Charged lepton electric dipole moment enhancement in the Lorentz violated extension of the standard model

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Abstract

We consider the Lorentz violated extension of the standard model. In this framework, there are terms that explicitly violate CP-symmetry. We examine the CPT-even $d_{\mu\nu}$-term to find the electric dipole moment of charged leptons. We show that the form factors besides the momentum transfer, depend on a new Lorentz-scalar, constructing by $d_{\mu\nu}$ and the four momenta of the lepton, as well. Such an energy dependence of the electric dipole form factor leads to an enhancement of the lepton electric dipole moment at high energy, even at the zero momentum transfer. We show that at $\frac{|d|p^2}{m_l^2} \sim 1$ the electric dipole moment of the charged lepton can be as large as $10^{-14} e\ cm$.

Keywords: electric dipole moment, Lorentz violated extension of the standard model

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1 Introduction

As the electron is a fundamental particle, discovering the nonzero electron electric dipole moment (eEDM), can unambiguously provide an experimental test on new physics. In fact, electric dipole moment (EDM) for fundamental particles violates CP symmetry. Although in the standard model there is no term which explicitly violates the CP, through the CKM-phase, a tiny EDM can be produced for all charged leptons. Therefore, to have the eEDM, comparable with the experimental bounds, one needs a new theory beyond the standard model. The new sources of CP-violations in such theories might have the same origin as the SM. For instance, in the SUSY the electron EDM originates in new CP-violating phases. In contrast, there might be theories with explicit CP-violating terms. In the framework of the standard model extension (SME), introduced by D. Colladay and V. Alan Kostelecky [1]-[2], there is such terms. The phenomenological aspects of the SME have been extensively considered by many authors in terrestrial [3]-[16] and astrophysical systems [17]-[27], for more than a decade. The bounds on the LV-parameters are collected in [28]. Here we examine the CPT-even $d_{\mu\nu}$-term that violates the CP symmetry to find the charged lepton EDM. The electric dipole form factor, as well as the others, depends not only on the momentum transfer, but also on this new constant tensor that violates the Lorentz symmetry. Therefore, one can expect new effects at the zero momentum transfer. In fact, the form factors should depend on the scalars constructed by $d_{\mu\nu}$ and four momenta of particles. Therefore, even at the zero momentum transfer, the form factors may depend on the energy of the particle and some enhancement for the particle's EDM with the energy can occur.

In Sec. II, the QED part of the SME and subsequently, the electromagnetic form factors and their impacts on the charged lepton EDM are introduced. In Sec. III, we explore the one-loop correction on the lepton-photon vertex, in the extended QED, and consequently, the lepton EDM, in high and low energy limits, are obtained. In Sec. IV, some concluding remarks are given. In appendix A some useful identities is introduced. The detail calculations of the vertex correction is given in appendix B.

2 Electromagnetic form factors

In the QED part of the SME the Lagrangian for a free particle is parameterized as [1]-[2]

$$\mathcal{L} = \bar{\psi}(i\Gamma_\mu \partial^\mu - M)\psi, \tag{1}$$

where

$$\Gamma_\mu = \gamma_\mu + c_{\nu\mu}\gamma^\nu - d_{\nu\mu}\gamma^\nu\gamma^5 + e_\mu + if_\mu\gamma^5 + \frac{1}{2}g_{\lambda\nu\mu}\sigma^{\lambda\nu},$$
$$M = m + a_\mu\gamma^\mu - b_\mu\gamma^\mu\gamma^5 + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu} + im_5\gamma^5. \quad (2)$$

As in [29]-[30] was noted, the violating Lorentz parameters in Γ_μ are appeared in the Lagrangian along with a momentum factor and therefore, at high energy limit, are more important than the LV-parameters which is given in the mass term M. Furthermore, in Γ_μ, at the lowest order in the Lorentz violating parameters, only f_μ and $d_{\mu\nu}$ can produce EDM for point particles. In this article, we are looking for some enhancement, at high energy limit, on the EDM of the charged leptons. For this purpose, since f_μ is unphysical [31], we restrict ourselves to the parameter $d_{\mu\nu}$. It should be noted that although the particle Lorentz transformation symmetry is broken, the Lagrangian (1) is fully covariant under the observer Lorentz transformations [1]-[2]. Therefore, under the observer Lorentz transformation, $d_{\mu\nu}$ behaves as a new Lorentz quantity. Consequently, the most general form for the electromagnetic current between Dirac leptons, consistent with the Lorentz covariance and the Ward identity, can be written as follows

$$<\psi(p)|J_\mu^{EM}|\psi(p')> = \bar{u}(p')\mathcal{G}_\mu(q^2)u(p), \quad (3)$$

where $q_\mu = p'_\mu - p_\mu$ and

$$\mathcal{G}_\mu(q^2) = F_1\Big[\gamma_\mu + \gamma_5\gamma^\nu d_{\nu\mu}\Big] + F_2\, i\frac{\sigma_{\mu\nu}q^\nu}{2m} + F_3\Big[(q_\mu - \frac{q^2}{2m}\gamma_\mu)\gamma_5 + \frac{q^2}{2m}d_{\nu\mu}\gamma^\nu\Big]$$
$$+ F_4\,\sigma_{\mu\nu}\frac{q^\nu}{2m}\gamma_5 + \mathcal{F}_d, \quad (4)$$

in which m is the charged lepton mass and F_i's $i = 1-4$ are the usual electric charge, magnetic dipole, anapole (axial charge) and electric dipole form factors, respectively. Meanwhile, $\mathcal{F}_d$ stands for all the new terms in the current that vanishes at $d = 0$. This part contains the new form factors which can be defined, for a symmetric and traceless $d_{\mu\nu}$, as follows

$$\mathcal{F}_d = (iF_5 + F_6\gamma_5)[d_{\mu\alpha}\sigma^{\alpha\nu} - d^{\nu\alpha}\sigma_{\alpha\mu}]\frac{q_\nu}{2m}$$
$$+ (F_7 + F_8\gamma_5)[q\cdot d\cdot\gamma q_\mu - q^2 d_{\mu\alpha}\gamma^\alpha]. \quad (5)$$

All the form factors are Lorentz scalars and depend on the scalars q^2, $p\cdot d\cdot p'$, $p'\cdot d\cdot p$, $p\cdot d\cdot p$ and $p'\cdot d\cdot p'$. One can easily see that the electric dipole form factor F_4 leads to a nonzero EDM for a charged lepton as

$$d_e = -\frac{F_4|_{q^2=0}}{2m}. \quad (6)$$

It should be noted that in the ordinary standard model $F_3\Big[(q_\mu - \frac{q^2}{2m}\gamma_\mu)\gamma_5\Big]$ shows the anapole term in the matrix element of a conserved four-current for a free spin-$\frac{1}{2}$ fermion [32]. Meanwhile, in the SME the Dirac equation is modified (see (13)) and therefore the current conservation leads to a new term for the anapole as given in (4).

In the Lorentz conserving QED only virtual quarks, in the loops, can violate the CP-symmetry that in turn, leads to a tiny nonzero value for F_4. However, in the LV counterpart of the QED not only F_4 is nonzero, even at the leading order, but also it depends on the new scalars such as $p\cdot d\cdot p'$ that can enhance the lepton's EDM at the high energy limit. It should be noted that the other new form factors given in (5) have also some contribution to the lepton's EDM as well. For instance, the F_6-term can couple to an external field $A_\mu=(\phi,0,0,0)$ as

$$\mathcal{G}^{(6)}_\mu(q^2)A^\mu = iF_6\gamma_5(d_{\mu\alpha}\sigma^{\alpha\nu} - d^{\nu\alpha}\sigma_{\alpha\mu})\frac{q_\nu A^\mu}{2m} = \frac{iF_6}{2m}\gamma_5(d_{00}\sigma^{0i} + d^{ij}\sigma_{0j} + d_{0j}\sigma^{ji})q_iA_0. \quad (7)$$

Meanwhile, in the limit p and $p'\ll m$ one has

$$u_(p) \simeq \sqrt{m}\begin{pmatrix} (1-\frac{\mathbf{p}\cdot\sigma}{2m})\ \xi \\ (1+\frac{\mathbf{p}\cdot\sigma}{2m})\ \xi \end{pmatrix}, \quad (8)$$

therefore, the spin dependent part of the current, at the zero momentum transfer and up to the first order of the LV-parameter, can be easily casted into

$$\bar{u}(p')\mathcal{G}^{(6)}_0(q^2)u(p) \simeq -F_6(0)[d_{00}\bar{\xi}\sigma^i\xi + d_{ji}\bar{\xi}\sigma^j\xi]q_i. \quad (9)$$

It should be noted that since $\mathcal{G}^{(6)}_\mu$ depends on the LV-parameter, then the spinors in the current, at the first order of the LV-parameter d, are the free ones. In the high energy limit, the spinors can be given as

$$u_(p) \simeq \frac{\sqrt{2E}}{2}\begin{pmatrix} (1-\hat{p}\cdot\sigma)\ \xi \\ (1+\hat{p}\cdot\sigma)\ \xi \end{pmatrix}, \quad (10)$$

though the spin dependent part of the current does not change. Therefore, the form factor F_6 leads to the electric dipole interaction as

$$-d_e\cdot\mathcal{E} = \frac{eF_6|_{q^2=0}}{m}(d_{00}S\cdot\mathcal{E} + S_i d_{ij}\mathcal{E}_j). \quad (11)$$

Before calculating the form factors, some comments are in order. As (5) shows, the Lorentz vector $\mathcal{F}_d$ is constructed by the Lorentz tensor $d_{\mu\nu}$. Therefore, up to the first order of $d_{\mu\nu}$, only the form factors F_1-F_4 depend on the LV-parameter. In fact, at the leading order, all the new form factors are d-independent and, at the zero recoil, they are of the order of $\frac{\alpha}{2\pi}$. Thus, the form factors such as the F_6, see Eq.(11), lead to $d_e \sim \frac{\alpha}{2\pi}\frac{e|d|}{2m_e}$ or $|d| \sim 10^{-14}$ for the eEDM of the order of $10^{-27}e\ cm$. Meanwhile, at the leading order, $F_4|_{q^2=0} \sim \frac{\alpha}{2\pi}\frac{p_i d_{ij} p_j}{m_e^2}$ that in turn results in $d_e \sim \frac{\alpha}{2\pi}\frac{e|d|p^2}{2m_e^3}$. In the other words, in the relativistic limit, there is an enhancement on the eEDM through the form factor F_4. It should be noted that, in any case it is assumed that $\frac{|d|p^2}{m_e^2} \leq 1$ and the LV parameter $d_{\mu\nu}$ is symmetric and can be taken traceless [1]-[2].

3 Charged Lepton EDM in the standard model extension

To obtain the F_i's in the electromagnetic current, we examine the lepton-photon vertex in the QED part of the SME. In this section, as a crosscheck, we assume both symmetric and antisymmetric parts of $d_{\mu\nu}$ are nonzero, however, at the end we show that the lepton EDM as a physical quantity depends only on the symmetric part of $d_{\mu\nu}$. The effective lagrangian for the only non vanishing LV-parameter $d_{\mu\nu}$ is

$$\mathcal{L}^{CPT-even}_{electron} = \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu\psi - m\bar{\psi}\psi + \frac{i}{2}\, d_{\mu\nu}\bar{\psi}\gamma_5\gamma^\mu \overleftrightarrow{D}^\nu\psi. \quad (12)$$

The Lagrangian (12) leads to the equation of motion for a free lepton as

$$(\not{p} - m + d_{\mu\nu}p^\nu\gamma_5\gamma^\mu)u(p) = 0. \quad (13)$$

Meanwhile, the modified Gordon identities can be obtained as follows

$$\bar{u}\gamma_\mu u = \bar{u}\frac{(p+p')_\mu}{2m}u + \bar{u}\frac{i\sigma_{\mu\nu}q^\nu}{2m}u + \bar{u}i\frac{\sigma_{\mu\alpha}d^{\alpha\nu}(p+p')_\nu}{2m}\gamma_5 u + \bar{u}\frac{d_{\mu\nu}q^\nu}{2m}\gamma_5 u, \quad (14)$$

and

$$\bar{u}\gamma_\mu\gamma_5 u = \bar{u}\frac{q_\mu\gamma_5}{2m}u + \bar{u}\frac{i\sigma_{\mu\nu}(p+p')^\nu\gamma_5}{2m}u + \bar{u}i\frac{\sigma_{\mu\alpha}d^{\alpha\nu}q_\nu}{2m}u + \bar{u}\frac{d_{\mu\alpha}(p+p')^\alpha}{2m}u. \quad (15)$$

Therefore, to the leading order of the LV-parameter $d^{\mu\nu}$, the lepton-photon vertex $ie(\gamma^\mu + d^{\nu\mu}\gamma_5\gamma_\nu)$ can be written as

$$\bar{u}(\gamma_\mu + \gamma_5\gamma^\nu d_{\nu\mu})u = \bar{u}(\frac{(p+p')_\mu}{2m} + \frac{i\sigma_{\mu\nu}q^\nu}{2m})u + \bar{u}[\frac{(d_{\mu\nu}-d_{\nu\mu})q^\nu}{2m} + i\frac{(\sigma_{\mu\alpha}d^{\alpha\nu} + \sigma^{\nu\alpha}d_{\alpha\mu})(p+p')_\nu}{2m}]\gamma_5 u. \quad (16)$$

In (16) the antisymmetric tensor $d^A_{\mu\nu} = (d_{\mu\nu} - d_{\nu\mu})$ can couple to an electric field as $d^A_{i0}\mathcal{E}_i$ which is a constant and, as is expected, it has not any contribution to the EDM. Meanwhile, to avoid a nonstandard time derivatives in the canonical quantization procedure of the fermion fields, Γ_0 in (2) must be equal to γ_0 or $d_{\mu 0} = 0$ [29]-[30]. In fact, to support $\Gamma_0 = \gamma_0$ in (1), one needs a field redefinition $\psi = A\chi$ [33]-[34] where its existence was shown in [35] and is given in [36] for $\Gamma_0 = c_{\nu 0}\gamma^\nu$. In our case, to leading order of $d_{\mu\nu}$, we introduce $A = 1 + \frac{1}{2}d_{\mu 0}\gamma^0\gamma^\mu\gamma_5$. Therefore, the lagrangian (12) in terms of the new field χ transforms into

$$\mathcal{L}^{CPT-even}_{electron} = \frac{i}{2}\bar{\chi}\tilde{\eta}_{\mu\nu}\gamma^\mu \overleftrightarrow{D}^\nu \chi - \tilde{m}\bar{\chi}\chi, \tag{17}$$

where

$$\begin{aligned} \tilde{\eta}_{\mu\nu} &= \eta_{\mu\nu} + \mathcal{D}_{\mu\nu}\gamma_5, \\ \mathcal{D}_{\mu\nu} &= d_{\mu\nu} - \eta_{\mu\nu}d_{00} + \eta_{0\mu}d_{\nu 0} - \eta_{0\nu}d_{\mu 0}, \\ \tilde{m} &= m(1 + i d_{\alpha 0}\sigma^{\alpha 0}\gamma_5). \end{aligned} \tag{18}$$

One can easily see that $\mathcal{D}_{\mu 0} = 0$. Here, for simplicity, we assume $d_{\mu 0} = 0$ then $\mathcal{D}_{\mu\nu} = d_{\mu\nu}$, $\tilde{m} = m$ and the fermion propagator for the new field is

$$S_F(p) = \frac{1}{\tilde{\eta}_{\mu\nu}\gamma^\mu p^\nu - m}. \tag{19}$$

Since the electromagnetic current, at the tree level, has not any contribution on the lepton EDM, then to find a nonzero value for the EDM we consider the one loop correction on the lepton-photon vertex in the framework of the QED part of SME. As is shown in Fig. 1, there are five places which are affected by the LV parameter d. To evaluate the one loop correction in the QED extension (QEDE), one has

$$\Gamma^\mu_{QEDE} = \int \frac{d^4k}{(2\pi)^4}\frac{-ig_{\rho\alpha}}{(p-k)^2}\bar{u}(p')(-ie\Gamma^\alpha)S_F(k')\Gamma^\mu S_F(k)(-ie\Gamma^\rho)u(p), \tag{20}$$

in which $\Gamma^\mu_{QEDE} = \bar{u}(p')\mathcal{G}^\mu(q^2)u(p)$, S_F is given in (19) and $\Gamma^\mu = (\gamma^\mu + d^{\nu\mu}\gamma_5\gamma_\nu)$. Replacing S_F with its expansion up to the first order of d cast the vertex function into

$$\begin{aligned} \Gamma^\mu_{QEDE} &= \int \frac{d^4k}{(2\pi)^4}\frac{-ie^2}{(p-k)^2}\bar{u}(p')\{\Gamma^\alpha\frac{(\not{k}'+m)}{k'^2-m^2}\Gamma^\mu\frac{(\not{k}+m)}{k^2-m^2}\Gamma_\alpha \\ &+ \Gamma^\alpha\frac{(\not{k}'+m)}{k'^2-m^2}\Gamma^\mu\frac{(\not{k}+m)}{k^2-m^2}\gamma\cdot d\cdot k\gamma_5\frac{(\not{k}+m)}{k^2-m^2}\Gamma_\alpha \\ &+ \Gamma^\alpha\frac{(\not{k}'+m)}{k'^2-m^2}\gamma\cdot d\cdot k'\gamma_5\frac{(\not{k}'+m)}{k'^2-m^2}\Gamma^\mu\frac{(\not{k}+m)}{k^2-m^2}\Gamma_\alpha\}u(p). \end{aligned} \tag{21}$$

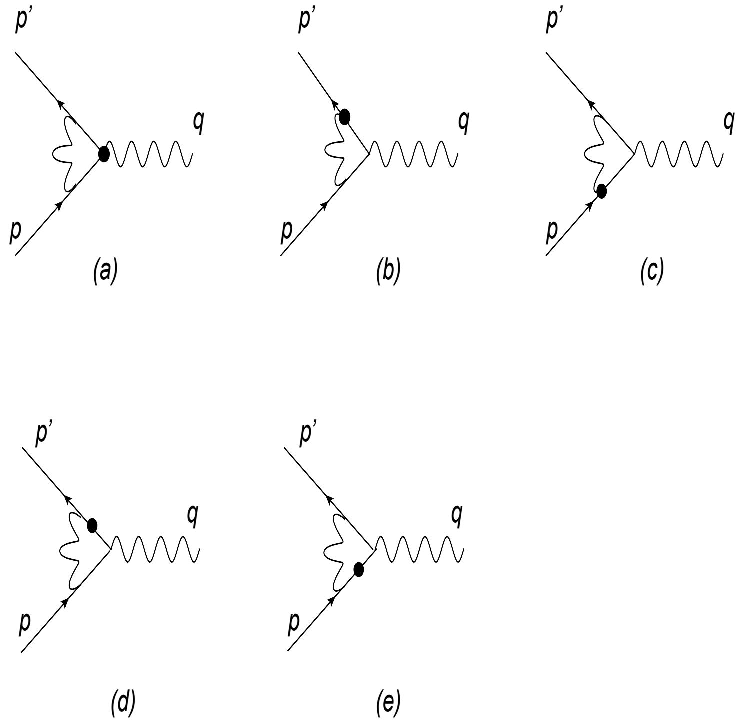

Figure 1: The one loop diagrams for lepton-photon vertex in the extended QED up to the first order of the lorentz violation parameter. The solid circle on each diagram shows the first order LV-contribution from the extended QED. a-c represent the LV-correction on the vertex while d and e show the corrected propagators.

As is already mentioned, the most important term for the EDM, in the high energy limit, is F_4. In fact, this form factor at the zero recoil depends on the scalar $p \cdot d \cdot p$ which enhances the value of the EDM in the higher energies. Therefore, to evaluating the vertex function, we only retain those terms which are proportional to $p \cdot d \cdot p$. To simplify (21), we introduce two identities as follows

$$\begin{aligned} \Gamma^{\alpha} d_{\mu\nu} \gamma^{\nu} \gamma_5 \Gamma_{\alpha} &= \gamma^{\alpha} d_{\mu\nu} \gamma^{\nu} \gamma_5 \gamma_{\alpha} \\ &= 2 d_{\mu\nu} \gamma^{\nu} \gamma_5, \end{aligned} \tag{22}$$

and

$$\begin{aligned} \Gamma^{\alpha} \Gamma_{\mu} \Gamma_{\alpha} &= \Gamma^{\alpha} \gamma_{\mu} \Gamma_{\alpha} - \Gamma^{\alpha} d_{\mu\nu} \gamma^{\nu} \gamma_5 \Gamma_{\alpha} \\ &= 2\gamma_{\alpha}(d_{\mu\alpha} + d_{\alpha\mu}) - 2 d_{\mu\nu} \gamma^{\nu} \gamma_5 - 2\gamma_{\mu}(1 + d^{\alpha}_{\alpha}). \end{aligned} \tag{23}$$

Then, the expression for F_4, after manipulating some algebra that is given in appendix B, can be obtained at the zero recoil and up to the first order of d, as in

$$F_4 = \frac{275\alpha}{18\pi} \left\{ \frac{p.d^S.p}{m^2} \right\}, \tag{24}$$

in which d^S is the symmetric part of the LV parameter $d_{\mu\nu}$. Consequently, one finds

$$d_e = 7 \times 10^{-13} \frac{p \cdot d^S \cdot p}{m_e^2} \; e\; cm\,, \tag{25}$$

for the electron's EDM and

$$d_\mu = 3 \times 10^{-15} \frac{p \cdot d^S \cdot p}{m_\mu^2} \; e\; cm\,, \tag{26}$$

for the muon's EDM. One should note that, at the low energy limit where the EDM of the electron as a stable particle is measured, the correction given in (25) in comparison with (11) is irrelevant. In contrast, the heavy charged leptons due to their short lifetimes should be measured in apparatus like the storage ring, as is suggested in [38] for the charged leptons and in [39] for the other heavy charged particles. Therefore, for instance, (26) can be used to put an upper bound on the LV-parameter d for the muon. In the storage ring, muons are in the xy plane therefore, besides $p_z = 0$ both p_x and p_y, in average, are equal to zero. Therefore, at the high energy limit (26) leads to

$$d_\mu = 3 \times 10^{-15} \frac{p_0^2(d_{xx} + d_{yy})}{2m_\mu^2} \; e\; cm\,. \tag{27}$$

To compare (27) with different experiments, it is convenient to use the standard Sun-centered inertial reference frame [40]-[41]. Denote a non rotating basis by $(X; Y; Z)$, with Z parallel to the earths axis along the north direction and the X and Y axes lying in the plane of the earths equator. Thus, the quantity $d_{xx} + d_{yy}$ in this frame is

$$\begin{aligned} d_{xx} + d_{yy} &= (1 - \sin^2\chi\cos^2\Omega t)d_{XX} - \frac{1}{2}\sin^2\chi\sin 2\Omega t(d_{XY} + d_{YX}) \\ &- \frac{1}{2}\sin 2\chi\cos\Omega t(d_{XZ} + d_{ZX}) - \frac{1}{2}\sin 2\chi\sin\Omega t(d_{YZ} + d_{ZY}) \\ &+ (1 - \sin^2\chi\sin^2\Omega t)d_{YY} + \sin^2\chi d_{ZZ}, \end{aligned} \tag{28}$$

where χ is the geographic colatitude of the experiment location. As (28) shows the μEDM is a time dependent quantity. Meanwhile, the time average of (28) leads to

$$d_{xx} + d_{yy} = (d_{XX} + d_{YY}) - \frac{1}{2}\sin^2\chi(d_{XX} + d_{YY} - 2d_{ZZ}), \tag{29}$$

where for measurements made at different χ one has

$$\delta(d_{xx} + d_{yy}) = \frac{1}{2}(\sin^2\chi_1 - \sin^2\chi_2)(d_{XX} + d_{YY} - 2d_{ZZ}). \tag{30}$$

The experimental bound on the μEDM is about $1.8\times10^{-19}\,e\,cm$ [42] where $\chi=49.1$ for the E821 experiment and the muon energy is of the order of $3\,GeV$. Therefore, (27) and (29) results in

$$d_\mu \;=\; 1.2\times10^{-12}[0.71(d_{XX}+d_{YY})+0.57d_{ZZ}]\;\; e\;cm\,, \tag{31}$$

or

$$[0.71(d_{XX}+d_{YY})+0.57d_{ZZ}] < 1.5\times10^{-7}, \tag{32}$$

which is the first bound on the combination of d_{ii} components of the Lorentz violation parameter d for muon. One should note that to see the enhancement on the eEDM at the high energy limit one needs to examine an indirect experiment such as $e^-e^+\to l^-l^+$ at the LEP. As was shown in [43], the EDM of leptons about $10^{-17}\,e\,cm\sim10^{-3}\,GeV^{-1}$ may have some measurable contribution on the $e^-e^+\to l^-l^+$ which is comparable to the interference term coming from the one Z-boson exchange channel. In fact, besides the ordinary one photon exchange diagram, there are diagrams at the lowest order in which one of the vertices is replaced by the electric-dipole one. Therefore, for the non vanishing interference term, there is an extra power of the momentum in the amplitude and the fractional correction with respect to the ordinary QED is of the order of d_lE where d_l is the lEDM. This correction is about 20 percent for $d_l\sim10^{-3}\,GeV^{-1}$ and $E\sim200\,GeV$. Unfortunately, the interference term is zero and the fractional correction is of the order of $(d_lE)^2\sim .02$. Consequently, the LV-parameter $d=8.9\times10^{-17}$ for the electron leads to a few percent fractional correction to the $e^-e^+\to l^-l^+$. Meanwhile, since $m_\mu\sim200m_e$ then to have the same order of magnitude correction, through the μEDM, the LV-parameter d for the muon should be 9×10^{-10}.

4 Conclusion

We examined the electric dipole moment of the charged fermions in the QED part of the SME. Besides the ordinary form factors there are a lot of new form factors in the SME framework, see (4). In addition to the q^2, the ordinary form factors, up to the first order of the LV parameter, depend on new Lorentz scalars such as $p.d.p$, see (24). Meanwhile, the new form factors, to the leading order of the LV-parameter d, depend only on the q^2, see (11). Therefore, the ordinary form factors in contrast with the QED counterpart, at the zero momentum transfer, depend on the energy of the particles, see (24). The energy dependence of the form factors lead to an enhancement of the electric dipole moment of leptons at high energy limit, see (25). In fact, at the high energy limit, but low enough to satisfy $\frac{|d|p^2}{m_e^2} \leq 1$, the eEDM can be as large as $\sim 10^{-14}\ e\,cm$, see (25). Consequently, the LEP data can be used to put bounds on d, via the enhanced EDM, of the order of 9×10^{-17} and 9×10^{-10} for the electron and muon, respectively. Using the storage ring data for the muon, a bound on $[0.71(d_{XX} + d_{YY}) + 0.57 d_{ZZ}] \sim 1.5 \times 10^{-7}$ has been obtained for the mu-lepton. In fact, this is the first bound on the components $|d_{ij}|$ of the muon [28].

5 Appendix A

Here we introduce some useful identities. The Dirac equation in the SME is

$$(\not{p} - m + d_{\mu\nu} p^\nu \gamma_5 \gamma^\mu) u(p) \;=\; 0, \tag{33}$$

and

$$\bar{u}(p)(\not{p} - m + d_{\mu\nu} p^\nu \gamma_5 \gamma^\mu) \;=\; 0. \tag{34}$$

These equations can be easily casted into

$$\bar{u}(p')(\not{q})u(p) \;=\; -\bar{u}(p')(\gamma_5 \gamma.d.q)u(p), \tag{35}$$

and

$$\bar{u}(p')(\not{q}\gamma_5)u(p) \;=\; \bar{u}(p')(2m\gamma_5 + \gamma.d.q)u(p). \tag{36}$$

Also one has

$$p^2 u(p) \;=\; (m^2 - 2m d_{\mu\nu} p^\mu \gamma_5 \gamma^\nu + 2p.d.p\gamma_5)u(p), \tag{37}$$

and

$$\bar{u}(p)p^2 \;=\; \bar{u}(p)(m^2 - 2m d_{\mu\nu} p^\mu \gamma_5 \gamma^\nu - 2p.d.p\gamma_5). \tag{38}$$

The Gordon identity for a Dirac particle in a LV-background $d_{\mu\nu}$ can be obtained as

$$\bar{u}\gamma_\mu u = \bar{u}\frac{(p+p')_\mu}{2m}u + \bar{u}\frac{i\sigma_{\mu\nu}q^\nu}{2m}u + \bar{u}i\frac{\sigma_{\mu\alpha}d^{\alpha\nu}(p+p')_\nu}{2m}\gamma_5 u + \bar{u}\frac{d_{\mu\nu}q^\nu}{2m}\gamma_5 u, \tag{39}$$

and

$$\bar{u}\gamma_\mu\gamma_5 u = \bar{u}\frac{q_\mu\gamma_5}{2m}u + \bar{u}\frac{i\sigma_{\mu\nu}(p+p')^\nu\gamma_5}{2m}u + \bar{u}i\frac{\sigma_{\mu\alpha}d^{\alpha\nu}q_\nu}{2m}u + \bar{u}\frac{d_{\mu\nu}(p+p')^\nu}{2m}u. \tag{40}$$

Some other useful identities are

$$\bar{u}[\sigma^{\mu\nu}q_\nu\gamma_5]u = i\bar{u}(p+p')^\mu\gamma_5 u + i\bar{u}d^{\mu\nu}q_\nu u - \bar{u}\sigma^{\mu\alpha}d_{\alpha\nu}(p+p')^\nu u, \tag{41}$$

and

$$\bar{u}[\sigma^{\mu\nu}(p+p')_\nu]u = i\bar{u}q^\mu u + i\bar{u}d^{\mu\nu}(p+p')_\nu\gamma_5 u - \bar{u}\gamma_5\sigma^{\mu\alpha}d_{\alpha\nu}q^\nu u. \tag{42}$$

6 Appendix B

In this appendix we give the details of the vertex function calculations. As a crosscheck, we assume both symmetric and antisymmetric parts of $d_{\mu\nu}$ are nonzero, however, at the end we show that the lepton EDM as a physical quantity depends only on the symmetric part of $d_{\mu\nu}$. To this end, the equation (21) can be written as follows

$$\Gamma^{\mu}_{QEDE} = \Gamma_1 + \Gamma_2 + \Gamma_3, \quad (43)$$

where

$$\Gamma_1 = \int \frac{d^4k}{(2\pi)^4}\frac{-ie^2}{(p-k)^2}\bar{u}(p')\{\Gamma^{\alpha}\frac{(\not k' + m)}{k'^2 - m^2}\Gamma^{\mu}\frac{(\not k + m)}{k^2 - m^2}\Gamma_{\alpha}\}u(p), \quad (44)$$

$$\Gamma_2 = \int \frac{d^4k}{(2\pi)^4}\frac{-ie^2}{(p-k)^2}\bar{u}(p')\{\Gamma^{\alpha}\frac{(\not k' + m)}{k'^2 - m^2}\Gamma^{\mu}\frac{(\not k + m)}{k^2 - m^2}\gamma\cdot d\cdot k\gamma_5\frac{(\not k + m)}{k^2 - m^2}\Gamma_{\alpha}\}u(p), \quad (45)$$

and

$$\Gamma_3 = \int \frac{d^4k}{(2\pi)^4}\frac{-ie^2}{(p-k)^2}\bar{u}(p')\{\Gamma^{\alpha}\frac{(\not k' + m)}{k'^2 - m^2}\gamma\cdot d\cdot k'\gamma_5\frac{(\not k' + m)}{k'^2 - m^2}\Gamma^{\mu}\frac{(\not k + m)}{k^2 - m^2}\Gamma_{\alpha}\}u(p). \quad (46)$$

Now, we use the identities (22) and (23) to simplify Γ_i's as follows

$$\begin{aligned}
\Gamma_1 &= \int \frac{d^4k}{(2\pi)^4}\frac{2ie^2}{(p-k)^2(k'^2-m^2)(k^2-m^2)} \\
& \bar{u}(p')\{\not k\gamma_\mu \not k' - d^{\alpha}_{\alpha}\not k'\gamma_\mu\not k\gamma_5 - d_{\mu\alpha}\not k\gamma_\mu\not k'\gamma_5 - d^s_{\alpha\beta}k^{\beta}\not k'\gamma_\mu\gamma^{\alpha}\gamma_5 \\
&+ d^s_{\alpha\mu}\not k'\not k\gamma^{\alpha}\gamma_5 - d^s_{\alpha\beta}k'^{\beta}\gamma_\mu\not k\gamma^{\alpha}\gamma_5 - 2m(k'+k)_\mu - 2md_{\mu\beta}q^{\beta}\gamma_5 \\
&- 2mi\sigma_{\alpha\beta}d^{\alpha\beta}q_\mu\gamma_5 - m\not q d^A_{\mu\alpha}\gamma^{\alpha}\gamma_5 + mq^{\beta}d^A_{\beta\alpha}\gamma_\mu\gamma^{\alpha}\gamma_5 - m^2 d^{\alpha}_{\alpha}\gamma_\mu\gamma_5 \\
&+ m^2(\gamma_\mu + d_{\mu\alpha}\gamma^{\alpha}\gamma_5) + m^2 d^s_{\alpha\mu}\gamma^{\alpha}\gamma_5\}u(p), \quad (47)
\end{aligned}$$

$$\begin{aligned}
\Gamma_2 &= \int \frac{d^4k}{(2\pi)^4}\frac{-2ie^2}{(p-k)^2(k'^2-m^2)(k^2-m^2)^2} \\
& \bar{u}(p')\{\gamma\cdot d\cdot k\gamma^{\mu}\not k'(k^2+m^2) - 2\not k\gamma^{\mu}\not k' k\cdot d^s\cdot k - 2mk^2 d_{\mu\alpha}k^{\alpha} \\
&- 2m(\gamma\cdot d\cdot k\not k'\gamma^{\mu}\not k + \not k\gamma^{\mu}\not k'\gamma\cdot d\cdot k) + 4m(k'+k)_\mu k\cdot d^s\cdot k \\
&+ 2m^2(\gamma\cdot d\cdot k\not k\gamma^{\mu} - \gamma_\mu k\cdot d^s\cdot k) - 2m^3 d_{\mu\alpha}k^{\alpha}\}\gamma_5 u(p), \quad (48)
\end{aligned}$$

and

$$\begin{aligned}
\Gamma_3 &= \int \frac{d^4k}{(2\pi)^4}\frac{-2ie^2}{(p-k)^2(k'^2-m^2)^2(k^2-m^2)} \\
& \bar{u}(p')\{\not k\gamma^{\mu}\gamma\cdot d\cdot k'(k'^2+m^2) - 2\not k\gamma^{\mu}\not k' k'\cdot d^s\cdot k' \\
&+ 2m(k'^2+m^2)d_{\mu\alpha}k'^{\alpha} + 2m(\not k\gamma\cdot d\cdot k'\not k'\gamma^{\mu} + \gamma^{\mu}\not k'\gamma\cdot d\cdot k'\not k) \\
&- 4mk'\cdot d^s\cdot k' q_\mu - 8mk'\cdot d^s\cdot k' k_\mu - 2m^2 k'\cdot d^s\cdot k'\gamma^{\mu} \\
&+ 2m^2\gamma^{\mu}\not k'\gamma\cdot d\cdot k'\}\gamma_5 u(p), \quad (49)
\end{aligned}$$

Now, we evaluate the integrals using the standard procedures. We use the method of Feynman parameters to rewrite the denominators as follows

$$\frac{1}{(p-k)^2(k'^2-m^2)(k^2-m^2)^2} = \int dxdydz\delta(x+y+z-1)\frac{6x}{D^4}, \tag{50}$$

and

$$\frac{1}{(p-k)^2(k'^2-m^2)^2(k^2-m^2)} = \int dxdydz\delta(x+y+z-1)\frac{6y}{D^4}, \tag{51}$$

where $D = l^2 - \Delta + i\epsilon$ and

$$\Delta = (1-z)^2m^2 - xyq^2 \quad , \quad l = k - zp + yq. \tag{52}$$

Here we are interested in the momentum dependent part of the F_4 form factor. Meanwhile, Eq.(41) shows that the F_4 comes as the coefficient of $(p+p')^\mu\gamma_5$. Therefore, we only retain those momentum dependent terms, in Γ_1 to Γ_3, which are proportional to $(p+p')^\mu\gamma_5$. One can see that only Γ_2 and Γ_3 have such terms which after performing the integrals on the momenta they can be obtained as follows

$$\begin{aligned}
\Gamma_{2\,p\cdot d\cdot p} &= \frac{2e^2}{(4\pi^2)}\int dxdydz\delta(x+y+z-1)\frac{x}{\Delta^2}\bar{u}(p')\{-2[-my(1-y) \\
&+ m(z+y)(z-2y+2)][y^2q\cdot d^s\cdot q - 2zyq\cdot d^s\cdot p + z^2p\cdot d^s\cdot p] \\
&- 2m[2y^2zp'\cdot d^s\cdot p' + 2(z+y)^2zp\cdot d^s\cdot p - 4yz(z+y)p'\cdot d^s\cdot p] \\
&+ 4m[y^2q\cdot d^s\cdot q - 2yzq\cdot d^s\cdot p + z^2p\cdot d^s\cdot p]z\}(p+p')^\mu\gamma_5u(p),
\end{aligned} \tag{53}$$

where at $q^2=0$ is

$$\begin{aligned}
\Gamma_{2\,p\cdot d\cdot p} &= \frac{2e^2}{(4\pi^2)}\int dxdydz\delta(x+y+z-1)\frac{x}{\Delta^2(q^2=0)} \\
&\quad \bar{u}(p')\{-2m[-y(1-y) + (z+y)(z-2y+2)]z^2 \\
&- 2m[2y^2z + 2(z+y)^2z - 4yz(z+y)] \\
&+ 4mz^3\}p\cdot d^s\cdot p(p+p')^\mu\gamma_5u(p),
\end{aligned} \tag{54}$$

and

$$\begin{aligned}
\Gamma_{3\,p\cdot d\cdot p} &= \frac{2e^2}{(4\pi^2)}\int dxdydz\delta(x+y+z-1)\frac{y}{\Delta^2}\bar{u}(p')\{ \\
&- 2m[y(z-y+1) + z(z-2y+2)][z^2p\cdot d^s\cdot p + (1-y)^2q\cdot d^s\cdot q \\
&+ 2z(1-y)q\cdot d^s\cdot q] + 2m[2(z+y)(1-y)^2p'\cdot d^s\cdot p' \\
&+ 2(z+y-1)^2(1-y)p\cdot d^s\cdot p + 4z(1-y)(z+y-1)p'\cdot d^s\cdot p] \\
&- 4mz((1-y)q+zp)\cdot d^s\cdot((1-y)q+zp)\}(p+p')^\mu\gamma_5u(p),
\end{aligned} \tag{55}$$

where at $q^2 = 0$ one has

$$\begin{aligned}\Gamma_{3\,p\cdot d\cdot p} &= \frac{2e^2}{(4\pi^2)}\int dxdydz\delta(x+y+z-1)\frac{y}{\Delta^2(q^2=0)}\bar{u}(p')\{-2m[y(z-y+1)\\ &+ z(z-2y+2)]z^2 + 2m[2(z+y)(1-y)^2 + 2(z+y-1)^2(1-y)\\ &+ 4z(1-y)(z+y-1)] - 4mz^3\}p\cdot d^s\cdot p(p+p')^\mu\gamma_5 u(p), \qquad (56)\end{aligned}$$

where the subscript $p \cdot d \cdot p$ stands for the momentum dependent parts of the form factors. It should be noted that in our manipulations we retained both the symmetric and the antisymmetric parts of $d_{\mu\nu}$. However, as is expected the results only depend on the symmetric part of $d_{\mu\nu}$. Now the total contribution on the EDM form factor can be found by adding (54) and (56) as

$$\begin{aligned}\Gamma_{2\,p\cdot d\cdot p} + \Gamma_{3\,p\cdot d\cdot p} &= \frac{2e^2}{(4\pi^2)}\int dxdydz\delta(x+y+z-1)\frac{2m}{((1-z)^2m^2)^2}\bar{u}(p')\{\\ &+ (x-y)z^2y(1-y) - z^3y^2 - z^3(x+y)(z-2y+2)\\ &- z^2xy(z-2y+2) + 2y[3z(1-y)(z+y-1) + (1-y)^2]\\ &- 2xz^3\}p\cdot d^s\cdot p(p+p')^\mu\gamma_5 u(p), \qquad (57)\end{aligned}$$

which after performing the integrals on the Feynman parameters, leads to

$$\Gamma_{2\,p\cdot d\cdot p} + \Gamma_{3\,p\cdot d\cdot p} = -\frac{2e^2}{(4\pi^2)}\bar{u}(p')(\frac{275}{18m^3})p\cdot d^s\cdot p(p+p')^\mu\gamma_5 u(p) + IR, \qquad (58)$$

in which IR stands for the infrared terms. By comparing (58) and (41), one can easily see that

$$F_4 = -\frac{275\alpha}{18\pi}\frac{p.d^s.p}{m^2}. \qquad (59)$$

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