

# ON A CONJECTURE OF H. WU

ROBERT TREGER

In an email (January, 2015), Sullivan has asked the author how much one can do for coverings of compact complex manifolds. In this short note, we derive a conjecture of H. Wu from the theorem in [4], provided the fundamental group  $\pi_1(X)$  is residually finite. The author is grateful to Dennis Sullivan for the question.

**Conjecture (H. Wu).** *The universal covering  $U$  of a compact Kahler manifold  $X$  of dimension  $n$  with negative sectional curvature is a bounded domain in  $\mathbf{C}^n$ .*

**Proposition.** *Let  $U$  be the universal covering of a compact Kahler manifold  $X$  of dimension  $n$  with negative sectional curvature and residually finite  $\pi_1(X)$ . Then  $U$  is a bounded domain in  $\mathbf{C}^n$ .*

*Proof.* We consider  $U$  with the Kahler metric induced from  $X$ .

According to Wu [5] (see also [2, Problem B(ii), p. 45]),  $U$  is Stein hence  $\pi_1(X)$  is large, i.e.,  $U$  contains no proper submanifolds of positive dimension.

According to Ballmann and Eberlein [1],  $\pi_1(X)$  is nonamenable.

We will show  $X$  is projective. According to Gromov,  $X$  is Kahler hyperbolic [3, (0.3.A), p. 265]. The canonical bundle  $\mathcal{K}_X$  is quasiample [3, (0.4.C), p. 267; (3.2.B), (3.2.B') p. 287]. Thus  $X$  is Moishezon and Kahler hyperbolic hence  $X$  is projective. The same follows from a theorem of Kodaira as follows. Since the sectional curvature of  $X$  is negative, the Ricci form of the volume form is negative. Hence  $\mathcal{K}_X$  is ample by Kodaira.

Now, we can apply the theorem in [4, Introduction].

## REFERENCES

- [1] W. Ballmann, P. Eberlein, *Fundamental group of manifolds of nonpositive curvature*, J. Diff. Geometry **25** (1987), 1–22.
- [2] P. A. Griffiths, *Complex-analytic properties of certain Zariski open sets on algebraic varieties*, Ann. of Math. **94** (1971), 21–51.
- [3] M. Gromov, *Kahler Hyperbolicity and  $L_2$ -Hodge theory*, J. Diff. Geometry **33** (1991), 263–292.
- [4] R. Treger, *Metrics on universal covering of projective variety*, arXiv:1209.3128v5.[math.AG].
- [5] H. Wu, *Negatively curved Kahler manifolds*, Notices Amer. Math. Soc. **14** (1967), 515.

PRINCETON, NJ 08540

*E-mail address:* roberttreger117@gmail.com