A Possible Reason for $M_H \simeq 126 \text{ GeV}$

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It is speculated that a possible reason for the scalar mass $M_H \simeq 126$ GeV is equality of the lifetimes for vacuum decay and instanton-induced proton decay.

INTRODUCTION

In the standard model (SM) of particle theory there exists what has been called "the scandal of the fermion masses" meaning that none of the twelve quark and lepton masses have been satisfactorily explained by theory. On the other hand, the masses of the spin-one bosons, the photon, gluon, W^{\pm} and Z are well understood in terms of symmetry breaking. Now, there is one additional bo-

son, with spin-zero, and the issue is whether its mass can be explained.

Surely the most dramatic discovery [1, 2] in experimental high-energy physics so far this century is that of the Brout-Englert-Higgs [3-7] boson (H) which completes the discovery of all the particles in the minimal standard model. Equally dramatic is the non-discovery of any other particle in the mass range which has so far been investigated. Given what we now know, it is very reasonable to assume there are no other particles at low energies, and that all that exists is the $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model with three quark-lepton families and only one scalar whose mass is measured as $M_H \simeq 126$ GeV.

At large scales, classical gravity is well described by general relativity and we assume that this is the theory which governs the behavior of the universe as a whole. Although large black holes are known to exist, we will ignore the effects of possible virtual black holes in microscopic processes because there is no fully satisfactory theory of quantum gravity at small scales.

In the present note, we shall offer a speculation which provides a possible reason for $M_H \simeq 126$ GeV, as well as saying something about the fate of dark energy in the extreme future of the Universe.

In gauge field theory, an important formal development was the discovery of the instanton solution [8]. For example, this was once proposed [9] as the key to understanding the QCD of strong interactions. In this note, the instantons of electroweak interactions will play a central role.

LIFETIME τ_{EW} OF METASTABLE VACUUM

The decay of the metastable electroweak vacuum into the true ground state was first studied as a function of M_H in [10–17], including only boson loops. Another kind of instability was first emphasized in [18–21] where the presence of heavy fermion loops could make the effective potential unbounded from below. The conditions for vacuum stability were further improved by summing over leading logs in the effective potential [22–26]. It was also emphasized in various subsequent works such as e.g. [27] that the so-called unbounded potential is only an artifact since near the energy scale where instability is supposed to occur the top quark Yukawa coupling becomes small [24] and/or new physics enters (see e.g. [28]) and the potential acquires a true vacuum at a value of the field much larger than the electroweak scale.

The discovery of the H scalar with mass $M_H \simeq 126$ GeV has precipitated a careful reevaluation of the electroweak vacuum stability and the result is especially interesting, that the vacuum is not perfectly stable. The result depends mainly on the mass of the top quark M_t and on M_H . This metastability of the vacuum depends sensitively on the values of these parameters. With the observed values $M_t = 173.36 \pm 0.65 \pm 0.3$ GeV and $M_H = 125.66 \pm 0.34$ GeV, the vacuum is metastable with an extremely long lifetime discussed below. If we keep one of the two fixed and vary the other, the sensitivity from [29] is seen to be that the vacuum would become absolutely stable if M_t were reduced to 171 GeV, or if M_H were increased to 130 GeV.

This proximity to criticality is sufficiently striking that it is likely informing us profoundly about theoretical physics. It can be speculated that there is a statistical basis provided by a multiverse interpretation but such an idea seems untestable. Here we aim to exploit only facts which are on a firm basis.

In [29] two different cosmological futures are considered: the Λ CDM model and a CDM model where the dark energy disappears. For these cases the lifetime of the electroweak vacuum for the observed values of M_H

and M_t is respectively

$$10^{400} < \tau_{EW} < 10^{700}$$
 years (1)

for the Λ CDM model, and

$$10^{100} < \tau_{EW} < 10^{400}$$
 years (2)

for the CDM model.

LIFETIME τ_p OF PROTON

According to the results of the Super-Kamiokande experiment [30], the lower bound on some proton decay partial lifetimes are $\tau(p \to e^+\pi^0) \geq 8.2 \times 10^{33}$ years and $\tau(p \to \mu^+\pi^0) \geq 6.6 \times 10^{33}$ years. In grand unified theories, e.g. [31], the lifetime was initially predicted as $\tau(p \to e^+\pi^0) \sim 10^{31}$ years which disagrees with experiment. We shall assume there is no grand unification which induces proton decay. As stated in the Introduction, we shall neglect also the effects of virtual black holes which could [32] otherwise give $\tau_p \sim 10^{50}$ years.

Perturbatively, the standard model conserves baryon number (B) and lepton number (L) but, as first noted by 't Hooft [33, 34], non-perturbative instantons induce violations of B and L while preserving (B-L). This leads to instanton-induced proton decay. The rate is suppressed by a factor

$$exp(-4\pi/\alpha_2) = exp(-4\pi\sin^2\Theta_W/\alpha_{em})$$
 (3)

The lifetime for proton decay τ_p is proportional to the inverse of this rate and so

$$\tau_p \propto exp(+4\pi \sin^2 \Theta_W/\alpha_{em})$$
$$\simeq exp(+371) \simeq 10^{+160} \tag{4}$$

The determinant of fermion zero modes in [34] will affect only the prefactor so that to a zeroth approximation

$$\tau_p \sim 10^{160}$$
 years (5)

which is such a long lifetime that the difference from absolute stability may prima facie seem only academic. Nevertheless, we shall argue that the finite lifetime in Eq.(5) is very important. A word of caution is in order here. Note that even if the prefactor were calculated accurately for the amplitude of the proton decay, it would not so drastically change the estimate Eq.(5) to bring it outside the range permitted by Eq. (2). It is thus very reasonable to expect that the instanton-induced proton decay lifetime lies comfortably within the CDM range. As a result, conservatively, we expect

$$10^{100} < \tau_p > 10^{400} \text{years}$$
 (6)

(By enlarging the SM gauge group, it might be possible to shorten considerably the instanton-induced proton lifetime [35]. However, we limit ourselves strictly to the minimal SM as we have stressed in the second paragraph of this paper.)

A POSSIBLE REASON

Both τ_{EW} and τ_p are related to electroweak instantons, and both represent major decay processes in the extreme future of the Universe. Based on the above observations, we make the following conjecture

$$\tau_{EW}(M_H) = \tau_p \,, \tag{7}$$

where τ_{EW} has a sensitive dependence on the Higgs mass M_H . Since τ_p is essentially fixed by the $SU(2)_L$ gauge coupling by Eq.(4), the equality Eq.(7) amounts to a determination of M_H in terms of the proton lifetime. Comparison of τ_{EW} (CDM) with Eq.(5) then shows that the value $M_H \simeq 126$ GeV is required for this equality. If M_H were increased to e.g. 130 GeV, τ_{EW} would become infinite. If M_H were reduced to e.g. 122 GeV, one would find $\tau_{EW} \ll \tau_p$. The fact that Eq.(7) requires τ_{EW} (CDM) rather that τ_{EW} (Λ CDM) from [29] suggests that the dark energy will disappear before cosmic time $t=10^{160}$ years, although a more careful analysis of Eq. (7) could sharpen this prediction about the extreme future of the Universe.

Needless to say, the deep reason behind the conjecture (7) is beyond our grasp at the present time and it would be very interesting if one could be found in the future.

As alluded to above, the true minimum is located at a value of the Higgs field orders of magnitude larger than for our metastable vacuum which is located at the electroweak scale. In the stable vacuum, quarks and leptons (and W^{\pm} , Z) will be supermassive and bound states very small compared to in the known universe, leading inevitably to a preponderance of black holes. On the other hand, if the protons have decayed long before the phase transition to the true ground state, as in the Λ CDM model, there will be no remaining quarks, only leptons e^{\pm} , ν and photons γ and, assuming the dark matter does not change, far less production of black holes. Such might not be the case if the proton decays when the phase transition to the true ground state occurs. This type of discussion may underwrite a rationale for the conjectured equality in Eq.(7).

The true ground state is stabilized by an incompletely understood mechanism and, as we have argued for the CDM case in a far distant future, it has vanishing vacuum energy since the dark energy has disappeared. Once gravity is included, and therefore absolute potential energy acquires significance, the decay is from the present vacuum which is dominated by dark energy with a relatively very tiny energy density to one with vanishing energy density, and it is tempting then to speculate even further that the mechanism that stabilizes the true vacuum also sets the energy gap between the two vacua equal to the dark energy density.

DISCUSSION

Our Eq.(7) equates the two longest time scales associated with the standard model. It has the advantage of using only facts which are known. We could hold M_H fixed, and vary M_t , but M_H is the more fundamental because the scalar is an exceptional particle in the standard model, being the only one with zero spin. If our present discussion is correct, it implies that the boson masses, rather than the fermion masses, are the more tractable.

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