Measurement of Neutrino Masses from Relative Velocities

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We present a new technique to measure neutrino masses using their flow field relative to dark matter. Present day streaming motions of neutrinos relative to dark matter and baryons are several hundred km/s, comparable with their thermal velocity dispersion. This results in a unique dipole anisotropic distortion of the matter-neutrino cross power spectrum, which is observable through the dipole distortion in the cross correlation of different galaxy populations. Such a dipole vanishes if not for this relative velocity and so it is a clean signature for neutrino mass. We estimate the size of this effect and find that current and future galaxy surveys may be sensitive to these signature distortions.

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Introduction.—Neutrinos are now established to be massive, and the mass differences have been measured, but the mass hierarchy and absolute mass values remain unknown [1]. Precision large scale structure data can be used to measure or constrain the sum of neutrino masses, as cosmic neutrinos with finite masses slightly suppress the growth of structure on scales below the neutrino thermal free-streaming scale [2–5]. But the challenge of this method is to conclusively disentangle the complex and poorly understood baryonic effects as many processes can lead to power suppression on small scales. In this Letter, we present an astrophysical effect which provides a new way to measure the neutrino masses by using a distinct signature in current or future galaxy surveys.

We consider the relative velocity between cold dark matter (CDM) and neutrinos. Neutrinos decoupled early in the history of the Universe when they were still relativistic, but their energy gradually decreased as the Universe expanded until they behaved as nonrelativistic particles. At this point they can cluster under the action of gravity. Nevertheless, due to their low masses the neutrinos can travel relatively large distances (even at low redshifts), and be perturbed by the underlying gravitational potential along their trajectories. The large scale structures can induce a significant bulk relative velocity field between CDM and neutrinos, with typical velocities comparable to the neutrino thermal velocity dispersion. As we shall show below, such a bulk relative velocity field will cause a local dipole asymmetry in the CDM-neutrino cross-correlation function. The concept of dipole asymmetry in correlation functions was discussed in Ref. [6] recently. The CDM-neutrino cross correlation may be inferred from the cross-correlation of different galaxy populations, and such a dipole asymmetry provides a distinctive and robust signature of neutrino mass, since such dipole anisotropy would be absent if not for this effect.

In this Letter, we delineate the principle of this method,

make an analytical estimate of the size of this effect, and then forecast the detectability of this effect in a simplified galaxy bias model.

The relative velocity.—We treat CDM and neutrinos as two fluids [7] interacting with each other through gravity. The CDM particles and neutrinos are collisionless, nevertheless much of their behavior in gravitational fields can still be modeled with the introduction of an "effective pressure," which takes into account the velocity dispersion or thermal motion of the particles [7]. In the fluid approximation, the effect of the thermal motion is included in this effective pressure and only the bulk motion is considered. The two fluids have different effective pressure, so they acquire different densities and velocities even though they are under the action of the same gravitational field. We use the moving background perturbation theory (MBPT) [8] to calculate analytically the evolution of the density perturbations and velocities of the two fluids, the details of this calculation are given in the Supplemental Material [9]. The basic idea is to assume that within a certain volume of radius R, each fluid has a coherent bulk velocity, which can be expanded around a background velocity as $v_i(x,t) = v_i^{(bg)}(t) + u_i(x,t)$, where i refers to neutrino (
u) or cold dark matter (c). The background velocity $oldsymbol{v}_{i}^{(bg)}$ is a slowly varying velocity long mode. Linear perturbative calculation then can be applied within the region to obtain the cross-correlation of the two fluids.

Starting at a high redshift (we use z=15 in our calculation) when the relative bulk mach number is small, we evolve the MBPT equations down to lower redshifts, and obtain the relative velocity field $\boldsymbol{v}_{\nu c}(\boldsymbol{x},z)$. We estimate the variance of this relative velocity analytically by taking the ensemble average for the given distribution of primordial fluctuations:

$$\langle v_{\nu c}^2(z)\rangle = \int \frac{dk}{k} \Delta_{\zeta}^2(k) \left[\frac{\theta_{\nu}(k,z) - \theta_{c}(k,z)}{k} \right]^2. \tag{1}$$

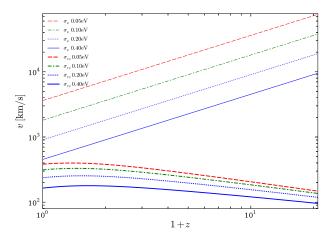


FIG. 1: Redshift evolution of the neutrino velocity dispersion (the thin lines on top) and the neutrino-CDM relative velocity (thick lines at the bottom) for different neutrino masses.

where Δ_{ζ}^2 is the primordial curvature perturbation spectrum, and $\theta \equiv \nabla \cdot \boldsymbol{v}$ is the velocity divergence. We plot the evolution of $\sqrt{\langle v_{\nu c}^2 \rangle}$ (σ_{rv}) and the neutrino thermal velocity dispersion σ_{ν} for four neutrino masses in Fig. 1. The thermal velocity dispersion of the neutrinos decreases as the Universe expands. On the other hand, the bulk relative velocity as represented by $\sqrt{\langle v_{\nu c}^2 \rangle}$ grows to its maximum at 0 < z < 1, then begins to decay. At low redshifts it is comparable with the thermal velocity dispersion.

The relative velocity correlation function $\xi_{v\nu c}(r) \equiv \langle v_{\nu c}(x) v_{\nu c}(x+r) \rangle$ for four redshifts are shown in Fig.2. The bulk velocity correlation functions for different neutrino masses are almost identical at very high redshifts, but become increasingly differentiated at low redshifts, as the correlation functions of the lighter neutrinos have larger amplitudes and longer correlation lengths. The coherent scales R, which is defined as the scale at which the correlation function $\xi_{v\nu c}$ drops to half of its maximum value, are 14.5, 10.3, 7.0, and 4.6 Mpc/h, respectively, for the four neutrino masses at z=0. However, the neutrinos are not visible, so we cannot use this correlation function to measure neutrino mass directly.

Power spectra and correlation functions.—Because of the bulk relative velocity between the CDM and neutrinos, the reflection symmetry along the direction of the flow is broken locally, and within a velocity coherent region the cross-correlation contains a dipole term,

$$\xi_{c\nu}(\mathbf{r}, \mathbf{v}_{\nu c}^{(bg)}) = \xi_{c\nu 0}(r, v_{\nu c}^{(bg)}) + \mu \xi_{c\nu 1}(r, v_{\nu c}^{(bg)}), \quad (2)$$

where $\mu = r \cdot v_{\nu c}^{(bg)}$. This also appears as an imaginary part in the CDM-neutrino cross power spectrum: $P_{c\nu}(k,v_{\nu c}^{(bg)},\mu) = P_{c\nu 0}(k,v_{\nu c}^{(bg)}) + i\mu P_{c\nu 1}(k,v_{\nu c}^{(bg)})$. [We can see this by noting that when taking the Hermite conjugate of $P_{c\nu}$, the imaginary part changes sign and so the angular dependent part is antisymmetric in " $c\nu$," i.e., $\xi_{\nu c}(r,v_{\nu c}^{(bg)}) = \xi_{c\nu 0}(r,v_{\nu c}^{(bg)}) - \mu \xi_{c\nu 1}(r,v_{\nu c}^{(bg)})$.] This imaginary term would otherwise be zero if not for the relative flow between neutrinos and CDM.

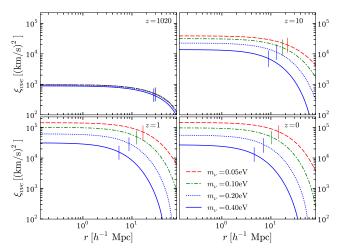


FIG. 2: The relative flow correlation function $\xi_{vvc}(r)$ at different redshifts. The amplitude and scale of the relative flow depends on neutrino mass. The tick marks the correlation length.

This effect is similar to gravitational redshift [10], which breaks the reflection symmetry along the line of sight, and causes an imaginary part in the cross power spectrum between two types of galaxies.

Taking $\sqrt{\langle v_{\nu c}^2 \rangle}$ as the representative value for the background velocity, we calculate the induced density correlations using MBPT. Figure 3 shows the monopole and the absolute value of the dipole (most parts of it are negative) terms of the CDM-neutrino cross power spectrum as well as the CDM autopower spectrum for four different neutrino masses. The oscillations in $P_{c\nu 1}$ (dotted line) are due to the sharp sound horizon which is an artifact of the fluid approximation of neutrinos in our calculation. (We have verified that the oscillation period is inversely proportionate to the effective sound speed, so it is due to the (false) acoustic oscillation in the fluid. Real neutrinos are not a collisional fluid, and the effective sound speed is actually a superposition of different sound speeds, so we do not expect the true cross power spectrum to exhibit these oscillations.) We have thus smoothed the dipole power spectrum and obtained an average $\bar{P}_{c\nu 1}$, which is shown as the solid line, for the different neutrino masses the power spectra are different and distinguishable. Figure 4 shows, respectively, the CDM autocorrelation function, the neutrino autocorrelation function, and the monopole and dipole part of CDMneutrino cross correlation functions. We find that the neutrino autocorrelation grows as the neutrino mass increases, since the more massive neutrinos tend to form more structures. The dipole term of the cross power spectrum have a broad peak or hump, its amplitude also grows with the neutrino mass. The scales of the peaks in the correlation function decrease with neutrino mass, and are located at 16, 11, 7, and 5 Mpc/h, respectively, for the four neutrino masses.

We have taken a single value of $v^{(bg)} = \sqrt{\langle v_{\nu c}^2 \rangle}$ for each neutrino mass. For a given background velocity value, the dipole correlation depends on the neutrino mass value, as is shown in the equations in the Supplemental Material [9]. But

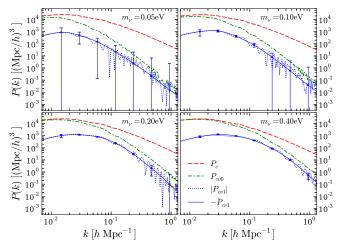


FIG. 3: The power spectra of CDM and neutrinos. The CDM autopower P_c , neutrino monopole $P_{c\nu 0}$ and dipole $P_{c\nu 1}$, and smoothed dipole term $\bar{P}_{c\nu 1}$ are plotted.

in fact the bulk relative velocity varies from point to point in space. A more rigorous treatment would require a consideration of the distribution of the bulk velocity. The fact that both the typical value of bulk velocity and the dipole correlation for a given background velocity depend on the neutrino mass enhances the sensitivity for this technique. Below for simplicity we will consider only the typical values.

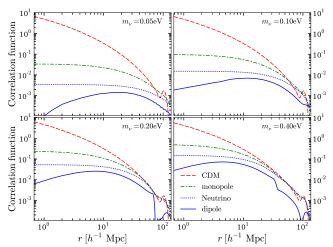


FIG. 4: The correlation functions, including CDM and neutrino autocorrelations, and the monopole and dipole part of their cross correlations. The dipole is $\Delta_{\rm corr} \equiv \xi_{c\nu}|_{\mu=-1}^{\mu=1}$.

Observability.—Neither the neutrinos nor the dark matter can be observed directly, but as their densities affect galaxy densities, their cross power can be inferred from the cross power of galaxies of different populations, provided that the biases of the two populations have different dependences on neutrinos and dark matter. Galaxies are known to be biased relative to each other [11]. The 21cm HIPASS galaxies typically have a bias of $b_c \sim 0.7$ [12] relative to the dark matter, whereas the bias for luminous red galaxies is typically greater

than 1. For a galaxy population, we assume its density contrast is related to the dark matter and neutrino density contrasts δ_c, δ_ν as $\delta_g = b_c f_c \delta_c + b_\nu f_\nu \delta_\nu$, where $f_c = \Omega_c/(\Omega_c + \Omega_\nu)$ and $f_\nu = \Omega_\nu/(\Omega_c + \Omega_\nu)$ [13]. Since the halo mass scale $10^{12} \sim 10^{13} M_\odot$ is smaller than the neutrino free streaming and coherent scales, we expect the neutrino bias to be insensitive to halo mass. This can also be seen by deriving the halo bias via the peak-background split formalism or the extended Press-Schechter formalism (see, e.g., Ref. [14]) with neutrino fluctuations only affecting the large scale background density. For the following calculations, we choose b_ν to be 1 but emphasize that an effect will be present as long as b_ν is the same for both galaxy populations, regardless of the particular value. The precise value could be calculated with the more elaborate treatment as prescribed in Ref. [15].

If we consider the cross-correlation of two galaxy populations denoted by α, β , and use b_{α}, b_{β} to denote b_c for α, β , then

$$\xi_{\alpha\beta} = \langle \delta_{\alpha}\delta_{\beta} \rangle = b_{\alpha}b_{\beta}f_c^2\xi_c + (b_{\alpha} + b_{\beta})f_cf_{\nu}\xi_{c\nu} + f_{\nu}^2\xi_{\nu}.$$

Now consider the μ dependence of $\xi_{\alpha\beta}$: because the cross correlation function is antisymmetric in " $c\nu$," a dipole $\mu(b_{\alpha}-b_{\beta})f_cf_{\nu}\xi_{c\nu1}$ appears. The observability of this dipole depends on the relative bias, $\Delta b \equiv b_{\alpha}-b_{\beta}$. The known spread in formation bias provides a lower bound on $\Delta b \gtrsim 0.5$. For sensitivity estimation, we will adopt $\Delta b = 1$. The actual error bar of the inferred neutrino mass will depend on the product of Δb and galaxy number density n_{α} .

For this measurement, the bulk velocity field can be reconstructed from the observed density field,

$$\boldsymbol{v}_{\nu c}(\boldsymbol{k}) = \delta_g(\boldsymbol{k}) \frac{[T_{\theta,\nu}(k) - T_{\theta,c}(k)]}{T_{\delta,g}(k)} \frac{i\boldsymbol{k}}{k^2}.$$
 (3)

Here $T_{\theta,\nu}(k), T_{\theta,c}(k)$ are the velocity-divergence transfer functions for neutrino and dark matter respectively, and $T_{\delta,g}(k)$ is the density transfer function, which depends on the unknown neutrino mass. In practice, one can iterate the reconstruction with different trial masses m_{ν} , until a self-consistent relative velocity field $v_{\nu c}$ and dipole value is found. At the high sampling densities considered here, the fractional error in $v_{\nu c}$ is comparable to the error in the CDM density field δ . The shot noise is much smaller than the sample variance, making the error on the velocity field negligible at the scales of interest.

The correlation function provides a local operational procedure to measure the dipole,

$$\xi_{\alpha\beta}(r,\mu) = \frac{1}{N} \sum_{\boldsymbol{x}} \sum_{\substack{|\Delta \boldsymbol{x}| \sim r \\ \hat{v}_{\nu c} \cdot \hat{\Delta} \boldsymbol{x} \sim \mu}} \delta_{\alpha}(\boldsymbol{x}) \delta_{\beta}(\boldsymbol{x} + \Delta \boldsymbol{x}), \quad (4)$$

where N is appropriate normalization. The dipole term can be extracted from this anisotropic correlation as in Eq.(2). Taking the Fourier transform then yields the power spectrum dipole. The error bar is easier to specify for the power spectrum than the correlation function, since k bins are statistically

TABLE I: The forecasted error on neutrino mass with a survey of $V_s=1.0h^{-3}{\rm Gpc}^3$, $n_g=2.4\times 10^{-2}h^3{\rm Mpc}^{-3}$ and with current survey data, modeled with SDSS and 2dF as $V_s=0.2h^{-3}{\rm Gpc}^3$, $n_gV_s=1\times 10^6$. Note that substantial uncertainties exist due to unknown galaxy neutrino bias, which is a nuisance parameter that we marginalize over.

	current (SDSS)		future	
m_{ν} (eV)	$\sigma_{m_{ u}}$	relative error	$\sigma_{m_ u}$	relative error
0.05	0.045	0.90	0.0042	0.084
0.10	0.044	0.44	0.0041	0.041
0.20	0.079	0.40	0.0074	0.037
0.40	0.097	0.24	0.0091	0.023

independent. The transformation from real space to redshift space does not change our error estimate because the dipole is orthogonal to the effect of redshift distortion, which is a quadrupole distortion.

In Fig. 3, we plot the expected error bars of the angular-dependent CDM-neutrino cross power spectrum for a survey with volume $V_s=1.0h^{-3}{\rm Gpc}^3$ and $n_g\Delta b=2.4\times 10^{-2}h^3{\rm Mpc}^{-3}$. This corresponds to an all-sky survey out to redshift z<0.2, comparable to the sloan digital sky survey (SDSS) main sample volume, but with a tenfold higher galaxy sampling density, about the density of HIPASS galaxies [16]. The two populations of galaxies could be, for example, a deep optical survey and an HI survey at low redshifts. Alternatively, the second tracer might be obtained by a nonlinear weighting of the same density field such as the cosmic tide field [17].

We proceed to calculate the error on the neutrino mass measurement using a Fisher matrix estimate. We use five k bins $(k=0.059,\,0.12,\,0.24,\,0.47,\,0.94\,h/\mathrm{Mpc})$ in Fig. 3. Modes with smaller k are not used because MBPT is not a very good approximation unless the background velocity comes from scales larger than the k mode. We fit for two parameters: a multiplicative (relative) galaxy bias Δb , treated as a nuisance parameter, and a neutrino mass, and marginalize the result over the relative bias. The result is given in Table I for the four different neutrino masses. Existing galaxy redshift data may result in a detection for optimistic neutrino mass and bias parameters. Future surveys can measure the neutrino masses precisely.

Discussions.—The neutrino mass measurement method proposed here differs from the one based on small scale power spectrum suppression, and it is more robust to scale-dependent galaxy biasing. In the approach based on power suppression, if for some reason there is a weak scale-dependent variation of bias at the level of $\sim 1\%$, it can completely swamp the neutrino signal. In our dipole cross correlation approach, the measured signal arises only from the relative velocity effect. If the galaxy bias were to depend on scale, the impact on the inferred neutrino mass would only be proportionate to any such changes, unlike for total power measurements where any uncertainty in bias is amplified by 2 orders of magnitude or more.

For the cases we considered, the correlation function peaks occur at scales $(16,11,7,5\,\text{Mpc/h})$ comparable to the relative

velocity field coherency scales (14.5, 10.3, 7.0, 4.6 Mpc/h); this is not unexpected as it is the coherence of the bulk velocity which induces such correlation. However, for the analytical MBPT calculation we used here, it does pose a problem, because strictly speaking the MBPT approximation is valid only for scales below the coherence scale. The nonlinear effects become significant for $k \gtrsim 0.1 h/\mathrm{Mpc}$. Nevertheless, the essence of large scale velocity modulation and the expected physical effect (the dipole structure) is still captured in the calculation, though quantitatively it may not be very accurate at the largest scales. This can be remedied with numerical simulations. We will study this in a future paper; preliminary results, however, show that the result is generally consistent with the analytical one.

In our Fisher analysis, we have treated the galaxy relative bias as a nuisance parameter. As described above, the sensitivity to this effect depends on $n_g \Delta b$ and so the galaxy density needed to detect this dipole depends on the bias. In any given detection of the dipole, Δb is immediately known, and thus the error on the neutrino mass would also be known. The uncertainty in the bias, and thus the error, is proportionate to the significance of the detection, i.e. for a 10σ detection, there is an additional 10% uncertainty in the error itself.

In the above we have considered a single neutrino mass. In fact, unlike the power spectrum suppression effect, which is sensitive only to the sum of the neutrino masses, the dipole effect discussed here can in principle be used to measure the mass of a single neutrino. For multiple neutrinos, the different mass eigenstates will have different bulk velocity directions for each of them, which at least in theory can be solved independently by repeating this procedure once for each mass. In practice this may be difficult, but if one or two neutrino masses are dominant and degenerate, then the procedure discussed in this Letter is already sufficient. For an inverted neutrino mass hierarchy, the effect would be twice as large and enhance the possibility of detection.

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