 1.207 \times 10^{-7}$ and $n_s = 0.950$. The parenthesized values are written just for completeness.

X	$\lambda_2^{(1)}$	$n_s^{(1)}$	$\lambda_2^{(2)}$	$n_s^{(2)}$
2.0×10^4	0.675×10^{-7}	0.948	0.914×10^{-7}	0.975
1.8×10^4	0.592×10^{-7}	0.937	1.189×10^{-7}	1.000
1.6×10^4	0.555×10^{-7}	0.932	1.468×10^{-7}	1.022
1.4×10^4	0.531×10^{-7}	0.928	1.807×10^{-7}	1.044
1.2×10^4	0.515×10^{-7}	0.926	2.247×10^{-7}	1.069
1.0×10^4	0.503×10^{-7}	0.924	2.861×10^{-7}	1.099
1.0×10^3	0.481×10^{-7}	0.920	3.837×10^{-6}	1.678
1.0×10^2	0.481×10^{-7}	0.920	(4.133×10^{-5})	(3.334)
1.0×10^1	0.481×10^{-7}	0.920	(4.229×10^{-4})	(8.501)

Table 2: Sample parameters sets for the $\mathcal{N} = 50$ case. The maximal value of X to achieve $\sqrt{A_s} = 4.69 \times 10^{-5}$ is $X = 2.051 \times 10^4$ with $\lambda_2 = 0.774 \times 10^{-7}$ and $n_s = 0.960$. The parenthesized values are written just for completeness.

X	$\lambda_2^{(1)}$	$n_s^{(1)}$	$\lambda_2^{(2)}$	$n_s^{(2)}$
2.4×10^4	0.469×10^{-7}	0.957	0.635×10^{-7}	0.979
2.2×10^4	0.417×10^{-7}	0.949	0.795×10^{-7}	0.997
2.0×10^4	0.393×10^{-7}	0.944	0.952×10^{-7}	1.012
1.8×10^4	0.376×10^{-7}	0.941	1.130×10^{-7}	1.027
1.4×10^4	0.356×10^{-7}	0.938	1.622×10^{-7}	1.061
1.0×10^4	0.344×10^{-7}	0.935	2.500×10^{-7}	1.108
1.0×10^3	0.334×10^{-7}	0.933	3.229×10^{-6}	1.628
1.0×10^2	0.334×10^{-7}	0.933	(3.454×10^{-5})	(3.135)
1.0×10^1	0.334×10^{-7}	0.933	(3.528×10^{-4})	(7.851)

Table 3: Sample parameters sets for the $\mathcal{N} = 60$ case. The maximal value of X to achieve $\sqrt{A_s} = 4.69 \times 10^{-5}$ is $X = 2.461 \times 10^4$ with $\lambda_2 = 0.538 \times 10^{-7}$ and $n_s = 0.965$. The parenthesized values are written just for completeness.