

Maximal zero textures of the inverse seesaw with broken $\mu\tau$ symmetry *

Biswajit Adhikary^{a†}, Ambar Ghosal^{b‡} and Probir Roy^{b§}

a) Department of Physics, Gurudas College, Narkeldanga, Kolkata-700054, India

b) Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India

Abstract

The inverse neutrino seesaw, characterised by only one source of lepton number violation at an ultralight $O(\text{keV})$ scale and observable new phenomena at TeV energies accessible to the LHC, is considered. Maximal zero textures of the 3×3 lighter and heavier Dirac mass matrices of neutral leptons, appearing in the Lagrangian for such an inverse seesaw, are studied within the framework of $\mu\tau$ symmetry in a specified weak basis. That symmetry ensures the identity of the positions of maximal zeros of the heavy neutrino mass matrix and its inverse. It then suffices to study the maximal zeros of the lighter Dirac mass matrix and those of the inverse of the heavier one since they come in a product. The observed absence of any unmixed neutrino flavour and the assumption of no strictly massless physical neutrino state allow only eight 4-zero \times 4-zero, eight 4-zero \times 6-zero and eight 6-zero \times 4-zero combinations. The additional requirement of leptogenesis is shown to eliminate the last sixteen textures. The surviving eight 4-zero \times 4-zero textures are subjected to the most general explicit $\mu\tau$ symmetry breaking terms in the Lagrangian in order to accommodate the nonzero value of θ_{13} in the observed range. A full diagonalisation is then carried out. On numerical comparison with all extant and relevant neutrino (antineutrino) data, seven of these eight combination textures in five neutrino matrix forms are found to be allowed, leading to five distinct neutrino mass matrices. Two of these permit only a normal (and the other three only an inverted) mass ordering of the light neutrinos.

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[†]biswajit.adhikary@saha.ac.in

[‡]ambar.ghosal@saha.ac.in

[§]probir.roy@saha.ac.in

1 Introduction

Type-I seesaw [1, 2, 3] has been the most popularly used mechanism so far to generate light neutrino masses. Unfortunately, all new phenomena in this scenario involve three heavy right chiral $SU(2)_L \times U(1)_Y$ singlet neutrinos ν_{kR} ($k = e, \mu, \tau$ and R a chiral index) with a Majorana mass matrix M , the smallest eigenvalue of which is bounded [4] from below by the leptogenesis constraint to $M_{lightest} \geq 10^8$ GeV. Thus they are beyond the foreseeable reach of laboratory experiments. Alternative ways exist of circumventing this. In particular, there is the inverse seesaw mechanism [5, 6, 7] to which we direct attention. Three additional left chiral $SU(2)_L \times U(1)_Y$ singlet fields s_L are introduced here along with their 3×3 Majorana mass matrix μ characterised by only ultralight eigenvalues in the keV scale. This mass matrix constitutes the sole source of lepton number violation since three other 3×3 lepton mass matrices, which are introduced, are of a Dirac type: M_ℓ between \bar{l}_L and l_R (l being a charged lepton), M_D between $\bar{\nu}_L$ and ν_R and M between \bar{s}_L and ν_R . Here M_D can be chosen with nonzero elements of the order of the known charged fermion masses, while those of M can be taken to be at the TeV scale with phenomenological consequences of interest to the LHC. Thus we have $O(\mu) \ll O(m_D) \ll O(M)$. Leptogenesis has been shown to be [8] realistically possible in this scenario.

The key feature of this inverse seesaw, as shown below, is that the product

$$F \equiv M_D M^{-1} \tag{1.1}$$

plays a role in the light neutrino mass formula which is analogous to that of the Dirac mass matrix M_D in type-I seesaw. We also consider the question of texture. A fruitful approach to the problem of light neutrino masses and mixing angles has been based on the idea [9] of maximal texture zeros. In the present case these vanishing elements are postulated in the matrices M_D and M^{-1} . Texture statements being basis dependent, one needs to choose a weak basis [10] in order to make such a statement. We choose one in which the flavoured charged leptons l and the singlet fields s_L are mass diagonal with real and positive entries. Such a choice can be consistently made since these two sets of fields do not have any mutual interaction at the Lagrangian level. In such a weak basis, in analogy with the type-I seesaw case [9], four and six turn out to be the number of maximal zeros allowed respectively in M_D and M^{-1} which are the two factors in the matrix F . In order to simplify the possible textures, we assume $\mu\tau$ symmetry (with a small breaking), motivated by the near-maximal and small values of θ_{23} and θ_{13} respectively. The further assumption of no strictly massless neutrino, bolstered by the observed fact of a nontrivial mixing for every neutrino flavor, allows only eight 4-zero \times 6-zero, eight 6-zero \times 4-zero and eight 4-zero \times 4-zero combinations. Of these, the first sixteen cannot effect leptogenesis and hence are ruled out. Of the remaining eight, seven combination textures, leading to five distinct neutrino matrix forms, are found to be allowed. Two of these allow only a normal (and the other three only an inverted) mass ordering of the light neutrinos after comparison with all available neutrino and antineutrino data.

In the rest of the paper, Sec. 2 is devoted to a discussion of maximal zero textures with $\mu\tau$ symmetry within the framework of the inverse seesaw. Section 3 shows how the requirement of leptogenesis constrains these textures. The effect of $\mu\tau$ symmetry breaking on the allowed texture combinations is discussed in Sec.4. The diagonalisation of the resultant neutrino mass matrices with broken $\mu\tau$ symmetry is carried out in Sec. 5. The numerical analysis in comparing the predictions of these surviving mass matrices and a global set of neutrino and antineutrino data is briefly presented in Sec.6. In Sec. 7 we summarise our conclusions. In the Appendix some relevant expressions, used in the text, are given explicitly.

2 Inverse seesaw, maximal zeros and $\mu\tau$ symmetry

The Lagrangian mass terms, required to facilitate the inverse seesaw, can be written as follows:

$$\begin{aligned}
-\mathcal{L}^{\text{mass}} &= \overline{l_{kL}}(M)_{ll'}l_{k'R} + \overline{\nu_{kL}}(M_D)_{kk'}\nu_{k'R} + \overline{s_{kL}}M_{kk'}\nu_{k'R} \\
&\quad + \frac{1}{2}\overline{s_{kL}}\mu_{kk'}s_{k'R}^C + h.c. \\
&= \overline{l_{kL}}(M)_{ll'}l_{k'R} + \frac{1}{2}\begin{pmatrix} \overline{\nu_L} & \overline{(\nu^C)_L} & \overline{s_L} \end{pmatrix} \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M^T \\ 0 & M & \mu \end{pmatrix} \begin{pmatrix} (\nu^C)_R \\ \nu_R \\ (s^C)_R \end{pmatrix} + h.c.
\end{aligned} \tag{2.1}$$

The magnitudes of the elements of the three mass matrices introduced above obey a hierarchy $O(\mu) \ll O(M_D) \ll O(M)$, as mentioned in the Introduction. The inverse seesaw is then effected by the 3×3 block matrix in the second RHS term of (2.1), leading to the light neutrino mass terms

$$-\mathcal{L}^{\nu\text{-mass}} = \overline{\nu_L}M_\nu(\nu^C)_R + h.c. \tag{2.2}$$

In (2.2), the effective light neutrino mass matrix M_ν is given by the inverse seesaw formula

$$M_\nu \simeq M_D M^{-1} \mu (M^T)^{-1} M_D^T \equiv F \mu F^T. \tag{2.3}$$

The interplay among the widely different energy scales involved in (2.3) is manifest in the relation

$$\left(\frac{M_\nu}{0.1\text{eV}}\right) \sim \left(\frac{M_D}{10^2\text{GeV}}\right)^2 \left(\frac{\mu}{1\text{keV}}\right) \left(\frac{M}{10^4\text{GeV}}\right)^{-2}, \tag{2.4}$$

where M_ν , M_D , μ and M are typical elements of the corresponding matrices. In our chosen weak basis, we have

$$M_\ell = \text{diag}(m_e, m_\mu, m_\tau), \quad \mu = \text{diag}(\mu_1, \mu_2, \mu_3). \tag{2.5}$$

Two additional inputs are now introduced to restrict the form of M_ν . First, there is the observed fact that none of the three known light neutrinos is free from flavour mixing. This means that any

block diagonal form of M_ν is inadmissible. The same statement can be made about the matrix F of (1.1) on account of (2.3). Indeed, if any row in a texture of F is orthogonal to both the other rows, one neutrino family decouples – disallowing that texture. Second, we assume that no neutrino is strictly massless because of the lack of any fundamental principle (unlike for a photon) suggesting that such be the case. Since that leads to a nonvanishing $\det M_\nu$, (2.3) implies that both $\det M_D$ and $\det \mu$ have to be nonvanishing. Thus not only is each μ_i nonzero for $i = 1, 2, 3$, neither M_ν nor M_D can have a vanishing row/column or a quartet of zeros at the four corners of a rectangular array. The same statement can be made about the matrix F of (2.4). The analysis, detailed in Ref. [11]-[15] for M_D , now holds *mutatis mutandis* for F . Therefore, the maximum number of zeros that F can accommodate is four. Moreover, there are seventy two such allowed textures of F with four vanishing entries.

We propose to reduce the number of these seventy-two four zero textures of F with the aid of $\mu\tau$ symmetry. This symmetry¹ postulated only in the neutrino sector, stipulates an invariance of all neutrino terms in the Lagrangian under the interchange of the flavor indices $2 \leftrightarrow 3$. This automatically requires $\mu_2 = \mu_3$ and moreover implies that

$$\Phi_{22} = \Phi_{33}, \quad \Phi_{12} = \Phi_{13}, \quad \Phi_{21} = \Phi_{31}, \quad \Phi_{32} = \Phi_{23}, \quad (2.6)$$

where Φ is any of the matrices M_D , M , M^{-1} , F . We then deconstruct the surviving textures in terms of those of M_D and M^{-1} . The $\mu\tau$ symmetric forms of M_D and M can be written as

$$M_D = \begin{pmatrix} A_1 & A_2 & A_2 \\ B_1 & B_2 & B_3 \\ B_1 & B_3 & B_2 \end{pmatrix}, \quad M = \begin{pmatrix} r_1 & r_2 & r_2 \\ s_1 & s_2 & s_3 \\ s_1 & s_3 & s_2 \end{pmatrix}, \quad (2.7)$$

where the entries $A_{1,2}$, $B_{1,2,3}$, $r_{1,2}$ and $s_{1,2,3}$ are unknown dimensional complex quantities in general.

It was shown [9] in the context of type-I seesaw that the number of allowed 4-zero Yukawa textures gets drastically reduced from seventy two to four on the imposition of $\mu\tau$ symmetry. In case of the inverse seesaw, an identical statement holds for the matrix F . The four textures, divided as in the earlier case into two categories A and B , are

$$F^{A1} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_2 & 0 \\ 0 & 0 & b_2 \end{pmatrix}, \quad F^{A2} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}, \\ F^{B1} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \quad F^{B2} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}, \quad (2.8)$$

¹For a review and original references, see Ref. [16]

where $a_{1,2}$ and $b_{1,2}$ can be given in terms of $A_{1,2}$, $B_{1,2}$, $r_{1,2}$. Once again, the two textures of Category A yield an identical M_ν and the same goes for the two textures of Category B .

An interesting consequence of $\mu\tau$ symmetry is that the positions of the maximal zeros of any texture of M and M^{-1} become identical. We utilise this in the deconstruction of each allowed 4-zero texture of F in terms of the maximally allowed zero textures of M_D and M . A straightforward but tedious amount of algebra leads to the deconstruction listed in Table 1. Two allowed 4-zero \times 4-zero textures, two allowed 4-zero \times 6-zero textures and two allowed 6-zero \times 4-zero textures follow for *each* of F^{A1} , F^{A2} , F^{B1} and F^{B2} .

3 Constraints from the requirement of leptogenesis

For the inverse seesaw, Dirac leptogenesis [8] occurs at an energy scale where the μ term, required to be $O(\text{keV})$, can be neglected. Gauge symmetry is preserved and lepton number is conserved at that stage. The relevant part of the Lagrangian for leptogenesis (LG) is

$$-\mathcal{L}^{\text{LG}} = \frac{\sqrt{2}}{v} \overline{\psi_{kL}} (M_D)_{kk'} \tilde{\Phi} \nu_{k'R} + \overline{s_{kL}} M_{kk'} \nu_{k'R} + h.c., \quad (3.1)$$

where $\psi_L \equiv \begin{pmatrix} \nu_L \\ l \end{pmatrix}$ and $\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ are weak isospin doublet lepton and Higgs fields respectively and $v \approx 246$ GeV. With the definition of a Dirac spinor field $N \equiv s_L \oplus \nu_R$, we have

$$-\mathcal{L}^{\text{LG}} = \frac{\sqrt{2}}{v} \overline{\psi_{kL}} (M_D)_{kk'} \tilde{\Phi} N_{k'R} + \overline{N_{kL}} M_{kk'} N_{k'R} + h.c.. \quad (3.2)$$

The CP asymmetry in the decays of N and \bar{N} is characterised in their mass basis by the parameter ε_{il} defined as

$$\varepsilon_{il} \equiv \frac{\Gamma(N_i \rightarrow l^- \phi^+, \nu_l \phi^0) - \Gamma(\bar{N}_i \rightarrow l^+ \phi^-, \nu_l^C \phi^{0*})}{\Gamma(N_i \rightarrow l^- \phi^+, \nu_l \phi^0) + \Gamma(\bar{N}_i \rightarrow l^+ \phi^-, \nu_l^C \phi^{0*})}. \quad (3.3)$$

The singlet neutrino Dirac mass matrix M can be put into a diagonal form by a biunitary transformation

$$U_L^\dagger M U_R = \text{diag}(M_1, M_2, M_3). \quad (3.4)$$

The important unitary matrix for leptogenesis is U_R which diagonalises $M^\dagger M$:

$$U_R^\dagger M^\dagger M U_R = \text{diag}(M_1^2, M_2^2, M_3^2). \quad (3.5)$$

In the mass basis of N , where it is denoted by a widehat, $\widehat{N}_{iR} = (U_R^\dagger)_{il} N_{lR}$, i.e. $N_{lR} = (U_R)_{li} \widehat{N}_{iR}$. In this basis of N , M_D is modified to

$$\widehat{M}_D = M_D U_R. \quad (3.6)$$

On account of the Dirac nature of the mass matrix M , the CP asymmetry, as generated from the interference of the tree and one loop self energy diagrams, will be

$$\varepsilon_{il} = \frac{M_i^2}{4\pi v^2 \widehat{h}_{ii}} \sum_j \frac{M_i^2 - M_j^2}{(M_i^2 - M_j^2)^2 + \Gamma_i^2 M_i^2} \text{Im}[\widehat{M}_{Dlj} \widehat{M}_{Dli}^* \widehat{h}_{ji}], \quad (3.7)$$

where $\widehat{h} = \widehat{M}_D^\dagger \widehat{M}_D = U_R^\dagger M_D^\dagger M_D U_R \equiv U_R^\dagger h U_R$ and Γ_i is the total width of N_i .

There are four allowed 4-zero and two allowed 6-zero textures of M :

$$\begin{aligned} & \begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}, & \begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & 0 & s_2 \\ 0 & s_2 & 0 \end{pmatrix}, \\ & \begin{pmatrix} r_1 & 0 & 0 \\ s_1 & s_2 & 0 \\ s_1 & 0 & s_2 \end{pmatrix}, & \begin{pmatrix} r_1 & 0 & 0 \\ s_1 & 0 & s_2 \\ s_1 & s_2 & 0 \end{pmatrix}; \\ & \begin{pmatrix} r_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}, & \begin{pmatrix} r_1 & 0 & 0 \\ 0 & 0 & s_2 \\ 0 & s_2 & 0 \end{pmatrix}. \end{aligned} \quad (3.8)$$

Each pair of M occurring in the three lines of the above equation leads to the following three respective forms for the matrix $H \equiv M^\dagger M$:

$$\begin{aligned} H &= \begin{pmatrix} |r_1|^2 & r_1^* r_2 & r_1^* r_2 \\ r_1 r_2^* & |r_2|^2 + |s_2|^2 & |r_2|^2 \\ r_1 r_2^* & |r_2|^2 & |r_2|^2 + |s_2|^2 \end{pmatrix}, \\ H &= \begin{pmatrix} |r_1|^2 + 2|r_2|^2 & r_2^* s_2 & r_2^* s_2 \\ r_2 s_2^* & |s_2|^2 & 0 \\ r_2 s_2^* & 0 & |s_2|^2 \end{pmatrix}, \\ H &= \begin{pmatrix} |r_1|^2 & 0 & 0 \\ 0 & |s_2|^2 & 0 \\ 0 & 0 & |s_2|^2 \end{pmatrix}. \end{aligned} \quad (3.9)$$

The top two H matrices of (3.9) can be diagonalised by U_R which can be defined by one unknown angle ϕ_{12} and one phase ψ :

$$U_R = \begin{pmatrix} c_{12}^\phi e^{-i\psi} & s_{12}^\phi e^{-i\psi} & 0 \\ -\frac{s_{12}^\phi}{\sqrt{2}} & \frac{c_{12}^\phi}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-s_{12}^\phi}{\sqrt{2}} & \frac{c_{12}^\phi}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (3.10)$$

where $c_{12}^\phi \equiv \cos \phi_{12}$, $s_{12}^\phi \equiv \sin \phi_{12}$. The mass eigenvalues M_i as well as the angle ϕ_{12} and the phase angle ψ of U_R are given in Table 3 for the two different pairs of 4-zero M 's. The H matrix

corresponding to the 6-zero textures is diagonal. Hence, for them, the diagonalising matrix will be $U_R = \mathbf{I}$ and the eigenvalues of H will be the diagonal elements $M_1^2 = |r_1|^2$, $M_{2,3}^2 = |s_2|^2$.

Now that we have the U_R 's for all possible $M^\dagger M$ matrices, we can construct the matrices $\widehat{M}_D = M_D U_R$ and \widehat{h} , which are relevant to leptogenesis, for all possible combinations of M_D and M given in Tables 1 and 2. We mainly focus on the determination of $\text{Im}(\widehat{M}_{Dlj} \widehat{M}_{Dli}^* \widehat{h}_{ji})$, cf. (3.7), and test whether this quantity is nonzero. We observe that, for each $\mu\tau$ symmetric 4-zero \times 6-zero combination, $U_R = \mathbf{I}$, $\widehat{M}_D = M_D$, $\widehat{h} = h$ and the said quantity vanishes. The same is true for all $\mu\tau$ symmetric 6-zero \times 4-zero combinations. So, not one of the allowed textures of the 4-zero \times 6-zero and 6-zero \times 4-zero combinations is able to create a nonvanishing lepton asymmetry which we require. Hence all these texture combinations are ruled out. Only the allowed 4-zero \times 4-zero combinations lead to $\varepsilon_{il} \neq 0$ for $i = 1, 2$ and all l . They are just the ones to survive the requirement of leptogenesis.

4 Effect of $\mu\tau$ symmetry breaking on the surviving textures

The symmetry, arising from $\mu\tau$ interchange, produces a maximal atmospheric mixing angle $\theta_{23} = \pi/4$ as well as a vanishing reactor angle $\theta_{13} = 0$. In the light of the recent results of reactor and short baseline experiments measuring $\theta_{13} \simeq 8^\circ$, it is clear the $\mu\tau$ symmetry is broken. The scale of this breaking can be estimated in terms of parameters $\{\epsilon\} \sim 15\%$ from the magnitude of the dimensionless quantity $\sin \theta_{13} \simeq 0.15$. We realise the breaking of $\mu\tau$ symmetry in M_D and in μ for the surviving 4-zero \times 4-zero combinations of M_D and M in terms of two complex parameters $\epsilon_1 e^{i\phi_1}$, $\epsilon_2 e^{i\phi_2}$ where $\epsilon_{1,2}$ and $\phi_{1,2}$ are real. We assume that the TeV scale M remains unbroken in a $\mu\tau$ symmetric form. In addition, we keep the positions of zeros in M_D , M and μ in tact. This is since we do not want to change the texture pattern. In this set up, $\mu\tau$ symmetry is broken in the diagonal matrix μ by means of a parameter δ introduced via

$$\mu^\delta = \text{diag}(\mu_1, \mu_2, \mu_2(1 + \delta)). \quad (4.1)$$

The four allowed forms of M_D , now designated $M_D^{\epsilon_1, \epsilon_2}$, look like

$$\begin{aligned} & \begin{pmatrix} A_1 & A_2 & A_2(1 + \epsilon_1 e^{i\phi_1}) \\ 0 & B_2(1 + \epsilon_2 e^{i\phi_2}) & 0 \\ 0 & 0 & B_2 \end{pmatrix}, & \begin{pmatrix} A_1 & A_2 & A_2(1 + \epsilon_1 e^{i\phi_1}) \\ 0 & 0 & B_3(1 + \epsilon_2 e^{i\phi_2}) \\ 0 & B_3 & 0 \end{pmatrix}, \\ & \begin{pmatrix} A_1 & 0 & 0 \\ B_1(1 + \epsilon_1 e^{i\phi_1}) & B_2(1 + \epsilon_2 e^{i\phi_2}) & 0 \\ B_1 & 0 & B_2 \end{pmatrix}, & \begin{pmatrix} A_1 & 0 & 0 \\ B_1(1 + \epsilon_1 e^{i\phi_1}) & 0 & B_3(1 + \epsilon_2 e^{i\phi_2}) \\ B_1 & B_3 & 0 \end{pmatrix}. \end{aligned} \quad (4.2)$$

The top two of the four matrices in (4.2) occur in the 4-zero \times 4-zero parts of Category *A* in Table 1. The bottom two matrices above occur in the 4-zero \times 4-zero parts of Category *B* in Table 2. Using the inverse see-saw formula (2.3) with broken $\mu\tau$ symmetric mass matrices μ^δ , $M_D^{\epsilon_1, \epsilon_2}$ and an unbroken M , we have found four distinct forms of $M_\nu^{\epsilon_1, \epsilon_2, \delta}$ for the four 4-zero \times 4-zero combinations of category *A* as given in (4.3)-(4.6) Those combinations, each with a broken $\mu\tau$ symmetric M_D and a $\mu\tau$ symmetric M , are named as *A1*, *A2*, *A3* and *A4* in respective order. We have used three real parameters k_1 , k_2 and k_3 , three phases $e^{i\alpha}$, $e^{i\beta}$ and $e^{i\gamma}$, and an overall complex mass parameter m_0 . The definitions of these quantities are given in Table 4. We have also been able to remove phase factor $e^{i\gamma}$ from the neutrino mass matrices of all categories by redefining the neutrino field ν_e . For Category *A*, then, we have four forms of $M_\nu^{\epsilon_1, \epsilon_2, \delta}$ as follows :

$$m_0 \left(\begin{array}{ccc} \left[\begin{array}{c} k_1^2 e^{2i\alpha} + (k_2 e^{i\beta} - k_3)^2 \\ + \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\}^2 (1 + \delta) \end{array} \right] & (k_2 e^{i\beta} - k_3)(1 + \epsilon_2 e^{i\phi_2}) & \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\} (1 + \delta) \\ (k_2 e^{i\beta} - k_3)(1 + \epsilon_2 e^{i\phi_2}) & (1 + \epsilon_2 e^{i\phi_2})^2 & 0 \\ (k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3)(1 + \delta) & 0 & 1 + \delta \end{array} \right), \quad (4.3)$$

$$m_0 \left(\begin{array}{ccc} \left[\begin{array}{c} k_1^2 e^{2i\alpha} + (k_2 e^{i\beta} - k_3)^2 (1 + \delta) \\ + \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\}^2 \end{array} \right] & \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\} (1 + \epsilon_2 e^{i\phi_2}) & (k_2 e^{i\beta} - k_3)(1 + \delta) \\ \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\} (1 + \epsilon_2 e^{i\phi_2}) & (1 + \epsilon_2 e^{i\phi_2})^2 & 0 \\ (k_2 e^{i\beta} - k_3)(1 + \delta) & 0 & 1 + \delta \end{array} \right), \quad (4.4)$$

$$m_0 \left(\begin{array}{ccc} \left[\begin{array}{c} k_1^2 e^{2i\alpha} + (k_2 e^{i\beta} - k_3)^2 \\ + \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\}^2 (1 + \delta) \end{array} \right] & \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\} (1 + \epsilon_2 e^{i\phi_2})(1 + \delta) & k_2 e^{i\beta} - k_3 \\ \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\} (1 + \epsilon_2 e^{i\phi_2})(1 + \delta) & (1 + \delta)(1 + \epsilon_2 e^{i\phi_2})^2 & 0 \\ k_2 e^{i\beta} - k_3 & 0 & 1 \end{array} \right), \quad (4.5)$$

and

$$m_0 \left(\begin{array}{ccc} \left[\begin{array}{c} k_1^2 e^{2i\alpha} + (k_2 e^{i\beta} - k_3)^2 (1 + \delta) \\ + \{k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3\}^2 \end{array} \right] & (k_2 e^{i\beta} - k_3)(1 + \epsilon_2 e^{i\phi_2})(1 + \delta) & k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3 \\ (k_2 e^{i\beta} - k_3)(1 + \epsilon_2 e^{i\phi_2})(1 + \delta) & (1 + \delta)(1 + \epsilon_2 e^{i\phi_2})^2 & 0 \\ k_2 e^{i\beta} (1 + \epsilon_1 e^{i\phi_1}) - k_3 & 0 & 1 \end{array} \right). \quad (4.6)$$

For category *B*, four combinations of a broken $\mu\tau$ symmetric M_D and $\mu\tau$ symmetric M are named as *B1*, *B2*, *B3* and *B4* in respective order. *B1* and *B2* lead to one form of $M_\nu^{\epsilon_1, \epsilon_2, \delta}$ whereas *B3*

and $B4$ to another form of $M_\nu^{\epsilon_1, \epsilon_2, \delta}$. Those are given below respectively as

$$m_0 \left(\begin{array}{ccc} k_3^2 & \begin{bmatrix} k_3 k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_3 k_2 e^{i\beta} \end{bmatrix} & k_3 \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} \\ \begin{bmatrix} k_3 k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_3 k_2 e^{i\beta} \end{bmatrix} & \begin{bmatrix} \{k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_2 e^{i\beta} (1 + \epsilon_2 e^{i\phi_2})\}^2 \\ + (1 + \epsilon_2 e^{i\phi_1})^2 \end{bmatrix} & \begin{bmatrix} \{k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_2 e^{i\beta} (1 + \epsilon_2 e^{i\phi_2})\} \\ \times \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} \end{bmatrix} \\ k_3 \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} & \begin{bmatrix} \{k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_2 e^{i\beta} (1 + \epsilon_2 e^{i\phi_2})\} \\ \times \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} \end{bmatrix} & \{k_1 e^{i\alpha} - k_2 e^{i\beta}\}^2 + 1 + \delta \end{array} \right), \quad (4.7)$$

$$m_0 \left(\begin{array}{ccc} k_3^2 & \begin{bmatrix} k_3 k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_3 k_2 e^{i\beta} \end{bmatrix} & k_3 \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} \\ \begin{bmatrix} k_3 k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_3 k_2 e^{i\beta} \end{bmatrix} & \begin{bmatrix} \{k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_2 e^{i\beta} (1 + \epsilon_2 e^{i\phi_2})\}^2 \\ + (1 + \delta)(1 + \epsilon_2 e^{i\phi_1})^2 \end{bmatrix} & \begin{bmatrix} \{k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_2 e^{i\beta} (1 + \epsilon_2 e^{i\phi_2})\} \\ \times \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} \end{bmatrix} \\ k_3 \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} & \begin{bmatrix} \{k_1 e^{i\alpha} (1 + \epsilon_1 e^{i\phi_1}) \\ -k_2 e^{i\beta} (1 + \epsilon_2 e^{i\phi_2})\} \\ \times \{k_1 e^{i\alpha} - k_2 e^{i\beta}\} \end{bmatrix} & \{k_1 e^{i\alpha} - k_2 e^{i\beta}\}^2 + 1 \end{array} \right). \quad (4.8)$$

So far, then, there are six distinct allowed forms of $M_\nu^{\epsilon_1, \epsilon_2, \delta}$

It is to be pointed out, that apart from 4-zero \times 4-zero combinations we can break $\mu\tau$ symmetry also in the 4-zero \times 6-zero and 6-zero \times 4-zero combinations. A question may then arise. Though the latter combinations with $\mu\tau$ symmetry are ruled out from the leptogenesis requirement, are they relevant after $\mu\tau$ symmetry breaking? We have observed that, for the 4-zero \times 6-zero combinations, $M^\dagger M$ is diagonal. So, $M_D^{\epsilon_1, \epsilon_2}$ retains the same four zero structure in the mass basis of N as in the $\mu\tau$ symmetric cases. Hence, the lepton asymmetry still vanishes for those combinations even after $\mu\tau$ symmetry breaking. For broken $\mu\tau$ symmetric 6-zero \times 4-zero combinations, we have again calculated $\text{Im}(\widehat{M}_{Dlj} \widehat{M}_{Dli}^* \widehat{h}_{ji})$ and have found a vanishing lepton asymmetry in each case. So, given the requirement of leptogenesis, only the 4-zero \times 4-zero combinations are left for study even after $\mu\tau$ symmetry breaking.

5 Exact diagonalisation of the neutrino mass matrix

We have written all neutrino mass matrices in the full fledged form without any approximation. So, we can perform an exact analysis by diagonalising them. Recently, an exact diagonalisation method was introduced [17] to find out the masses, mixing angles and phases in terms of the neutrino mass matrix elements. We use that methodology next to determine the physical observables. We just present the results here, for details one can see [17].

In order to find out the squared masses, mixing angles and the Dirac CP phase, we have constructed the matrices $h = M_\nu^{\epsilon_1, \epsilon_2, \delta} (M_\nu^{\epsilon_1, \epsilon_2, \delta})^\dagger$ for all the above six neutrino mass matrices and have diagonalised them. The general form of the eigenvalues of h are

$$\begin{aligned}\lambda_1 &= -\frac{b}{3a} - \frac{2\sqrt[3]{r}}{3\sqrt[3]{2a}} \cos \theta, \\ \lambda_2 &= -\frac{b}{3a} + \frac{\sqrt[3]{r}}{3\sqrt[3]{2a}} (\cos \theta - \sqrt{3} \sin \theta), \\ \lambda_3 &= -\frac{b}{3a} + \frac{\sqrt[3]{r}}{3\sqrt[3]{2a}} (\cos \theta + \sqrt{3} \sin \theta).\end{aligned}\tag{5.1}$$

where the definitions of a , b , r and θ in terms of the elements of h are given in the Appendix. Now the identification of λ_1 , λ_2 and λ_3 with the squared masses m_i^2 is done in order to compare with experimental data. To determine the mixing angles, we have used the eigenvectors for the eigenvalues m_i^2 and have constructed the diagonalising matrix in terms of the elements of h . The elements of the diagonalising U matrix can be written as

$$\begin{aligned}U_{1i} &= \frac{(h_{22} - m_i^2)h_{13} - h_{12}h_{23}}{N_i}, \\ U_{2i} &= \frac{(h_{11} - m_i^2)h_{23} - h_{12}^*h_{13}}{N_i}, \\ U_{3i} &= \frac{|h_{12}|^2 - (h_{11} - m_i^2)(h_{22} - m_i^2)}{N_i},\end{aligned}\tag{5.2}$$

with $i=1,2,3$. The N_i 's are normalization constants given by

$$\begin{aligned}|N_i|^2 &= |(h_{22} - m_i^2)h_{13} - h_{12}h_{23}|^2 + \\ &|(h_{11} - m_i^2)h_{23} - h_{12}^*h_{13}|^2 + \\ &\{|h_{12}|^2 - (h_{11} - m_i^2)(h_{22} - m_i^2)\}^2.\end{aligned}\tag{5.3}$$

The elements of the U matrix can have unwanted overall phases but their moduli can be equated to the moduli of the elements of the mixing matrix (without Majorana phases) :

$$U^{\text{MIXING}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}\tag{5.4}$$

with $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ and δ_{CP} being the Dirac phase. So, using $|U_{ij}^{\text{MIXING}}| = |U_{ij}|$, we have the expressions for the three mixing angles as

$$\tan \theta_{23} = \frac{|U_{23}|}{|U_{33}|}, \quad (5.5)$$

$$\tan \theta_{12} = \frac{|U_{12}|}{|U_{11}|}, \quad (5.6)$$

$$\sin \theta_{13} = |U_{13}|. \quad (5.7)$$

The δ_{CP} phase can be obtained by using the expression for $h_{12}h_{23}h_{31}$:

$$\text{Im}(h_{12}h_{23}h_{31}) = \frac{1}{8}(m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2) \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta_{CP}. \quad (5.8)$$

Eqn.(5.8) can be inverted to obtain the Dirac CP phase. An exact knowledge of the masses, mixing angles and the Dirac CP phase in terms of the mass elements of the neutrino mass matrix enables us to obtain expressions for the Majorana phases, neglecting terms with $(c_{23}^2 - s_{23}^2)s_{13}$, s_{13}^2 and their higher powers. Thus we can write

$$\tan \theta_j = \frac{Y'_j W_j - W'_j Y_j}{X_j W'_j - W_j X'_j}, \quad (5.9)$$

where $j = 1, 2$ and $\theta_1 = \alpha_M$, $\theta_2 = \beta_M$. The quantities X_j , X'_j , Y_j , Y'_j , W_j and W'_j , with $j = 1$ and 2 , are given by

$$\begin{aligned} X_1 &= A_i - \{D_r \sin(\beta_M - \alpha_M) + D_i \cos(\beta_M - \alpha_M) + F_r \sin 2(\beta_M - \alpha_M) + F_i \cos 2(\beta_M - \alpha_M) + E_i\}, \\ X'_1 &= \{D_r \cos(\beta_M - \alpha_M) - D_i \sin(\beta_M - \alpha_M) + F_r \cos 2(\beta_M - \alpha_M) - F_i \sin 2(\beta_M - \alpha_M) + E_r\} - A_r, \\ Y_1 &= A_r + \{D_r \cos(\beta_M - \alpha_M) - D_i \sin(\beta_M - \alpha_M) + F_r \cos 2(\beta_M - \alpha_M) - F_i \sin 2(\beta_M - \alpha_M) + E_r\}, \\ Y'_1 &= A_i + \{D_r \sin(\beta_M - \alpha_M) + D_i \cos(\beta_M - \alpha_M) + F_r \sin 2(\beta_M - \alpha_M) + F_i \cos 2(\beta_M - \alpha_M) + E_i\}, \\ W_1 &= B_r + C_r \cos(\beta_M - \alpha_M) - C_i \sin(\beta_M - \alpha_M), \\ W'_1 &= B_i + C_r \sin(\beta_M - \alpha_M) + C_i \cos(\beta_M - \alpha_M), \end{aligned} \quad (5.10)$$

$$\begin{aligned} X_2 &= A_i - \{D_i \cos(\beta_M - \alpha_M) - D_r \sin(\beta_M - \alpha_M) + E_i \cos 2(\beta_M - \alpha_M) - E_r \sin 2(\beta_M - \alpha_M) + F_i\}, \\ X'_2 &= \{D_r \cos(\beta_M - \alpha_M) + D_i \sin(\beta_M - \alpha_M) + E_r \cos 2(\beta_M - \alpha_M) + E_i \sin 2(\beta_M - \alpha_M) + F_r\} - A_r, \\ Y_2 &= A_r + \{D_r \cos(\beta_M - \alpha_M) + D_i \sin(\beta_M - \alpha_M) + E_r \cos 2(\beta_M - \alpha_M) + E_i \sin 2(\beta_M - \alpha_M) + F_r\}, \\ Y'_2 &= A_i + \{D_i \cos(\beta_M - \alpha_M) - D_r \sin(\beta_M - \alpha_M) + E_i \cos 2(\beta_M - \alpha_M) - E_r \sin 2(\beta_M - \alpha_M) + F_i\}, \\ W_2 &= C_r + B_r \cos(\beta_M - \alpha_M) + B_i \sin(\beta_M - \alpha_M), \\ W'_2 &= C_i + B_i \cos(\beta_M - \alpha_M) - B_r \sin(\beta_M - \alpha_M). \end{aligned} \quad (5.11)$$

In the above, the suffixes i and r stand for imaginary and real part respectively and

$$\beta_M - \alpha_M = \cos^{-1} \left[\frac{|(M_\nu^{\epsilon_1, \epsilon_2, \delta_{CP}})_{11}|^2 - c_{12}^4 m_1^2 - s_{12}^4 m_2^2}{2c_{12}^2 s_{12}^2 m_1 m_2} \right]. \quad (5.12)$$

Moreover, the complex quantities A , B , C , D , E and F are defined as

$$\begin{aligned} A &= m_3^2 [Z - 1], \\ B &= m_3 m_1 \left[Z s_{12}^2 (1 + t_{23}^4) t_{23}^{-2} + Z \sin 2\theta_{12} s_{13} e^{i\delta_{CP}} (1 - t_{23}^2) t_{23}^{-1} + 2s_{12}^2 \right], \\ C &= m_3 m_2 \left[Z c_{12}^2 (1 + t_{23}^4) t_{23}^{-2} + Z \sin 2\theta_{12} s_{13} e^{i\delta_{CP}} (t_{23}^2 - 1) t_{23}^{-1} + 2c_{12}^2 \right], \\ D &= m_1 m_2 \left[2Z c_{12}^2 s_{12}^2 + Z \sin 2\theta_{12} \cos 2\theta_{12} s_{13} e^{i\delta_{CP}} (t_{23}^2 - 1) t_{23}^{-1} - 2s_{12}^2 c_{12}^2 \right], \\ E &= m_1^2 \left[Z s_{12}^4 + Z s_{12}^2 \sin 2\theta_{12} s_{13} e^{i\delta_{CP}} (t_{23}^2 - 1) t_{23}^{-1} - s_{12}^4 \right], \\ F &= m_2^2 \left[Z c_{12}^4 - Z c_{12}^2 \sin 2\theta_{12} s_{13} e^{i\delta_{CP}} (t_{23}^2 - 1) t_{23}^{-1} - c_{12}^4 \right], \end{aligned} \quad (5.13)$$

where $t_{23} \equiv \tan \theta_{23}$ and $Z = [(M_\nu^{\epsilon_1, \epsilon_2, \delta_{CP}})_{23}]^2 [(M_\nu^{\epsilon_1, \epsilon_2, \delta_{CP}})_{22} (M_\nu^{\epsilon_1, \epsilon_2, \delta_{CP}})_{33}]^{-1}$.

6 Numerical Analysis

We analyse four $M_\nu^{\epsilon_1, \epsilon_2, \delta}$'s from A1, A2, A3, A4, and two $M_\nu^{\epsilon_1, \epsilon_2, \delta}$'s from the two pairs (B1,B2) and (B3,B4) to determine the admitted parameter space accommodating all experimental data from neutrino oscillation studies. We utilise the 3σ ranges given in Table 5 as inputs to constrain the parameter space. Each of the above six categories of textures contains ten parameters. First of all, *the texture A1 is ruled out due to its θ_{13} value being outside the allowed interval*. Except A1, each of the other five $M_\nu^{\epsilon_1, \epsilon_2, \delta}$'s admits a constrained parameter space. We follow a minimalistic approach in which we keep as many as parameters equal to zero as possible. The allowed ranges of the parameters are given in Table 6.

For each category, we present our predictions in Table 7 on the individual masses of the three neutrinos and their sum, $|m_{\nu\beta\beta}|$ relevant to $0\nu 2\beta$ decay, the CP violating parameters J_{CP} and δ_{CP} and the Majorana phases α_M and β_M . To this end, we note that the testability of each texture crucially depends on the nature of the light neutrino mass ordering which will hopefully be experimentally determined in the near future. Again, a composite analysis is performed, including cosmological and astrophysical experimental data, such as those from the recent PLANCK satellite, WMAP low- l polarization, gravitational lensing and the Hubble constant H_0 from Hubble space telescope data with priors. These imply a value of $\Sigma m_i < 1.11\text{eV}$ whereas the incorporation of the SDSS DR8 results with the above combination reduces this upper limit drastically to 0.23eV . Altogether, we consider a conservative range of the upper limit on Σm_i as $\Sigma m_i < (0.23 - 1.11)\text{eV}$ [19, 20]. In our analysis, the predicted values of Σm_i in all the cases are far below the lower limit

of the aforesaid range. Another prediction, $|m_{\nu_{\beta\beta}}|$, to be measured in neutrinoless double beta decay experiments, is also much less than the quoted limit $|m_{\nu_{\beta\beta}}| < (0.14 - 0.38)\text{eV}$ [21, 22, 23] presented by the EXO-200 collaboration [24]. Finally, we have also included our CP violating Jarlskog parameter J_{CP} and the Dirac CP phase δ_{CP} [25] in Table 7. The former can be extracted from experiments looking for CP violation with neutrino and antineutrino beams by measuring the difference in oscillation probabilities $P(\nu_{\mu} \rightarrow \nu_e) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)$. A detailed review of this is presented in Ref. [26]. An estimation of the latter on the basis of global analysis is given in Ref. [18] with the result $\delta_{CP} = \left(300_{-138}^{+66}\right)^{\circ}$ for 1σ range.

7 Concluding Summary

We have considered maximal zero textures in the context of the inverse seesaw mechanism. Lepton number violation is posited here through a keV scale Majorana mass matrix μ . The observed nontrivial mixing of every light neutrino flavor has been used. We have also assumed that none of the light neutrinos is massless. Consequently our allowed choices narrowed down to eight 4-zero \times 6-zero, eight 6-zero \times 4-zero and eight 4-zero \times 4-zero combinations in $F = M_D M^{-1}$ which controls neutrino masses and mixing angles through the relation $M_{\nu} = F \mu F^T$. The first sixteen texture combinations give rise to a null lepton asymmetry of the Universe and hence are excluded on that count. The remaining eight texture combinations can effect leptogenesis and are tested by all available and relevant neutrino oscillation data. Only seven combinations leading to five distinct neutrino matrix forms are seen to be allowed with strongly constrained parameter spaces. Two of these are seen to imply a normal mass ordering of the light neutrinos while the remaining three lead to an inverted one.

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A Appendix

Relevant expressions

$$\theta = \frac{1}{3} \tan^{-1} \left[\frac{3\sqrt{3}a\sqrt{\Delta}}{2b^3 - 9abc + 27a^2d} \right],$$

$$r = \sqrt{\left(3\sqrt{3}a\sqrt{\Delta}\right)^2 + (2b^3 - 9abc + 27a^2d)^2}, \quad (\text{A.1})$$

where

$$\begin{aligned}
a &= 1, \\
b &= -(h_{11} + h_{22} + h_{33}), \\
c &= h_{33}h_{11} + h_{33}h_{22} + h_{11}h_{22} - |h_{12}|^2 - |h_{13}|^2 - |h_{23}|^2, \\
d &= h_{11}|h_{23}|^2 + h_{33}|h_{12}|^2 + h_{22}|h_{13}|^2 - h_{11}h_{22}h_{33} - 2\text{Re}(h_{12}h_{23}h_{13}^*),
\end{aligned}
\tag{A.2}$$

and

$$\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2.
\tag{A.3}$$

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Table 1: Deconstructed maximal zero textures of F^{A1} and F^{A2} in category A .

F	Type	M_D	M	Relations
$F^{A1} =$ $\begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_2 & 0 \\ 0 & 0 & b_2 \end{pmatrix}$	4-zero \times 4-zero	$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = (A_2 r_1 - A_1 r_2) r_1^{-1} s_2^{-1}$ $b_2 = B_2 s_2^{-1}$
		$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = (A_2 r_1 - A_1 r_2) r_1^{-1} s_3^{-1}$ $b_2 = B_3 s_3^{-1}$
	4-zero \times 6-zero	$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = A_2 s_3^{-1}$ $b_2 = B_3 s_3^{-1}$
		$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = A_2 s_2^{-1}$ $b_2 = B_2 s_2^{-1}$
	6-zero \times 4-zero	$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = -A_1 r_2 r_1^{-1} s_2^{-1}$ $b_2 = B_2 s_2^{-1}$
		$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = -A_1 r_2 r_1^{-1} s_3^{-1}$ $b_2 = B_3 s_3^{-1}$
$F^{A2} =$ $\begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}$	4-zero \times 4-zero	$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = (A_2 r_1 - A_1 r_2) r_1^{-1} s_2^{-1}$ $b_2 = B_3 s_2^{-1}$
		$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = (A_2 r_1 - A_1 r_2) r_1^{-1} s_3^{-1}$ $b_2 = B_2 s_3^{-1}$
	4-zero \times 6-zero	$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = A_2 s_2^{-1}$ $b_2 = B_3 s_2^{-1}$
		$\begin{pmatrix} A_1 & A_2 & A_2 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = A_2 s_3^{-1}$ $b_2 = B_3 s_3^{-1}$
	6-zero \times 4-zero	$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = -A_1 r_2 r_1^{-1} s_2^{-1}$ $b_2 = B_2 s_2^{-1}$
		$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & r_2 & r_2 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $a_2 = -A_1 r_2 r_1^{-1} s_3^{-1}$ $b_2 = B_3 s_3^{-1}$

Table 2: Deconstructed maximal zero textures of F^{B1} and F^{B2} in category B .

F	Type	M_D	M	Relations	
$F^{B1} =$ $\begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}$	4-zero \times 4-zero	$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & 0 & B_3 \\ B_1 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & 0 & s_3 \\ s_1 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = (B_1 s_3 - B_3 s_1) r_1^{-1} s_3^{-1}$ $b_2 = B_3 s_3^{-1}$	
		$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & B_2 & 0 \\ B_1 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & s_2 & 0 \\ s_1 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = (B_1 s_2 - B_2 s_1) r_1^{-1} s_2^{-1}$ $b_2 = B_2 s_2^{-1}$	
	4-zero \times 6-zero	$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & 0 & B_3 \\ B_1 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = B_1 r_1^{-1}$ $b_2 = B_3 s_3^{-1}$	
		$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & B_2 & 0 \\ B_1 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = B_1 r_1^{-1}$ $b_2 = B_2 s_2^{-1}$	
	6-zero \times 4-zero	$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & 0 & s_3 \\ s_1 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = -B_3 s_1 r_1^{-1} s_3^{-1}$ $b_2 = B_3 s_3^{-1}$	
		$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & s_2 & 0 \\ s_1 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = -B_2 s_1 r_1^{-1} s_2^{-1}$ $b_2 = B_2 s_2^{-1}$	
	$F^{B2} =$ $\begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}$	4-zero \times 4-zero	$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & 0 & B_3 \\ B_1 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & s_2 & 0 \\ s_1 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = (B_1 s_2 - B_3 s_1) r_1^{-1} s_2^{-1}$ $b_2 = B_3 s_2^{-1}$
			$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & B_2 & 0 \\ B_1 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & 0 & s_3 \\ s_1 & s_2 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = (B_1 s_3 - B_2 s_1) r_1^{-1} s_3^{-1}$ $b_2 = B_2 s_3^{-1}$
4-zero \times 6-zero		$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & 0 & B_3 \\ B_1 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = B_1 r_1^{-1}$ $b_2 = B_3 s_2^{-1}$	
		$\begin{pmatrix} A_1 & 0 & 0 \\ B_1 & B_2 & 0 \\ B_1 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ 0 & 0 & s_3 \\ 0 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = B_1 r_1^{-1}$ $b_2 = B_2 s_3^{-1}$	
6-zero \times 4-zero		$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & 0 & B_3 \\ 0 & B_3 & 0 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & s_2 & 0 \\ s_1 & 0 & s_2 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = -B_3 s_1 r_1^{-1} s_2^{-1}$ $b_2 = B_3 s_2^{-1}$	
		$\begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{pmatrix}$	$\begin{pmatrix} r_1 & 0 & 0 \\ s_1 & 0 & s_3 \\ s_1 & s_3 & 0 \end{pmatrix}$	$a_1 = A_1 r_1^{-1}$ $b_1 = -B_2 s_1 r_1^{-1} s_3^{-1}$ $b_2 = B_2 s_3^{-1}$	

Table 3: Mass eigenvalues, ψ and ϕ_{12} for two categories of H from 4-zero M 's.

$M^\dagger M$	M_i^2	ψ and ϕ_{12}
$\begin{pmatrix} r_1 ^2 & r_1^* r_2 & r_1^* r_2 \\ r_1 r_2^* & r_2 ^2 + s_2 ^2 & r_2 ^2 \\ r_1 r_2^* & r_2 ^2 & r_2 ^2 + s_2 ^2 \end{pmatrix}$	$M_{2,1}^2 = \frac{ r_1 ^2 + 2 r_2 ^2 + s_2 ^2}{2}$ $\pm [(s_2 ^2 + 2 r_2 ^2 - r_1 ^2)^2 + 8 r_1 r_2]^{1/2}$ $M_3^2 = s_2 ^2$	$\psi = \arg(r_1 r_2^*)$ $\tan \phi_{12} = \frac{\sqrt{2} r_1 r_2 }{M_2^2 - r_1 ^2}$
$\begin{pmatrix} r_1 ^2 + 2 r_2 ^2 & r_2^* s_2 & r_2^* s_2 \\ r_2 s_2^* & s_2 ^2 & 0 \\ r_2 s_2^* & 0 & s_2 ^2 \end{pmatrix}$	$M_{2,1}^2 = \frac{ r_1 ^2 + 2 r_2 ^2 + s_2 ^2}{2}$ $\pm [(r_1 ^2 + 2 r_2 ^2 - s_2 ^2)^2 + 8 r_2 s_2]^{1/2}$ $M_3^2 = s_2 ^2$	$\psi = \arg(r_2 s_2^*)$ $\tan \phi_{12} = \frac{\sqrt{2} r_2 s_2^* }{M_2^2 - r_1 ^2 - 2r_2^2}$

Table 4: Definitions of parameters used in mass matrices for different categories.

Parameters	Definition of Parameters for Category			
	A1	A2	A3	A4
m_0	$B_2^2 \mu_2 / s_2^2$	$B_3^2 \mu_2 / s_3^2$	$B_3^2 \mu_2 / s_2^2$	$B_2^2 \mu_2 / s_3^2$
$k_1 e^{i(\alpha+\gamma)}$	$A_1 s_2 \sqrt{\mu_1 / \mu_2} / B_2 r_1$	$A_1 s_3 \sqrt{\mu_1 / \mu_2} / B_3 r_1$	$A_1 s_2 \sqrt{\mu_1 / \mu_2} / B_3 r_1$	$A_1 s_3 / \sqrt{\mu_1 / \mu_2} / B_2 r_1$
$k_2 e^{i(\beta+\gamma)}$	A_2 / B_2	A_2 / B_3	A_2 / B_3	A_2 / B_2
$k_3 e^{i\gamma}$	$A_1 r_2 / B_2 r_1$	$A_1 r_2 / B_3 r_1$	$A_1 r_2 / B_3 r_1$	$A_1 r_2 / B_2 r_1$
	B1	B2	B3	B4
m_0	$B_3^2 \mu_2 / s_3^2$	$B_2^2 \mu_2 / s_2^2$	$B_3^2 \mu_2 / s_2^2$	$B_2^2 \mu_2 / s_3^2$
$k_1 e^{i\alpha}$	$B_1 s_3 / B_3 r_1$	$B_1 s_2 / B_2 r_1$	$B_1 s_2 / B_3 r_1$	$B_1 s_3 / B_2 r_1$
$k_2 e^{i\beta}$	r_2 / r_1	r_2 / r_1	r_2 / r_1	r_2 / r_1
$k_3 e^{i\gamma}$	$A_1 s_3 \sqrt{\mu_1 / \mu_2} / B_3 r_1$	$A_1 s_2 \sqrt{\mu_1 / \mu_2} / B_2 r_1$	$A_1 s_2 \sqrt{\mu_1 / \mu_2} / B_3 r_1$	$A_1 s_3 \sqrt{\mu_1 / \mu_2} / B_2 r_1$

Table 5: Input experimental values [18]

Quantity	3σ ranges
Δ_{21}^2	$7.00 < \Delta_{21}^2 (10^5 \text{ eV}^{-2}) < 8.09$
$\Delta_{32}^2 < 0$	$-2.649 < \Delta_{32}^2 (10^3 \text{ eV}^{-2}) < -2.242$
$\Delta_{32}^2 > 0$	$2.195 < \Delta_{32}^2 (10^3 \text{ eV}^{-2}) < 2.625$
θ_{12}	$31.09^\circ < \theta_{12} < 35.89^\circ$
θ_{23}	$35.80^\circ < \theta_{23} < 54.80^\circ$
θ_{13}	$7.19^\circ < \theta_{13} < 9.96^\circ$
δ_D	Unconstrained

Parameters	Categories				
	A2	A3	A4	(B1,B2)	(B3,B4)
k_1	2.20-2.90	2.80-3.40	1.81-2.00	0.4-4.90	1.50-2.85
k_2	2.00-2.95	2.00-2.40	1.60-2.24	0.6-4.90	1.40-2.25
k_3	0.55-1.00	0.005-0.010	0.84-2.40	0.21-0.80	0.35-0.55
α (degree)	89-91	0-180	30-150	10-180	90-180
β (degree)	0	0-160	30-40	10-180	90-160
ϕ_1 (degree)	30-100	20-140	90-180	0-180	0
ϕ_2 (degree)	0-140	0	0-30	0-180	0
ϵ_1	0.06-0.12	0.06-0.15	0.12-0.15	0.0001-0.15	0.14-0.15
ϵ_2	0.03-0.11	0	0.03-0.12	0.0002-0.15	0.12-0.15
δ	0	0	0	0	0.03-0.09

Table 6: Allowed Parameter ranges

Observables	Categories				
	A2	A3	A4	(B1,B2)	(B3,B4)
Hierarchy	I	I	I	N	N
m_1 (eV)	0.050-0.060	0.049-0.055	0.053-0.061	0.003-0.068	0.017-0.026
m_2 (eV)	0.051-0.061	0.050-0.056	0.054-0.062	0.009-0.069	0.019-0.028
m_3 (eV)	0.016-0.023	0.017-0.021	0.026-0.035	0.048-0.086	0.051-0.058
Σm_i (eV)	0.115-0.134	0.116-0.132	0.134-0.158	0.061-0.224	0.09-0.113
$m_{\nu\beta\beta}$ (eV)	0.015-0.024	0.016-0.022	0.026-0.035	0.001-0.049	0.005-0.015
J_{CP}	0.026-0.036	0.002-0.035	0.002-0.03	3.7×10^{-5} -0.0311	0.001-0.03
δ_{CP} (degree)	55-90	3-85	3-67	0.1-85	21-90
α_M (degree)	-57-58	-13-9	-90-90	-90-90	-84-34
β_M (degree)	-90-90	-45-40	-90-90	-90-90	-63-10

Table 7: Predictions