

Generalized two-point tree-level amplitude $jf \rightarrow j'f'$ in a magnetized medium (extended version)

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The tree-level two-point amplitudes for the transitions $jf \rightarrow j'f'$, where f is a fermion and j is a generalized current, in a constant uniform magnetic field of an arbitrary strength and in charged fermion plasma, for the jf interaction vertices of the scalar, pseudoscalar, vector and axial-vector types have been investigated. The particular cases of a very strong magnetic field, and of the coherent scattering off the real fermions without change of their states (the “forward” scattering) have been analysed.

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I. INTRODUCTION

Nowadays, there exists rather keen interest to astrophysical objects with the scale of the magnetic field strength near the critical value of $B_e = m^2/e \simeq 4.41 \times 10^{13} \text{ G}$ ¹. This group of objects includes the radio pulsars and the so-called magnetars, which are the neutron stars featuring the magnetic field strengths from 10^{12} G (radio pulsars) to $4 \times 10^{14} \text{ G}$ (magnetars) [1–4]. The spectra analysis of these objects also provides an evidence for the presence of electron-positron plasma in the radio pulsars and magnetars environment, with the minimum magnetospheric plasma density being of the order of the Goldreich-Julian density [5]:

$$n_{GJ} \simeq 3 \cdot 10^{13} \text{ cm}^{-3} \left(\frac{B}{100B_e} \right) \left(\frac{10 \text{ s}}{P} \right). \quad (1)$$

It is well-known that strong magnetic field and/or plasma could have an essential influence on various quantum processes [6–9], because the external active medium catalyses the processes, by changing their kinematics and inducing new interactions. Therefore, the effects of magnetized plasma on microscopic physics should be incorporated in the magnetosphere models of strongly magnetized neutron stars. In the present paper we consider the two-point processes, because such reactions can have possible resonant behavior, and therefore they are very interesting for astrophysical applications [10].

The investigation of the two-point processes in an external active medium (electromagnetic field and/or plasma) has a rather long history. The most general expression for a two-vertex loop amplitude of the form $j \rightarrow f\bar{f} \rightarrow j'$ in a pure constant uniform magnetic field and in a crossed field was obtained previously in Ref. [11],

where all possible combinations of scalar, pseudoscalar, vector, and pseudovector interactions of the generalized currents j and j' with fermions were considered.

The typical example of a tree-level process with two vector vertices in the presence of magnetized plasma is the Compton scattering as a possible channel of the radiation spectra formation. This process was studied in a number of papers, see e.g. [12–17]), but the results were presented there in the form without taking account of the photon dispersion properties. In the recent paper [18] this neglect was corrected. The expression for the Compton scattering amplitude, with the initial and final electrons being on the lowest Landau level was presented in Ref. [18] in the explicit Lorentz invariant form. The other example of the Compton like process with the vector and axial-vector vertices, the photon transition into the neutrino pair, $\gamma \rightarrow \nu\bar{\nu}$, in the presence of magnetized plasma, was studied in Ref. [19]. However, the results in that paper were presented in rather cumbersome form with an implicit covariance. Those results would be poorly applicable for an analysis of the other photon-fermion scattering processes with the production of exotic particles, such as axion, neutralino, etc.

Thus, it is interesting to consider the tree-level two-point amplitude for the transition of the type $jf \rightarrow j'f'$ in a constant uniform magnetic field and charged fermion plasma, for different combinations of the vertices that were used in the paper [11]. Particularly, we generalize the results, obtained in Ref. [11] to the case of magnetized plasma, since such a situation looks the most realistic for astrophysical objects. Such a generalization was performed in part in Ref. [20] for the case of the photon polarization operator in a magnetized electron-positron plasma. The paper is organized as follows. In Sec. II, we calculate the scattering amplitudes for different spin states of the initial and final fermions and for generalized vertices of the scalar, pseudoscalar, vector or axial vector types. All the amplitudes are presented in the explicit Lorentz and gauge invariant forms. In Sec. III, we consider the particular case, when all the fermions occupy the ground Landau level (the strong field limit). A coherent scattering of neutral particles off the real fermions

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¹ We use natural units $c = \hbar = k_B = 1$, m_f is the fermion mass, and e_f is the fermion charge.

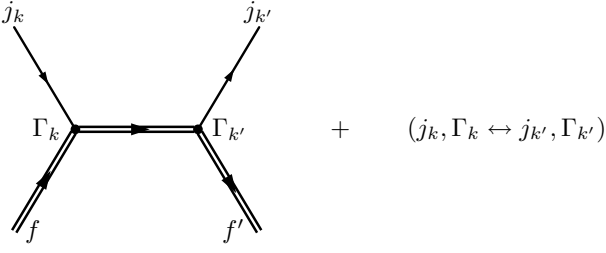


FIG. 1: The Feynman diagrams for the reaction $jf \rightarrow j'f'$. Double lines mean that the effect of the external field on the initial and final states is exactly taken into account.

without change of their states (“forward” scattering) is analysed in Sec. IV. Final comments and discussion of the obtained results and possible astrophysical applications are given in Sec. V.

II. THE SET OF EXPRESSIONS FOR THE AMPLITUDES

The generalized amplitude of the transition $jf \rightarrow j'f'$ will be analyzed by using the effective Lagrangian for the interaction of the current j with fermions in the form

$$\mathcal{L}(x) = \sum_k g_k [\bar{\psi}_f(x) \Gamma_k \psi_f(x)] j_k(x), \quad (2)$$

where the generalized index $k = S, P, V, A$ numbers the matrices Γ_k , $\Gamma_S = 1$, $\Gamma_P = \gamma_5$, $\Gamma_V = \gamma_\alpha$, $\Gamma_A = \gamma_\alpha \gamma_5$; $j_k(x)$ are the generalized currents (j_S , j_P , $j_{V\alpha}$ or $j_{A\alpha}$) or the photon polarization vectors, g_k are the coupling constants, and $\psi_f(x)$ are the fermion wave functions. The γ_5 matrix is defined as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

The S -matrix element in the tree approximation is described by the Feynman diagrams shown in Fig. 1 and has the form

$$S_{k'k}^{s's} = -g_k g_{k'} \int d^4X d^4Y j_k(X) j_{k'}(Y) \times \left[\bar{\Psi}_{p',\ell'}^{s'}(Y) \Gamma_{k'} \hat{S}(Y, X) \Gamma_k \Psi_{p,\ell}^s(X) \right]. \quad (3)$$

Here, $p^\mu = (E_\ell, \mathbf{p})$ and $p'^\mu = (E_{\ell'}, \mathbf{p}')$ are the four-momenta of the initial and final fermions correspondingly, $\Psi_{p,n}^s(X)$ are the fermion wave functions in the presence of external magnetic field, $X^\mu = (X_0, X_1, X_2, X_3)$.

There exist several descriptions of a procedure of obtaining the fermion wave functions in the presence of external magnetic field by solving the Dirac equation, see e.g. [21–27] and also [8, 9]. In the most cases, the solutions are presented in the form with the upper two components of the bispinor corresponding to the fermion states with the spin projections $1/2$ and $-1/2$ on the magnetic field direction. In this approach, we use a representation of the fermion wave functions as the eigenstates of the covariant operator $\hat{\mu}_z = m_f \Sigma_z - i\gamma_0 \gamma_5 [\boldsymbol{\Sigma} \times \hat{\mathbf{P}}]_z$ [23].

Here, $\hat{\mathbf{P}} = -i\nabla - e_f \mathbf{A}$ is the generalized momentum operator. We take the frame where the field is directed along the z axis, and the Landau gauge where the four-potential is: $A^\mu = (0, 0, xB, 0)$. It is convenient to use the notation $\beta = |e_f|B$, and to introduce the sign of the fermion charge as $\eta = e_f/|e_f|$.

Our choice of the Dirac equation solutions as the eigenfunctions of the operator $\hat{\mu}_z$ is caused by the following arguments. Calculations of the process widths with two or more vertices in an external magnetic field by the standard method, including the squaring the amplitude with all the Feynman diagrams and with summation or averaging over the fermion polarization states, contain significant computational difficulties. In this case, it is convenient to calculate partial contributions to the amplitude from the channels with different fermion polarization states and for each diagram separately, by direct multiplication of the bispinors and the Dirac matrices. The result, up to a total for both diagrams non-invariant phase will have an explicit Lorentz invariant structure. On the contrary, the amplitudes obtained with using the solutions for a fixed direction of the spin, do not have Lorentz invariant structure. Only the amplitude squared, summed over the fermion polarization states, is manifestly Lorentz-invariant.

The fermion wave functions having the form

$$\Psi_{p,n}^s(X) = \frac{e^{-i(E_n X_0 - p_y X_2 - p_z X_3)} U_n^s(\xi)}{\sqrt{4E_n M_n (E_n + M_n)(M_n + m_f) L_y L_z}}, \quad (4)$$

where

$$E_n = \sqrt{M_n^2 + p_z^2}, \quad M_n = \sqrt{m_f^2 + 2\beta n}, \quad (5)$$

are the solutions of the equation

$$\hat{\mu}_z \Psi_{p,n}^s(X) = s M_n \Psi_{p,n}^s(X), \quad s = \pm 1. \quad (6)$$

It is convenient to present the bispinors $U_n^s(\xi)$ in the form of decomposition over the solutions for negative and positive fermion charge, $U_{n,\eta}^s(\xi)$:

$$U_n^s(\xi) = \frac{1-\eta}{2} U_{n,-}^s(\xi) + \frac{1+\eta}{2} U_{n,+}^s(\xi), \quad (7)$$

where

$$U_{n,-}^s(\xi) = \begin{pmatrix} -i\sqrt{2\beta n} p_z V_{n-1}(\xi) \\ (E_n + M_n)(M_n + m_f) V_n(\xi) \\ -i\sqrt{2\beta n} (E_n + M_n) V_{n-1}(\xi) \\ -p_z (M_n + m_f) V_n(\xi) \end{pmatrix}, \quad (8)$$

$$U_{n,+}^s(\xi) = \begin{pmatrix} (E_n + M_n)(M_n + m_f) V_{n-1}(\xi) \\ -i\sqrt{2\beta n} p_z V_n(\xi) \\ p_z (M_n + m_f) V_{n-1}(\xi) \\ i\sqrt{2\beta n} (E_n + M_n) V_n(\xi) \end{pmatrix}, \quad (9)$$

$$U_{n,+}^{-}(\xi) = \begin{pmatrix} i\sqrt{2\beta n} p_z V_n(\xi) \\ (E_n + M_n)(M_n + m_f)V_{n-1}(\xi) \\ i\sqrt{2\beta n}(E_n + M_n)V_n(\xi) \\ -p_z(M_n + m_f)V_{n-1}(\xi) \end{pmatrix}, \quad (10)$$

$$U_{n,+}^{+}(\xi) = \begin{pmatrix} (E_n + M_n)(M_n + m_f)V_n(\xi) \\ i\sqrt{2\beta n} p_z V_{n-1}(\xi) \\ p_z(M_n + m_f)V_n(\xi) \\ -i\sqrt{2\beta n}(E_n + M_n)V_{n-1}(\xi) \end{pmatrix}, \quad (11)$$

$V_n(\xi)$ ($n = 0, 1, 2, \dots$) are the normalized harmonic oscillator functions, which are expressed in terms of Hermite polynomials $H_n(\xi)$ [28]:

$$V_n(\xi) = \frac{\beta^{1/4} e^{-\xi^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} H_n(\xi), \quad (12)$$

$$\xi = \sqrt{\beta} \left(X_1 - \eta \frac{p_y}{\beta} \right). \quad (13)$$

The currents j_k in Eq. (3) can be expressed through the amplitudes in the momentum space:

$$j_k(X) = \frac{e^{-i(qX)}}{\sqrt{2q_0V}} j_k(q). \quad (14)$$

We use the fermion propagator in the form of the sum over the Landau levels [9, 29]:

$$S(X, X') = \sum_{n=0}^{\infty} S_n(X, X'), \quad (15)$$

$$\begin{aligned} S_n(X, X') &= \frac{i}{2^n n!} \sqrt{\frac{\beta}{\pi}} \exp\left(-\beta \frac{X_1^2 + X_1'^2}{2}\right) \\ &\times \int \frac{dp_0 dp_y dp_z}{(2\pi)^3} \frac{e^{-i(p(X-X'))_{\parallel}}}{p_{\parallel}^2 - m^2 - 2\beta n + i\varepsilon} \\ &\times \exp\left\{-\frac{p_y^2}{\beta} - p_y [X_1 + X_1' - i(X_2 - X_2')]\right\} \\ &\times \left\{ [(p\gamma)_{\parallel} + m] [\Pi_- H_n(\xi) H_n(\xi')] \right. \\ &+ \Pi_+ 2n H_{n-1}(\xi) H_{n-1}(\xi') \\ &+ i 2n \sqrt{\beta} \gamma^1 [\Pi_- H_{n-1}(\xi) H_n(\xi') \\ &\left. - \Pi_+ H_n(\xi) H_{n-1}(\xi')] \right\}, \quad (16) \end{aligned}$$

where ξ and ξ' are defined similarly to Eq. (13).

Hereafter we use the following notations: four-vectors with the indices \perp and \parallel belong to the Euclidean $\{1, 2\}$ subspace and the Minkowski $\{0, 3\}$ subspace correspondingly. Then for arbitrary 4-vectors A_{μ}, B_{μ} one has

$$\begin{aligned} A_{\perp}^{\mu} &= (0, A_1, A_2, 0), \quad A_{\parallel}^{\mu} = (A_0, 0, 0, A_3), \\ (AB)_{\perp} &= (A\Lambda B) = A_1 B_1 + A_2 B_2, \\ (AB)_{\parallel} &= (A\tilde{\Lambda}B) = A_0 B_0 - A_3 B_3, \end{aligned}$$

where the matrices $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$, $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$ are constructed with the dimensionless tensor of the external magnetic field, $\varphi_{\mu\nu} = F_{\mu\nu}/B$, and the dual tensor, $\tilde{\varphi}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\varphi_{\rho\sigma}$. The matrices $\Lambda_{\mu\nu}$ and $\tilde{\Lambda}_{\mu\nu}$ are connected by the relation $\tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and play the role of the metric tensors in the perpendicular (\perp) and the parallel (\parallel) subspaces respectively.

After integration in Eq. (3) over d^4X and d^4Y we obtain

$$\mathcal{M}_{k'k}^{s's} = \frac{i(2\pi)^3 \delta^{(3)}(P - p' - q')}{\sqrt{2q_0V} 2q_0' V 2E_{\ell} L_y L_z 2E_{\ell'} L_y L_z} \mathcal{M}_{k'k}^{s's} \quad (17)$$

where $\delta^3(P - p' - q') = \delta(P_0 - E_{\ell'} - q_0')\delta(P_y - p_y' - q_y')\delta(P_z - p_z' - q_z')$, $P_{\alpha} = (p + q)_{\alpha}$, $\alpha = 0, 2, 3$, and the partial amplitudes $\mathcal{M}_{k'k}^{s's}$ can be presented in the following form:

$$\begin{aligned} \mathcal{M}_{k'k}^{s's} &= \frac{-\exp[-i\theta]}{2\sqrt{M_{\ell}M_{\ell'}(M_{\ell} + m_f)(M_{\ell'} + m_f)}} \quad (18) \\ &\times \left\{ \exp\left[\frac{i(q\varphi q')}{2\beta}\right] \left[\frac{q_y + iq_x}{\sqrt{q_{\perp}^2}}\right]^{-\ell} \left[\frac{q_y' - iq_x'}{\sqrt{q_{\perp}^{\prime 2}}}\right]^{-\ell'} \right. \\ &\times \sum_{n=0}^{\infty} \left(\frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_{\perp}^2 q_{\perp}^{\prime 2}}}\right)^n \frac{\mathcal{R}_{k'k}^{s's}}{P_{\parallel}^2 - m_f^2 - 2\beta n} \\ &+ (-1)^{\ell+\ell'} \exp\left[-\frac{i(q\varphi q')}{2\beta}\right] \left[\frac{q_y' + iq_x'}{\sqrt{q_{\perp}^{\prime 2}}}\right]^{-\ell} \left[\frac{q_y - iq_x}{\sqrt{q_{\perp}^2}}\right]^{-\ell'} \\ &\left. \times \sum_{n=0}^{\infty} \left(\frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_{\perp}^2 q_{\perp}^{\prime 2}}}\right)^n \frac{\mathcal{R}_{kk'}^{s's}}{P_{\parallel}^{\prime 2} - m_f^2 - 2\beta n} \right\}, \end{aligned}$$

where $\theta = (q_x - q_x')(p_y + p_y')/(2\beta)$ is the general phase for both diagrams in Fig. 1.

The main part of the problem is to calculate the values $\mathcal{R}_{k'k}^{s's}$ which are expressed via the following Lorentz covariants in the $\{0, 3\}$ -subspace

$$\begin{aligned} \mathcal{K}_{1\alpha} &= \sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_{\ell}M_{\ell'}}} \\ &\times \left\{ M_{\ell}(\tilde{\Lambda}p')_{\alpha} + M_{\ell'}(\tilde{\Lambda}p)_{\alpha} \right\}, \quad (19) \end{aligned}$$

$$\begin{aligned} \mathcal{K}_{2\alpha} &= \sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_{\ell}M_{\ell'}}} \\ &\times \left\{ M_{\ell}(\tilde{\varphi}p')_{\alpha} + M_{\ell'}(\tilde{\varphi}p)_{\alpha} \right\}, \quad (20) \end{aligned}$$

$$\mathcal{K}_3 = \sqrt{2 \left[(p\tilde{\Lambda}p') + M_\ell M_{\ell'} \right]}, \quad (21) \quad \text{where}$$

$$\mathcal{K}_4 = -\sqrt{\frac{2}{(p\tilde{\Lambda}p') + M_\ell M_{\ell'}}} (p\tilde{\varphi}p'). \quad (22)$$

$$\begin{aligned} \mathcal{I}_{n,\ell}(x) &= (-1)^{n-\ell} \mathcal{I}_{\ell,n}(x) \\ &= \sqrt{\frac{\ell!}{n!}} e^{-x/2} x^{(n-\ell)/2} L_\ell^{n-\ell}(x), \end{aligned} \quad (24)$$

The following integrals appear in the calculations:

$$\frac{1}{\sqrt{\pi}} \int dZ e^{-Z^2} H_n \left(Z + \frac{q_y + iq_x}{2\sqrt{\beta}} \right) \quad (23)$$

$$\begin{aligned} &\times H_\ell \left(Z - \frac{q_y - iq_x}{2\sqrt{\beta}} \right) \\ &= 2^{(n+\ell)/2} \sqrt{n! \ell!} \left[\frac{q_y + iq_x}{\sqrt{q_\perp^2}} \right]^{n-\ell} e^{q_\perp^2/(4\beta)} \mathcal{I}_{n,\ell} \left(\frac{q_\perp^2}{2\beta} \right), \end{aligned}$$

and $L_n^k(x)$ are the generalized Laguerre polynomials [28].

The results for $\mathcal{R}_{k'k}^{s's}$ are presented below. Hereafter we use the following definitions: $P_\alpha = (p+q)_\alpha$, $P'_\alpha = (p-q)_\alpha$, $\mathcal{I}_{n,\ell} \equiv \mathcal{I}_{n,\ell}(q_\perp^2/(2\beta))$ and $\mathcal{I}'_{n,\ell'} \equiv \mathcal{I}_{n,\ell'}(q_\perp'^2/(2\beta))$. For definiteness, we further consider the fermion with a negative charge, $\eta = -1$.

1. In the case when j and j' are scalar currents ($k, k' = S$) the calculation yields

$$\begin{aligned} \mathcal{R}_{SS}^{++} &= g_s g'_s j_s j'_s \left\{ 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \right. \\ &\quad \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_3 [\sqrt{\ell}(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f) \mathcal{I}'_{n,\ell} \mathcal{I}_{n-1,\ell-1}] \right\}; \end{aligned} \quad (25)$$

$$\begin{aligned} \mathcal{R}_{SS}^{+-} &= ig_s g'_s j_s j'_s \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_2 P) - m_f \mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_2 P) + m_f \mathcal{K}_4] \right. \\ &\quad \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_4 [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell} \mathcal{I}_{n-1,\ell-1}] \right\}; \end{aligned} \quad (26)$$

$$\begin{aligned} \mathcal{R}_{SS}^{-+} &= -ig_s g'_s j_s j'_s \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_2 P) + m_f \mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_2 P) - m_f \mathcal{K}_4] \right. \\ &\quad \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_4 [2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell} \mathcal{I}_{n-1,\ell-1}] \right\}; \end{aligned} \quad (27)$$

$$\begin{aligned} \mathcal{R}_{SS}^{--} &= g_s g'_s j_s j'_s \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \right. \\ &\quad \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_3 [\sqrt{\ell'}(M_\ell + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell} \mathcal{I}_{n-1,\ell-1}] \right\}. \end{aligned} \quad (28)$$

For second diagram we have the following replacement $P_\alpha \rightarrow P'_\alpha$, $\mathcal{I}_{m,n} \leftrightarrow \mathcal{I}'_{m,n}$.

2. In the case where j is scalar current and j' is pseudoscalar current ($k = S$, $k' = P$) we obtain

$$\begin{aligned} \mathcal{R}_{PS}^{++} &= g_s g'_p j_s j'_p \left\{ 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_2 P) + m_f \mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_2 P) - m_f \mathcal{K}_4] \right. \\ &\quad \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_4 [\sqrt{\ell}(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f) \mathcal{I}'_{n,\ell} \mathcal{I}'_{n-1,\ell-1}] \right\}; \end{aligned} \quad (29)$$

$$\begin{aligned} \mathcal{R}_{SP}^{++} = & -g_s g_p' j_s j_p' \left\{ 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_2 P') - m_f \mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_2 P') + m_f \mathcal{K}_4] \right. \\ & \left. \times \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_4 [\sqrt{\ell}(M_{\ell'} + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \right\}; \end{aligned} \quad (30)$$

$$\begin{aligned} \mathcal{R}_{PS}^{+-} = & i g_s g_p' j_s j_p' \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \right. \\ & \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_3 [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}; \end{aligned} \quad (31)$$

$$\begin{aligned} \mathcal{R}_{SP}^{+-} = & -i g_s g_p' j_s j_p' \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} + \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \right. \\ & \left. \times \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_3 [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + 2\beta\sqrt{\ell\ell'} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \right\}; \end{aligned} \quad (32)$$

$$\begin{aligned} \mathcal{R}_{PS}^{-+} = & -i g_s g_p' j_s j_p' \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \right. \\ & \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_3 [2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}; \end{aligned} \quad (33)$$

$$\begin{aligned} \mathcal{R}_{SP}^{-+} = & i g_s g_p' j_s j_p' \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} + \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \right. \\ & \left. \times \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_3 [2\beta\sqrt{\ell\ell'} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \right\}; \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{R}_{PS}^{--} = & g_s g_p' j_s j_p' \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_2 P) - m_f \mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_2 P) + m_f \mathcal{K}_4] \right. \\ & \left. \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_4 [\sqrt{\ell'}(M_\ell + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell}(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}; \end{aligned} \quad (35)$$

$$\begin{aligned} \mathcal{R}_{SP}^{--} = & -g_s g_p' j_s j_p' \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_2 P') + m_f \mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} - 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_2 P') - m_f \mathcal{K}_4] \right. \\ & \left. \times \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_4 [\sqrt{\ell'}(M_\ell + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - \sqrt{\ell}(M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \right\}. \end{aligned} \quad (36)$$

3. In the case where j is scalar current and j' is a vector current ($k = S, k' = V$) we obtain

$$\begin{aligned}
\mathcal{R}_{VS}^{++} = & g_s g'_v j_s \left\{ -2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
& - 2\beta\sqrt{n}(\mathcal{K}_1j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
& + \sqrt{\frac{2\beta}{q_\perp'^2}} (M_{\ell'} + m_f) [(q'\Lambda j') + i(q'\varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{n}(M_\ell + m_f)\mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} [(q'\Lambda j') - i(q'\varphi j')] \left[(M_\ell + m_f)[(\mathcal{K}_1P) + m_f\mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{\ell n} \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{37}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{SV}^{++} = & g_s g'_v j_s \left\{ -2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& - 2\beta\sqrt{n}(\mathcal{K}_1j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
& + \sqrt{\frac{2\beta\ell}{q_\perp'^2}} [(q'\Lambda j') + i(q'\varphi j')] \left[(M_{\ell'} + m_f) [(\mathcal{K}_1P') + m_f\mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - 2\beta\sqrt{\ell'n} \mathcal{K}_3 \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
& \left. - \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) [(q'\Lambda j') - i(q'\varphi j')] \left[\sqrt{\ell'} [(\mathcal{K}_1P') - m_f\mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - \sqrt{n}(M_{\ell'} + m_f)\mathcal{K}_3 \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} \right] \right\}; \tag{38}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{VS}^{+-} = & -ig_s g'_v j_s \left\{ \sqrt{2\beta\ell'}(M_\ell + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
& + \sqrt{2\beta\ell}(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n}(\mathcal{K}_2j') \left[(M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] - \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \\
& \times (M_{\ell'} + m_f) \left[(M_\ell + m_f)[(\mathcal{K}_2P) - m_f\mathcal{K}_4] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} + 2\beta\sqrt{n\ell} \mathcal{K}_4 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - 2\beta\sqrt{\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} + \sqrt{n}(M_\ell + m_f) \mathcal{K}_4 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{39}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{S\bar{V}}^{+-} = & \text{ig}_s g'_v j_s \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\varphi} j') \mathcal{K}_3 - (P' \tilde{\Lambda} j') \mathcal{K}_4 + m_f (\mathcal{K}_2 j') \right] \mathcal{I}_{n,\ell} \mathcal{I}'_{n,\ell} \right. \\
& + \sqrt{2\beta\ell'} (M_{\ell'} + m_f) \left[(P' \tilde{\varphi} j') \mathcal{K}_3 - (P' \tilde{\Lambda} j') \mathcal{K}_4 - m_f (\mathcal{K}_2 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& + \sqrt{2\beta n} (\mathcal{K}_2 j') \left[(M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + 2\beta \sqrt{\ell\ell'} \mathcal{I}_{n,\ell} \mathcal{I}'_{n-1,\ell-1} \right] + \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \\
& \times (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_2 P') + m_f \mathcal{K}_4] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - 2\beta \sqrt{n\ell'} \mathcal{K}_4 \mathcal{I}_{n,\ell} \mathcal{I}'_{n-1,\ell} \right] \\
& \left. + 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_2 P') - m_f \mathcal{K}_4] \mathcal{I}_{n,\ell} \mathcal{I}'_{n,\ell-1} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_4 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{40}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{V\bar{S}}^{-+} = & -\text{ig}_s g'_v j_s \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_4 + (P \tilde{\varphi} j') \mathcal{K}_3 + m_f (\mathcal{K}_2 j') \right] \mathcal{I}'_{n,\ell} \mathcal{I}_{n,\ell} \right. \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_4 + (P \tilde{\varphi} j') \mathcal{K}_3 - m_f (\mathcal{K}_2 j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n} (\mathcal{K}_2 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}'_{n,\ell} \mathcal{I}_{n-1,\ell-1} \right] \\
& - 2\beta \sqrt{\ell'} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_2 P) + m_f \mathcal{K}_4] \mathcal{I}'_{n,\ell-1} \mathcal{I}_{n,\ell} + \sqrt{n} (M_\ell + m_f) \mathcal{K}_4 \mathcal{I}'_{n,\ell-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_{\ell'} + m_f) \left[(M_\ell + m_f) [(\mathcal{K}_2 P) - m_f \mathcal{K}_4] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} - 2\beta \sqrt{n\ell} \mathcal{K}_4 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{41}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{S\bar{V}}^{-+} = & \text{ig}_s g'_v j_s \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P' \tilde{\varphi} j') \mathcal{K}_3 - (P' \tilde{\Lambda} j') \mathcal{K}_4 - m_f (\mathcal{K}_2 j') \right] \mathcal{I}_{n,\ell} \mathcal{I}'_{n,\ell} \right. \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\varphi} j') \mathcal{K}_3 - (P' \tilde{\Lambda} j') \mathcal{K}_4 + m_f (\mathcal{K}_2 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& + \sqrt{2\beta n} (\mathcal{K}_2 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n,\ell} \mathcal{I}'_{n-1,\ell-1} \right] \\
& + 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_2 P') - m_f \mathcal{K}_4] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_4 \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} \right] \\
& \left. + \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_2 P') + m_f \mathcal{K}_4] \mathcal{I}_{n,\ell} \mathcal{I}'_{n,\ell-1} - 2\beta \sqrt{n\ell'} \mathcal{K}_4 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{42}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{\bar{V}S}^- &= g_s g_v' j_s \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
&\quad - 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
&\quad + 2\beta\sqrt{n}(\mathcal{K}_1j') \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&\quad - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} [(q'\Lambda j') + i(q'\varphi j')] \left[(M_\ell + m_f)[(\mathcal{K}_1P) + m_f\mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell n} \mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
&\quad \left. + \sqrt{\frac{2\beta}{q_\perp'^2}} (M_{\ell'} + m_f) [(q'\Lambda j') - i(q'\varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} - \sqrt{n} (M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}. \tag{43}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{S\bar{V}}^- &= g_s g_v' j_s \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
&\quad - 2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&\quad + 2\beta\sqrt{n}(\mathcal{K}_1j') \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&\quad - \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) [(q'\Lambda j') + i(q'\varphi j')] \left[\sqrt{\ell'} [(\mathcal{K}_1P') - m_f\mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_3 \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
&\quad \left. + \sqrt{\frac{2\beta\ell}{q_\perp'^2}} [(q'\Lambda j') - i(q'\varphi j')] \left[(M_{\ell'} + m_f) [(\mathcal{K}_1P') + m_f\mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - 2\beta\sqrt{\ell'n} \mathcal{K}_3 \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} \right] \right\}. \tag{44}
\end{aligned}$$

4. In the case where j is scalar current and j' is a pseudovector current ($k = S, k' = A$) we obtain

$$\begin{aligned}
\mathcal{R}_{AS}^{++} = & -g_s g_a' j_s \left\{ 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{\frac{2\beta}{q_\perp'^2}} (M_{\ell'} + m_f) [(q'\Lambda j') + i(q'\varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{n}(M_\ell + m_f)\mathcal{K}_4 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} [(q'\Lambda j') - i(q'\varphi j')] \left[(M_\ell + m_f)[(\mathcal{K}_2P) - m_f\mathcal{K}_4] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} + 2\beta\sqrt{\ell n} \mathcal{K}_4 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{45}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{SA}^{++} = & -g_s g_a' j_s \left\{ 2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 + m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 - m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
& + \sqrt{\frac{2\beta\ell}{q_\perp'^2}} [(q'\Lambda j') + i(q'\varphi j')] \left[(M_{\ell'} + m_f) [(\mathcal{K}_2P') + m_f\mathcal{K}_4] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - 2\beta\sqrt{\ell'n} \mathcal{K}_4 \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
& \left. + \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) [(q'\Lambda j') - i(q'\varphi j')] \left[\sqrt{\ell'} [(\mathcal{K}_2P') - m_f\mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} - \sqrt{n}(M_{\ell'} + m_f)\mathcal{K}_4 \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} \right] \right\}; \tag{46}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{AS}^{+-} = & -ig_s g_a' j_s \left\{ \sqrt{2\beta\ell'}(M_\ell + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
& - \sqrt{2\beta\ell}(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n}(\mathcal{K}_1j') \left[(M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
& - \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} (M_{\ell'} + m_f) \left[(M_\ell + m_f)[(\mathcal{K}_1P) + m_f\mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{n\ell} \mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& \left. + 2\beta\sqrt{\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} - \sqrt{n}(M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{47}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{SA}^{+-} = & ig_s g_a' j_s \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
& - \sqrt{2\beta\ell'} (M_{\ell'} + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& - \sqrt{2\beta n} (\mathcal{K}_1 j') \left[(M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - 2\beta \sqrt{\ell\ell'} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
& - \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell'} - 2\beta \sqrt{n\ell'} \mathcal{K}_3 \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} \right] \\
& \left. + 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_3 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{48}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{AS}^{-+} = & -ig_s g_a' j_s \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_3 + (P \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
& - \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_3 + (P \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n} (\mathcal{K}_1 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
& - 2\beta \sqrt{\ell'} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{n} (M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& \left. + \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_{\ell'} + m_f) \left[(M_\ell + m_f) [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} + 2\beta \sqrt{n\ell} \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{49}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{SA}^{-+} = & ig_s g_a' j_s \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
& - \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& - \sqrt{2\beta n} (\mathcal{K}_1 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
& - 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_\ell + m_f) \mathcal{K}_3 \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} \right] \\
& \left. + \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} + 2\beta \sqrt{n\ell'} \mathcal{K}_3 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{50}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{AS}^- &= -g_s g_a' j_s \left\{ - (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
&\quad - 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
&\quad - 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&\quad + \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} \left[(q'\Lambda j') + i(q'\varphi j') \right] \left[(M_\ell + m_f)[(\mathcal{K}_2P) - m_f\mathcal{K}_4] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell n} \mathcal{K}_4 \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} \right] \\
&\quad \left. + \sqrt{\frac{2\beta}{q_\perp'^2}} (M_{\ell'} + m_f) \left[(q'\Lambda j') - i(q'\varphi j') \right] \left[\sqrt{\ell} [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} + \sqrt{n} (M_\ell + m_f)\mathcal{K}_4 \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \right\}; \tag{51}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{SA}^- &= -g_s g_a' j_s \left\{ - (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 - m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
&\quad - 2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 + m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&\quad - 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&\quad - \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) \left[(q'\Lambda j') + i(q'\varphi j') \right] \left[\sqrt{\ell'} [(\mathcal{K}_2P') + m_f\mathcal{K}_4] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_{\ell'} + m_f)\mathcal{K}_4 \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
&\quad \left. - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} \left[(q'\Lambda j') - i(q'\varphi j') \right] \left[(M_{\ell'} + m_f) [(\mathcal{K}_2P') - m_f\mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - 2\beta\sqrt{\ell'n} \mathcal{K}_4 \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} \right] \right\}; \tag{52}
\end{aligned}$$

5. In the case where j and j' are pseudoscalar currents ($k = k' = P$) we obtain

$$\begin{aligned}
\mathcal{R}_{PP}^{++} &= -g_p g_p' j_p j_p' \left\{ 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \times \right. \\
&\quad \left. \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_3 [\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}; \tag{53}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PP}^{+-} &= -ig_p g_p' j_p j_p' \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \times \right. \\
&\quad \left. \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_4 [(M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}; \tag{54}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PP}^{-+} &= ig_p g_p' j_p j_p' \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell'} (M_\ell + m_f) [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \times \right. \\
&\quad \left. \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - \sqrt{2\beta n} \mathcal{K}_4 [2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}; \tag{55}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PP}^{--} &= -g_p g_p' j_p j_p' \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \times \right. \\
&\quad \left. \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{n}\mathcal{K}_3 [\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right\}. \tag{56}
\end{aligned}$$

For second diagram we have the following replacement $P_\alpha \rightarrow P'_\alpha$, $\mathcal{I}_{m,n} \leftrightarrow \mathcal{I}'_{m,n}$.

6. In the case where j is pseudoscalar current and j' is a vector current ($k = P$, $k' = V$) we obtain

$$\begin{aligned}
\mathcal{R}_{VP}^{++} = & g_p g'_v j_p \left\{ 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{\frac{2\beta}{q_\perp'^2}} (M_{\ell'} + m_f) [(q'\Lambda j') + i(q'\varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_2P) - m_f\mathcal{K}_4]\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} + \sqrt{n}(M_\ell + m_f)\mathcal{K}_4\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} [(q'\Lambda j') - i(q'\varphi j')] \left[(M_\ell + m_f)[(\mathcal{K}_2P) + m_f\mathcal{K}_4]\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} + 2\beta\sqrt{\ell n} \mathcal{K}_4\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} \right] \right\}; \tag{57}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PV}^{++} = & -g_p g'_v j_p \left\{ 2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 - m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 + m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell-1} \right] \\
& + \sqrt{\frac{2\beta\ell}{q_\perp'^2}} [(q'\Lambda j') + i(q'\varphi j')] \left[(M_{\ell'} + m_f) [(\mathcal{K}_2P') - m_f\mathcal{K}_4]\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell} - 2\beta\sqrt{\ell'n} \mathcal{K}_4\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell} \right] \\
& \left. + \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) [(q'\Lambda j') - i(q'\varphi j')] \left[\sqrt{\ell'} [(\mathcal{K}_2P') + m_f\mathcal{K}_4]\mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell-1} - \sqrt{n}(M_{\ell'} + m_f)\mathcal{K}_4\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell-1} \right] \right\}; \tag{58}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{VP}^{+-} = & i g_p g'_v j_p \left\{ \sqrt{2\beta\ell'}(M_\ell + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} \right. \\
& - \sqrt{2\beta\ell}(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n}(\mathcal{K}_1j') \left[(M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'}\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] \\
& - \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} (M_{\ell'} + m_f) \left[(M_\ell + m_f)[(\mathcal{K}_1P) - m_f\mathcal{K}_3]\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} - 2\beta\sqrt{n\ell} \mathcal{K}_3\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1} \right] \\
& \left. + 2\beta\sqrt{\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_1P) + m_f\mathcal{K}_3]\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} - \sqrt{n}(M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} \right] \right\}; \tag{59}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{P\bar{V}}^+ &= ig_p g'_v j_p \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n,\ell} \mathcal{I}'_{n,\ell} \right. \\
&\quad - \sqrt{2\beta\ell'} (M_{\ell'} + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&\quad - \sqrt{2\beta n} (\mathcal{K}_1 j') \left[(M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - 2\beta \sqrt{\ell\ell'} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&\quad - \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell'} - 2\beta \sqrt{n\ell'} \mathcal{K}_3 \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} \right] \\
&\quad \left. + 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_3 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{60}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{\bar{V}P}^- &= ig_p g'_v j_p \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_3 + (P \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
&\quad - \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_3 + (P \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
&\quad + \sqrt{2\beta n} (\mathcal{K}_1 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&\quad - 2\beta \sqrt{\ell'} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{n} (M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
&\quad \left. + \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_{\ell'} + m_f) \left[(M_\ell + m_f) [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} + 2\beta \sqrt{n\ell} \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{61}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{P\bar{V}}^- &= ig_p g'_v j_p \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
&\quad - \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&\quad - \sqrt{2\beta n} (\mathcal{K}_1 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&\quad - 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_\ell + m_f) \mathcal{K}_3 \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} \right] \\
&\quad \left. + \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} + 2\beta \sqrt{n\ell'} \mathcal{K}_3 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{62}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{VP}^- &= g_p g'_v j_p \left\{ - (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
&- 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
&- 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&+ \sqrt{\frac{2\beta\ell'}{q_\perp^2}} [(q'\Lambda j') + i(q'\varphi j')] \left[(M_\ell + m_f)[(\mathcal{K}_2P) + m_f\mathcal{K}_4] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell n} \mathcal{K}_4 \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} \right] \\
&\left. + \sqrt{\frac{2\beta}{q_\perp^2}} (M_{\ell'} + m_f) [(q'\Lambda j') - i(q'\varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} + \sqrt{n} (M_\ell + m_f) \mathcal{K}_4 \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \right\}; \tag{63}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PV}^- &= -g_p g'_v j_p \left\{ - (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 + m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
&- 2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 - m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&- 2\beta\sqrt{n}(\mathcal{K}_2j') \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&- \sqrt{\frac{2\beta}{q_\perp^2}} (M_\ell + m_f) [(q'\Lambda j') + i(q'\varphi j')] \left[\sqrt{\ell'} [(\mathcal{K}_2P') - m_f\mathcal{K}_4] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_4 \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
&\left. - \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(q'\Lambda j') - i(q'\varphi j')] \left[(M_{\ell'} + m_f) [(\mathcal{K}_2P') + m_f\mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - 2\beta\sqrt{\ell'n} \mathcal{K}_4 \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} \right] \right\}; \tag{64}
\end{aligned}$$

7. In the case where j is pseudoscalar current and j' is a pseudovector current ($k = P$, $k' = A$) we obtain

$$\begin{aligned}
\mathcal{R}_{AP}^{++} &= -g_p g'_a j_p \left\{ - 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
&+ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
&- 2\beta\sqrt{n}(\mathcal{K}_1j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&+ \sqrt{\frac{2\beta}{q_\perp^2}} (M_{\ell'} + m_f) [(q'\Lambda j') + i(q'\varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{n}(M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
&\left. - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} [(q'\Lambda j') - i(q'\varphi j')] \left[(M_\ell + m_f)[(\mathcal{K}_1P) - m_f\mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{\ell n} \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}; \tag{65}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PA}^{++} = & g_p g_a' j_p \left\{ -2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j') \right] \mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j') \right] \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell-1} \\
& - 2\beta\sqrt{n}(\mathcal{K}_1j') \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell-1} \right] \\
& + \sqrt{\frac{2\beta\ell}{q_\perp'^2}} \left[(q'\Lambda j') + i(q'\varphi j') \right] \left[(M_{\ell'} + m_f) \left[(\mathcal{K}_1P') - m_f\mathcal{K}_3 \right] \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell} - 2\beta\sqrt{\ell'n}\mathcal{K}_3\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell} \right] \\
& \left. - \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) \left[(q'\Lambda j') - i(q'\varphi j') \right] \left[\sqrt{\ell'} \left[(\mathcal{K}_1P') + m_f\mathcal{K}_3 \right] \mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell-1} - \sqrt{n}(M_{\ell'} + m_f)\mathcal{K}_3\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell-1} \right] \right\}; \tag{66}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{AP}^{+-} = & ig_p g_a' j_p \left\{ \sqrt{2\beta\ell'}(M_\ell + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} \right. \\
& + \sqrt{2\beta\ell}(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n}(\mathcal{K}_2j') \left[(M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'}\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] - \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \\
& \times (M_{\ell'} + m_f) \left[(M_\ell + m_f)[(\mathcal{K}_2P) + m_f\mathcal{K}_4]\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} + 2\beta\sqrt{n\ell}\mathcal{K}_4\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - 2\beta\sqrt{\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_2P) - m_f\mathcal{K}_4]\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} + \sqrt{n}(M_\ell + m_f)\mathcal{K}_4\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} \right] \right\}; \tag{67}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PA}^{+-} = & ig_p g_a' j_p \left\{ \sqrt{2\beta\ell'}(M_\ell + m_f) \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 - m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell} \right. \\
& + \sqrt{2\beta\ell}(M_{\ell'} + m_f) \left[(P'\tilde{\varphi}j')\mathcal{K}_3 - (P'\tilde{\Lambda}j')\mathcal{K}_4 + m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell-1} \\
& + \sqrt{2\beta n}(\mathcal{K}_2j') \left[(M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell} + 2\beta\sqrt{\ell\ell'}\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell-1} \right] + \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \\
& \times (M_\ell + m_f) \left[(M_{\ell'} + m_f)[(\mathcal{K}_2P') - m_f\mathcal{K}_4]\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell} - 2\beta\sqrt{n\ell'}\mathcal{K}_4\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell} \right] \\
& \left. + 2\beta\sqrt{\ell} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_2P') + m_f\mathcal{K}_4]\mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell-1} - \sqrt{n}(M_{\ell'} + m_f)\mathcal{K}_4\mathcal{I}_{n,\ell'-1}\mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{68}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{AP}^{-+} = & ig_p g_a' j_p \left\{ \sqrt{2\beta\ell}(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} \right. \\
& + \sqrt{2\beta\ell'}(M_\ell + m_f) \left[(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} \\
& + \sqrt{2\beta n}(\mathcal{K}_2j') \left[2\beta\sqrt{\ell\ell'}\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] \\
& - 2\beta\sqrt{\ell'} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell} [(\mathcal{K}_2P) - m_f\mathcal{K}_4]\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} + \sqrt{n}(M_\ell + m_f)\mathcal{K}_4\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1} \right] \\
& \left. - \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} (M_{\ell'} + m_f) \left[(M_\ell + m_f)[(\mathcal{K}_2P) + m_f\mathcal{K}_4]\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} - 2\beta\sqrt{n\ell}\mathcal{K}_4\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} \right] \right\}; \tag{69}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PA}^- &= ig_p g_a' j_p \left\{ \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P' \tilde{\varphi} j') \mathcal{K}_3 - (P' \tilde{\Lambda} j') \mathcal{K}_4 + m_f (\mathcal{K}_2 j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
&+ \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\varphi} j') \mathcal{K}_3 - (P' \tilde{\Lambda} j') \mathcal{K}_4 - m_f (\mathcal{K}_2 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&+ \sqrt{2\beta n} (\mathcal{K}_2 j') \left[2\beta \sqrt{\ell\ell'} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + (M_\ell + m_f) (M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&+ 2\beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \left[\sqrt{\ell'} [(\mathcal{K}_2 P') + m_f \mathcal{K}_4] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_4 \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} \right] \\
&\left. + \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} (M_\ell + m_f) \left[(M_{\ell'} + m_f) [(\mathcal{K}_2 P') - m_f \mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - 2\beta \sqrt{n\ell'} \mathcal{K}_4 \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \right\}; \tag{70}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{AP}^- &= -g_p g_a' j_p \left\{ (M_\ell + m_f) (M_{\ell'} + m_f) \left[(P \tilde{\Lambda} j') \mathcal{K}_3 + (P \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. \\
&- 2\beta \sqrt{\ell\ell'} \left[(P \tilde{\Lambda} j') \mathcal{K}_3 + (P \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \\
&+ 2\beta \sqrt{n} (\mathcal{K}_1 j') \left[\sqrt{\ell'} (M_\ell + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell} (M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&- \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} [(q' \Lambda j') + i(q' \varphi j')] \left[(M_\ell + m_f) [(\mathcal{K}_1 P) - m_f \mathcal{K}_3] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta \sqrt{\ell n} \mathcal{K}_3 \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
&\left. + \sqrt{\frac{2\beta}{q_\perp'^2}} (M_{\ell'} + m_f) [(q' \Lambda j') - i(q' \varphi j')] \left[\sqrt{\ell} [(\mathcal{K}_1 P) + m_f \mathcal{K}_3] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} - \sqrt{n} (M_\ell + m_f) \mathcal{K}_3 \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} \right] \right\}. \tag{71}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{PA}^- &= g_p g_a' j_p \left\{ (M_\ell + m_f) (M_{\ell'} + m_f) \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} \right. \\
&- 2\beta \sqrt{\ell\ell'} \left[(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j') \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
&+ 2\beta \sqrt{n} (\mathcal{K}_1 j') \left[\sqrt{\ell'} (M_\ell + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - \sqrt{\ell} (M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1} \right] \\
&- \sqrt{\frac{2\beta}{q_\perp'^2}} (M_\ell + m_f) [(q' \Lambda j') + i(q' \varphi j')] \left[\sqrt{\ell'} [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} - \sqrt{n} (M_{\ell'} + m_f) \mathcal{K}_3 \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
&\left. + \sqrt{\frac{2\beta\ell}{q_\perp'^2}} [(q' \Lambda j') - i(q' \varphi j')] \left[(M_{\ell'} + m_f) [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} - 2\beta \sqrt{\ell' n} \mathcal{K}_3 \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} \right] \right\}. \tag{72}
\end{aligned}$$

8. Both vertices are vectors ($k = k' = V$):

$$\begin{aligned}
\mathcal{R}_{VV}^{++} = & g_v g_v' \left\{ 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) - m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \right. & (73) \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) + m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \\
& \times \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} + 2\beta\sqrt{n} [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] [\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \\
& - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} (M_{\ell'} + m_f) [(q'\Lambda j') + i(q'\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f) [(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f) [(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} (M_{\ell'} + m_f) [(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& + 2\beta\sqrt{\ell\ell'} [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta n} (\mathcal{K}_1j) \left[2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{2\beta n} (\mathcal{K}_1j') \left[(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} + 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \\
& + 2\beta\sqrt{n} \mathcal{K}_3 \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \right. \\
& \left. + \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \left. \right\};
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{VV}^{\pm\bar{v}} = & \text{ig}_v g'_v \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P) \right. \right. \\
& - m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \\
& \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_2P) + m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right. \\
& + \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j)] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j) + i(q'\varphi j)}{\sqrt{q_\perp'^2}} [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& - 2\beta\sqrt{n} (\mathcal{K}_2j) \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} - \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& + 2\beta\sqrt{n} (\mathcal{K}_2j') \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} - \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \\
& - \sqrt{2\beta n} \mathcal{K}_4 \left[2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \right. \\
& \left. - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell} \right] \Big\} ;
\end{aligned} \tag{74}$$

$$\begin{aligned}
\mathcal{R}_{VV}^{\bar{+}} = & \text{ig}_v g'_v \left\{ -\sqrt{2\beta\ell} (M_\ell + m'_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P) \right. \right. \\
& + m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \\
& \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_2P) - m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right. \\
& - \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] [2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q'\Lambda j') + i(q'\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_\perp^2}} [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2} \sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2} \sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& + 2\beta\sqrt{n} (\mathcal{K}_2j) \left[\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} - \sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& - 2\beta\sqrt{n} (\mathcal{K}_2j') \left[\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} - \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \\
& + \sqrt{2\beta n} \mathcal{K}_4 [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \\
& \left. - 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell} \right] \left. \right\};
\end{aligned} \tag{75}$$

$$\begin{aligned}
\mathcal{R}_{\bar{V}\bar{V}} = g_v g_v' & \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) \right. \right. \\
& + m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') \right. \\
& - (j\tilde{\Lambda}j')(\mathcal{K}_1P) - m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \left. \right] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} (M_\ell + m_f)[(q'\Lambda j') + i(q'\varphi j')][(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)]\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')]\mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell-1} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j') - i(q\varphi j')][(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)]\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} (M_\ell + m_f)[(q'\Lambda j) + i(q'\varphi j)][(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')]\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell} \\
& + 2\beta\sqrt{\ell\ell'} [(j\Lambda j') + i(j\varphi j')][(\mathcal{K}_1P) + m_f\mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell-1} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta n} (\mathcal{K}_1j) [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1}] \\
& - \sqrt{2\beta n} (\mathcal{K}_1j') [2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell-1} + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell}] \\
& + 2\beta\sqrt{n} \mathcal{K}_3 [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell-1} \\
& + \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell}] \left. \right\}.
\end{aligned} \tag{76}$$

For second diagram we have the following replacement $P_\alpha \rightarrow P'_\alpha$, $q_\alpha \leftrightarrow -q'_\alpha$, $j_\alpha \leftrightarrow j'_\alpha$, $\mathcal{I}_{m,n} \leftrightarrow \mathcal{I}'_{m,n}$.

9. In the case where j is a vector current and j' is a pseudovector current ($k = V, k' = A$) we obtain

$$\begin{aligned}
\mathcal{R}_{AV}^{++} = & g_v g_a' \left\{ 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P) \right. \right. \\
& + m_f[(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \\
& - (j\tilde{\Lambda}j')(\mathcal{K}_2P) - m_f[(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \left. \right] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}[(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} - \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} (M_{\ell'} + m_f)[(q'\Lambda j') + i(q'\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j)] \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} \\
& + \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f)[(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j')] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell-1} \\
& + \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f)[(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j)] \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} (M_{\ell'} + m_f)[(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j')] \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f)[(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell-1} \\
& + 2\beta\sqrt{\ell\ell'} [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta n} (\mathcal{K}_2j) \left[2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1} \right] \\
& + \sqrt{2\beta n} (\mathcal{K}_2j') \left[(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell-1} - 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell} \right] \\
& - 2\beta\sqrt{n} \mathcal{K}_4 \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell-1} \right. \\
& \left. - \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell} \right] \left. \right\};
\end{aligned} \tag{77}$$

$$\begin{aligned}
\mathcal{R}_{VA}^{++} = g_v g_a' & \left\{ 2\beta\sqrt{\ell\ell'} \left[(P'\tilde{\Lambda}j')(\mathcal{K}_2j) + (P'\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P') \right. \right. \\
& - m_f[(j\tilde{\Lambda}j')\mathcal{K}_4 - (j\tilde{\varphi}j')\mathcal{K}_3] \mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \left. \left[(P'\tilde{\Lambda}j')(\mathcal{K}_2j) + (P'\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \right. \\
& - (j\tilde{\Lambda}j')(\mathcal{K}_2P') + m_f[(j\tilde{\Lambda}j')\mathcal{K}_4 - (j\tilde{\varphi}j')\mathcal{K}_3] \left. \right] \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell-1} \\
& + 2\beta\sqrt{n}[(j\tilde{\Lambda}j')\mathcal{K}_4 - (j\tilde{\varphi}j')\mathcal{K}_3] \left[\sqrt{\ell'}(M_{\ell'} + m_f)\mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell} - \sqrt{\ell}(M_\ell + m_f)\mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell-1} \right] \\
& + \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j) + i(q\varphi j)] \left[(P'\tilde{\Lambda}j')\mathcal{K}_4 - (P'\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n,\ell'-1}\mathcal{I}'_{n,\ell} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} (M_\ell + m_f)[(q'\Lambda j') - i(q'\varphi j')] \left[(P'\tilde{\Lambda}j)\mathcal{K}_4 + (P'\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j) \right] \mathcal{I}_{n,\ell'}\mathcal{I}'_{n,\ell-1} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} (M_\ell + m_f)[(q'\Lambda j) - i(q'\varphi j)] \left[(P'\tilde{\Lambda}j')\mathcal{K}_4 - (P'\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j') \right] \mathcal{I}_{n-1,\ell'}\mathcal{I}'_{n-1,\ell-1} \\
& + \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j') + i(q\varphi j')] \left[(P'\tilde{\Lambda}j)\mathcal{K}_4 + (P'\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j) \right] \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n-1,\ell} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f)[(j\Lambda j') - i(j\varphi j')] \left[(\mathcal{K}_2P') - m_f\mathcal{K}_4 \right] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n,\ell'-1}\mathcal{I}'_{n,\ell-1} \\
& + 2\beta\sqrt{\ell\ell'} [(j\Lambda j') + i(j\varphi j')] \left[(\mathcal{K}_2P') + m_f\mathcal{K}_4 \right] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n-1,\ell'}\mathcal{I}'_{n-1,\ell} \\
& + \sqrt{2\beta n} (\mathcal{K}_2j') \left[2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n-1,\ell'}\mathcal{I}'_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n,\ell'-1}\mathcal{I}'_{n-1,\ell-1} \right] \\
& - \sqrt{2\beta n} (\mathcal{K}_2j) \left[(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'-1}\mathcal{I}'_{n,\ell-1} - 2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'}\mathcal{I}'_{n-1,\ell} \right] \\
& - 2\beta\sqrt{n} \mathcal{K}_4 \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'}\mathcal{I}'_{n,\ell-1} \right. \\
& \left. - \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'-1}\mathcal{I}'_{n-1,\ell} \right] \left. \right\};
\end{aligned} \tag{78}$$

$$\begin{aligned}
\mathcal{R}_{AV}^{+-} = & \text{ig}_v g'_a \left\{ -\sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) \right. \right. \\
& - m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') \right. \\
& \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_1P) - m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right. \\
& + \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
& - \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& - \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j) + i(q'\varphi j)}{\sqrt{q_\perp'^2}} [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& + \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& - 2\beta\sqrt{n} (\mathcal{K}_1j) \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} + \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& - 2\beta\sqrt{n} (\mathcal{K}_1j') \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} + \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \\
& + \sqrt{2\beta n} \mathcal{K}_3 \left[2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \right. \\
& \left. + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell} \right] \left. \right\};
\end{aligned}
\tag{79}$$

$$\begin{aligned}
\mathcal{R}_{VA}^{+-} = & \text{ig}_v g'_a \left\{ -\sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P'\tilde{\Lambda}j')(\mathcal{K}_1j) + (P'\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P') \right. \right. \\
& + m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 - (j\tilde{\varphi}j')\mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} + \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P'\tilde{\Lambda}j')(\mathcal{K}_1j) + (P'\tilde{\Lambda}j)(\mathcal{K}_1j') \right. \\
& \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_1P') + m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 - (j\tilde{\varphi}j')\mathcal{K}_4] \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right. \\
& + \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_3 - (j\tilde{\varphi}j')\mathcal{K}_4] [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + 2\beta\sqrt{\ell\ell'} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} [(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')] \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n,\ell} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp'^2}} [(q'\Lambda j') - i(q'\varphi j')] [(P'\tilde{\Lambda}j)\mathcal{K}_3 + (P'\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp'^2}} [(q'\Lambda j) - i(q'\varphi j)] [(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')] \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n-1,\ell-1} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q_\perp^2}} [(P'\tilde{\Lambda}j)\mathcal{K}_3 + (P'\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} \\
& + \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_1P') - m_f\mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n,\ell-1} \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_1P') + m_f\mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n-1,\ell} \\
& + 2\beta\sqrt{n} (\mathcal{K}_1j') \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} + \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \right] \\
& + 2\beta\sqrt{n} (\mathcal{K}_1j) \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} + \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell} \right] \\
& + \sqrt{2\beta n} \mathcal{K}_3 \left[2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell-1} \right. \\
& \left. + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell} \right] \left. \right\};
\end{aligned} \tag{80}$$

$$\begin{aligned}
\mathcal{R}_{AV}^- = & \text{ig}_v g_a \left\{ -\sqrt{2\beta\ell} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) \right. \right. \\
& - m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') \right. \\
& \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_1P) + m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right. \\
& - \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] [2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp'^2}} [(q'\Lambda j') + i(q'\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_\perp^2}} [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp'^2}} [(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_1P) - m_f \mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_1P) + m_f \mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& + 2\beta\sqrt{n} (\mathcal{K}_1j) \left[\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} + \sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& + 2\beta\sqrt{n} (\mathcal{K}_1j') \left[\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} + \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \\
& - \sqrt{2\beta n} \mathcal{K}_3 [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \\
& \left. + 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell} \right] \left. \right\};
\end{aligned} \tag{81}$$

$$\begin{aligned}
\mathcal{R}_{VA}^- = & \text{ig}_v g'_a \left\{ -\sqrt{2\beta\ell} (M_\ell + m_f) \left[(P' \tilde{\Lambda} j') (\mathcal{K}_1 j) + (P' \tilde{\Lambda} j) (\mathcal{K}_1 j') - (j \tilde{\Lambda} j') (\mathcal{K}_1 P') \right. \right. \\
& + m_f [(j \tilde{\Lambda} j') \mathcal{K}_3 - (j \tilde{\varphi} j') \mathcal{K}_4] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} - \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P' \tilde{\Lambda} j') (\mathcal{K}_1 j) + (P' \tilde{\Lambda} j) (\mathcal{K}_1 j') \right. \\
& - (j \tilde{\Lambda} j') (\mathcal{K}_1 P') - m_f [(j \tilde{\Lambda} j') \mathcal{K}_3 - (j \tilde{\varphi} j') \mathcal{K}_4] \left. \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} \\
& - \sqrt{2\beta n} [(j \tilde{\Lambda} j') \mathcal{K}_3 - (j \tilde{\varphi} j') \mathcal{K}_4] [2\beta\sqrt{\ell\ell'} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \\
& - \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j) + i(q\varphi j)] [(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 + m_f (\mathcal{K}_1 j')] \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n,\ell} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} [(P' \tilde{\Lambda} j) \mathcal{K}_3 + (P' \tilde{\varphi} j) \mathcal{K}_4 + m_f (\mathcal{K}_1 j)] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q' \Lambda j) - i(q' \varphi j)}{\sqrt{q_\perp'^2}} [(P' \tilde{\Lambda} j') \mathcal{K}_3 - (P' \tilde{\varphi} j') \mathcal{K}_4 - m_f (\mathcal{K}_1 j')] \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n-1,\ell-1} \\
& - \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j') + i(q\varphi j')] [(P' \tilde{\Lambda} j) \mathcal{K}_3 + (P' \tilde{\varphi} j) \mathcal{K}_4 - m_f (\mathcal{K}_1 j)] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} \\
& - \sqrt{2\beta\ell'} (M_\ell + m_f) [(j \Lambda j') - i(j \varphi j')] [(\mathcal{K}_1 P') + m_f \mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n,\ell-1} \\
& - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j \Lambda j') + i(j \varphi j')] [(\mathcal{K}_1 P') - m_f \mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n-1,\ell} \\
& - 2\beta\sqrt{n} (\mathcal{K}_1 j') [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} + \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1}] \\
& - 2\beta\sqrt{n} (\mathcal{K}_1 j) [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} + \sqrt{\ell'} (M_\ell + m_f) \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell}] \\
& - \sqrt{2\beta n} \mathcal{K}_3 [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell-1} \\
& + 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell}] \left. \right\};
\end{aligned} \tag{82}$$

$$\begin{aligned}
\mathcal{R}_{AV}^- &= g_v g_a' \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P) \right. \right. \\
&\quad \left. \left. - m_f[(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \right. \\
&\quad \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_2P) + m_f[(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} + 2\beta\sqrt{n}[(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right. \\
&\quad \times \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
&\quad - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} (M_\ell + m_f)[(q'\Lambda j') + i(q'\varphi j')][(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
&\quad + \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j) - i(q\varphi j)][(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
&\quad + \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
&\quad - \sqrt{\frac{2\beta\ell'}{q_\perp'^2}} (M_\ell + m_f)[(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
&\quad - 2\beta\sqrt{\ell\ell'} [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
&\quad + (M_\ell + m_f)(M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
&\quad - \sqrt{2\beta n} (\mathcal{K}_2j) [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1}] \\
&\quad + \sqrt{2\beta n} (\mathcal{K}_2j') [2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell}] \\
&\quad - 2\beta\sqrt{n} \mathcal{K}_4 [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \\
&\quad \left. - \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell}] \right\};
\end{aligned} \tag{83}$$

$$\begin{aligned}
\mathcal{R}_{V\bar{A}}^- &= g_v g_a' \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P' \tilde{\Lambda} j')(\mathcal{K}_2 j) + (P' \tilde{\Lambda} j)(\mathcal{K}_2 j') - (j \tilde{\Lambda} j')(\mathcal{K}_2 P') \right. \right. \\
&+ m_f [(j \tilde{\Lambda} j') \mathcal{K}_4 - (j \tilde{\varphi} j') \mathcal{K}_3] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell} - 2\beta \sqrt{\ell \ell'} \left[(P' \tilde{\Lambda} j')(\mathcal{K}_2 j) + (P' \tilde{\Lambda} j)(\mathcal{K}_2 j') \right. \\
&- (j \tilde{\Lambda} j')(\mathcal{K}_2 P') - m_f [(j \tilde{\Lambda} j') \mathcal{K}_4 - (j \tilde{\varphi} j') \mathcal{K}_3] \left. \right] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell-1} + 2\beta \sqrt{n} [(j \tilde{\Lambda} j') \mathcal{K}_4 - (j \tilde{\varphi} j') \mathcal{K}_3] \\
&\times [\sqrt{\ell'} (M_\ell + m_f) \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell} - \sqrt{\ell} (M_{\ell'} + m_f) \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell-1}] \\
&+ \sqrt{\frac{2\beta \ell'}{q_\perp^2}} (M_\ell + m_f) [(q \Lambda j) + i(q \varphi j)] [(P' \tilde{\Lambda} j') \mathcal{K}_4 - (P' \tilde{\varphi} j') \mathcal{K}_3 + m_f (\mathcal{K}_2 j')] \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n,\ell} \\
&- \sqrt{\frac{2\beta \ell}{q_\perp'^2}} (M_{\ell'} + m_f) [(q' \Lambda j') - i(q' \varphi j')] [(P' \tilde{\Lambda} j) \mathcal{K}_4 + (P' \tilde{\varphi} j) \mathcal{K}_3 + m_f (\mathcal{K}_2 j)] \mathcal{I}_{n,\ell'} \mathcal{I}'_{n,\ell-1} \\
&- \sqrt{\frac{2\beta \ell}{q_\perp'^2}} (M_{\ell'} + m_f) [(q' \Lambda j) - i(q' \varphi j)] [(P' \tilde{\Lambda} j') \mathcal{K}_4 - (P' \tilde{\varphi} j') \mathcal{K}_3 - m_f (\mathcal{K}_2 j')] \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n-1,\ell-1} \\
&+ \sqrt{\frac{2\beta \ell'}{q_\perp^2}} (M_\ell + m_f) [(q \Lambda j') + i(q \varphi j')] [(P' \tilde{\Lambda} j) \mathcal{K}_4 + (P' \tilde{\varphi} j) \mathcal{K}_3 - m_f (\mathcal{K}_2 j)] \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n-1,\ell} \\
&- 2\beta \sqrt{\ell \ell'} [(j \Lambda j') - i(j \varphi j')] [(\mathcal{K}_2 P') + m_f \mathcal{K}_4] \frac{(q \Lambda q') + i(q \varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n,\ell-1} \\
&+ (M_\ell + m_f)(M_{\ell'} + m_f) [(j \Lambda j') + i(j \varphi j')] [(\mathcal{K}_2 P') - m_f \mathcal{K}_4] \frac{(q \Lambda q') - i(q \varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n-1,\ell} \\
&+ \sqrt{2\beta n} (\mathcal{K}_2 j') [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q \Lambda j) - i(q \varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell} - 2\beta \sqrt{\ell \ell'} \frac{(q \Lambda j) + i(q \varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell-1}] \\
&- \sqrt{2\beta n} (\mathcal{K}_2 j) [2\beta \sqrt{\ell \ell'} \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'-1} \mathcal{I}'_{n,\ell-1} - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'} \mathcal{I}'_{n-1,\ell}] \\
&- 2\beta \sqrt{n} \mathcal{K}_4 [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q \Lambda j) - i(q \varphi j)}{\sqrt{q_\perp^2}} \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n-1,\ell'} \mathcal{I}'_{n,\ell-1} \\
&- \sqrt{\ell'} (M_\ell + m_f) \frac{(q \Lambda j) + i(q \varphi j)}{\sqrt{q_\perp^2}} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}_{n,\ell'-1} \mathcal{I}'_{n-1,\ell}] \left. \right\}.
\end{aligned} \tag{84}$$

10. In the case where j and j' are pseudovector currents ($k = k' = A$) we obtain

$$\begin{aligned}
\mathcal{R}_{AA}^{++} = & g_a g_a' \left\{ 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) + m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell} \right. \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) - m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \\
& \times \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell-1} + 2\beta\sqrt{n}[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \left[\sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell} + \sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} (M_{\ell'} + m_f)[(q'\Lambda j') + i(q'\varphi j')][(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)]\mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f)[(q\Lambda j) - i(q\varphi j)][(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')]\mathcal{I}'_{n,\ell'}\mathcal{I}_{n,\ell-1} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f)[(q\Lambda j') - i(q\varphi j')][(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)]\mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell-1} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp'^2}} (M_{\ell'} + m_f)[(q'\Lambda j) + i(q'\varphi j)][(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')]\mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n-1,\ell} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f)[(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n,\ell-1} \\
& + 2\beta\sqrt{\ell\ell'} [(j\Lambda j') - i(j\varphi j')][(\mathcal{K}_1P) - m_f\mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta n} (\mathcal{K}_1j) \left[2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1}\mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{2\beta n} (\mathcal{K}_1j') \left[(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1}\mathcal{I}_{n,\ell-1} + 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell} \right] \\
& + 2\beta\sqrt{n} \mathcal{K}_3 \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'}\mathcal{I}_{n,\ell-1} \right. \\
& \left. + \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'}\mathcal{I}_{n-1,\ell} \right] \left. \right\};
\end{aligned} \tag{85}$$

$$\begin{aligned}
\mathcal{R}_{AA}^{+-} = & ig_a g_a' \left\{ \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P) \right. \right. \\
& + m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} - \sqrt{2\beta\ell} (M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \\
& - (j\tilde{\Lambda}j')(\mathcal{K}_2P) - m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \left. \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} + \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right. \\
& \times \left. \left. [(M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - 2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \right. \right. \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \left. \left. [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \right. \right. \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j) + i(q'\varphi j)}{\sqrt{q_\perp'^2}} \left. \left. [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \right. \right. \\
& - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') + i(j\varphi j')] \left. \left. [(\mathcal{K}_2P) + m_f\mathcal{K}_4] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \right. \right. \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') - i(j\varphi j')] \left. \left. [(\mathcal{K}_2P) - m_f\mathcal{K}_4] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \right. \right. \\
& - 2\beta\sqrt{n} (\mathcal{K}_2j) \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} - \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right] \\
& + 2\beta\sqrt{n} (\mathcal{K}_2j') \left[\sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} - \sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell} \right] \\
& - \sqrt{2\beta n} \mathcal{K}_4 \left[2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \right. \\
& \left. \left. - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell} \right] \right\};
\end{aligned} \tag{86}$$

$$\begin{aligned}
\mathcal{R}_{AA}^- = & \text{ig}_a g_a' \left\{ -\sqrt{2\beta\ell} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') - (j\tilde{\Lambda}j')(\mathcal{K}_2P) \right. \right. \\
& - m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + \sqrt{2\beta\ell'} (M_\ell + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_2j) + (P\tilde{\Lambda}j)(\mathcal{K}_2j') \right. \\
& - (j\tilde{\Lambda}j')(\mathcal{K}_2P) + m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \left. \left. \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right. \right. \\
& - \sqrt{2\beta n} [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] [2\beta\sqrt{\ell\ell'} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1}] \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp'^2}} [(q'\Lambda j') + i(q'\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 - m_f(\mathcal{K}_2j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_\perp^2}} [(P\tilde{\Lambda}j)\mathcal{K}_4 - (P\tilde{\varphi}j)\mathcal{K}_3 + m_f(\mathcal{K}_2j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& + \frac{2\beta\sqrt{\ell\ell'}}{\sqrt{q_\perp'^2}} [(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& + \sqrt{2\beta\ell'} (M_\ell + m_f) [(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_2P) - m_f \mathcal{K}_4] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& - \sqrt{2\beta\ell} (M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_2P) + m_f \mathcal{K}_4] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& + 2\beta\sqrt{n} (\mathcal{K}_2j) [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} - \sqrt{\ell'} (M_\ell + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1}] \\
& - 2\beta\sqrt{n} (\mathcal{K}_2j') [\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} - \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell}] \\
& + \sqrt{2\beta n} \mathcal{K}_4 [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} \\
& - 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell}] \left. \right\};
\end{aligned} \tag{87}$$

$$\begin{aligned}
\mathcal{R}_{AA}^- = g_a g_a' & \left\{ (M_\ell + m_f)(M_{\ell'} + m_f) \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') - (j\tilde{\Lambda}j')(\mathcal{K}_1P) \right. \right. \\
& - m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \left[(P\tilde{\Lambda}j')(\mathcal{K}_1j) + (P\tilde{\Lambda}j)(\mathcal{K}_1j') \right. \\
& \left. \left. - (j\tilde{\Lambda}j')(\mathcal{K}_1P) + m_f[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell-1} \right. \\
& + 2\beta\sqrt{n}[(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \left[\sqrt{\ell'}(M_\ell + m_f)\mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell} + \sqrt{\ell}(M_{\ell'} + m_f)\mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell-1} \right] \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f)[(q'\Lambda j') + i(q'\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 - m_f(\mathcal{K}_1j)] \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j) - i(q\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1j')] \mathcal{I}'_{n,\ell'} \mathcal{I}_{n,\ell-1} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp^2}} (M_{\ell'} + m_f)[(q\Lambda j') - i(q\varphi j')] [(P\tilde{\Lambda}j)\mathcal{K}_3 - (P\tilde{\varphi}j)\mathcal{K}_4 + m_f(\mathcal{K}_1j)] \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell-1} \\
& - \sqrt{\frac{2\beta\ell'}{q_\perp^2}} (M_\ell + m_f)[(q'\Lambda j) + i(q'\varphi j)] [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1j')] \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n-1,\ell} \\
& + 2\beta\sqrt{\ell\ell'}[(j\Lambda j') + i(j\varphi j')] [(\mathcal{K}_1P) - m_f\mathcal{K}_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n,\ell-1} \\
& + (M_\ell + m_f)(M_{\ell'} + m_f) [(j\Lambda j') - i(j\varphi j')] [(\mathcal{K}_1P) + m_f\mathcal{K}_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2 q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{2\beta n} (\mathcal{K}_1j) [(M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell} + 2\beta\sqrt{\ell\ell'} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell-1}] \\
& + \sqrt{2\beta n} (\mathcal{K}_1j') [2\beta\sqrt{\ell\ell'} \frac{-(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n-1,\ell'-1} \mathcal{I}_{n,\ell-1} - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \mathcal{I}'_{n,\ell'} \mathcal{I}_{n-1,\ell}] + \\
& 2\beta\sqrt{n} \mathcal{K}_3 \left[\sqrt{\ell} (M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n-1,\ell'} \mathcal{I}_{n,\ell-1} + \right. \\
& \left. \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp'^2}} \mathcal{I}'_{n,\ell'-1} \mathcal{I}_{n-1,\ell} \right] \Big\}.
\end{aligned} \tag{88}$$

For second diagram we have the following replacement $P_\alpha \rightarrow P'_\alpha$, $q_\alpha \leftrightarrow -q'_\alpha$, $j_\alpha \leftrightarrow j'_\alpha$, $\mathcal{I}_{m,n} \leftrightarrow \mathcal{I}'_{m,n}$.

III. GROUND LANDAU LEVEL

The obtained results can be essentially simplified in several special cases. In the present section we consider the strong field limit, where the magnetic field strength B is the maximal physical parameter, namely, $\sqrt{eB} \gg \omega, E$, etc. In this case $n, \ell, \ell' = 0$, $M_\ell = M_{\ell'} = m_f$ and we obtain the following expressions for the amplitudes,

for different spin states of the initial and final fermions and for generalized vertices of the scalar, pseudoscalar, vector or axial vector types

$$\begin{aligned}
\mathcal{M}_{k'k}^- = & -\exp[-i\theta] \exp\left[-\frac{q_\perp^2 + q_\perp'^2}{4\beta}\right] \\
& \times \left\{ \frac{e^{i(q\varphi q')/(2\beta)} \mathcal{R}_{0k'k}^{(1)}}{P_\parallel^2 - m_f^2} + \frac{e^{-i(q\varphi q')/(2\beta)} \mathcal{R}_{0kk'}^{(2)}}{P_\parallel'^2 - m_f^2} \right\}, \tag{89}
\end{aligned}$$

where

$$\mathcal{R}_{0SS}^{(1)} = g_s g_s' j_s j_s' [(\mathcal{K}_1 P) + m_f \mathcal{K}_3]; \quad (90)$$

$$\mathcal{R}_{0SS}^{(2)} = g_s g_s' j_s j_s' [(\mathcal{K}_1 P') + m_f \mathcal{K}_3]; \quad (91)$$

$$\mathcal{R}_{0PS}^{(1)} = g_s g_p' j_s j_p' [(\mathcal{K}_2 P) - m_f \mathcal{K}_4]; \quad (92)$$

$$\mathcal{R}_{0SP}^{(2)} = -g_s g_p' j_s j_p' [(\mathcal{K}_2 P') + m_f \mathcal{K}_4]; \quad (93)$$

$$\mathcal{R}_{0VS}^{(1)} = g_s g_v' j_s [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1 j')]; \quad (94)$$

$$\mathcal{R}_{0SV}^{(2)} = g_s g_v' j_s [(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 + m_f(\mathcal{K}_1 j')]; \quad (95)$$

$$\mathcal{R}_{0AS}^{(1)} = g_s g_a' j_s [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2 j')]; \quad (96)$$

$$\mathcal{R}_{0SA}^{(2)} = g_s g_a' j_s [(P'\tilde{\Lambda}j')\mathcal{K}_4 - (P'\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2 j')]; \quad (97)$$

$$\mathcal{R}_{0PP}^{(1)} = -g_p g_p' j_p j_p' [(\mathcal{K}_1 P) - m_f \mathcal{K}_3]; \quad (98)$$

$$\mathcal{R}_{0PP}^{(2)} = -g_p g_p' j_p j_p' [(\mathcal{K}_1 P') - m_f \mathcal{K}_3]; \quad (99)$$

$$\mathcal{R}_{0VP}^{(1)} = -g_p g_v' j_p [(P\tilde{\Lambda}j')\mathcal{K}_4 + (P\tilde{\varphi}j')\mathcal{K}_3 + m_f(\mathcal{K}_2 j')]; \quad (100)$$

$$\mathcal{R}_{0PV}^{(2)} = g_p g_v' j_p [(P'\tilde{\Lambda}j')\mathcal{K}_4 - (P'\tilde{\varphi}j')\mathcal{K}_3 - m_f(\mathcal{K}_2 j')]; \quad (101)$$

$$\mathcal{R}_{0AP}^{(1)} = -g_p g_a' j_p [(P\tilde{\Lambda}j')\mathcal{K}_3 + (P\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1 j')]; \quad (102)$$

$$\mathcal{R}_{0PA}^{(2)} = g_p g_a' j_p [(P'\tilde{\Lambda}j')\mathcal{K}_3 - (P'\tilde{\varphi}j')\mathcal{K}_4 - m_f(\mathcal{K}_1 j')]; \quad (103)$$

$$\mathcal{R}_{0VV}^{(1)} = g_v g_v' \left\{ (P\tilde{\Lambda}j')(\mathcal{K}_1 j) + (P\tilde{\Lambda}j)(\mathcal{K}_1 j') - (j\tilde{\Lambda}j')(\mathcal{K}_1 P) + m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right\}; \quad (104)$$

$$\mathcal{R}_{0VV}^{(2)} = g_v g_v' \left\{ (P'\tilde{\Lambda}j')(\mathcal{K}_1 j) + (P'\tilde{\Lambda}j)(\mathcal{K}_1 j') - (j\tilde{\Lambda}j')(\mathcal{K}_1 P') + m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 - (j\tilde{\varphi}j')\mathcal{K}_4] \right\}; \quad (105)$$

$$\mathcal{R}_{0AV}^{(1)} = g_v g_a' \left\{ (P\tilde{\Lambda}j')(\mathcal{K}_2 j) + (P\tilde{\Lambda}j)(\mathcal{K}_2 j') - (j\tilde{\Lambda}j')(\mathcal{K}_2 P) - m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 + (j\tilde{\varphi}j')\mathcal{K}_3] \right\}; \quad (106)$$

$$\mathcal{R}_{0VA}^{(2)} = g_v g_a' \left\{ (P'\tilde{\Lambda}j')(\mathcal{K}_2 j) + (P'\tilde{\Lambda}j)(\mathcal{K}_2 j') - (j\tilde{\Lambda}j')(\mathcal{K}_2 P') + m_f [(j\tilde{\Lambda}j')\mathcal{K}_4 - (j\tilde{\varphi}j')\mathcal{K}_3] \right\}; \quad (107)$$

$$\mathcal{R}_{0AA}^{(1)} = g_a g_a' \left\{ (P\tilde{\Lambda}j')(\mathcal{K}_1 j) + (P\tilde{\Lambda}j)(\mathcal{K}_1 j') - (j\tilde{\Lambda}j')(\mathcal{K}_1 P) - m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 + (j\tilde{\varphi}j')\mathcal{K}_4] \right\}. \quad (108)$$

$$\mathcal{R}_{0AA}^{(2)} = g_a g_a' \left\{ (P'\tilde{\Lambda}j')(\mathcal{K}_1 j) + (P'\tilde{\Lambda}j)(\mathcal{K}_1 j') - (j\tilde{\Lambda}j')(\mathcal{K}_1 P') - m_f [(j\tilde{\Lambda}j')\mathcal{K}_3 - (j\tilde{\varphi}j')\mathcal{K}_4] \right\}. \quad (109)$$

We note that the obtained results for $\mathcal{M}_{\bar{\nu}V}^-$ and $\mathcal{M}_{V\bar{A}}^-$ exactly coincide with the amplitude of the photo-neutrino process from Ref. [30] (see also [18]) after taking account of the second diagram and of the momentum conservation law.

IV. FORWARD SCATTERING

For generalization of the results obtained in Ref. [11], to the case of magnetized plasma we consider the process of a coherent scattering of the generalized current j off the real fermions without change of their states (the ‘‘forward’’ scattering). In this case, under the generalized current j in the initial state we mean only the field operator of a single particle, while the generalized current j' in the final state could be both the field operator of a single particle and e.g. the neutrino current. In this case: $s = s'$, $q^\mu = q'^\mu$, $p^\mu = p'^\mu$, $\mathcal{K}_{1\alpha} = 2(p\tilde{\Lambda})_\alpha$, $\mathcal{K}_{2\alpha} = 2(\tilde{\varphi}p)_\alpha$, $\mathcal{K}_3 = 2M_\ell$, $\mathcal{K}_4 = 0$. We obtain the following results for the amplitudes:

$$\mathcal{M}_{k'k} = -\frac{\beta}{2\pi^2} \sum_{\ell, n=0}^{\infty} \int \frac{dp_z}{E_\ell} f_f(E_\ell) \left\{ \frac{\mathcal{D}_{k'k}^{(1)}}{(p+q)_\parallel^2 - m_f^2 - 2\beta n} + \frac{\mathcal{D}_{kk'}^{(2)}}{(p-q)_\parallel^2 - m_f^2 - 2\beta n} \right\}, \quad (110)$$

where $f_f(E_\ell) = [1 + \exp(E_\ell - \mu_f)/T]^{-1}$ is the fermion distribution function, T and μ_f are the temperature and

the chemical potential of plasma correspondingly,

$$\mathcal{D}_{SS}^{(1)} = g_s g'_s j_s j'_s \left\{ [(q\tilde{\Lambda}p) + 2\beta\ell + 2m_f^2] \quad (111)$$

$$\times (\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2) - 4\beta\sqrt{n\ell}\mathcal{I}_{n,\ell}\mathcal{I}_{n-1,\ell-1} \right\};$$

$$\mathcal{D}_{SS}^{(2)} = \mathcal{D}_{SS}^{(1)}(q \rightarrow -q);$$

$$\mathcal{D}_{SP}^{(1)} = \mathcal{D}_{PS}^{(2)} = g_s g'_p j_s j'_p (q\tilde{\varphi}p) [\mathcal{I}_{n,\ell}^2 - \mathcal{I}_{n-1,\ell-1}^2]; \quad (112)$$

$$\mathcal{D}_{VS}^{(1)} = g_s g'_v j_s m_f \left\{ [2(p\tilde{\Lambda}j') + (q\tilde{\Lambda}j')] [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2]$$

$$- \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \quad (113)$$

$$+ [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n-1,\ell} \mathcal{I}_{n-1,\ell-1}]$$

$$- \sqrt{\frac{2\beta n}{q_\perp^2}} [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell-1}$$

$$+ [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \};$$

$$\mathcal{D}_{SV}^{(2)} = g_s g'_v j_s m_f \left\{ [2(p\tilde{\Lambda}j') - (q\tilde{\Lambda}j')] [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2]$$

$$+ \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \quad (114)$$

$$+ [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n-1,\ell} \mathcal{I}_{n-1,\ell-1}]$$

$$+ \sqrt{\frac{2\beta n}{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell-1}$$

$$+ [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \};$$

$$\mathcal{D}_{AS}^{(1)} = g_s g'_a j_s m_f [2(p\tilde{\varphi}j') + (q\tilde{\varphi}j')] \quad (115)$$

$$\times [\mathcal{I}_{n,\ell}^2 - \mathcal{I}_{n-1,\ell-1}^2];$$

$$\mathcal{D}_{SA}^{(2)} = \mathcal{D}_{AS}^{(1)}(q \rightarrow -q);$$

$$\mathcal{D}_{PP}^{(1)} = -g_p g'_p j_p j'_p \left\{ [(q\tilde{\Lambda}p) + 2\beta\ell] [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2] \right. \\ \left. - 4\beta\sqrt{n\ell}\mathcal{I}_{n,\ell}\mathcal{I}_{n-1,\ell-1} \right\}; \quad (116)$$

$$\mathcal{D}_{PP}^{(2)} = \mathcal{D}_{PP}^{(1)}(q \rightarrow -q); \\ \mathcal{D}_{VP}^{(1)} = \mathcal{D}_{PV}^{(2)} \\ = -g_p g'_v j_p m_f (q\tilde{\varphi}j') [\mathcal{I}_{n,\ell}^2 - \mathcal{I}_{n-1,\ell-1}^2]; \quad (117)$$

$$\mathcal{D}_{AP}^{(1)} = -g_p g'_a j_p m_f \left\{ (q\tilde{\Lambda}j') [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2] \quad (118)$$

$$+ \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1}$$

$$+ [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n-1,\ell} \mathcal{I}_{n-1,\ell-1}]$$

$$- \sqrt{\frac{2\beta n}{q_\perp^2}} [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell-1}$$

$$+ [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \};$$

$$\mathcal{D}_{PA}^{(2)} = -g_p g'_a j_p m_f \left\{ (q\tilde{\Lambda}j') [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2] \quad (119)$$

$$- \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1}$$

$$- [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n-1,\ell} \mathcal{I}_{n-1,\ell-1}]$$

$$- \sqrt{\frac{2\beta n}{q_\perp^2}} [(q\Lambda j') - i(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell-1}$$

$$+ [(q\Lambda j') + i(q\varphi j')] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \};$$

$$\begin{aligned}
\mathcal{D}_{VV}^{(1)} &= g_v g_v' \left\{ [(p\tilde{\Lambda}j)(P\tilde{\Lambda}j') + (P\tilde{\Lambda}j)(p\tilde{\Lambda}j') - (j\tilde{\Lambda}j')[2\beta\ell + (p\tilde{\Lambda}q)]] [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2] \right. \\
&+ 4\beta\sqrt{n\ell} (j\tilde{\Lambda}j') \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell-1} - \sqrt{\frac{2\beta\ell}{q_{\perp}^2}} [(P\tilde{\Lambda}j)[(q\Lambda j') + i(q\varphi j')] + (P\tilde{\Lambda}j')[[(q\Lambda j) - i(q\varphi j)]] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \\
&- \sqrt{\frac{2\beta\ell}{q_{\perp}^2}} [(P\tilde{\Lambda}j)[(q\Lambda j') - i(q\varphi j')] + (P\tilde{\Lambda}j')[[(q\Lambda j) + i(q\varphi j)]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n-1,\ell} \\
&- \sqrt{\frac{2\beta n}{q_{\perp}^2}} [(p\tilde{\Lambda}j)[(q\Lambda j') - i(q\varphi j')] + (p\tilde{\Lambda}j')[[(q\Lambda j) + i(q\varphi j)]] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \\
&- \sqrt{\frac{2\beta n}{q_{\perp}^2}} [(p\tilde{\Lambda}j)[(q\Lambda j') + i(q\varphi j')] + (p\tilde{\Lambda}j')[[(q\Lambda j) - i(q\varphi j)]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell-1} + [2\beta\ell + (p\tilde{\Lambda}q)] \\
&\times \left. [[(j\Lambda j') + i(j\varphi j')] \mathcal{I}_{n,\ell-1}^2 + [(j\Lambda j') - i(j\varphi j')] \mathcal{I}_{n-1,\ell}^2] + \frac{4\beta\sqrt{n\ell}}{q_{\perp}^2} [(q\Lambda j)(q\Lambda j') - (q\varphi j)(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell} \right\}; \\
\mathcal{D}_{VV}^{(2)} &= \mathcal{D}_{VV}^{(1)}(q \rightarrow -q, j \leftrightarrow j');
\end{aligned} \tag{120}$$

$$\begin{aligned}
\mathcal{D}_{AV}^{(1)} &= g_v g_a' \left\{ [(P\tilde{\Lambda}j)(j'\tilde{\varphi}p) + (P\tilde{\Lambda}j')(j\tilde{\varphi}p) - (j\tilde{\Lambda}j')(q\tilde{\varphi}p) - m_f^2(j\tilde{\varphi}j')] [\mathcal{I}_{n,\ell}^2 - \mathcal{I}_{n-1,\ell-1}^2] \right. \\
&+ \sqrt{\frac{2\beta\ell}{q_{\perp}^2}} [(P\tilde{\varphi}j)[(q\Lambda j') + i(q\varphi j')] + (P\tilde{\varphi}j')[[(q\Lambda j) - i(q\varphi j)]] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \\
&- \sqrt{\frac{2\beta\ell}{q_{\perp}^2}} [(P\tilde{\varphi}j)[(q\Lambda j') - i(q\varphi j')] + (P\tilde{\varphi}j')[[(q\Lambda j) + i(q\varphi j)]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n-1,\ell} \\
&+ \sqrt{\frac{2\beta n}{q_{\perp}^2}} [(p\tilde{\varphi}j)[(q\Lambda j') - i(q\varphi j')] + (p\tilde{\varphi}j')[[(q\Lambda j) + i(q\varphi j)]] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \\
&- \sqrt{\frac{2\beta n}{q_{\perp}^2}} [(p\tilde{\varphi}j)[(q\Lambda j') + i(q\varphi j')] + (p\tilde{\varphi}j')[[(q\Lambda j) - i(q\varphi j)]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell-1} \\
&\left. + (p\tilde{\varphi}q) [[(j\Lambda j') + i(j\varphi j')] \mathcal{I}_{n,\ell-1}^2 - [(j\Lambda j') - i(j\varphi j')] \mathcal{I}_{n-1,\ell}^2] \right\};
\end{aligned} \tag{121}$$

$$\begin{aligned}
\mathcal{D}_{VA}^{(2)} = & g_v g_a' \left\{ [(P' \tilde{\Lambda} j)(j' \tilde{\varphi} p) + (P' \tilde{\Lambda} j')(j \tilde{\varphi} p) + (j \tilde{\Lambda} j')(q \tilde{\varphi} p) - m_f^2 (j \tilde{\varphi} j')] [\mathcal{I}_{n,\ell}^2 - \mathcal{I}_{n-1,\ell-1}^2] \right. \\
& - \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(P' \tilde{\varphi} j')[(q\Lambda j) + i(q\varphi j)] + (P' \tilde{\varphi} j)[(q\Lambda j') - i(q\varphi j')]] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \\
& + \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(P' \tilde{\varphi} j')[(q\Lambda j) - i(q\varphi j)] + (P' \tilde{\varphi} j)[(q\Lambda j') + i(q\varphi j')]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{\frac{2\beta n}{q_\perp^2}} [(p \tilde{\varphi} j')[(q\Lambda j) - i(q\varphi j)] + (p \tilde{\varphi} j)[(q\Lambda j') + i(q\varphi j')]] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \\
& + \sqrt{\frac{2\beta n}{q_\perp^2}} [(p \tilde{\varphi} j')[(q\Lambda j) + i(q\varphi j)] + (p \tilde{\varphi} j)[(q\Lambda j') - i(q\varphi j')]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell-1} \\
& \left. - (p \tilde{\varphi} q) [(j\Lambda j') - i(j\varphi j')] \mathcal{I}_{n,\ell-1}^2 - [(j\Lambda j') + i(j\varphi j')] \mathcal{I}_{n-1,\ell}^2 \right\};
\end{aligned} \tag{122}$$

$$\begin{aligned}
\mathcal{D}_{AA}^{(1)} = & g_a g_a' \left\{ [(P \tilde{\Lambda} j)(p \tilde{\Lambda} j') + (p \tilde{\Lambda} j)(P \tilde{\Lambda} j') - (j \tilde{\Lambda} j')(M_\ell^2 + m^2 + (p \tilde{\Lambda} q))] [\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2] \right. \\
& + 4\beta\sqrt{n\ell} (j \tilde{\Lambda} j') \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell-1} - \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(P \tilde{\Lambda} j)[(q\Lambda j') + i(q\varphi j')] + (P \tilde{\Lambda} j')[[(q\Lambda j) - i(q\varphi j)]] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \\
& - \sqrt{\frac{2\beta\ell}{q_\perp^2}} [(P \tilde{\Lambda} j)[(q\Lambda j') - i(q\varphi j')] + (P \tilde{\Lambda} j')[[(q\Lambda j) + i(q\varphi j)]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{\frac{2\beta n}{q_\perp^2}} [(p \tilde{\Lambda} j)[(q\Lambda j') - i(q\varphi j')] + (p \tilde{\Lambda} j')[[(q\Lambda j) + i(q\varphi j)]] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \\
& - \sqrt{\frac{2\beta n}{q_\perp^2}} [(p \tilde{\Lambda} j)[(q\Lambda j') + i(q\varphi j')] + (p \tilde{\Lambda} j')[[(q\Lambda j) - i(q\varphi j)]] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell-1} + (M_\ell^2 + m^2 + (p \tilde{\Lambda} q)) \\
& \times \left. [[(j\Lambda j') + i(j\varphi j')] \mathcal{I}_{n,\ell-1}^2 + [(j\Lambda j') - i(j\varphi j')] \mathcal{I}_{n-1,\ell}^2] + \frac{4\beta\sqrt{n\ell}}{q_\perp^2} [(q\Lambda j)(q\Lambda j') - (q\varphi j)(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell} \right\}; \\
\mathcal{D}_{AA}^{(2)} = & \mathcal{D}_{AA}^{(1)}(q \rightarrow -q, j \leftrightarrow j').
\end{aligned} \tag{123}$$

We notice, that the expressions for amplitudes \mathcal{M}_{VS} , \mathcal{M}_{VP} , \mathcal{M}_{VV} and \mathcal{M}_{AV} are manifestly gauge invariant.

V. DISCUSSION

In this paper, we have calculated the tree-level two-point amplitudes for the transitions $jf \rightarrow j'f'$ in a constant uniform magnetic field of an arbitrary strength, and in charged fermion plasma, for generalized vertices of the scalar, pseudoscalar, vector or axial types. It is remarkable, that all the amplitudes obtained are manifestly Lorentz invariant, due to the choice of the Dirac equation solutions as the eigenfunctions of the covariant

operator $\hat{\mu}_z$. In this case, partial contributions to the amplitude from the channels with different fermion polarization states are calculated separately, by direct multiplication of the bispinors and the Dirac matrices. This approach is an alternative to the method where the amplitudes squared are calculated, summed over the fermion polarization states, with using the fermion density matrices, see, e.g. [31, 32].

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