Comment on "Texture Zeros and WB Transformations in the Quark Sector of the Standard Model"

Samandeep Sharma, Priyanka Fakay, Gulsheen Ahuja*, Manmohan Gupta
Department of Physics, Centre of Advanced Study, P.U., Chandigarh, India.

**gulsheen@pu.ac.in*

February 16, 2018

Abstract

Recently, using specific Weak Basis transformations, Y. Giraldo [Phys. Rev. D 86, 093021 (2012)] has constructed some texture 5 and 4 zero quark mass matrices and examined their compatibility with the quark mixing data. In this comment, we have re-analyzed these to bring forth certain important issues regarding their viability which need to be taken note of.

In the context of flavor physics, texture specific mass matrices have provided valuable information to understand the flavor mixing data [1]. The relationship of these mass matrices with the Weak Basis (WB) transformations have been discussed by several authors [2, 3, 4]. In particular, using WB transformations, recently Y. Giraldo [4] has made an attempt to explicitly construct texture five zero and texture four zero quark mass matrices. While examining the compatibility of these mass matrices with the mixing data, we come across certain issues which need to be taken note of.

To begin with, we re-analyze a particular "class" of texture 5 zero non-Fritzsch like quark mass matrices, discussed in section V of [4] and shown to be compatible with the latest mixing data, e.g.,

$$M_U = P^\dagger \begin{pmatrix} 0 & 0 & |C_U| \\ 0 & A_U & |B_U| \\ |C_U| & |B_U| & \tilde{B_U} \end{pmatrix} P, \qquad M_D = \begin{pmatrix} 0 & |C_D| & 0 \\ |C_D| & 0 & |B_D| \\ 0 & |B_D| & A_D \end{pmatrix}. \tag{1}$$

This, however, appears to be somewhat in conflict with the earlier analyses of similar matrices [1], therefore making it necessary to re-examine the compatibility of the above mentioned mass matrices with the recent data.

The essentials of the methodology usually used to carry out the analysis include diagonalizing the mass matrices M_U and M_D by unitary transformations and obtaining a Cabibbo-Kobayashi-Maskawa (CKM) matrix from these transformations.

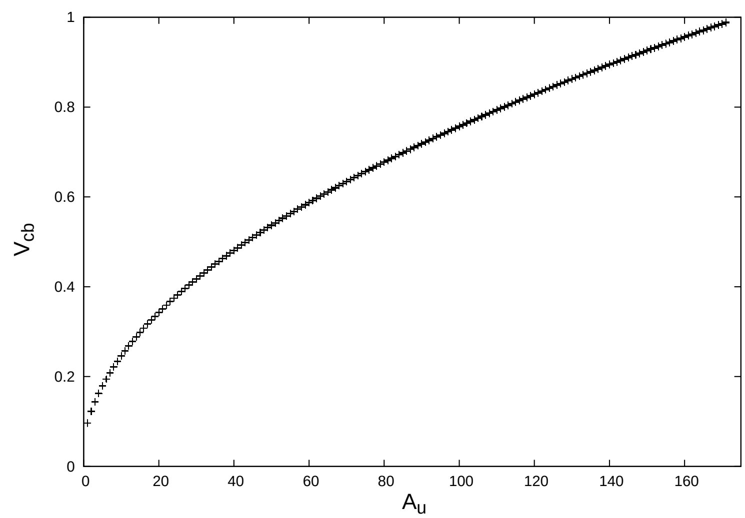

Figure 1: Plot showing the dependence of CKM matrix element V_{cb} on A_U, the free parameter of the mass matrix M_U. The values of A_U are in GeV units.

To ensure the viability of the considered mass matrices, this CKM matrix should be compatible with the quark mixing data, for details regarding this we refer the readers to [1]. Following this methodology for the above mentioned matrices considered by Giraldo, the CKM matrix so obtained is given by

$$V_{\rm CKM} = \begin{pmatrix} 0.9741-0.9744 & 0.2247-0.2260 & 0.0048-0.0119 \\ 0.0161-0.0833 & 0.0019-0.1461 & 0.0960-0.9890 \\ 0.0015-0.2288 & 0.0011-0.9755 & 0.0020-0.9999 \end{pmatrix}. \quad (2)$$

A look at this matrix immediately reveals that the ranges of some of the CKM elements, in particular of $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$ and $|V_{cb}|$, show no overlap with those obtained by recent global analyses [5]. This, therefore, leads one to conclude that the texture 5 zero non-Fritzsch like quark mass matrices considered in [4] are not compatible with the recent quark mixing data.

To re-emphasize our conclusion, we have examined the dependence of one of the CKM matrix element $|V_{cb}|$, obtained from the mass matrices considered here, solely on A_U, the free parameter of the mass matrix M_U. This variation has been plotted in Figure (1) from which one can easily find that for all possible values of A_U, the values of $|V_{cb}|$ are much larger than the allowed range $0.0407-0.0423$ [5], clearly ruling out these mass matrices.

The above conclusion is in direct conflict with that of Ref. [4], however it is not difficult to understand how a fit for quark mixing matrix elements as well as the Jarlskog's rephasing invariant parameter J has been obtained in [4]. It can be seen easily that this has been achieved by introducing additional phases in the unitary matrices used for diagonalizing the mass matrices M_U and M_D which cannot be rephased away in the mixing matrix and hence can be considered physical. To emphasize that these phases are physical or represent additional parameters, we reconstruct the complex mixing matrix, corresponding to the one given in equation (2), e.g.,

$$V_{\rm CKM} = \begin{pmatrix} 0.3447 + 0.911361i & 0.02595 + 0.2233i & 0.00893 - 0.00069i \\ -0.02391 - 0.050292i & 0.01658 - 0.01157i & 0.16240 - 6.4312 \times 10^{-5} i \\ 0.00427 - 0.00755i & -0.0230 + 0.0272i & 0.9932 - 0.0039i \end{pmatrix}. \tag{3}$$

It may be mentioned that the above matrix has been constructed using some typical values of input parameters, e.g.,

$$A_U = 4.0\ GeV, \quad \phi_1 = 110^0, \quad \phi_2 = 10^0, \tag{4}$$

which are within the ranges of these required to reproduce the matrix in equation (2).

Using the relation [5]

$$J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln} = {\rm Im}(V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}), \tag{5}$$

for the CKM matrix in equation (3) the value of Jarlskog's parameter J is 5.564×10^{-6}. Even on allowing full variation to all the input parameters, one gets $|J| \leq 5.6428 \times 10^{-6}$, clearly outside the range given by PDG 2012, i.e., $(2.80-3.16)\times 10^{-5}$. This is in contrast with the results given in [4], therefore, the phases introduced in are additional and physical ones and their addition leads to an ad hoc fitting of the mixing matrix.

To further verify that additional phases introduced in [4] are physical and cannot be rephased away, we have investigated the implications of the mass matrices constructed in [4] on other CP violating parameters as well. In this context, we try to find the value of CP violating parameter ϵ_k using the following rephasing invariant expression given by Buras *et al.* [6]

$$|\epsilon_K| = \kappa_\epsilon \frac{G_F^2 F_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_k} B_K {\rm Im}\lambda_t [{\rm Re}\lambda_c(\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)) - {\rm Re}\lambda_t \eta_2 S_0(x_t)], \tag{6}$$

where η_1, η_2, η_3 are the perturbative QCD corrections, $S_0(x_i)$ are Inami-Lim functions, $x_i = m_i^2/M_W^2$, and $\lambda_i = V_{id}V_{is}^{*}$, $i = c, t$.

To this end, in Table 1 we present specific cases of texture 5 and 4 zero mass matrices considered by [4] and the corresponding ϵ_k values found here. Interestingly, a look at the table shows that these ϵ_k values are much lower than its experimental value i.e. $(2.228 \pm 0.011) \times 10^{-3}$ [5]. This can be understood by examining the CKM matrices corresponding to the numerical mass matrices presented in Table 1. For example, for the first set of mass matrices presented in row (a) of the table, we get the following numerical CKM matrix

$$\begin{pmatrix} -0.484 - 0.845i & 0.151 + 0.167i & -0.0028 - 0.002i \\ -0.22519 - 0.0012i & -0.952 + 0.202i & 0.029 + 0.029i \\ 0.0034 - 0.008i & 0.0223 - 0.0336i & 0.9991 - 0.0051i \end{pmatrix}. \tag{7}$$

	M_U	M_D	ϵ_k
a	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & -92.3618+157.694i \\ \mathbf{0} & 5748.17 & 28555.1+5911.83i \\ -92.3618-157.694i & 28555.1-5911.83i & 166988 \end{pmatrix}$,	$\begin{pmatrix} \mathbf{0} & 13.9899 & \mathbf{0} \\ 13.9899 & \mathbf{0} & 424.808 \\ \mathbf{0} & 424.808 & 2796.9 \end{pmatrix}$	1.07038×10^{-3}
b	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & 123.038-285.496i \\ \mathbf{0} & 1430.03 & 18632.8-2336.25i \\ 123.038+285.496i & 18632.8+2336.25i & 170033 \end{pmatrix}$,	$\begin{pmatrix} \mathbf{0} & 13.2473 & \mathbf{0} \\ 13.2473 & \mathbf{0} & 425.817 \\ \mathbf{0} & 425.817 & 2796.6 \end{pmatrix}$	0.90177×10^{-3}
c	$\begin{pmatrix} \mathbf{0} & 4543.2 & \mathbf{0} \\ 4543.2 & -171468 & 9388.13 \\ \mathbf{0} & 9388.13 & 7.34102 \end{pmatrix}$,	$\begin{pmatrix} \mathbf{0} & 123.93+10.0184i & \mathbf{0} \\ 123.93-10.0184i & -2829.92 & 267.035+1.39152i \\ \mathbf{0} & 267.035-1.39152i & 29.738 \end{pmatrix}$	1.418×10^{-3}

Table 1: Specific cases of texture 5 and 4 zero mass matrices constructed by [4] and their corresponding ϵ_k values found here.

For the purpose of comparison, we have also constructed here the CKM matrix using the latest values [5] of Wolfenstein parameters, e.g.,

$$\begin{pmatrix} 0.9746 & 0.2254 & 0.0012-0.0032i \\ -0.2254 & 0.9746 & 0.0412 \\ 0.0081-0.0032i & -0.0412 & 1 \end{pmatrix}. \qquad (8)$$

A look at the above matrices clearly reveals that most of the elements of mixing matrix given in equation (7) contain sizeable imaginary parts as compared to those in equation (8). In principle, one could say that this may not have any implication in view of the facility of rephasing invariance of CKM matrix. However, this is not the case as can be found by subjecting the mixing matrix presented in equation (7) to CP violating parameter ϵ_k.

We would also like to mention a couple of more points, e.g., for the case of texture 4 zero quark mass matrices, well known to be compatible with the mixing data [1], while carrying out the calculations pertaining to these in [4] again arbitrary phase factors have been incorporated and some of the CP violating parameters such as Sin2β and J have been reproduced. Further, one finds that while constructing the non parallel texture 4 zero quark mass matrices, [4] assumes the (1,1)th entry to be zero because of large uncertainty in it. This zero does not reflect a WB choice and thus the non parallel texture 4 zero structure constructed by [4] is effectively texture 3 zero only which re-emphasizes the conclusion in [2] that starting with the most general quark mass matrices it is not possible to obtain more than three texture zeroes by any WB transformation. Further, it also needs to be mentioned that while constructing parallel texture 4 zero matrices, [4] does not start with the most general mass matrices, rather it starts with a special weak basis wherein the mass matrix M_U has been taken to be diagonal and only the matrix M_D is considered to be most general.

To summarize, we have re-analyzed specific cases of texture 5 and 4 zero quark mass matrices considered in [4]. Texture 5 zero mass matrices have been shown to be incompatible with the recent mixing data, in contrast with the findings of [4] wherein additional phases have been incorporated while showing the compatibility of these mass matrices. For some of the cases of texture 5 and 4 zero mass matrices considered in [4], we find that even after incorporating additional phases we are not able to reproduce the CP violating parameter ϵ_k. In conclusion, we would like to emphasize that one needs to be careful in analyzing the implications of Weak Basis transformations on textures.

Acknowledgements
G.A. would like to acknowledge DST, Government of India (Grant No: SR/FTP/PS-017/2012) for financial support. P.F., S.S., G.A. acknowledge the Chairperson, Department of Physics, P.U., for providing facilities to work.

References

[1] M. Gupta, and G. Ahuja, Int. J. Mod. Phys. A **27**, 1230033 (2012).

[2] G. C. Branco, D. Emmanuel-Costa and R. G. Felipe, Phys. Lett. B **477**, 147 (2000).

[3] D. Emmanuel-Costa and C. Simoes, Phys. Rev. D **79**, 073006 (2009).

[4] Y. Giraldo, Phys. Rev. D **86**, 093021 (2012).

[5] J. Beringer *et al.*, Particle Data Droup, Phys. Rev. D **86**, 010001 (2012).

[6] A. J. Buras, hep-ph/0101336.