

Gluon bremsstrahlung by heavy quarks - its effects on transport coefficients and equilibrium distribution

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The effects of gluon radiation by charm quarks on the transport coefficients *e.g.* drag, longitudinal and transverse diffusion and shear viscosity have been studied within the ambit of perturbative quantum chromodynamics (pQCD) and kinetic theory. We found that while the soft gluon radiation has substantial effects on the transport coefficients of the charm quarks in the quark gluon plasma its effects on the equilibrium distribution function is insignificant.

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I. INTRODUCTION

Recently the study of Quark Gluon Plasma (QGP), expected to be created in heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies, intrigues the scientific community with its multifarious interesting aspects. Therefore, in order to understand different properties of QGP we need to probe it. Among many, one of the efficient probes is the charm quark (CQ) produced in the early hard collisions of the partons from the colliding nuclei. Generally the transport coefficients are sensitive to the interaction of the probes with the medium. Hence, estimation of the various transport coefficients of QGP by using CQ is a field of high contemporary interest. Moreover, strong coupling of the probes with the medium may bring the probe in equilibrium with the bulk matter, so that the probe follow a momentum distribution similar to that of the constituents of the medium. In the present work, we will consider the QGP as the thermal medium of light quarks, their antiparticles and gluons and the CQs as probes. This will enable us to estimate the drag, diffusion and shear viscous coefficients of QGP and the nature of the equilibrium distribution of the CQs. The reasons behind choosing CQ as a probe are: (i) being created from the early hard collisions, CQ can experience the hot/dense medium from its birth. The CQ distribution function is different from that of the medium particle and being heavier than the constituent particles of QGP, it does not get equilibrated quickly and hence it qualifies to act as the Brownian particle. (ii) The probability of the production of CQ (with mass M) inside the thermal medium of temperature T ($T \ll M$) is very less, hence, the probability of annihilation of CQs in the QGP is also small. Therefore, the CQs witness the entire evolution of the bath. The probability of creation and annihilation of bottom quarks is even smaller, therefore, the present work can be extended to the bottom quarks also. While propagating through the QGP the CQ interacts with the medium particle via two dominant processes: a) collisional or elastic interaction and b) inelastic interaction like gluon bremsstrahlung. In earlier works, while calculating the momentum diffusion coefficients, either

the gluon radiation by the CQ have been ignored [1–3, 5] or it is calculated for non-relativistic CQ [18, 19] or estimated by first determining the drag from radiative energy loss [6–8] and then using the Einstein relation between drag and diffusion coefficients [4].

In this work, we calculate, using pQCD, the transverse and longitudinal diffusion coefficients of the CQ undergoing radiative loss by emitting gluons while propelling through the QGP. We consider that the emitted gluons, being soft, get absorbed in the medium, thereby resulting in transporting the energy from the fast moving CQ to the slow moving constituents of the bath. This transportation of momentum is reflected in the momentum diffusion coefficients of the CQ in QGP.

The values of the drag and diffusion coefficients can be used to characterize the distribution function of the probe. Therefore, these transport coefficients can be utilized to understand the departure of the CQ distribution from the thermal distribution of the bath particles. The shape of the equilibrium distribution function of CQ have been studied using a generalized Einstein relation derived in Ref.[9]. We revisit this relation by including both the collisional and the radiative transport coefficients of CQ.

The present work is organized as follows. In the next section, we discourse the formalism of Fokker Planck Equation (FPE) and the procedure to evaluate transport coefficients for collisional and radiative processes. In section III, the equilibrium distribution (f_{eq}^{CQ}) of CQ is elaborated in the context of bremsstrahlung process. The impact of the radiative transport coefficients on the f_{eq}^{CQ} is particularly highlighted. In section IV, the η/s of QGP has been estimated using CQ transverse diffusion coefficients with particular emphasis on the radiative processes. Section V is dedicated to summary and conclusions.

II. FORMALISM AND TRANSPORT COEFFICIENTS

Heavy Quark propagates as a Brownian particle in the QGP medium. The ensemble of Brownian particles immersed in the thermal medium can be characterized by the single particle distribution function, $f(\vec{x}, \vec{p}, t)$. The

time evolution of f is governed by the master equation, a simplified version of which is the Fokker Planck Equation(FPE).

The form of the master equation or the Boltzmann transport equation(BTE) governing the CQ distribution f is given by:

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial}{\partial \vec{p}} \right] f(\vec{x}, \vec{p}, t) = \left[\frac{\partial f}{\partial t} \right]_{\text{collisions}} \quad (1)$$

In the absence of external force, \vec{F} and for a homogeneous plasma, we can write BTE as follows:

$$\left[\frac{\partial f}{\partial t} \right]_{\text{collisions}} = \int d^3 \vec{k} [w(\vec{p} + \vec{k}, \vec{k}) f(\vec{p} + \vec{k}) - w(\vec{p}, \vec{k}) f(\vec{p})]. \quad (2)$$

where $w(\vec{p}, \vec{k})$ is the rate of collision of CQ changing its momentum from \vec{p} to $\vec{p} - \vec{k}$. Considering only the soft scattering of CQ with the bath particles, we reduce the integro-differential Eq. 2 to the FPE:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[A_i(\vec{p}) f + \frac{\partial}{\partial p_j} [B_{ij}(\vec{p}) f] \right], \quad (3)$$

where the kernels are defined as

$$A_i = \int d^3 \vec{k} w(\vec{p}, \vec{k}) k_i, \quad (4)$$

and

$$B_{ij} = \frac{1}{2} \int d^3 \vec{k} w(\vec{p}, \vec{k}) k_i k_j. \quad (5)$$

where A_i and B_{ij} are the drag and the diffusion coefficients of CQ. Our motivation is to find out these coefficients due to the elastic and inelastic interactions of the CQs with the bath particle within the ambit of pQCD.

A. Transport co-efficient for collisional process

First, we concentrate on the two-body elastic processes. While propagating inside the plasma, the CQ(Q) encounters the following interactions with the bath particle: $Q(p) + g(q) \rightarrow Q(p') + g(q')$, $Q(p) + q(q) \rightarrow Q(p') + q(q')$ and $Q(p) + \bar{q}(q) \rightarrow Q(p') + \bar{q}(q')$, where the quantities within the bracket denote momenta of the CQ, quark (q), antiquark (\bar{q}) and gluon (g). Therefore, A_i and B_{ij} are written in terms of invariant amplitude squared [1] as:

$$\begin{aligned} A_i &= \frac{1}{2E_p} \int \frac{d^3 q}{(2\pi)^3 2E_q} \int \frac{d^3 q'}{(2\pi)^3 2E_{q'}} \\ &\times \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{1}{\gamma} \sum |M|_{2 \rightarrow 2}^2 (2\pi)^4 \delta^4(p + q - p' - q') \\ &\times \hat{f}(\mathbf{q})(1 \pm \hat{f}(\mathbf{q}'))(p - p')_i \\ &= \ll (p - p')_i \gg \end{aligned} \quad (6)$$

$$(7)$$

Similarly,

$$B_{ij} = \frac{1}{2} \ll (p' - p)_i (p' - p)_j \gg \quad (8)$$

Since, A_i and B_{ij} only depend on three momentum, \vec{p} and background temperature, T, we can write them as:

$$A_i = p_i A \quad (9)$$

and

$$B_{ij} = (\delta_{ij} - \frac{p_i p_j}{p^2}) B_{\perp}(p, T) + \frac{p_i p_j}{p^2} B_{\parallel}(p, T). \quad (10)$$

where, B_{\perp} and B_{\parallel} are transverse and longitudinal diffusion coefficients respectively. Using Eqs. 7, 8, 9 and 10 we get:

$$A(p) = \ll 1 \gg - \frac{\ll \vec{p} \cdot \vec{p}' \gg}{p^2}, \quad (11)$$

$$B_{\perp}(p) = \frac{1}{4} \left[\ll p'^2 \gg - \frac{\ll (\vec{p} \cdot \vec{p}')^2 \gg}{p^2} \right], \quad (12)$$

$$B_{\parallel}(p) = \frac{1}{2} \left[\frac{\ll (\vec{p} \cdot \vec{p}')^2 \gg}{p^2} - 2 \ll \vec{p} \cdot \vec{p}' \gg + p^2 \ll 1 \gg \right]. \quad (13)$$

Using Eqs. 11,12 and 13 with the matrix elements of those elastic processes mentioned above, the drag, transverse and longitudinal diffusion coefficients can be estimated.

The expression for the transport coefficient($X(\vec{p}, T)$) can be schematically written as:

$$X(p) = \int \text{phase space} \times \text{interaction} \times \text{transport part} \quad (14)$$

In case of drag(diffusion), transport part involves momentum(square of the momentum) transfer of CQ with the bath particle. The evaluation of the drag and diffusion coefficients with collisional loss are elaborated in Refs. [6, 7]. Therefore, we refer to these references for details and do not repeat the discussions here.

B. Transport co-efficient for radiative process

In view of Eq. 14, we can estimate radiative transport coefficients. There, the phase space factor will be that for three-body scattering, the interaction will involve three-body invariant amplitude squared and the transport part will remain same. The $X(\vec{p}, T)$ for the radiative process, $Q(p) + \text{parton}(q) \rightarrow Q(p') + \text{parton}(q') + \text{gluon}(k_5)$, (where parton stands for light quarks, anti-quarks and gluons and $k_5 = (E_5, k_{\perp}, k_z)$) can be written as:

$$X = \frac{1}{2E_p} \int \frac{d^3 q}{(2\pi)^3 2E_q} \int \frac{d^3 q'}{(2\pi)^3 2E_{q'}} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}}$$

$$\begin{aligned}
& \times \int \frac{d^3 k_5}{(2\pi)^3 2E_5} \frac{1}{\gamma} \sum |M|_{2 \rightarrow 3}^2 (2\pi)^4 \delta^4(p + q - p' - q' - k_5) \\
& \times \hat{f}(E_q)(1 \pm \hat{f}(E_{q'}))(1 + \hat{f}(E_5)) \\
& \times \theta(\tau - \tau_F)\theta(E_p - E_5)
\end{aligned} \tag{15}$$

where τ_F is the formation time of the emitted gluon, step functions are introduced in Eq. 15 to take into accounts the LPM effects and to prohibit the emission of gluons with energy greater than E_p . In order to calculate $\sum |M|_{2 \rightarrow 3}^2$, the necessary Mandelstam variables are defined as:

$$s = (p + q)^2, \quad s' = (p' + q')^2, \tag{16}$$

$$t = (p - p')^2, \quad t' = (q - q')^2, \tag{17}$$

$$u = (p - q')^2, \quad u' = (q - p')^2, \tag{18}$$

with

$$s + t + u + s' + t' + u' = 4M^2. \tag{19}$$

In this work, we will consider the case of soft gluon emission, i.e. when $k_5 \rightarrow 0$, which implies $s' \rightarrow s, t' \rightarrow t, u' \rightarrow u$. For the kinematic region,

$$\sqrt{s} \gg \sqrt{|t|} \gg E_5 \tag{20}$$

the invariant amplitude squared for $2 \rightarrow 3$ process can be expressed in terms of $2 \rightarrow 2$ process multiplied with the emitted gluon spectrum as:

$$|M|_{2 \rightarrow 3}^2 = |M|_{2 \rightarrow 2}^2 \times 12g_s^2 \frac{1}{k_\perp^2} \left(1 + \frac{M^2}{s} e^{2y}\right)^{-2} \tag{21}$$

The last term in Eq. 21 is the dead cone factor [10] and y denotes the rapidity of the emitted gluon. Following the prescription given in Eq. 21, we have the radiative X :

$$\begin{aligned}
X_{rad} &= X_{coll} \times \int \frac{d^3 k_5}{(2\pi)^3 2E_5} 12g_s^2 \frac{1}{k_\perp^2} \\
&\times \left(1 + \frac{M^2}{s} e^{2y}\right)^{-2} [1 + \hat{f}(E_5)] \\
&\times \theta(\tau - \tau_F)\theta(E_p - E_5)
\end{aligned} \tag{22}$$

After having calculated the radiative transport coefficients, we find out total or effective transport coefficient as the sum of the collisional and radiative contribution, i.e.,

$$X_{eff} = X_{coll} + X_{rad}. \tag{23}$$

where X_{coll} and X_{rad} are the transport coefficients for the collisional and radiative processes respectively. In order to obtain the effective transport coefficient, we have added the collisional and radiative parts with the view in mind that though the invariant amplitude of three body scattering can be expressed in terms of the two body scattering, the process of collision and radiation are taking place inside the thermal medium independently.

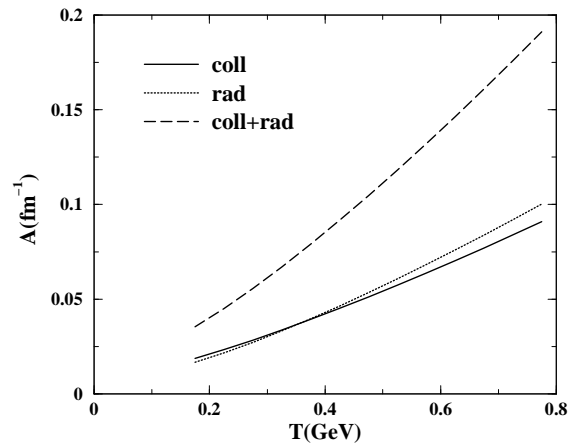


FIG. 1: Temperature dependence of the drag coefficient of CQ of momentum, $p = 5$ GeV.

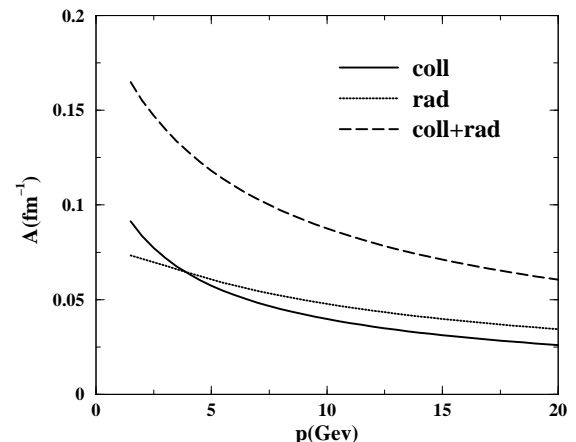


FIG. 2: Momentum dependence of the drag coefficient of CQ for bath temperature, $T = 525$ MeV

In Fig. 1 we display the temperature dependence of the drag of CQ with momentum, $p = 5$ GeV. At low T , although, the drag for radiative loss is comparable to the collisional loss, at high temperature the radiative drag tend to dominate. The difference between total and collisional transport coefficients broadens with increasing temperature. Even at temperatures attainable in RHIC, this distinction is significant enough to have a pronounced effect on certain experimental observables like nuclear modification factor, elliptic flow of CQ etc. In the temperature range that may be achieved at LHC collision conditions the radiative contributions to the drag may surpass the collisional contributions. Therefore, radiative processes will play more dominant role at LHC than at RHIC. For a CQ (mass, $M = 1.3$ GeV) with $p = 5$ GeV and $T = 300$ MeV, the value of drag coefficient attains a value almost double the value for collisional case when radiation is included. At 600 MeV temperature,

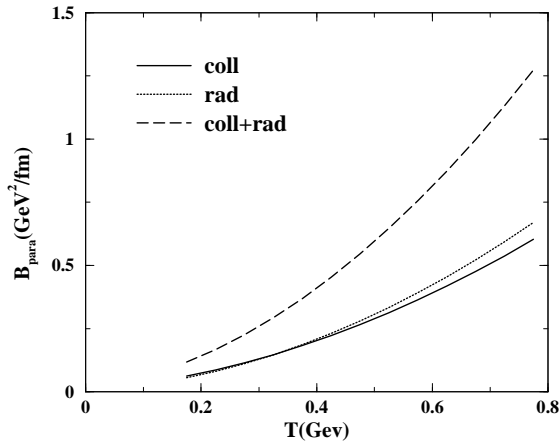


FIG. 3: Temperature dependence of the longitudinal diffusion coefficient of CQ of momentum, $p = 5$ GeV.

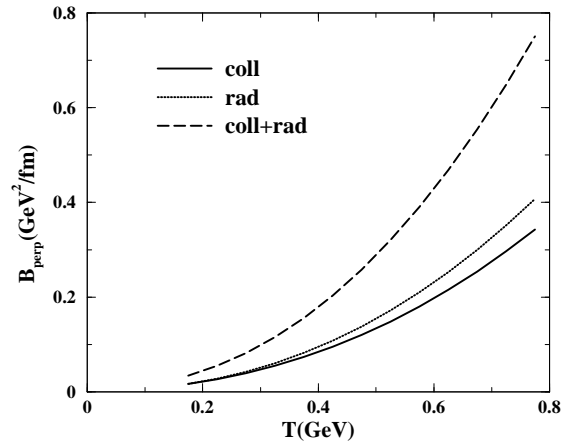


FIG. 5: Temperature dependence of the transverse diffusion coefficient of CQ of momentum, $p = 5$ GeV.

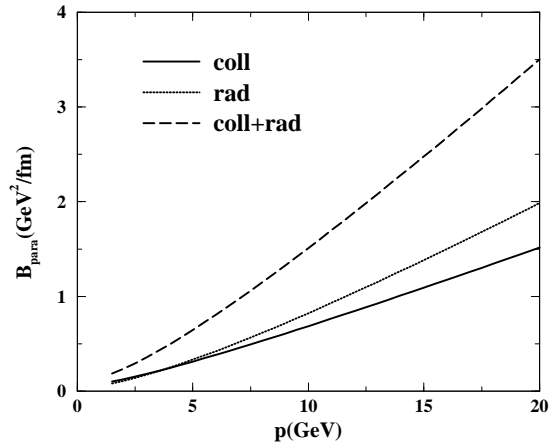


FIG. 4: Momentum dependence of the longitudinal diffusion coefficient of CQ for bath temperature, $T = 525$ MeV

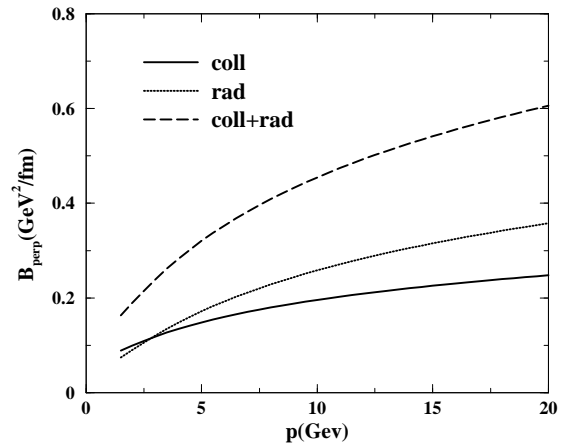


FIG. 6: Momentum dependence of the transverse diffusion coefficient of CQ for bath temperature, $T = 525$ MeV

total drag becomes 2.12 times the collisional drag. The variation of drag with p at $T = 525$ MeV is depicted in Fig. 2. The dominance of radiative processes, in spite of dead cone suppression, is evident from the results for p beyond 5 GeV.

In Fig. 3 and 4, the variation of longitudinal diffusion coefficient with temperature and momentum are displayed respectively. Similar to drag, the contributions from radiative processes dominate over the collisional processes for higher T and p .

For $T = 300$ MeV, the radiative and collisional loss have similar contributions to B_{\perp} , but for $T = 600$ MeV the radiative part exceeds the collisional part (Fig 5). It is interesting to note the qualitative change in the momentum dependence of B_{\perp} from B_{\parallel} at fixed T (Fig. 6). The variation of B_{\perp} with p is slower than B_{\parallel} . In this case again the domination of the radiative transport coefficient over its collisional counterpart is evident. Though

the nature of the momentum dependence of the diffusion coefficients is different from that of drag, it is always true that, save at very low momentum of the CQ, radiative contribution is more than the elastic contribution at $T = 525$ MeV for the momentum range considered here. Accordingly, for a relativistic CQ the effect of radiation becomes imperative and should be included in the analysis of experimental data from nuclear collisions at RHIC and LHC. This statement can be kept on a firmer ground if we quote some quantitative results comparing radiative and collisional contributions to the transport coefficients. Drag coefficient of a CQ having momentum 10 GeV is $0.038 fm^{-1}$ in case of elastic loss, whereas the radiative contribution is $0.047 fm^{-1}$. Radiative B_{\perp} is about 1.33 times its collisional counterpart. In case of longitudinal diffusion coefficient, radiative contribution is 1.2 times the elastic one.

III. EQUILIBRIUM DISTRIBUTION OF CHARM QUARK

Having calculated the diffusion coefficients of CQs including radiative effects, we would like to investigate the fate of the equilibrium distribution function of a CQ undergoing elastic as well as radiative processes. A generalized Einstein relation involving the three transport coefficients, *i.e.* drag, transverse and longitudinal diffusion coefficients is obtained in [9] to establish the shape of the distribution of CQ after it gets equilibrated due to its collisional and radiative interaction with the medium. In Ref. [9] the radiative process was not taken into account. We would like to explore the role of the radiative processes in the characterisation of the equilibrium distribution and to check whether CQ becomes a part of the thermal medium abiding by the same class of statistics, which is the Boltzmann-Jüttner distribution, followed by the bath particles.

We discuss the generalized Einstein relation by examining the Fokker Planck equation in the absence of any external force in a homogeneous QGP (Eq. 24).

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left(A_i f + \frac{\partial}{\partial p_j} B_{ij} f \right) = -\vec{\nabla}_p \cdot \vec{\varphi} \quad (24)$$

A relationship among the transport coefficients can be derived by demanding that $\partial f / \partial t$ is zero, *i.e.* the probability current, $\vec{\varphi}$ vanishes when Eq. 24 is satisfied by the equilibrium distribution function, f_{eq}^{CQ} .

Using the following form of f_{eq}^{CQ} ,

$$f_{eq}^{CQ}(p; T, q) = N \exp[-\Phi(p; T, q)] \quad (25)$$

the desired relation can be found out, where, N is the normalization factor; T, q are parameters needed to specify the shape of the distribution. Using Eqs. 9 and 10 and the fact that f_{eq}^{CQ} depends only on the magnitude of momentum for spatially homogeneous case, we arrive at the general Einstein relation:

$$A(p, T) = \frac{1}{p} \frac{d\Phi}{dp} B_{||}(p, T) - \frac{1}{p} \frac{dB_{||}}{dp} - \frac{2}{p^2} [B_{||}(p, T) - B_{\perp}(p, T)] \quad (26)$$

This relation is valid for any momentum of CQ and can be reduced to the well-known Einstein relation, $D = \gamma MT$, in the non-relativistic limit, where $A = \gamma$ and $B_{\perp} = B_{||} = D$, *i.e.* $B_{ij} = D\delta_{ij}$ and $\Phi = p^2 / (2MT)$.

From Eq. 26, it is quite conspicuous that if the three drag/diffusion coefficients are known then one can infer the correct equilibrium distribution function obeyed by CQ and ascertain whether or not CQ will fall under Boltzmann-Jüttner class of statistics. It is clear from the variation of $d\Phi/dp$ (calculated from Eq. 26) with p (Fig. 7) that $d\Phi/dp$ deviates significantly from $d/dp(\sqrt{p^2 + m^2}/T)$, *i.e.* CQ seems to be away from the Boltzmann-like distribution. In principle, we should have

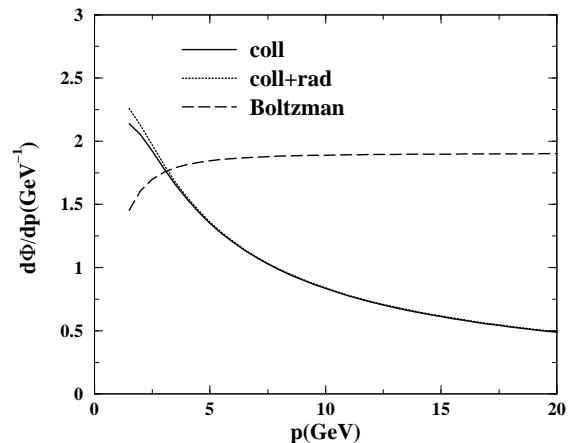


FIG. 7: For CQ propagating in QGP having temperature, $T = 525$ MeV.

ascertained the precise form of Φ from Eq. 26, had we been able to include all the non-perturbative contributions to calculate A, B_{\perp} and $B_{||}$. To study the equilibrium distribution and its deviation from the Boltzmann-Jüttner distribution quantitatively, we consider Tsallis distribution [11] for which Φ is given by:

$$\Phi_{Ts} = \frac{1}{1-q} \ln [1 - (1-q)E(p)/T_T] , \quad (27)$$

where T_T (temperature like) and q are parameters. The Φ_{Ts} reduces to the Boltzmann distribution in the limit $q \rightarrow 1$ and $T_T \rightarrow T$ (T is the temperature of the heat bath). The value of T_T and q will designate the form of f_{eq}^{CQ} . Putting Eq. 27 into Eq. 26, we get[9]

$$T_T + (q-1)E = \frac{dE}{dp} \frac{B_{||}}{pA + \frac{dB_{||}}{dp} + \frac{2}{p}(B_{||} - B_{\perp})} \quad (28)$$

Our aim is to calculate the right hand side of Eq. 28 and to determine the value of T_T and q by studying the variation of $T_T + (q-1)E$ with E and parameterizing the variation by straight line. First we consider the elastic processes only. The dependence of $T_T + (q-1)E$ on E for CQ of mass, $M = 1.3$ GeV propagating inside a heat bath having temperature, $T = 525$ MeV is plotted in Fig. 8 considering A, B_{\perp} and $B_{||}$ for collisional loss only. We get $q = 1.101$ and $T_T = 184$ MeV. Φ_{Ts} with these value of T_T and q is far from being that of the Boltzmann-Jüttner statistics (long-dashed line). Results displayed in Fig. 8 also indicates that the inclusion of radiative effects on the drag and diffusion coefficients does not make any noteworthy change on the shape of the equilibrium distribution of CQ. With the total transport coefficients, the values of T_T and q do not get altered. As a matter of fact, this effect is not quite unexpected. By looking at Eq. 28, we might conclude that it is not the magnitude of the transport coefficients, rather their ratio which decides

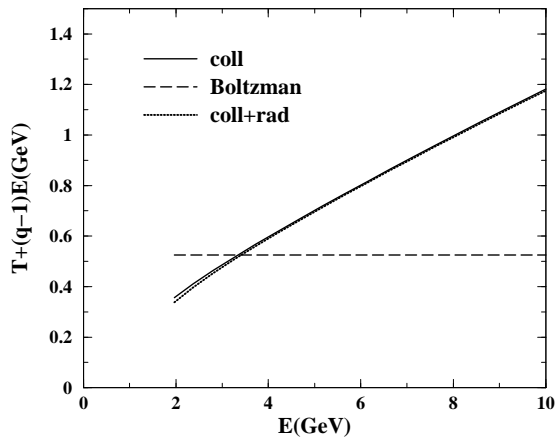


FIG. 8: Plot of RHS of Eq. 28 vs E for collisional as well as total transport coefficients at $T = 525$ MeV. Long dashed line: expected for Boltzmann-Jüttner distribution

the shape of the equilibrium distribution. Therefore, it is not surprising at all that although the value of the relaxation time of CQ is dictated by the magnitude of the drag coefficient (in which the radiative contribution is substantial), the shape is largely independent of the magnitude of the transport coefficients. In turn, this means that the nature of the underlying interaction of CQ with the bath particles, i.e. whether it suffers only elastic collisions or it undergoes bremsstrahlung also, has got to do very little with the ultimate shape of f_{eq}^{CQ} . This conclusion remains unaltered even when we increase the bath temperature, T to 725 MeV. At $T = 725$ MeV, the value of the slope of

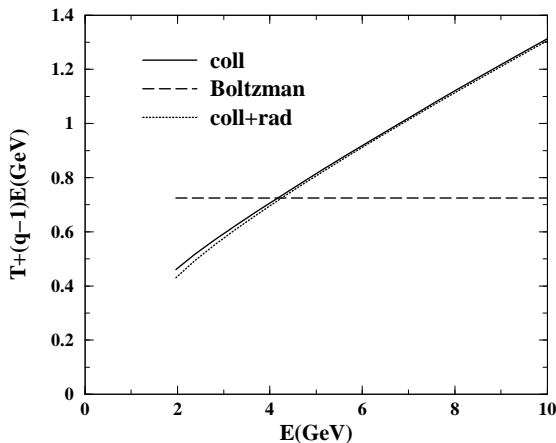


FIG. 9: Plot of RHS of Eq. 28 vs E for collisional as well as total transport coefficients at $T = 725$ MeV. Long dashed line: expected for Boltzmann-Jüttner distribution

the straight line remains unchanged, i.e. the value of the parameter q comes out to be 1.095 which is almost same as that at $T = 525$ MeV. But, the value of the other parameter of the Tsallis distribution, T_T is found out to be

335 MeV. At this temperature of the heat bath too, the incorporation of the radiative drag/diffusion coefficients hardly has any bearing as far as the shape of f_{eq}^{CQ} is concerned. In Fig. 8 and Fig. 9, the long-dashed horizontal lines represent the Boltzmann-Jüttner distribution ($q = 1$ and $T_T = T$) which is obeyed by the constituent of QGP. In order to expect the probe particle, the CQ, to become a part of the system, i.e. to follow the same statistics as that of the bath particles, one should have found out the parameters q and T_T of the Tsallis distribution to be 1 and T respectively. Instead, we notice that q and the ratio, T/T_T (this ratio is 2.85 at $T = 525$ MeV and 2.164 at $T = 725$ MeV) are never equal to unity. Therefore, it may be concluded that though, the CQ may equilibrate while propagating through QGP, it may not share the same distribution with the bath particles, i.e. with the light quarks and gluons for a wide range of CQ energies and bath temperatures.

IV. SHEAR VISCOSITY(η) TO ENTROPY DENSITY(s) RATIO OF QGP PROBED BY CHARM QUARK

The value of shear viscosity(η) to entropy density(s) ratio, η/s , play a pivotal role in deciding the nature of QGP, i.e. whether the medium behaves like a weakly coupled gas or a strongly coupled liquid. In this work we evaluate η/s by calculating the transport parameter, \hat{q} , which is a measure of the squared average momentum exchange between the probe and the bath particles per unit length [13–15]. The \hat{q} , which has been found to be ~ 1 GeV²/fm in Ref. [15], can be related to the transverse diffusion coefficient of CQ which is calculated here. When a CQ with a certain momentum propagates in QGP, a transverse momentum exchange with the bath particles occurs. Hence, the momentum of the energetic CQ is shared by the low momentum (on the average) bath particles which is expressed through the transverse diffusion coefficients. The transverse diffusion coefficients causes the minimization of the momentum (or velocity) gradient in the system. Therefore, it must be related to the shear viscous coefficients of the system which drive the system towards depletion of the velocity gradient. The transverse momentum diffusion coefficient, B_{\perp} can be written as:

$$B_{\perp} = \frac{1}{2} \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_{ij} \quad (29)$$

By Eq. 8 and using the notation $(p' - p)_i = k_i$,

$$\begin{aligned} B_{\perp} &= \frac{1}{2} \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) \frac{1}{2} \ll k_i k_j \gg \\ &= \frac{1}{4} \ll \vec{k}^2 - \frac{(\vec{p} \cdot \vec{k})^2}{p^2} \gg \end{aligned}$$

If we take \vec{p} to be along z-axis,

$$B_{\perp} = \frac{1}{4} \ll \vec{k}^2 - k_z^2 \gg$$

$$\begin{aligned}
&= \frac{1}{4} \ll k_{\perp}^2 \gg \\
&= \frac{1}{4} \hat{q}
\end{aligned}
\tag{30}$$

With this definition of \hat{q} , we calculate η/s of QGP from the following expression[13]:

$$\frac{\eta}{s} \approx 1.25 \frac{T^3}{\hat{q}}
\tag{31}$$

Therefore,

$$4\pi \frac{\eta}{s} \approx 1.25\pi \frac{T^3}{B_{\perp}}.
\tag{32}$$

Eq. 32 indicates that the η/s can be estimated from B_{\perp} . From the analysis of the experimental data [15] it was found that $4\pi \frac{\eta}{s} = 1.4 \pm 0.4$ which may be compared with the AdS/CFT bound $4\pi \frac{\eta}{s} \geq 1$ [17]. We display $4\pi \frac{\eta}{s}$ against T when CQ undergoes both collisional and radiative processes.

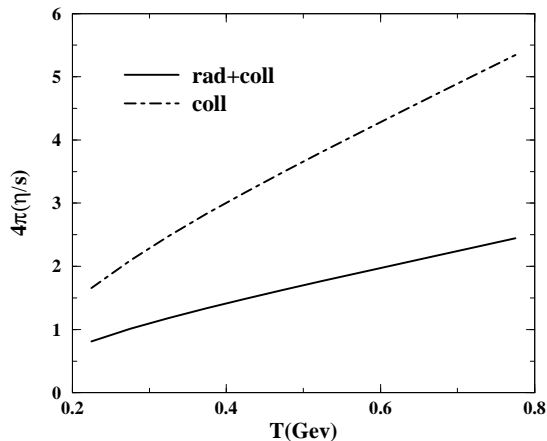


FIG. 10: For a CQ with momentum, $\langle p_T \rangle = 5$ Gev propagating in QGP of temperature, T

From the results shown in Fig. 10, it should be noted that the value of η/s changes substantially with the inclusion of the radiative effects. The inclusion of the radiative loss in B_{\perp} brings the theoretical values closer to the experimental findings[16]. This highlights the importance of the radiative loss of the CQ in QGP.

V. SUMMARY AND CONCLUSION

Transport coefficients *i.e.* drag, transverse and longitudinal diffusion coefficients of CQ propagating in QGP have been evaluated using pQCD by including both the elastic collision of CQ with the constituent particles of the bath along with soft gluon radiation (which gets absorbed in the medium subsequently). Radiative drag/diffusion coefficients are seen to exceed the collisional ones for high bath temperature and large CQ momentum. A relation between the transverse diffusion coefficients (B_{\perp}) and η/s is established. We obtain reasonable value of η/s for the QGP when the contributions from the gluon bremsstrahlung of the CQ is added with the collisional contributions. We also investigate the dependence of the shape of the equilibrium distribution function of CQ on the three transport coefficients. We find that the incorporation of radiation does not alter the shape of the equilibrium distribution significantly, owing to the fact that the shape counts on the ratios of the transport coefficients instead of their absolute values. The present work has been performed for CQ. However, its extension for bottom quark is straight forward, where the mass of the charm quark has to be replaced by that of the bottom quark.

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