Variations of α and G from nonlinear multidimensional gravity

K.A. Bronnikov[1] **and M.V. Skvortsova**[2]

Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow 119361, Russia;
Institute of Gravitation and Cosmology, PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia

Received February 10, 2013

To explain the recently reported large-scale spatial variations of the fine structure constant α, we apply some models of curvature-nonlinear multidimensional gravity. Under the reasonable assumption of slow changes of all quantities as compared with the Planck scale, the original theory reduces to a multi-scalar field theory in four dimensions. On this basis, we consider different variants of isotropic cosmological models in both Einstein and Jordan conformal frames. One of the models turns out to be equally viable in both frames, but in the Jordan frame the model predicts simultaneous variations of α and the gravitational constant G, equal in magnitude. Large-scale small inhomogeneous perturbations of these models allow for explaining the observed spatial distribution of α values.

1 Introduction

The long-standing problem of possible space-time variations of the fundamental physical constants (FPC) is now actively discussed on both theoretical and observational grounds, and in particular, in connection with more or less confidently observed variations of the fine-structure constant α in space and time [1, 2]. The first data on temporal changes of α, such that α was in the past slightly smaller than now (the relative change $\delta\alpha/\alpha$ is about 10^{-5}), appeared in [1] from observations of mostly the Northern sky at the Keck telescope (the Hawaiian islands). In 2010, an analysis of new data obtained at the VLT (Very Large Telescope), located in Chile, and their comparison with the Keck data led to a conclusion on spatial variations of α, i.e., on its dependence on the direction of observations. According to VLT observations in the Southern sky, α was in the past slightly larger than now. This anisotropy has a dipole nature [2] and has been termed "the Australian dipole" [3]. The dipole axis is located at a declination of $-61 \pm 9^\circ$ and at a right ascension of $17.3 \pm 0,6$ hours. The deflection of α value at an arbitrary point r of space from its modern value α_0, measured on Earth, is, at a confidence level of 4.1σ,

$$\delta\alpha/\alpha_0 = (1.10 \pm 0.25) \times 10^{-6}\, r \cos\psi, \qquad (1)$$

where ψ is the angle between the direction of observation and the dipole axis, while the distance r is measured in billions of light years [2].

On the other hand, recent laboratory experiments have given the tightest constraints on α variations on Earth in the modern epoch [4]

$$(d\alpha/dt)/\alpha = (-1.6 \pm 2.3) \times 10^{-17} \text{ per year}. \qquad (2)$$

This result is of the same order of magnitude as the tightest constraints obtained previously from an isotopic composition analysis of the decay products in the natural nuclear reactor that operated in the Oklo region (Gabon) about 2 billion years ago. Unlike the laboratory data, the Oklo results [5] and, in particular, the tightest constraint [6]

$$d(\ln\alpha)/dt = (-0.4 \pm 0.5) \times 10^{-17}/\text{yr} \qquad (3)$$

rely on the assumption that during these 2 billion years the value of α changed uniformly, if changed at all. This assumption looks rather natural but actually follows from nowhere.

The observed space-time distribution of α values is illustrated in Fig. 1: on Earth, at least since the Oklo times, α is constant on the level of $\sim 10^{-17}$ per year, whereas according to the quasar data for about 10 billion years a variation rate can be about 10^{-15} per year. Meanwhile, one cannot exclude the opportunity that the variations are purely spatial in nature whereas the time dependence is related to the finiteness of the velocity of light: being located at a fixed point and at fixed time, we receive signals from distant regions of the Universe emitted at earlier cosmological epochs, and it is therefore impossible to separate spatial and temporal dependences of the parameters.

[1] e-mail: kb20@yandex.ru
[2] e-mail: milenas577@mail.ru

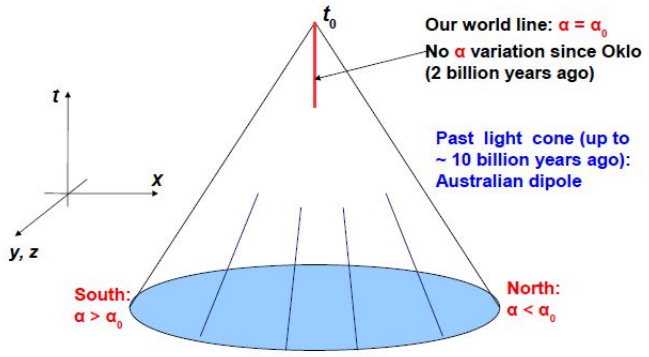

Figure 1: The empirical data on variations of α

A number of theoretical models have been constructed in order to explain these variations [8–14]. In these approaches the variability of α is explained in the framework of general relativity with the aid of scalar fields whose existence, self-interaction and coupling to the electromagnetic field were postulated "by hand". In [11], it was shown that in $F(R)$ gravity it is possible to obtain a static effective (gravitational) domain wall with spatially varying α by postulating a certain nonminimal interaction between the electromagnetism and gravity. In [15] it was shown that scalar fields and their interaction law with electromagnetism leading to variations of α naturally follow from curvature-nonlinear multidimensional gravity. It was noted that an advantage of multidimensional gravity in the treatment of FPC variations is that all such variations are explained in a unified way from spatial and temporal variations of the size of extra dimensions [16, 17].

In [15], in the approach to nonlinear multidimensional gravity formulated in [18], a simple model was built, explaining the observed variations of α. Using the methodology of [18], a particular multidimensional theory was reduced to a scalar field theory in 4 dimensions, which resulted in a cosmological model with accelerated expansion, and certain initial conditions were chosen, slightly different from homogeneity and isotropy. The results were obtained in the Einstein conformal frame, in which, by construction, the gravitational constant does not change.

The present paper continues this study. We show that if we consider the Jordan frame as the physical (observational) one [20,21], then the model built in [15] predicts a very large cosmic acceleration contrary to observations. We suggest another simple model which also fairly well describes the present accelerated stage along with variations of α, but is equally viable in the Einstein and Jordan frames. In the latter, α and G evolve according to the same law, being inversely proportional to the volume of extra dimensions. Unlike those of α, variations of G have not been discovered so far, there are only upper bounds. The tightest constraint following from the results of lunar laser ranging is [19]

$$\dot{G}/G = (2 \pm 7) \times 10^{-13}\,\mathrm{yr}^{-1}. \qquad (4)$$

Since α variations are at most $\sim 10^{-15}$ per year, similar variations of G, if any, are in agreement with (4).

Let us note that the methodology of [18], allowing for a transition from a broad class of multidimensional theories of gravity with higher derivatives to Einstein gravity with effective scalar fields, was successfully applied to obtain a unified description of the early inflation and modern acceleration of the Universe [22]; it has provided a possible explanation of the origin of the Higgs field and solutions of some other physical and cosmological problems [23–25].

The paper is organized as follows. Section 2 briefly describes the general formalism to be used. In this framework, in Section 3 we consider two variants of isotropic cosmological models; one of them, obtained previously [15], is viable only in the Einstein picture, while the other is equally viable in the Einstein and Jordan pictures. In Section 4 we consider small large-scale inhomogeneous perturbations of these models and show that each of them is able to account for the observed spatial variations of α. Section 5 is a brief conclusion.

2 Basic equations

Consider a $(D = 4 + d_1)$-dimensional manifold with the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \mathrm{e}^{2\beta(x)} b_{ab} dx^a dx^b \qquad (5)$$

where the extra-dimensional metric components b_{ab} are independent of x^μ, the observable four space-time coordinates.[3]

[3] Our sign conventions are: the metric signature $(+---)$; the curvature tensor $R^\sigma{}_{\mu\rho\nu} = \partial_\nu \Gamma^\sigma_{\mu\rho} - \ldots$, $R_{\mu\nu} = R^\sigma{}_{\mu\sigma\nu}$, so that the Ricci scalar $R > 0$ for de Sitter space-time and the matter-dominated cosmological epoch; the system of units $8\pi G = c = 1$.

The D-dimensional Riemann tensor has the nonzero components

$$
\begin{aligned}
R^{\mu\nu}{}_{\rho\sigma} &= \overline{R}^{\mu\nu}{}_{\rho\sigma}, \\
R^{\mu a}{}_{\nu a} &= \delta^a_b\, B^\mu_\nu, \qquad B^\mu_\nu := \mathrm{e}^{-\beta}\nabla_\nu(\,\mathrm{e}^{\beta}\beta^\mu), \\
R^{ab}{}_{cd} &= \mathrm{e}^{-2\beta}\overline{R}^{ab}{}_{cd} + \delta^{ab}{}_{cd}\beta_\mu\beta^\mu, \qquad (6)
\end{aligned}
$$

where capital Latin indices cover all D coordinates, the bar marks quantities obtained from $g_{\mu\nu}$ and b_{ab} taken separately, $\beta_\mu \equiv \partial_\mu\beta$ and $\delta^{ab}{}_{cd} \equiv \delta^a_c\delta^b_d - \delta^a_d\delta^b_c$. The nonzero components of the Ricci tensor and the scalar curvature are

$$
\begin{aligned}
R^\nu_\mu &= \overline{R}^\nu_\mu + d_1\, B^\nu_\mu, \\
R^b_a &= \mathrm{e}^{-2\beta}\overline{R}^b_a + \delta^b_a[\Box\beta + d_1(\partial\beta)^2], \\
R &= \overline{R}[g] + \mathrm{e}^{-2\beta}\overline{R}[b] + 2d_1\Box\beta \\
&\qquad + d_1(d_1+1)(\partial\beta)^2, \qquad (7)
\end{aligned}
$$

where $(\partial\beta)^2 \equiv \beta_\mu\beta^\mu$, $\Box = \nabla^\mu\nabla_\mu$ is the d'Alembert operator while $\overline{R}[g]$ and $\overline{R}[b]$ are the Ricci scalars corresponding to $g_{\mu\nu}$ and b_{ab}, respectively. Let us also present, using similar notations, the expressions for two more curvature invariants, the Ricci tensor squared and the Kretschmann scalar $\mathcal{K} = R^{ABCD}R_{ABCD}$ (where capital Latin indices cover all D coordinates):

$$
\begin{aligned}
R_{AB}R^{AB} &= \overline{R}_{\mu\nu}\overline{R}^{\mu\nu} + 2d_1\overline{R}_{\mu\nu}B^{\mu\nu} + d_1^2 B_{\mu\nu}B^{\mu\nu} \\
&\quad + \mathrm{e}^{-4\beta}\overline{R}_{ab}\overline{R}^{ab} + 2\,\mathrm{e}^{-2\beta}\overline{R}[b][\Box\beta + d_1(\partial\beta)^2] \\
&\quad + d_1[\Box\beta + d_1(\partial\beta)^2]^2, \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K} &= \overline{\mathcal{K}}[g] + 4d_1 B_{\mu\nu}B^{\mu\nu} + \mathrm{e}^{-4\beta}\overline{\mathcal{K}}[b] \\
&\quad + 4\,\mathrm{e}^{-2\beta}\overline{R}[b](\partial\beta)^2 + 2d_1(d_1-1)[(\partial\beta)^2]^2. \qquad (9)
\end{aligned}
$$

Suppose now that b_{ab} describes a compact d_1-dimensional space of nonzero constant curvature, i.e., a sphere ($K = 1$) or a compact d_1-dimensional hyperbolic space ($K = -1$) with a fixed curvature radius r_0 normalized to the D-dimensional analogue m_D of the Planck mass, i.e., $r_0 = 1/m_D$ (we use the natural units, with the speed of light c and Planck's constant $\hbar$ equal to unity). We have

$$
\begin{aligned}
\overline{R}^{ab}{}_{cd} &= K\, m_D^2\, \delta^{ab}{}_{cd}, \\
\overline{R}^b_a &= K\, m_D^2\,(d_1-1)\delta^b_a, \\
\overline{R}[b] &= K\, m_D^2\, d_1(d_1-1) = R_b. \qquad (10)
\end{aligned}
$$

The scale factor $b(x) \equiv \mathrm{e}^{\beta}$ in (5) is thus kept dimensionless; R_b has the meaning of a characteristic curvature scale of the extra dimensions.

In this geometry, we deal with a sufficiently general curvature-nonlinear theory of gravity with the action

$$
\begin{aligned}
S &= \frac{1}{2}m_D^{D-2}\int \sqrt{{}^Dg}\, d^Dx\,(L_g + L_m), \\
L_g &= F(R) + c_1 R^{AB}R_{AB} + c_2\mathcal{K}, \qquad (11)
\end{aligned}
$$

where $F(R)$ is an arbitrary smooth function of the D-dimensional scalar curvature R, c_1 and c_2 are constants, L_m is a matter Lagrangian and ${}^Dg = |\det(g_{MN})|$.

We suppose that b_{ab} describes a compact d_1-dimensional space of nonzero constant curvature, i.e., a sphere ($K = 1$) or a compact d_1-dimensional hyperbolic space ($K = -1$) with a unit curvature radius; we use the system of units in which the speed of light c, the Planck constant $\hbar$ and the D-dimensional Planck length are equal to unity. Thus all quantities are now expressed in (D-dimensional) Planck units.

The field equations of the full theory (11) are very complicated. Let us simplify the theory in the following way:

(a) Integrate out the extra dimensions and express everything in terms of 4D variables and $\beta(x)$; we have, in particular,

$$
\begin{aligned}
R &= R_4 + \phi + f_1, \\
f_1 &= 2d_1\Box\beta + d_1(d_1+1)(\partial\beta)^2, \qquad (12)
\end{aligned}
$$

where R_4 is the 4D scalar curvature, $(\partial\beta)^2 = g^{\mu\nu}\partial_\mu\beta\partial_\nu\beta$, and we have introduced the effective scalar field

$$
\phi(x) = R_b\,\mathrm{e}^{-2\beta(x)} = Kd_1(d_1-1)\,\mathrm{e}^{-2\beta(x)} \qquad (13)
$$

The sign of ϕ coincides with $K = \pm 1$, the sign of curvature in the d_1 extra dimensions.

The action in four dimensions has the form

$$
S = \frac{1}{2}\mathcal{V}[d_1]\int \sqrt{{}^4g}\, d^4x\ \mathrm{e}^{d_1\beta}\,[L_g + L_m], \qquad (14)
$$

where ${}^4g = |\det(g_{\mu\nu})|$ and $\mathcal{V}[d_1]$ is the volume of a compact d_1-dimensional space of unit curvature.

(b) Suppose that all quantities are slowly varying, i.e., consider each derivative ∂_μ as an expression containing a small parameter ε; neglect all quantities of orders higher than $O(\varepsilon^2)$ (see [18, 25]).

(c) Perform a conformal mapping leading to the Einstein conformal frame, where the 4-curvature appears to be minimally coupled to the scalar ϕ.

In the decomposition (12), both terms f_1 and R_4 are regarded small in our approach, which actually means that all quantities, including the 4D curvature, are small as compared with the D-dimensional Planck scale. The only term which is not small is ϕ, and we can use a Taylor decomposition of the function $F(R) = F(\phi + R_4 + f_1)$:

$$F(R) = F(\phi + R_4 + f_1) \simeq F(\phi) + F'(\phi)\cdot(R_4 + f_1) + ..., \quad (15)$$

with $F'(\phi) \equiv dF/d\phi$. In (14), we obtain, up to $O(\varepsilon^2)$,

$$L_g = F'(\phi)R_4 + F(\phi) + F'(\phi)f_1 + c_*\phi^2 + 2c_1\phi\Box\beta + 2(c_1d_1 + 2c_2)(\partial\beta)^2 \quad (16)$$

with $c_* = c_1/d_1 + 2c_2/[d_1(d_1-1)]$.

The action (14) with (16) is typical of a scalar-tensor theory (STT) of gravity in a Jordan frame. To study the dynamics of the system, it is helpful to pass on to the Einstein frame. Applying the conformal mapping

$$g_{\mu\nu} \mapsto \widetilde{g}_{\mu\nu} = |f(\phi)|g_{\mu\nu}, \quad f(\phi) = \mathrm{e}^{d_1\beta}F'(\phi), \quad (17)$$

after a lengthy calculation, we obtain the action in the Einstein frame as [15, 18]

$$S = \frac{1}{2}\mathcal{V}[d_1]\int\sqrt{\widetilde{g}}\,(\mathrm{sign}\,F')L,$$

$$L = \widetilde{R}_4 + K_{\mathrm{E}}(\phi)(\partial\phi)^2 - 2V_{\mathrm{E}}(\phi) + \widetilde{L}_m, \quad (18)$$

$$\widetilde{L}_m = (\mathrm{sign}\,F')\frac{\mathrm{e}^{-d_1\beta}}{F'(\phi)^2}L_m; \quad (19)$$

$$K_{\mathrm{E}}(\phi) = \frac{1}{4\phi^2}\left[6\phi^2\left(\frac{F''}{F'}\right)^2 - 2d_1\phi\frac{F''}{F'} + \frac{1}{2}d_1(d_1+2) + \frac{4(c_1+c_2)\phi}{F'}\right], \quad (20)$$

$$-2V_{\mathrm{E}}(\phi) = (\mathrm{sign}\,F')\frac{\mathrm{e}^{-d_1\beta}}{F'(\phi)^2}[F(\phi) + c_*\phi^2], \quad (21)$$

where the tilde marks quantities obtained from or with $\widetilde{g}_{\mu\nu}$; the indices are raised and lowered with $\widetilde{g}_{\mu\nu}$; everywhere $F = F(\phi)$ and $F' = dF/d\phi$; e^{β} is expressed in terms of ϕ using (13).

Let us consider the electromagnetic field $F_{\mu\nu}$ as matter in the initial Lagrangian, putting

$$L_m = \alpha_1^{-1}F_{\mu\nu}F^{\mu\nu}, \quad (22)$$

where α_1 is a constant. After reduction to four dimensions this expression acquires the factor $\mathrm{e}^{d_1\beta}$ arising from the metric determinant: $\sqrt{{}^{D}g} = \sqrt{{}^{4}g}\,\mathrm{e}^{d_1\beta}$. In the subsequent transition to the Einstein picture the expression $\sqrt{{}^{4}g}F_{\mu\nu}F^{\mu\nu}$ remains the same (due to conformal invariance of the electromagnetic field), hence the Lagrangian (19) takes the form

$$\widetilde{L}_m = \alpha_1^{-1}\,\mathrm{e}^{d_1\beta}F_{\mu\nu}F^{\mu\nu}, \quad (23)$$

and for the effective fine structure constant α we obtain

$$\frac{\alpha}{\alpha_0} = \mathrm{e}^{d_1(\beta_0-\beta)}, \quad (24)$$

where α_0 and β_0 are values of the respective quantities at a fixed space-time point, for instance, where and when the observation is taking place.

3 Isotropic cosmologies

3.1 Equations for small ϕ

Depending on the choice of $F(R)$, the constants c_1 and c_2 and the matter Lagrangian, the theory (11) can lead to a great variety of cosmological models. Some of them were discussed in [18], mostly those related to minima $V = V_{\min}$ of the effective potential (21) at nonzero values of ϕ. Such minima correspond to stationary states of the scalar ϕ, and consequently of the volume factor of extra dimensions that determines the effective FPC values. If $V_{\min} > 0$, it can play the role of a cosmological constant that launches an accelerated expansion of the Universe.

Here, as in [15], we focus on another minimum of the potential V_{E}, existing for generic choices of the function $F(R)$ with $F' > 0$ and located at $\phi = 0$. If $\phi \to 0$ at late times, this corresponds to growing rather than stabilized extra dimensions: $b = \mathrm{e}^{\beta} \sim 1/\sqrt{|\phi|} \to \infty$. Such a model can still be of interest if the growth is sufficiently slow and the size b does not reach detectable values by now. We can recall that the admissible range of such growth comprises as many as 16 orders of magnitudes if the D-dimensional Planck length $l_D = 1/m_D$ coincides with the 4D one, i.e., about 10^{-33} cm: the upper bound corresponds to lengths about 10^{-17} cm or energies of the order of a few TeV. This estimate certainly changes if there is no such coincidence.

Assuming small values of ϕ, we should still take care of not violating our general assumptions,

namely, the requirement that $|\phi|$ is still very large as compared to 4D quantities. It is really so since

$$|\phi| = \frac{d_1(d_1-1)}{b^2},$$

where $b \lesssim 10^{16}$, hence $|\phi| \gtrsim d_1^2 \cdot 10^{-32}$, whereas the quantity $\widetilde{R}_4$, if identified with the curvature of the modern Universe, is $\sim 10^{-122}$ in Planck units (that is, close to (the Hubble time)$^{-2}$, see also Eq. (37) below.

Let us check whether it is possible to describe the modern stage of the Universe evolution by an asymptotic form of the solution for small ϕ as a spatially flat cosmology with the 4D Einstein-frame metric

$$d\widetilde{s}_4^2 = dt^2 - a^2(t)d\vec{x}^2, \tag{25}$$

where $a(t)$ is the Einstein-frame scale factor. At small ϕ, assuming a smooth function $F(\phi)$, we can restrict ourselves to the first three terms of its Taylor decomposition:[4]

$$F(\phi) = -2\Lambda_D + F_1\phi + F_2\phi^2, \tag{26}$$

where Λ_D is the initial cosmological constant. For simplicity, we suppose[5] $F_1 = 0$, $F_2 = 1$. It is then convenient to rewrite the Lagrangian (18) at small ϕ in terms of β instead of ϕ:

$$L = \widetilde{R}_4 + 2K_0(\partial\beta)^2 - 2V(\beta) + \widetilde{L}_m, \tag{27}$$

with

$$K_0 = \frac{1}{4}[d_1^2 - 2d_1 + 12 + 4(c_1+c_2)]. \tag{28}$$

Neglecting the gravitational influence of the electromagnetic field (that is, considering only vacuum models), one can write down the independent components of the Einstein and scalar field equations with the unknowns $\beta(t)$ and $a(t)$ as follows:

$$3\frac{\dot{a}^2}{a^2} = K_0\dot{\beta}^2 + V(\beta), \tag{29}$$

$$2K_0\Big(\ddot{\beta} + 3\frac{\dot{a}}{a}\dot{\beta}\Big) = -\frac{dV}{d\beta}. \tag{30}$$

The form of the potential $V(\beta)$ depends on further assumptions. If $\Lambda_D \neq 0$, we have

$$V = V_1(\beta) = V_{10}\,\mathrm{e}^{-2\overline{d}\beta},$$
$$V_{10} = \frac{\Lambda_D}{4d_1^2(d_1-1)^2}, \qquad \overline{d} = \frac{d_1-4}{2}. \tag{31}$$

[4] We assume for certainty $\phi > 0$, hence by (13), $K = +1$, but everything can be easily reformulated for $\phi < 0$.

[5] The theory is insensitive to multiplying the action by a constant, and we use this freedom to fix $F_2 = 1$.

and $d_1 > 4$ is required here. This model was considered in [15], and we here discuss it for comparison with another model, in which $\Lambda_D = 0$ (or, more generally, $\Lambda_D \ll \phi^2$).

Under the assumption $\Lambda_D = 0$ we have

$$V = V_2(\beta) = V_{20}\,\mathrm{e}^{-d_1\beta},$$
$$V_{20} = -\frac{1+c_*}{8} = -\frac{1}{8}\bigg[1 + \frac{c_1}{d_1} + \frac{c_2}{d_1(d_1-1)}\bigg]. \tag{32}$$

3.2 Model 1: $F(R) = -2\Lambda + R^2$

Eqs. (29) and (30) correspond to a scalar field with an exponential potential and can be solved exactly, but the solution looks rather involved, and for our purpose more preferable is the comparatively simple approximate solution obtainable in the slow-rolling approximation that should be acceptable at late times. Let us suppose that

$$|\ddot{\beta}| \ll 3\frac{\dot{a}}{a}\dot{\beta}, \qquad K_0\dot{\beta}^2 \ll V(\beta), \tag{33}$$

and neglect the corresponding terms in Eqs. (29) and (30). Then, expressing the quantity $\dot{a}/a$ from (29) and substituting it into (30) with $V = V_1$, we obtain an expression for $\dot{\beta}$ whose integration gives

$$\mathrm{e}^{\overline{d}\beta} = \frac{\overline{d}^2}{K_0}\sqrt{\frac{V_{10}}{3}}(t+t_1), \tag{34}$$

where t_1 is an integration constant. For the scale factor $a(t)$ we have $\dot{a}/a = p/(t+t_1)$ whence

$$a = a_1(t+t_1)^p, \quad a_1 = \mathrm{const}, \quad p = \frac{K_0}{\overline{d}^2}. \tag{35}$$

One can verify that the slow-rolling conditions (33) hold as long as $3p \gg 1$, or in terms of the input parameters of the theory,

$$3p = 3\frac{d_1^2 - 2d_1 + 12 + 4(c_1+c_2)}{(d_1-4)^2} \gg 1. \tag{36}$$

Let us assume that this condition holds.

A further interpretation of the results depends on which conformal frame is regarded physical (observational) [20,21], and this in turn depends on the manner in which fermions appear in the (so far unknown) underlying unification theory involving all interactions.

Let us make some estimates assuming that the observational picture is Einstein's. The inverse of

the modern value of the Hubble parameter (the Hubble time) is estimated as[6]

$$t_H = \frac{1}{H_0} = \frac{a_0}{\dot a_0} \approx 4,4 \times 10^{17}\,\mathrm{s} \approx 8 \times 10^{60}\, t_{\mathrm{pl}}, \quad (37)$$

where t_{pl} is the Planck time. From (35) it follows that $H_0 = p/(t_0 + t_1)$, whence

$$t_* := t_0 + t_1 = p t_H \gg t_H. \quad (38)$$

With $p \gg 1$, the power-law expansion is close to exponential, and the model satisfies the observational constraints on the factor w in the effective equation of state $p = w\rho$ of dark energy: at $w = \mathrm{const}$ we have $a \sim t^{2/(3+3w)}$, consequently, $w = -1 + 2/(3p)$ is a number close to -1.

The "internal" scale factor $b(t) = \mathrm{e}^{\beta}$ grows much slower than $a(t)$:

$$b(t) = b_0\Big(\frac{t+t_1}{t_*}\Big)^{1/\bar d}, \quad b_0 = \left(\frac{1}{H_0}\sqrt{\frac{V_1}{3}}\right)^{1/\bar d}. \quad (39)$$

Using the expression for V_{10} in (31), one can estimate the initial parameter Λ_D in terms of the present size b_0 of the extra factor space: in Planck units,

$$\Lambda_D = 12 H_0^2 d_1^2 (d_1-1)^2 b_0^{d_1-4} \approx \frac{3}{16} d_1^2 (d_1-1)^2 b_0^{d_1-4} \times 10^{-120}. \quad (40)$$

As already mentioned, b_0 should be in the range $1 \ll b_0 \lesssim 10^{16}$ in Planck units. The estimate (40) shows that the present model makes much easier the well-known "cosmological constant problem" (the difficulty of explaining why in standard cosmology $\Lambda_{\mathrm{standard}} \sim 10^{-122}$ in Planck units). For example, if (in the admissible range) $b_0 = 5\times 10^{14}$ and $d_1 = 12$, it follows $\Lambda_D \approx 12.76$, without any indication of fine tuning.

Other initial parameters, c_1 and c_2, should not be too large: as estimated in [15], they should not exceed 10^{11}, otherwise our basic assumptions can be violated. But the smallness of the observed variations of α indicates that they should not be too small. Indeed, according to (24),

$$\frac{\alpha}{\alpha_0} = \left(\frac{t+t_1}{t_0+t_1}\right)^{-2d_1/(d_1-4)} \approx 1 - \frac{2d_1}{d_1-4}\frac{t-t_0}{t_*}, \quad (41)$$

so that $\dot\alpha/\alpha \sim 10^{-10}/p$ per year. By the empirical data, this quantity cannot be larger than about 10^{-17} per year. This leads to the constraint $p \gtrsim 10^7$. The allowed range of c_1 and c_2 (assuming that they are both positive and have the same order of magnitude)

$$10^6 < c_{1,2} \ll 10^{11} \quad (42)$$

is wide enough, and there is no fine tuning. Moreover, we shall see that the inequality $c_{1,2} > 10^6$ is substantially relaxed in the perturbed model.

3.3 Model 1 in Jordan's frame

It is of interest how the same model looks in the Jordan frame corresponding to the initial D-dimensional action. To obtain it, we return to the transformation (17) and find

$$ds_{\mathrm{J}}^2 = ds_{\mathrm{E}}^2/f, \qquad f = \mathrm{e}^{d_1\beta} F'(\phi). \quad (43)$$

Since $F' = 2\phi \sim \mathrm{e}^{-2\beta}$, with the solution (39) we have

$$f = \mathrm{const}\cdot (t+t_1)^{2(d_1-2)/(d_1-4)}. \quad (44)$$

We apply this transformation to the cosmological metric (25), putting

$$ds_{\mathrm{J}} = d\tau^2 - a_{\mathrm{J}}^2(\tau) d\vec{x}^2 = \frac{1}{f}[dt^2 - a^2(t) d\vec{x}^2], \quad (45)$$

where τ and $a_{\mathrm{J}}(\tau)$ are the cosmological time and the scale factor in the Jordan frame, respectively. Then $d\tau = dt/\sqrt{f}$ and $a_{\mathrm{J}}(\tau) = a(t)/\sqrt{f}$. Integrating, we find

$$\tau - \tau_1 = -\bar d\, t_*^{1+1/\bar d}(t+t_1)^{-1/\bar d}, \quad \tau_1 = \mathrm{const}. \quad (46)$$

Substituting (46) into the solution (35), (39), we obtain

$$a_{\mathrm{J}}(\tau) = a_{1*}(\tau_1 - \tau)^{-\bar d p + \bar d + 1}, \quad a_{1*} = \mathrm{const}, \quad (47)$$

$$b(\tau) = b_0 \frac{\tau_1 - \tau_0}{\tau_1 - \tau}, \quad (48)$$

where b_0 and τ_0 are the present-time values of b and τ. It is a big-rip cosmology: both scale factors a and b blow up at a certain time τ_1 in the future. Moreover, comparing the behavior of $a_{\mathrm{J}}(\tau)$ with the dependence $a \sim \tau^{2/(3+3w)}$, corresponding to a model with a constant-w perfect fluid ($p = w\rho$), we see that now the effective equation-of-state parameter is

$$w = -1 - \frac{2}{3}(\bar d p - \bar d - 1). \quad (49)$$

[6] As usual in cosmology, here and henceforth the subscript "0" refers to the present epoch.

The solution (35), (39) is valid under the assumption $p \gg 1$, therefore the parameter (49) is much smaller than -1, and we have to conclude that the Jordan-frame version of Model 1 contradicts the observations.

We conclude that this model can be viable only if the Einstein frame is interpreted as the observational one.

3.4 Model 2: $F(R) = R^2$

Let us now try to build a cosmological model with a purely quadratic function $F(R)$, that is, $F(R) = R^2$ (see footnote 5). As before, the Einstein-frame 4D scale factor $a(t)$ and the effective scalar field $\beta(t)$ obey the same equations (29) and (30), but with V_1 replaced by V_2. For the model to be viable, we need a positive potential V and therefore assume $V_{20} > 0$ which is possible under a proper choice of c_1 and c_2.

The solution in the slow-rolling approximation (33), obtained in the same way as before, has the form

$$a(t) = a_2(t+t_2)^q, \qquad q = \frac{1}{d_1^2}[d_1^2 - 2d_1 + 12 + 4(c_1+c_2)]; \tag{50}$$

$$b(t) \equiv \mathrm{e}^{\beta} = b_0\left(\frac{t+t_2}{t_{**}}\right)^{2/d_1}, \quad b_0 = \left(\frac{t_{**}\sqrt{V_{20}}}{\sqrt{3}q}\right)^{2/d_1}, \tag{51}$$

where a_2 and t_2 are integration constants and $t_{**} = t_0 + t_2$, t_0 being the present time. Identifying the present Hubble parameter H_0 with the present value of $\dot{a}/a$ according to (50), we obtain $t_* = 1/H_0 = qt_H$, where t_H is the Hubble time. And, as before, we easily verify that the slow-rolling conditions (33) hold as long as $q \gg 1$. Under this condition the model adequately describes the present state of the Universe since the effective equation-of-state parameter is $w = -1 + 2/(3q)$, as in Sec. 3.2.

This model does not contain an initial cosmological constant like Λ_D, and instead of the estimate (40) we have a constraint on d_1 that follows from the expression for b_0 in (51). Indeed, since $t_{**} = q/H_0$, we have

$$b_0 = \left(\frac{\sqrt{V_{20}}}{\sqrt{3}H_0}\right)^{2/d_1} \lesssim 10^{16}, \tag{52}$$

while (see (37) $1/H_0 = t_H \sim 10^{60}$ in Planck units, hence, assuming that $V_{20} \sim 1$, we must have $d_1 \geq 8$.

The constraints on the input parameters c_1 and c_2 are now different from (42) because, while the condition $q \gg 1$ requires $c_1 + c_2 \gg 1$, to have $V_{20} \sim 1$, we must require $c_* < -1$.

For time variations of α we now have

$$\frac{\alpha}{\alpha_0} = \left(\frac{b}{b_0}\right)^{-d_1} = \left(\frac{t+t_2}{t_*}\right)^{-2} \approx 1 - 2\frac{t-t_0}{t_*},$$

$$\dot{\alpha}/\alpha_0 \approx -2H_0/q = -2/(qt_H). \tag{53}$$

The empirical bounds on $\dot{\alpha}$ require $q \gtrsim 10^7$, which leads to the constraint $c_1 + c_2 \gtrsim 10^6$. This can be combined with $c_* < -1$ if $c_2 \gtrsim 2\times 10^6$ and $c_1 \lesssim -10^6$. And, as in the first model, these conditions are substantially relaxed for the perturbed configuration, see Section 4.

3.5 Model 2 in Jordan's frame

In the transformation (17) we now have

$$ds_{\mathrm{J}}^2 = \frac{ds_{\mathrm{E}}^2}{f(\phi)}, \quad f(\phi) = \mathrm{const}\cdot(t+t_2)^{2-4/d_1}. \tag{54}$$

Applying this transformation to the metric (25), we once again obtain Eq. (45) where τ and $a_{\mathrm{J}}(\tau)$ are the cosmological time and the scale factor in Jordan's frame, respectively. Integrating $d\tau = dt/\sqrt{f}$ and substituting it to $a_{\mathrm{J}}(\tau) = a(t)/\sqrt{f}$, we find

$$\tau + \tau_2 = \int \frac{dt}{(t+t_2)^{1-2/d_1}} \sim (t+t_2)^{2/d_1}, \tag{55}$$

$$a_{\mathrm{J}}(\tau) = a_{2*}(\tau+\tau_2)^s, \quad s = \tfrac{1}{2}d_1(q-1)+1; \tag{56}$$

$$b(\tau) = b_0\frac{\tau+\tau_2}{\tau_*}, \quad \tau_* = \tau_0 + \tau_2, \tag{57}$$

where τ_2 and a_{2*} are integration constants, b_0 and τ_0 are the present-day values of b and τ, and b_0 is still given by (52) leading to the same constraint $d_1 \geq 8$.

The viability of this Jordan picture as an accelerated cosmology is provided by the condition $s > q \gg 1$ in (47). We should also consider time variations of α and the gravitational constant G: the latter is constant by definition in the Einstein frame but changes in Jordan's by the same law as α,

$$G/G_0 = \alpha/\alpha_0 = \mathrm{e}^{d_1(\beta_0-\beta)}. \tag{58}$$

Variations of α on Earth are restricted to the 17th significant digit, see (2), (3). The "Australian dipole" (1) testifies to variations on the level of 10^{-15} per year. The observational constraints on G are much weaker, see (4). Consequently, if we constrain the parameters of our model by variations of α, the observational bounds on G variations hold automatically and need not be considered separately.

The Hubble parameter is here $H_0 = s/\tau_*$, for $d\alpha/d\tau$ we have the expression $\alpha_0(d_1/s)H_0$, and since $d_1/s \approx 2/q$ for $d_1 \geq 8$, the estimate (53) still remains valid and leads to the same constraints on c_1 and c_2 as in the Einstein frame.

4 x-dependent perturbations and varying α

4.1 General relations

We have discussed the properties of homogeneous models which cannot account for spatial variation of any physical quantity.

Let us now try to describe variations of α by taking into account spatial perturbations of the effective scalar field and the metric. We take the Einstein-frame metric more general than (25),

$$ds^2_{\rm E} = {\rm e}^{2\gamma}dt^2 - {\rm e}^{2\lambda}dx^2 - {\rm e}^{2\eta}(dy^2 + dz^2), \qquad (59)$$

where γ, λ, η are functions of x and t. We will not discuss the reasons why the metric perturbation has a distinguished direction, only mentioning a possible weak inhomogeneity at the beginning of the inflationary period and the opportunity of domain walls that can be thick on the cosmological scale.

The conditions that the metric (59) only slightly differs from (25) are

$$\gamma = \delta\gamma(x,t), \quad \lambda = \ln a(t) + \delta\lambda(x,t),$$
$$\eta = \ln a(t) + \delta\eta(x,t),$$

where all "deltas" are small. We also replace the effective scalar field $\beta(t)$ with $\beta(t) + \delta\beta(x,t)$. Then the relevant Einstein-scalar equations corresponding to the Lagrangian (27) can be written as follows (preserving only terms linear in the "deltas"):

$$\delta\ddot\beta + \frac{3\dot a}{a}\delta\dot\beta + \dot\beta(\delta\dot\lambda - \delta\dot\gamma) - \frac{\delta\beta''}{a^2} + \frac{\delta(V_\beta\,{\rm e}^{2\gamma})}{2K_0} = 0, \qquad (60)$$

$$\frac{\dot a}{a}(\delta\dot\lambda - \delta\dot\gamma) = \delta(V\,{\rm e}^{2\gamma}), \qquad (61)$$

$$\frac{\dot a}{a}\delta\gamma' = K_0\dot\beta\,\delta\beta', \qquad (62)$$

where we choose the gauge (i.e,, the reference frame in perturbed space-time) $\delta\eta \equiv 0$, dots and primes stand for $\partial/\partial t$ and $\partial/\partial x$, respectively. We also denote $V_\beta = dV/d\beta$.

Without loss of generality, (62) leads to

$$\delta\gamma = \frac{K_0}{H}\dot\beta\delta\beta, \qquad (63)$$

where, as before, $H = \dot a/a$. Substituting this $\delta\gamma$ to (60) and taking the difference $\delta\dot\lambda - \delta\dot\gamma$ from (61), we arrive at the following single wave equation for $\delta\beta$:

$$\delta\ddot\beta + \frac{3\dot a}{a}\delta\dot\beta - \frac{1}{a^2}\delta\beta'' + \delta\beta\left[\frac{2\dot\beta^2}{H^2}VK_0 + \frac{2\dot\beta}{H}V_\beta + \frac{1}{2K_0}V_{\beta\beta}\right] = 0 \qquad (64)$$

with an arbitrary constant K_0 and an arbitrary potential $V(\beta)$. Assuming that the background quantities $a(t)$ and $\beta(t)$ are known, it remains to find a solution for $\delta\beta$ which, being added to the background $\beta(t)$, would be able to account for the observed picture of variations of α.

Since the background is x-independent, we can separate the variables and assume

$$\delta\beta = y(t)\sin k(x + x_0)$$

where k has the meaning of a wave number. Then $y(t)$ obeys the equation

$$\ddot y + \frac{3\dot a}{a}y + \frac{k^2 y}{a^2} + \left[\frac{2\dot\beta^2}{H^2}VK_0 + \frac{2\dot\beta}{H}V_\beta + \frac{1}{2K_0}V_{\beta\beta}\right]y = 0 \qquad (65)$$

Since Eq. (65) is itself approximate and describes only a restricted period of time close to the present epoch. it is reasonable to seek its solution in the form of a Taylor series near $t = t_0$,

$$y(t) = y_0 + y_1(t - t_0) + \frac{1}{2}y_2(t - t_0)^2 + \ldots, \qquad (66)$$

with $y_i = {\rm const}$. Then y_0 and y_1 can be fixed at will as initial conditions. Even more than that, for a certain neighborhood of $t = t_0$ we can simply suppose $y = y_0 + y_1(t - t_0)$. Actually, in our models this approximation is good enough for $t - t_0 \ll t_*$.

In this approximation we obtain the following expression for variations of α:

$$\frac{\alpha}{\alpha_0} \approx 1 + d_1[\beta_0 - \beta(t) - (y_0 + y_1(t-t_0))\sin k(x+x_0)] + O(\epsilon^2), \quad (67)$$

where $O(\epsilon^2)$ means $O((t-t_0)^2/t_*^2)$ and $\beta(t)$ is the background solution in a particular model.

4.2 The two models in Einstein's frame

Model 1. The function $\beta(t)$ is given by (39). Assuming that the observer is located at $x = 0$ and requiring $\alpha/\alpha_0 = 1 + O(\epsilon^2)$ at $x = 0$, we obtain the condition

$$y_1 \sin(kx_0) = -1/(\bar{d}\, t_*). \quad (68)$$

This explains very small, if any, variations of α on Earth at present and since the Oklo times. To account for the "Australian dipole", we need an approximately linear dependence $\alpha(x)$ on the past light cone of the point $t = t_0$, $x = 0$.

At small enough k (that is, assuming a very long wave of perturbations), so that $kx \ll 1$ and $kx \sim (t - t_0)/t_*$, a substitution of (68) into (67) leads to [15]

$$\alpha/\alpha_0 \approx 1 - d_1 y_0 \sin(kx_0) + d_1 y_0\, kx \cos(kx_0) + O(\epsilon^2), \quad (69)$$

so that a time dependence is eliminated from the expression for α up to $O(\epsilon^2)$ while the x dependence is linear, as required. (Though, since the measurement errors are rather large, the x dependence should not necessarily be strictly linear.)

This model contains the input theoretical parameters d_1, c_1, c_2, and the constants k, x_0, y_0, y_1, which can be ascribed to slightly inhomogeneous initial conditions at primordial inflation. Their choice enables us to explain the spatial variations of α in agreement with the observations [2]. Actually, there are only two conditions imposed on them: (68) and the relationship identifying (69) with the expression (1) at $r = x$ and $\cos\psi = 1$, i.e., on the dipole axis. We obtain (in Planck units)

$$d_1 y_0 k \cos(kx_0) \approx -2 \times 10^{-66}. \quad (70)$$

The small constant shift of the α value at $x = 0$ against the background does not change the interpretation of these results.

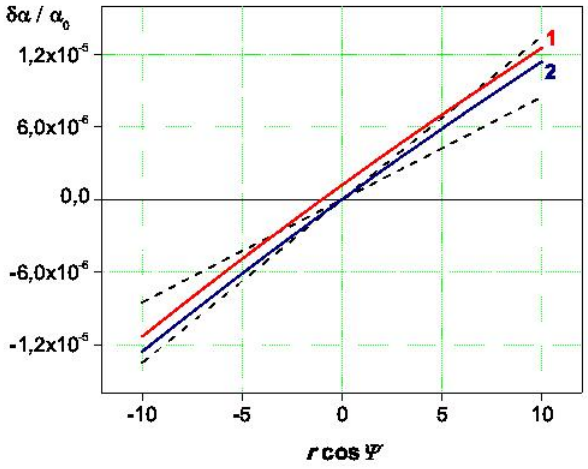

Figure 2: The r dependence of $\delta\alpha/\alpha_0$; the distance r is measured in billions of light years (BLY) (from [15]). The dashed lines correspond to Eq. (1), the solid lines to Eq. (67) at the parameter values $d_1 = 12$, $p = 10^7$, $y_0 = -2\times10^{-5}$, $y_1 = -10^{-7}$ (bill. years)$^{-1}$, $k = 0.005$ (BLY)$^{-1}$. Line 1 corresponds to $x_0 = 1$ BLY, line 2 to $x_0 = 0.01$ BLY.

The constraint on the input parameters c_1 and c_2 due to slow variations of α on Earth is now cancelled since this condition is provided by the equality (68). We should only provide the validity of the approximation in which we work, that is, $p \gg 1$; it holds fairly well if $c_1 + c_2 \gtrsim 1000$. Hence the inequality (42) is replaced by a much weaker one:

$$1000 \lesssim c_1 \sim c_2 \ll 10^{11}. \quad (71)$$

Fig. 2 presents the distance dependence of $\delta\alpha/\alpha$ (see (67)) for some values of the parameters.

Model 2. Now the background function $\beta(t)$ is given by (51), and as a result,

$$\frac{\alpha}{\alpha_0} \approx 1 - d_1 \sin[k(x+x_0)]\,[y_0 + y_1(t-t_0)] - \frac{2(t-t_0)}{t_*} + O(\epsilon^2), \quad (72)$$

To eliminate variations of α at $x = 0$, we require

$$y_1 \sin(kx_0) = -2/(d_1\, t_*). \quad (73)$$

Then, for $x \ll t_*$, and $kx \ll 1$, we obtain on the past light cone an expression coinciding with (69):

$$\frac{\alpha}{\alpha_0} \approx 1 - d_1 y_0 \sin(kx_0) + d_1 y_0\, kx \cos(kx_0). \quad (74)$$

Fig. 2, showing a comparison of our models with the Australian dipole (1), thus applies to both kinds of models, with and without the term Λ_D in $F(R)$ in the initial action.

4.3 Model 2 in Jordan's frame

What changes in Jordan's frame if we use the same solution for $\beta(t,x)$? The conformal mapping does not affect the expression for $\alpha(x,t)$ as well as the light cone, only the scales along all coordinate axes change, causing slightly different relationships in terms of the Jordan-frame cosmic time τ. We now assume $y = y_0 + y_1(\tau - \tau_0)$ and obtain

$$\frac{\alpha}{\alpha_0} = 1 - d_1[y_0 + y_1(\tau-\tau_0)\sin k(x+x_0)] - \frac{d_1(\tau-\tau_0)}{\tau_*} + O(\epsilon^2), \qquad (75)$$

Zero variations at $x = 0$ are now provided by

$$y_1 \sin(kx_0) = -1/\tau_*. \qquad (76)$$

Under this condition, for $\tau - \tau_0 \ll \tau_*$, $kx \ll 1$, we have again the same relation (69) or (74) for purely spatial variations of α. Since it is the Jordan picture, the same variations are predicted for the gravitational constant G.

5 Conclusion

Continuing the study begun in [15], we have considered some cosmological models that follow from curvature-nonlinear multidimensional gravity in the slow-change (on the Planck scale) approximation, which after reduction to four dimensions turn into multi-scalar-tensor gravity. We have shown that the recently reported variations of the fine structure constant α in space and time [1,2] can be explained in the framework of such models chosen as slightly perturbed isotropic models: both an accelerated expansion and variations of α follow from the behavior of the scalar field of multidimensional origin. The agreement with observations is provided by the choice of initial data, which can be interpreted as random values of the extra-dimensional metric at the inflationary stage of the Universe. Thus spatial and temporal variations of α can be manifestations of the multidimensional space-time geometry.

The models described here are very simple, approximate and tentative but still work fairly well at times not too far from the present epoch. They do not take into account other kinds of matter than dark energy; their inclusion must lead to a better description on a wider time interval. A possible existence of more than one extra factor space, which will lead to more than one effective scalar field in four dimensions, can hopefully lead to a viable model covering the whole classical evolution of the Universe beginning with primordial inflation.

We showed that the model considered in [15] is viable only in Einstein's conformal frame and suggested another model, viable in both Einstein's and Jordan's pictures. In the latter, along with α, the gravitational constant G varies at the same rate. Such variations are in agreement with the observational constraints, see (4). Let us note that the prediction of simultaneous variations of different physical constants due to a common cause, namely, variation of the size of extra dimensions, is a common feature of multidimensional theories. Thus, for example, a possible future discovery of G variations qualitatively different from those of α, can put to doubt not only models of the kind considered here but the whole paradigm of multidimensional gravity. On the contrary, a discovery of parallel evolution of different constants would be a strong argument in favor of extra dimensions.

Acknowledgments

We thank M.I. Kalinin for helpful discussions.

References

[1] J. K. Webb et al., Further evidence for cosmological evolution of the fine structure constant. Phys. Rev. Lett. **87**, 091301 (2001).

[2] J. K. Webb et al., Evidence for spatial variation of the fine structure constant. Phys. Rev. Lett. **107**, 191101 (2011); ArXiv: 1008.3907.

[3] J. C. Berengut and V. V. Flambaum, Astronomical and laboratory searches for space-time variation of fundamental constants. J. Phys. Conf. Ser. **264**, 012010 (2011); Arxiv: 1009.3693.

[4] T. Rosenband et al., Observation of the 1S0¿3P0 Clock Transition in $^{27}Al^{+}$. Phys. Rev. Lett. **98**, 220801 (2007).

[5] A. I. Shlyakhter, Direct test of the constancy of fundamental nuclear constants. Nature **260**, 340 (1976).

[6] Y. Fujii et al. The nuclear interaction at Oklo 2 billion years ago. Nucl. Phys. **B573**. 377 (2000).

[7] T. Chiba. The constancy of the constants of Nature: Updates. Prog. Theor. Phys. **126**, 993–1019 (2011); ArXiv: 1111.0092.

[8] T. Chiba and M. Yamaguchi, Runaway domain wall and space-time varying α. JCAP **1103**, 044 (2011); ArXiv: 1102.0105.

[9] K. A. Olive, M. Peloso, and J.-P. Uzan, The wall of fundamental constants. Phys. Rev. D **83**, 043509 (2011); ArXiv: 1011.1504.

[10] K. A. Olive, M. Peloso, and A. J. Peterson, Where are the walls? ArXiv: 1204.4391.

[11] K. Bamba, S. Nojiri, S.D. Odintsov, ArXiv: 1107.2538.

[12] J. D. Barrow and S. Z. W. Lip, A generalized theory of varying alpha. ArXiv: 1110.3120.

[13] A. Mariano and L. Perivolaropoulos, Is there correlation between fine structure and dark energy cosmic dipoles? ArXiv: 1206.4055.

[14] A. Mariano and L. Perivolaropoulos, CMB maximum temperature asymmetry axis: alignment with other cosmic asymmetries. ArXiv: 1211.5915.

[15] K. A. Bronnikov, V. N. Melnikov, S. G. Rubin, and I. V. Svadkovsky, Nonlinear multidimensional gravity and the Australian dipole, ArXiv: 1301.3098.

[16] V. N. Melnikov, Multidimensional classical and quantum cosmology and gravitation. Exact solutions and variations of constants. In: Cosmology and Gravitation, ed. M. Novello, Editions Frontieres, Singapore, 1994, p. 147.

[17] V. N. Melnikov, Gravity and cosmology as key problems of the millennium. In: *Albert Einstein Century Int. Conf.*, eds. J.-M. Alimi and A. Fuzfa, AIP Conf. Proc. 861, 2006, p. 109.

[18] K. A. Bronnikov and S. G. Rubin, Self-stabilization of extra dimensions. Phys. Rev. **D 73**, 124019 (2006).

[19] J. Mueller and L. Biskupek. Variations of the gravitational constant from lunar laser ranging data. Class. Quantum Grav. **24**, 4533–4538 (2007).

[20] K. A. Bronnikov and V. N. Melnikov, On observational predictions from multidimensional gravity. Gen. Rel. Grav. **33**, 1549 (2001).

[21] K. A. Bronnikov and V. N. Melnikov. Conformal frames and D-dimensional gravity, gr-qc/0310112; in: *Proc. 18th Course of the School on Cosmology and Gravitation: The Gravitational Constant. Generalized Gravitational Theories and Experiments* (30 April–10 May 2003, Erice), Ed. G.T. Gillies, V.N. Melnikov and V. de Sabbata, (Kluwer, Dordrecht/Boston/London, 2004) pp. 39–64.

[22] K. A. Bronnikov, S. G. Rubin, and I. V. Svadkovsky, Multidimensional world, inflation and modern acceleration Phys. Rev. D **81**, 084010 (2010).

[23] S. V. Bolokhov, K. A. Bronnikov, and S. G. Rubin, Extra dimensions as a source of the electroweak model, Phys. Rev. D **84**, 044015 (2011).

[24] S. G. Rubin and A. S. Zinger, The Universe formation by a space reduction cascade with random initial parameters, Gen. Rel. Grav. **44**, 2283 (2012); ArXiv: 1101.1274.

[25] K. A. Bronnikov and S. G. Rubin, Black Holes, Cosmology and Extra Dimensions (World Scientific, Singapore, 2012).