

## Mixing angle of $K_1$ axial vector mesons

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Analyses of various experimental measurements all indicate that the mixing angle  $\theta_{K_1}$  of  $K_1(1270)$  and  $K_1(1400)$  is in the vicinity of  $33^\circ$  or  $57^\circ$ . However, whether  $\theta_{K_1}$  is greater or less than  $45^\circ$  is still quite controversial. For example, there were two very recent studies of the strong decays of  $K_1$  mesons. One group claimed that  $\theta_{K_1} \approx 60^\circ$ , while the other group obtained  $\theta_{K_1} = (33.6 \pm 4.3)^\circ$ . Since the determination of the mixing angles  $\alpha_{3P_1}$  and  $\alpha_{1P_1}$  with the former (latter) being the mixing angle of  $f_1(1285)$  ( $h_1(1170)$ ) and  $f_1(1420)$  ( $h_1(1380)$ ) in the flavor basis through mass relations depends on  $\theta_{K_1}$ , we show that  $\theta_{K_1} \approx 57^\circ$  is ruled out as it leads to a too large deviation from ideal mixing in the  $^1P_1$  sector, inconsistent with the lattice calculation of  $\alpha_{1P_1}$  and the observation of strong decays of  $h_1(1170)$  and  $h_1(1380)$ . We find that for  $\theta_{K_1} \approx (28 - 30)^\circ$ , the corresponding  $\alpha_{3P_1}$  and  $\alpha_{1P_1}$  agree well with all lattice and phenomenological analyses. This again reinforces the statement that  $\theta_{K_1} \sim 33^\circ$  is much more favored than  $57^\circ$ .

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## 1. Introduction

The mixing of the flavor-SU(3) singlet and octet states of vector and tensor mesons to form mass eigenstates is of fundamental importance in hadronic physics. According to the Appelquist-Carazzone decoupling theorem, in a vectorial theory, as the mass of a particle gets large compared with a relevant scale, say,  $\Lambda_{QCD} \simeq 300$  MeV, one can integrate this particle out and define a low-energy effective field theory applicable below this scale [1]. Evidently, even though  $m_s$  is not  $\gg \Lambda_{QCD}$ , there is still a nearly complete decoupling for the case of vector mesons, namely,  $\rho(770)$  and  $\omega(892)$  states. A similar situation of near-ideal mixing occurs for the  $J^{PC} = 2^{++}$  tensor mesons  $f_2(1275)$ ,  $f_2'(1525)$  and the  $J^{PC} = 3^{--}$  mesons  $\omega_3(1670)$ ,  $\phi_3(1850)$  and this can also be understood in terms of approximate decoupling of the light  $u\bar{u} + d\bar{d}$  state from the heavier  $s\bar{s}$  state.

In the quark model, two nonets of  $J^P = 1^+$  axial-vector mesons are expected as the orbital excitation of the  $q\bar{q}$  system. In terms of the spectroscopic notation  $^{2S+1}L_J$ , there are two types of  $P$ -wave axial-vector mesons, namely,  $^3P_1$  and  $^1P_1$ . These two nonets have distinctive  $C$  quantum numbers for the corresponding neutral mesons,  $C = +$  and  $C = -$ , respectively. Experimentally, the  $J^{PC} = 1^{++}$  nonet consists of  $a_1(1260)$ ,  $f_1(1285)$ ,  $f_1(1420)$  and  $K_{1A}$ , while the  $1^{+-}$  nonet contains  $b_1(1235)$ ,  $h_1(1170)$ ,  $h_1(1380)$  and  $K_{1B}$ . The non-strange axial vector mesons, for example, the neutral  $a_1(1260)$  and  $b_1(1235)$  cannot have a mixing because of the opposite  $C$ -parities. On the contrary,  $K_{1A}$  and  $K_{1B}$  are not the physical mass eigenstates  $K_1(1270)$  and  $K_1(1400)$  and they are mixed together due to the mass difference of strange and light quarks. Following the common convention we write

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \sin \theta_{K_1} & \cos \theta_{K_1} \\ \cos \theta_{K_1} & -\sin \theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}. \quad (1.1)$$

Various phenomenological studies indicate that the  $K_{1A}$ - $K_{1B}$  mixing angle  $\theta_{K_1}$  is around either  $33^\circ$  or  $57^\circ$ ,<sup>1</sup> but there is no consensus as to whether this angle is greater or less than  $45^\circ$ .

We have shown in [2] that the mixing angle  $\theta_{K_1}$  can be pinned down based on the observation that when the  $f_1(1285)$ - $f_1(1420)$  mixing angle  $\theta_{P_1}$  and the  $h_1(1170)$ - $h_1(1380)$  mixing angle  $\theta_{P_1}$  are determined from the mass relations, they depend on the masses of  $K_{1A}$  and  $K_{1B}$ , which in turn depend on  $\theta_{K_1}$ . Since nearly ideal mixing occurs for vector, tensor and  $3^{--}$  mesons except for pseudoscalar mesons where the axial anomaly plays a unique role, this feature is naively expected to hold also for axial-vector mesons. Lattice calculations of  $\theta_{P_1}$  and the phenomenological analysis of the strong decays of  $h_1(1170)$  and  $h_1(1380)$  will enable us to discriminate the two different solutions for  $\theta_{K_1}$ . In this talk we will elaborate on this in more detail.

## 2. Mixing of axial-vector mesons

There exist several estimations on the mixing angle  $\theta_{K_1}$  in the literature. From the early experimental information on masses and the partial rates of  $K_1(1270)$  and  $K_1(1400)$ , Suzuki found

<sup>1</sup>As discussed in [2] and many early publications, the sign ambiguity of  $\theta_{K_1}$  can be removed by fixing the relative sign of the decay constants of  $K_{1A}$  and  $K_{1B}$ . We shall choose the convention of decay constants in such a way that  $\theta_{K_1}$  is always positive.

two possible solutions  $\theta_{K_1} \approx 33^\circ$  and  $57^\circ$  [3]. A similar constraint  $35^\circ \lesssim \theta_{K_1} \lesssim 55^\circ$  was obtained in Ref. [4] based solely on two parameters: the mass difference between the  $a_1(1260)$  and  $b_1(1235)$  mesons and the ratio of the constituent quark masses. An analysis of  $\tau \rightarrow K_1(1270)v_\tau$  and  $K_1(1400)v_\tau$  decays also yielded the mixing angle to be  $\approx 37^\circ$  or  $58^\circ$  [5].<sup>2</sup> Another determination of  $\theta_{K_1}$  comes from the  $f_1(1285)$ - $f_1(1420)$  mixing angle  $\theta_{3P_1}$  to be introduced shortly below which can be reliably estimated from the analysis of the radiative decays  $f_1(1285) \rightarrow \phi\gamma, \rho^0\gamma$  [6]. A recent updated analysis yields  $\theta_{3P_1} = (19.4^{+4.5}_{-4.6})^\circ$  or  $(51.1^{+4.5}_{-4.6})^\circ$  [7].<sup>3</sup> As we shall see below, the mixing angle  $\theta_{3P_1}$  is correlated to  $\theta_{K_1}$ . The corresponding  $\theta_{K_1}$  is found to be  $(31.7^{+2.8}_{-2.5})^\circ$  or  $(56.3^{+3.9}_{-4.1})^\circ$ . Therefore, all the analyses yield a mixing angle  $\theta_{K_1}$  in the vicinity of either  $33^\circ$  or  $57^\circ$ .

However, there is no consensus as to whether  $\theta_{K_1}$  is greater or less than  $45^\circ$ . It was found in the non-relativistic quark model that  $m_{K_{1A}}^2 < m_{K_{1B}}^2$  [10, 11, 12] and hence  $\theta_{K_1}$  is larger than  $45^\circ$ . Interestingly,  $\theta_{K_1}$  turned out to be of order  $34^\circ$  in the relativized quark model of [13]. Based on the covariant light-front model [14], the value of  $51^\circ$  was found by the analysis of [15]. From the study of  $B \rightarrow K_1(1270)\gamma$  and  $\tau \rightarrow K_1(1270)v_\tau$  within the framework of light-cone QCD sum rules, Hatanaka and Yang advocated that  $\theta_{K_1} = (34 \pm 13)^\circ$  [16]. There existed two recent studies of strong decays of  $K_1(1270)$  and  $K_1(1400)$  mesons with different approaches. One group obtained  $\theta_{K_1} \approx 60^\circ$  based on the  $^3P_0$  quark-pair-creation model for  $K_1$  strong decays [17], while the other group found  $\theta_{K_1} = (33.6 \pm 4.3)^\circ$  using a phenomenological flavor symmetric relativistic Lagrangian [18]. In short, there is a variety of different values of the mixing angle cited in the literature. It is the purpose of this work to pin down  $\theta_{K_1}$ .

We next consider the mixing of the isosinglet  $1^3P_1$  states,  $f_1(1285)$  and  $f_1(1420)$ , and the  $1^1P_1$  states,  $h_1(1170)$  and  $h_1(1380)$  in the quark flavor and octet-singlet bases:

$$\begin{pmatrix} |f_1(1285)\rangle \\ |f_1(1420)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{3P_1} & \sin \theta_{3P_1} \\ -\sin \theta_{3P_1} & \cos \theta_{3P_1} \end{pmatrix} \begin{pmatrix} |f_1\rangle \\ |f_8\rangle \end{pmatrix} = \begin{pmatrix} \cos \alpha_{3P_1} & \sin \alpha_{3P_1} \\ -\sin \alpha_{3P_1} & \cos \alpha_{3P_1} \end{pmatrix} \begin{pmatrix} |f_q\rangle \\ |f_s\rangle \end{pmatrix}, \quad (2.1)$$

and

$$\begin{pmatrix} |h_1(1170)\rangle \\ |h_1(1380)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{1P_1} & \sin \theta_{1P_1} \\ -\sin \theta_{1P_1} & \cos \theta_{1P_1} \end{pmatrix} \begin{pmatrix} |h_1\rangle \\ |h_8\rangle \end{pmatrix} = \begin{pmatrix} \cos \alpha_{1P_1} & \sin \alpha_{1P_1} \\ -\sin \alpha_{1P_1} & \cos \alpha_{1P_1} \end{pmatrix} \begin{pmatrix} |h_q\rangle \\ |h_s\rangle \end{pmatrix}, \quad (2.2)$$

where  $f_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ ,  $f_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ ,  $f_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ ,  $f_s = s\bar{s}$  and likewise for  $h_1$ ,  $h_8$ ,  $h_q$  and  $h_s$ . The mixing angle  $\alpha$  in the flavor basis is related to the singlet-octet mixing angle  $\theta$  by the relation  $\alpha = 35.3^\circ - \theta$ . Therefore,  $\alpha$  measures the deviation from ideal mixing. Applying the Gell-Mann Okubo relations for the mass squared of the octet states

$$m_8^2(^3P_1) \equiv m_{3P_1}^2 = \frac{1}{3}(4m_{K_{1A}}^2 - m_{a_1}^2), \quad m_8^2(^1P_1) \equiv m_{1P_1}^2 = \frac{1}{3}(4m_{K_{1B}}^2 - m_{b_1}^2), \quad (2.3)$$

we obtain the following mass relations for the mixing angles  $\theta_{1P_1}$  and  $\theta_{3P_1}$  (for details, see [2])

$$\tan \theta_{3P_1} = \frac{m_{3P_1}^2 - m_{f_1'}^2}{\sqrt{m_{3P_1}^2(m_{f_1}^2 + m_{f_1'}^2 - m_{3P_1}^2) - m_{f_1}^2 m_{f_1'}^2}},$$

<sup>2</sup>Note that the mixing angle results in [5] based on CLEO [8] and OPEL [9] data differ from the the ones obtained in the CLEO paper [8].

<sup>3</sup>From the same radiative decays, it was found  $\theta_{3P_1} = (56^{+4}_{-5})^\circ$  in [6]. This has led some authors (e.g. [10]) to claim that  $\theta_{K_1} \sim 59^\circ$ . However, another solution, namely,  $\theta_{3P_1} = (14.6^{+4}_{-5})^\circ$  corresponding to a smaller  $\theta_{K_1}$ , was missed in [6].

**Table 1:** The values of the  $f_1(1285)$ - $f_1(1420)$  and  $h_1(1170)$ - $h_1(1380)$  mixing angles in the quark flavor (upper) and octet-singlet (lower) bases calculated using Eq. (2.4) for some representative  $K_{1A}$ - $K_{1B}$  mixing angle  $\theta_{K_1}$ .

$\theta_{K_1}$	$57^\circ$	$51^\circ$	$45^\circ$	$33^\circ$	$30^\circ$	$28^\circ$
$\alpha_{3P_1}$	$16.5^\circ$	$9.6^\circ$	$2.4^\circ$	$-13.7^\circ$	$-18.9^\circ$	$-23.5^\circ$
$\alpha_{1P_1}$	$-53.0^\circ$	$-44.6^\circ$	$-21.1^\circ$	$-6.4^\circ$	$-3.8^\circ$	$-2.4^\circ$
$\theta_{3P_1}$	$52^\circ$	$45^\circ$	$38^\circ$	$22^\circ$	$16^\circ$	$12^\circ$
$\theta_{1P_1}$	$-18^\circ$	$-9^\circ$	$14^\circ$	$29^\circ$	$32^\circ$	$33^\circ$

$$\tan \theta_{1P_1} = \frac{m_{1P_1}^2 - m_{h'_1}^2}{\sqrt{m_{1P_1}^2 (m_{h_1}^2 + m_{h'_1}^2 - m_{1P_1}^2) - m_{h_1}^2 m_{h'_1}^2}}, \quad (2.4)$$

where  $f_1$  and  $f'_1$  ( $h_1$  and  $h'_1$ ) are the short-handed notations for  $f_1(1285)$  and  $f_1(1420)$  ( $h_1(1170)$  and  $h_1(1380)$ ), respectively, and

$$\begin{aligned} m_{K_{1A}}^2 &= m_{K_1(1400)}^2 \cos^2 \theta_{K_1} + m_{K_1(1270)}^2 \sin^2 \theta_{K_1}, \\ m_{K_{1B}}^2 &= m_{K_1(1400)}^2 \sin^2 \theta_{K_1} + m_{K_1(1270)}^2 \cos^2 \theta_{K_1}. \end{aligned} \quad (2.5)$$

It is clear that the mixing angles  $\theta_{3P_1}$  and  $\theta_{1P_1}$  depend on the masses of  $K_{1A}$  and  $K_{1B}$  states, which in turn depend on the  $K_{1A}$ - $K_{1B}$  mixing angle  $\theta_{K_1}$ . Table 1 exhibits the values of  $\alpha_{3P_1}$ ,  $\theta_{3P_1}$  and  $\alpha_{1P_1}$ ,  $\theta_{1P_1}$  calculated using Eq. (2.4) for some representative values of  $\theta_{K_1}$ .

### 3. Discussion

We see from Table 1 that the  $K_{1A}$ - $K_{1B}$  mixing angle  $\theta_{K_1} \approx 57^\circ$  corresponds to  $\alpha_{1P_1} = -53^\circ$  which is too far away from ideal mixing for the  $^1P_1$  sector. Indeed, it is in violent disagreement with the lattice result  $\alpha_{1P_1} = \pm(3 \pm 1)^\circ$  obtained by the Hadron Spectrum Collaboration [19]. Since only the modes  $h_1(1170) \rightarrow \rho\pi$  and  $h_1(1380) \rightarrow K\bar{K}^*, \bar{K}K^*$  have been seen so far, this implies that the quark content is primarily  $s\bar{s}$  for  $h_1(1380)$  and  $q\bar{q}$  for  $h_1(1170)$ . Indeed, if  $\theta_{K_1} = 57^\circ$ , we will have  $h_1(1170) = 0.60n\bar{n} - 0.80s\bar{s}$  and  $h_1(1380) = 0.80n\bar{n} + 0.60s\bar{s}$  with  $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ . It is obvious that the large  $s\bar{s}$  content of  $h_1(1170)$  and  $n\bar{n}$  content of  $h_1(1380)$  cannot explain why only the strong decay modes  $h_1(1170) \rightarrow \rho\pi$  and  $h_1(1380) \rightarrow K\bar{K}^*, \bar{K}K^*$  have been seen thus far. Therefore, it is evident that  $\theta_{K_1} \approx 57^\circ$  is ruled out.

Can we conclude that  $\theta_{K_1}$  is less than  $45^\circ$ ? Let's examine the mixing angle  $\alpha_{3P_1}$ . There are some information available. First, the radiative decay  $f_1(1285) \rightarrow \phi\gamma$  and  $\rho\gamma$  yields  $\alpha_{3P_1} = \pm(15.8_{-4.6}^{+4.5})^\circ$  [7]. An updated lattice calculation gives  $\alpha_{3P_1} = \pm(27 \pm 2)^\circ$  [20]. A study of  $B_{d,s} \rightarrow J/\psi f_1(1285)$  decays by LHCb leads to  $\alpha_{3P_1} = \pm(24.0_{-2.6-0.8}^{+3.1+0.6})^\circ$  [21]. Hence,  $\alpha_{3P_1}$  lies in the range  $\pm(15 \sim 27)^\circ$ . Unlike the  $^1P_1$  sector, the deviation of  $f_1(1285)$ - $f_1(1420)$  mixing from the ideal one is sizable. Nevertheless, the quark content is still primarily  $s\bar{s}$  for  $f_1(1420)$  and  $q\bar{q}$  for  $f_1(1285)$ . Indeed,  $K^*\bar{K}$  and  $K\bar{K}\pi$  are the dominant modes of  $f_1(1420)$  whereas  $f_1(1285)$  decays mainly to the  $\eta\pi\pi$  and  $4\pi$  states. It is clear from Table 1 that when  $\theta_{K_1} \approx (28 - 30)^\circ$ , the corresponding

$\alpha_{3P_1}$  and  $\alpha_{1P_1}$  agree well with all lattice and phenomenological analyses. This in turn reinforces the statement that  $\theta_{K_1} \sim 33^\circ$  is much more favored than  $57^\circ$ .

Two remarks are in order: (i) The  $K_1$  mixing angle  $\theta_{K_1} \approx 57^\circ$  leads to acceptable  $\alpha_{3P_1}$  but too large  $\alpha_{1P_1}$ . (ii) In the octet-singlet basis, the mixing angles are of order  $\theta_{3P_1} \sim 15^\circ$  and  $\theta_{1P_1} \sim 32^\circ$ .

#### 4. Conclusions

The  $K_1$  mixing angle  $\theta_{K_1} \approx 57^\circ$  is ruled out as it will lead to a too large deviation from ideal mixing in the  $^1P_1$  sector, inconsistent with the observation of strong decays of  $h_1(1170)$  and  $h_1(1380)$  and a recent lattice calculation of  $\theta_{1P_1}$ . We found when  $\theta_{K_1} \approx (28 - 30)^\circ$ , the corresponding  $\alpha_{3P_1}$  and  $\alpha_{1P_1}$  agree well with all lattice and phenomenological analyses. This again implies that  $\theta_{K_1} \sim 33^\circ$  is much more favored than  $57^\circ$ .

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