# Invisible  $K_L$  decays as a probe of new physics

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The decay  $K_L \rightarrow invisible$  has never been experimentally tested. In the Standard Model (SM) its branching ratio for the decay into two neutrinos is helicity suppressed and predicted to be  $Br(K_L \to \nu\bar{\nu}) \lesssim 10^{-10}$ . We consider several natural extensions of the SM, such as two-Higgs-doublet (2HDM), 2HDM and light scalar, and mirror dark matter models, those main feature is that they allow to avoid the helicity suppression factor and lead to an enhanced  $Br(K_L \to invisible)$ . For the decay  $K_L \rightarrow \nu \bar{\nu}$  the smallness of the neutrino mass in the considered 2HDM model is explained by the smallness of the second Higgs doublet vacuum expectation value. The small nonzero value of the second Higgs isodoublet can arise as a consequence of nonzero quark condensate. We show that taking into account the most stringent constraints from the  $K \to \pi + invisible$  decay, this process could be in the region of  $Br(K_L \to invisible) \simeq 10^{-8} - 10^{-6}$ , which is experimentally accessible. In some scenarios the  $K_L \rightarrow invisible$  decay could still be allowed while the  $K \rightarrow \pi + invisible$ decay is forbidden. The results obtained show that the  $K_L \rightarrow invisible$  decay is a clean probe of new physics scales well above 100 TeV, that is complementary to rare  $K \to \pi + invisible$  decay, and provide a strong motivation for its sensitive search in a near future experiment.

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### I. INTRODUCTION

In the Standard Model (SM) the branching ratios of the  $K^+ \rightarrow \pi^+ + invisible$  and  $K_L \rightarrow \pi^0 + invisible$ decays are predicted to be [\[1\]](#page-4-0)

<span id="page-0-0"></span>
$$
Br(K_L \to \pi^0 \nu \bar{\nu}) = (2.6 \pm 0.4) \times 10^{-11}, \quad (1)
$$

<span id="page-0-1"></span>
$$
Br(K^{+} \to \pi^{+} \nu \bar{\nu}) = (8.5 \pm 0.7) \cdot 10^{-11}, \quad (2)
$$

with the invisible final state represented by neutrino pairs. A strong comparison between experiment and theory is possible due to the accuracy of both the measurements and the SM calculations of these observables. A discrepancy would signal the presence of physics beyond the Standard Model (BSM) making the precision measurements of these decays an effective probe to search for it, see e.g. [\[1](#page-4-0)[–6\]](#page-4-1).

The branching ratio of the  $K_L \rightarrow invisible$  decay in the SM is predicted to be very small compared to those of Eq.[\(1\)](#page-0-0) and Eq.[\(2\)](#page-0-1) for  $\nu$  masses laying in the sub-eV region favored by observations of  $\nu$  oscillations [\[7](#page-4-2)]. Indeed, the  $K_L$  has zero spin, and it cannot decay into two massless neutrinos, as it contradicts to momentum and angular momentum conservation simultaneously. For the case of massive  $\nu s$  their spins in the  $K_L$  rest frame must be opposite and, therefore, one of them is forced to have the "wrong" helicity. This results in the  $K_L \rightarrow \nu \overline{\nu}$ decay rate being proportional to the  $\nu$  mass squared  $\Gamma(K_L \rightarrow \nu \overline{\nu}) \propto \left( \frac{m_{\nu}}{m_{K_L}} \right)$  $\Big)^2 \leq 10^{-17}$  assuming  $m_{\nu} \leq 1$ eV. However, if one take the direct experimental upper limit on the  $\nu_{\tau}$  mass  $m_{\nu_{\tau}} < 18.2$  MeV [\[7](#page-4-2)], the predicted branching ratio, calculated at the quantum loop level is  $|8|$ 

$$
Br(K_L \to \nu \overline{\nu}) \simeq 10^{-10} \tag{3}
$$

Therefore, an observed  $Br(K_L \to invisible) \gg 10^{-10}$ would unambiguously signal the presence of BSM physics.

The decay  $K_L \rightarrow invisible$  has never been experimentally tested. Since long ago it was recognized that this decay "would be interesting to explore, but its detection looks essentially impossible. New ingenious experimental ideas are required" [\[8](#page-4-3)]. Recently, an approach for performing such kind of experiments by using the  $K^+n \to K^0p$  (or  $K^-p \to \overline{K}^0n$ ) charge-exchange reaction as a source of well tagged  $K^{0}$ 's has been reported [\[9\]](#page-4-4). At the same time the first experimental bound  $Br(K_L \to invisible) \lesssim 6.3 \cdot 10^{-4}$  has been set from existing experimental data. It has been shown, that compared to this limit, the expected sensitivity of the proposed search is at least two orders of magnitude higher -  $Br(K_L \to invisible) \lesssim 10^{-6}$  per  $\simeq 10^{12}$  incident kaons. It could be further improved by utilizing a more careful design of the experiment, thus making the region  $Br(K_L \rightarrow invisible) \simeq 10^{-8} - 10^{-6}$ , or even below, experimentally accessible [\[9\]](#page-4-4).

Being motivated by these considerations, we discuss in this work several natural extensions of the SM and show that taking into account the most stringent constraints from the measured  $K^+ \rightarrow \pi^+ + invisible$  decay rate, the decay  $K_L \rightarrow invisible$  could occur at the level  $Br(K_L \to invisible) \simeq 10^{-8} - 10^{-6}$ . The main feature of the considered models, that leads to an enhanced branching ratio for  $K_L \rightarrow invisible$ , compared to  $K^+ \rightarrow \pi^+ + invisible$ , is that they allow to avoid the helicity suppression factor  $\left(\frac{m_{\nu}}{m_{K_L}}\right)$  $\int^2$  in the SM, while profiting from its larger phase-space due to the decay into two light weakly interacting particles. In addition, there might be the case when  $K_L \rightarrow invisible$  could still be kinematically allowed, while  $K^+ \to \pi^+ + invisible$  is forbidden. Additional motivation to search for the  $K_L$ (and  $K<sub>S</sub>$ ) invisible decay is related to precision tests of the  $K^0 - \overline{K}^0$  system by using the Bell-Steinberger uni-tarity relation [\[9](#page-4-4)]. This relation connects  $CP$  and  $CPT$ violation in the mass matrix to  $CP$  and  $CPT$  violation in all decay channels of neutral kaons and is a powerful tool for testing  $CPT$  invariance with neutral kaons [\[10\]](#page-4-5). The question of how much the invisible decays of  $K_S$ or  $K_L$  can influence the precision of the Bell-Steinberger analysis still remains open [\[11](#page-4-6)]. All this makes the future searches for this decay mode very interesting and complementary to the study of the  $K \to \pi + invisible$ decays.

### II.  $K_L \rightarrow \nu \bar{\nu}$  DECAY IN MODEL WITH ADDITIONAL SCALAR DOUBLET

Consider now the  $K_L \rightarrow \nu \bar{\nu}$  decay in the two-Higgsdoublet model (2HDM) with an additional heavy Higgs doublet  $H_2$ . This type of 2HDM models can introduce flavor-changing neutral currents, provide explanations of the origin of Dark Matter and CP violation , see e.g. Ref.[\[12](#page-4-7)]. The interaction of the heavy isodoublet field  $H_2$ with quarks, leptons and the standard Higgs isodoublet H leading to the  $K_L \rightarrow \nu \bar{\nu}$  decay has the form

$$
L_{int} = h_{2\tau} \bar{L}_{\tau} \tilde{H}_{2} \nu_{\tau_R} + h_{2d_{LSR}} \bar{Q}_{1L} H_{2} s_R
$$
  
+  $\delta m_{HH_2}^2 H^+ H_2 + h.c. - M_{H_2}^2 H_2^+ H_2$ , (4)

where  $L_{\tau} = (\nu_{\tau_L}, \tau_L), Q_{1L} = (u_L, d_L), H_2 = (H_2^+, H_2^0),$  $\tilde{H}_2 = ((H_2^0)^*, -\tilde(H_2^+)^*)$  and  $h_{2\tau}, h_{2d_Ls_R}$  are Yukawa coupling constants. Note that in general the second Higgs isodoublet  $H_2$  will have nonzero Yukawa interactions with other quark and lepton fields but since we are interested mainly in the  $K_L \rightarrow \nu \bar{\nu}$  decay we have written explicitly only the Yukawa interactions important for us. In considered model the neutrinos acquire nonzero Dirac masses  $m_{\nu_{\tau}} = h_{2\tau} < H_2 >$  due to nonzero vacuum expectation value of the second Higgs isodoublet  $H_2$  $\langle H_2 \rangle = \frac{\delta m_{HH_2}^2}{M_{H_2}^2} \langle H \rangle \langle \langle H \rangle = 174 \text{ GeV}$  and the smallnes of the Dirac neutrino masses is a consequence of the  $H_2$  > smallnes. The smallnes of  $H_2$  is due to the assumed large value of  $M_{H_2}$  or(and) small value of  $\delta m_{HH_2}^2$ [\[35](#page-5-0)]. For instance, for  $m_{\nu_\tau} = 0.1 \text{ eV}$ ,  $h_{2\tau} = 0.1$ and  $M_{H_2} = 10^5$  GeV we find that  $\frac{\delta m_{HH_2}^2}{M_{H_2}^2} = 0.6 \cdot 10^{-11}$ and  $\delta m_{HH_2}^2 = 0.06 \; GeV^2$ . It is interesting to note that for  $\delta m_{HH_2}^2 = 0$  the  $\langle H_2 \rangle = 0$  at classical level but the spontaneous symmetry breaking of  $SU_L(3) \otimes SU_R(3)$ chiral symmetry in QCD leads to nonzero vacuum expectation values for the Higgs fields[\[14\]](#page-4-8). Really, for monzero Yukawa interaction  $L_{H_2Q_1d} = h_{2d_Ld_R}\bar{Q}_{1L}H_2d_R + h.c$ . due to nonzero vacuum expectation value of quark condensate  $\langle \bar{d}d \rangle = -\frac{f_{\pi}^2 m_{\pi}^2}{(m_u + m_d)} (f_{\pi} = 93 \text{ MeV})$  the field  $\langle H_2 \rangle$  acquires monzero vacuum expectation value  $\langle H_1 \rangle$  $H_2 \geq \frac{\bar{\zeta}d\bar{\zeta}}{2h_2d_{\bar{\zeta}}d_{\bar{\zeta}}}$  $\frac{}{2h_{2d_Ld_R}M_{H_2}^2}$ . Numerically for  $h_{2\tau} = h_{2d_Ld_R} = 1$ 

and  $m_{\nu_{\tau}} = 0.1 \text{ eV}$  we find that  $M_{H_2} \sim O(10^4) \text{ GeV}$ . So in this model with  $\delta m_{HH_2}^2 = 0$  the vacuum expectation value  $\langle H_2 \rangle = 0$  at tree level but the nonzero quark condensate leads to the appearance of small vacuum expectation value  $\langle H_2 \rangle \neq 0$  for the second Higgs isodoublet that explains the smallnes of the neutrino masses.

For the case of nonzero neutrino Majorana mass  $m_{\nu_{\tau_{\rm{B}}}}$ we assume that the mass  $m_{\nu_{\tau_R}}$  is small so the decay  $K_L \rightarrow \nu_\tau \bar{\nu_\tau}$  is kinematically allowed. Again, as in the previous case we assume that the Dirac neutrino mass arises due to nonzero  $\langle H_2 \rangle$  vacuum expectation value and the smallness of the see saw  $m_{\nu_{\tau_R}} = \frac{m_{D\nu_{\tau}}^2}{m_{\nu_{\tau_R}}}$  neutrino mass is again explained due to the smallness of  $\langle H_2 \rangle$ . The Lagrangian (4) contains  $\Delta S = 1$  neutral flavour changing terms but for heavy doublet  $H_2$  it is not dangerous. The effective four fermion Lagrangian describing the decay  $K_L \rightarrow \nu_\tau \bar{\nu_\tau}$  has the form

$$
L_{eff} = \frac{1}{M_X^2} \bar{d}_L s_R \bar{\nu}_{\tau_L} \nu_{\tau_R} + h.c. \,, \tag{5}
$$

where

$$
\frac{1}{M_X^2} = \frac{h_{2d_{LSR}} h_{2\tau}}{M_{H_2}^2} \,. \tag{6}
$$

As it has been mentioned before we assume the existence of small Dirac or Majorana neutrino mass  $\nu_{\tau}$ . The decay rate of the invisible decay  $K_L \rightarrow \nu_\tau \bar{\nu}_\tau$  is determined by formula

$$
\Gamma(K_L \to \nu_{L\tau} \bar{\nu}_{R\tau}, \nu_{R\tau} \bar{\nu}_{L\tau}) = \frac{M_{K_L}^5}{16\pi M_X^4}
$$
\n
$$
(\frac{F_K}{2(m_d + m_s)})^2 K(m_\nu^2 / M_{K_L}^2), \tag{7}
$$

where  $K(x) = (1-4x)^{1/2}$  for Dirac neutrino with a mass  $m_{\nu_{\tau}}$  and  $K(x) = (1-x)^2$  for Majorana neutrino  $\nu_{\tau_R}$  with a mass  $m_{\nu_{\tau_R}}$ . Here  $F_K \approx 160 \; MeV$  is kaon decay constant and  $m_s$ ,  $m_d$  are the masses of s- and d-quarks[\[36\]](#page-5-1). For  $Br(K_L \to \nu_\tau \bar{\nu_\tau}) = 10^{-6}$  we can test the value of  $M_X$ up to [\[37](#page-5-2)]

$$
M_X \lesssim 0.6 \cdot 10^5 \; GeV \tag{8}
$$

for small Dirac or Majorana neutrino mass  $m_{\nu_{\tau}} \ll M_{K_L}$ .

It should be noted that the existence of  $\Delta S = 1$  neutral flavour changing interaction (5) leads to additional contribution to rare decays  $K_L \to \pi^0 \nu \bar{\nu}$  and  $K^+ \to \pi^+ \nu \bar{\nu}$ . The current experimental values are [\[15\]](#page-4-9), [\[16](#page-4-10)]

$$
Br(K_L \to \pi^0 \nu \bar{\nu}) < 2.6 \times 10^{-8} , \qquad (9)
$$

$$
Br(K^+ \to \pi^+ \nu \bar{\nu}) = (17.3^{+11.5}_{-10.5}) \cdot 10^{-11}, \qquad (10)
$$

with the SM predictions of [\(1\)](#page-0-0) and [\(2\)](#page-0-1), respectively. The measured value (10) for the  $Br(K^+ \to \pi^+ \nu \bar{\nu})$  allows to set more stringent constraints. Therefore, we restrict ourselves to the calculation of the BSM contribution only to this decay channel by using the effective Lagrangian (5). This leads to the following formula for the differential  $K^+ \to \pi^+ \nu \bar{\nu}$  decay width:

$$
\frac{d\Gamma^{BSM}(K^+ \to \pi^+ \nu \bar{\nu})}{dq^2} = \frac{1}{(2\pi)^3} \cdot \frac{1}{32M_{K^+}^3} \cdot \frac{(q^2 - m_{\nu_{\tau,R}}^2)^2}{q^2 M_{\mathcal{X}}^4}
$$

$$
\cdot \sqrt{(M_{K^+}^2 + M_{\pi^+}^2 - q^2)^2 - 4M_{K^+}^2 M_{\pi^+}^2}
$$

$$
\cdot \left[ \frac{f_0(q^2)(M_{K^+}^2 - m_{\pi^+}^2)}{2(-m_d + m_s)} \right]^2 \tag{11}
$$

The form factor  $f_0(q^2)$  is determined in standard way as

$$
\langle \pi | \bar{d}\gamma_{\mu} s | K \rangle = f_{+}(q^{2}) (P_{K} + P_{\pi})^{\mu} + f_{-}(q^{2}) (P_{K} - P_{\pi})^{\mu} =
$$
\n(12)

$$
f_+(q^2)[(P_K+P_\pi)^{\mu}-\frac{M_K^2-M_\pi^2}{q^2}q^{\mu}]+f_0(q^2)\frac{M_K^2-M_\pi^2}{q^2}q^{\mu}
$$
,

where  $q^{\mu} = (P_K - P_{\pi})^{\mu}$  and  $m_{\nu_R}^2 \le (M_{K^+} - M_{\pi^+})^2$ . The form factors  $f_+$  and  $f_0$  are related to the exchange of  $1^$ and  $0^+$ , respectively. The following relation holds:

$$
f_{+}(0) = f_{0}(0) , f_{0}(q^{2}) = f_{+}(q^{2}) + \frac{q^{2}}{M_{K}^{2} - M_{\pi}^{2}} f_{-}(q^{2}).
$$
\n(13)

In our calculations we use standard linear parametrization for the form factor  $f_0(q^2)$ , namely

$$
f_0(q^2) = f_0(0)(1 + \lambda_0 \frac{q^2}{M_{\pi^+}^2}).
$$
 (14)

Numerically we take  $f_0(0) = 0.96[18]$  $f_0(0) = 0.96[18]$  and  $\lambda_0 = -0.06$ [\[19\]](#page-4-12).

It is convenient to represent the result in terms of the ratio  $\beta^{-1} \equiv \frac{Br(K_L \to \nu \bar{\nu})}{Br(K^+ \to \pi^+ \nu \bar{\nu})}$  because the ratio  $\beta$  does not depend on unknown value of  $M_X$ . Also  $\beta$  does not depend on the values of quark masses  $m_d$ ,  $m_s$ . For the case of massless neutrino we find that

$$
\beta \approx 2 \cdot 10^{-3} \,. \tag{15}
$$

Note that the smallness of the  $\beta$  is mainly due to the 3-body phase space smallnes in comparison with 2-body phase space. From the difference between the theoretical and experimental values (2) and (10), respectively, by summing up errors of (10) in quadrature we find that the BSM contribution to the  $Br(K^+ \to \pi^+ \nu \bar{\nu})$  is less than

$$
Br^{BSM}(K^+ \to \pi^+ \nu \bar{\nu}) \lesssim 2.1 \cdot 10^{-10} \,. \tag{16}
$$

From the limit (16) and the estimate (15) we find that for massless neutrinos

$$
Br(K_L \to \nu \bar{\nu}) \lesssim 10^{-7} \tag{17}
$$

The estimates (15, 17) are valid for small  $m_{\nu_R} \ll M_{\pi^+}$ Majorana msss of righthanded neutrino. For higher  $m_{\nu_R}$  values the limit (17) is more weak and for the case  $M_{K_L} \geq$  $m_{\nu_R} \geq M_{K^+} - M_{\pi^+}$  when the decay  $K^+ \to \pi^+ \nu_{\tau_L} \bar{\nu}_{\tau_R}$  is kinematically prohibited, but the decay  $K_L \rightarrow \nu \bar{\nu}$  is still allowed, the restriction from  $K^+ \to \pi^+ \nu \bar{\nu}$  decay does not work.

The measured  $(K_L - K_S)$  mass difference strongly re-stricts [\[20\]](#page-4-13) the effective  $\Delta S = 2$  interaction

$$
L_{\bar{s}d\bar{s}d} = \frac{1}{\Lambda_{\bar{s}d\bar{s}d}^2} \bar{s_R} d_L \bar{s_R} d_L + h.c. \quad . \tag{18}
$$

Namely [\[20](#page-4-13)]

$$
\Lambda_{\bar{s}d\bar{s}d} \ge 1.8 \cdot 10^7 \; GeV. \tag{19}
$$

For the model (2) with the the additional Higgs doublet  $H_2 = (H_2^+, H_{2,1}^0 + iH_{2,2}^0)$  we find that

$$
\frac{1}{\Lambda_{\bar{s}d\bar{s}d}^2} = |h_{2d_Ls_R}|^2 \left| \frac{1}{M_{H_{2,1}^0}^2} - \frac{1}{M_{H_{2,2}^0}^2} \right| \sim \frac{|h_{2d_Ls_R}|^2}{M_{H_2}^2} \cdot \frac{\delta m_{HH_2}^2}{M_{H_2}^2} \cdot (20)
$$

Using the bound (19) we can restrict the parameter  $\delta m_{HH_2}^2$ . For instance, for  $M_{H_2} = 10^5$  GeV,  $h_{2d_Ls_R} = 1$ we find  $\delta m_{HH_2}^2 \leq 0.3 \cdot 10^6 \; GeV^2$  that is much more weak than the estimate of  $\delta m_{HH_2}^2$  coming from the neutrino mass.

In general case we can have additional flavour changing Yukawa interaction  $h_{2s_Ld_R}\overline{Q}_{2L}H_2d_R + h.c$  ( $Q_{2L}$  =  $(c<sub>L</sub>, s<sub>L</sub>)$  in the Lagrangian (4) that leads to the tree level flavour changing  $\Delta S = 2$  effective interaction  $L_{eff} =$  $\frac{h_{2dL^{s}R}h_{2sL}^{*d}a}{M_{H2}^{2}}(\bar{d}_L s_R \bar{d}_R s_L + h.c.)$  We can simultaneously avoid the  $\Delta S = 2$  bound  $\Lambda_{\Delta s=2} \equiv (h_{2d_L s_R} h_2^*)$  $^{*}_{2s_Ld_R}$ )<sup>-1/2</sup>.  $M_{H_2} > 1.8 \cdot 10^7$  GeV and obtain phenomenologically interesting values for  $Br(K_L \to \nu\bar{\nu})$  for small quark Yukawa coupling constants  $h_{2d_Ls_R}$ ,  $h_{2s_Ld_R}$ , relatively light second Higgs doublet and not small lepton Yukawa coupling constant  $h_{2\tau}$ . For instance, for  $h_{2d_{LSR}} =$  $h_{2s_Ld_R} = (1/300)^2$ ,  $h_{2\tau} = 1$  and  $M_{H_2} = 300 \text{ GeV}$  we find that  $\Lambda_{\Delta s=2} = 2.7 \cdot 10^7 \ GeV$  and  $Br(K_L \to \nu \bar{\nu}) =$  $0.4 \cdot 10^{-6}$ . The existence of relatively light with a mass  $M_{H_2} = 300 \text{ GeV}$  second Higgs doublet does not contradict the LHC data. The best way to look for the second Higgs isodoublet at the LHC is the use of the reaction  $pp \rightarrow Z^* / gamma^* \rightarrow H_2^+ H_2^- \rightarrow \tau^+ \tau^- \nu \bar{\nu}$ . So the signature is two  $\tau$  leptons plus nonzero  $E_{miss}^T$  in final state that coincides with the signature used for the search for direct production of stau leptons at the LHC.

## III.  $K_L \rightarrow \phi \phi$  DECAY IN MODEL WITH ADDITIONAL SCALAR DOUBLET AND SCALAR SINGLET  $\phi$

Consider now the  $K_L \rightarrow \phi \phi$  decay in the extension of the SM with heavy Higgs doublet  $H_2$  and light neutral scalar singlet field  $\phi$ . The Yukawa interaction of the heavy isodoublet  $H_2$  with quarks and the interaction

of the  $\phi$  field with Higgs isodoublets  $H_2$  and  $H$ (Higgs isodoublet of the SM) has the form

$$
L_I = h_{2d_Ls_R}\bar{Q}_{1L}s_RH_2 + \lambda(H_2^+H)\phi^2 + \delta m_{HH_2}^2H^+H_2^2(1) + h.c. - M_{H_2}^2H_2^+H_2,
$$

where  $Q_{1L} = (u_L, d_L)$ ,  $H_2 = (H_2^+, H_2^0)$  and  $h_{2d_Ls_R}$ ,  $\lambda$  are Yukawa and Higgs couplings. After electroweak  $SU_L(2)\otimes U(1)$  symmetry breaking trilinear term describing transition  $H_2 \to \phi \phi$ 

$$
L_{H_2\phi\phi} = \lambda < H > H_2^+ \phi^2 + h.c. \tag{22}
$$

arises. The effective Lagrangian

$$
L_{eff} = \frac{1}{M_X} \bar{d}_L s_R \phi^2 + h.c. \,, \tag{23}
$$

$$
\frac{1}{M_X} = \frac{h_{2d_L s_R} \lambda < H >}{M_{H_2}^2} \tag{24}
$$

describes invisible decay  $K_L \to \phi \phi$ . Here we assume that the mass of  $\phi$  is less than  $M_{K_L}/2$ . The decay rate of the invisible decay  $K_L \rightarrow \phi \phi$  is determined by formula

$$
\Gamma(K_L \to \phi \phi) = \frac{M_{K_L}^3}{8\pi M_X^2} \left(\frac{F_K}{2(m_d + m_s)}\right)^2 K(m_\phi^2 / M_{K_L}^2),\tag{25}
$$

where  $K(x) = (1 - 4x)^{1/2}$ . For  $Br(K_L \to \phi \phi) = 10^{-6}$ and  $m_{\phi} \ll M_{K_L}$  we can test the value of  $M_X$  up to

$$
M_X \lesssim 10^{10} \; GeV \,. \tag{26}
$$

For  $\lambda = 1$  and  $h_{2d_{LSR}} = 1$  the mass of the second Higgs isodoublet can be tested up to  $M_{H_2} \leq 10^6$  GeV.

The bound (16) allows to restrict the  $K_L \rightarrow \phi \phi$  decay in full analogy with previous model. Namely, in the model with the effective Lagrangian (22) the  $K_L \rightarrow \phi \phi$ decay width is determined by the expression

$$
\frac{d\Gamma^{BSM}(K^{+} \to \pi^{+}\phi\phi)}{dq^{2}} = \frac{1}{(2\pi)^{3}} \cdot \frac{1}{32M_{K_{L}}^{3}} \cdot \frac{2}{M_{X}^{2}}
$$

$$
\cdot \sqrt{\left[ (M_{K^{+}}^{2} + M_{\pi^{+}}^{2} - q^{2})^{2} - 4M_{K^{+}}^{2}M_{\pi^{+}}^{2} \right] \left( 1 - \frac{4m_{\phi}^{2}}{q^{2}} \right)} \cdot \frac{\left[ \frac{f_{0}(q^{2})(M_{K^{+}}^{2} - m_{\pi^{+}}^{2})}{2(-m_{d} + m_{s})} \right]^{2}}{2(-m_{d} + m_{s})}.
$$
 (27)

It is convenient to use the ratio  $\beta^{-1} \equiv \frac{\Gamma(K_L \to \phi \phi)}{\Gamma(K^+ \to \pi^+ \nu \bar{\nu})}$ because the ratio  $\beta$  does not depend on unknown value of  $M_X$  and on the values of quark masses  $m_d$ ,  $m_s$ . For the case  $m_{\phi} \ll M_{\pi^+}$  we find that

$$
\beta \approx 10^{-2} \,. \tag{28}
$$

As in the previous model the smallness of the  $\beta$  is mainly due to the 3-body phase space smallnes in comparison with 2-body phase space.

From the (16) and (28) we find

$$
Br(K_L \to \phi \phi) \lesssim 2 \cdot 10^{-8} \,. \tag{29}
$$

For not very light  $\phi$ -particle the limit on  $Br(K_L \to \phi\phi)$ will be not so stringent as the bound (29), moreover, for  $\phi$ particle mass  $M_{K_L}/2 \ge m_\phi \ge (M_{K^+} - M_{\pi^+})/2$  the decay  $K^+ \to \pi^+ \nu \bar{\nu}$  is kinematically prohibited while the decay  $K_L \rightarrow \phi \phi$  is allowed. Therefore the bound (29) derived from the decay width of the  $K^+ \to \pi^+ \nu \bar{\nu}$  decay does not work for  $K_L \rightarrow \phi \phi$  decay mode. Note, that such sub-Gev scalar  $\phi$  could be a good dark matter candidate [\[21\]](#page-4-14). As in the previous model the bound from the  $K_L - K_S$  mass difference leads to the bound on the unknown parameter  $\delta m_{HH_2}^2$  at the level  $\delta m_{HH_2}^2 \leq 30 \; GeV^2$  for  $\frac{M_{H_2}}{h_{2d_Ls_R}} =$  $10^4$  GeV.

### IV.  $K_L \rightarrow invisible$  DECAY IN MODEL WITH MIRROR WORLD

Finally, we discuss the  $K_L$  oscillations into a hidden sector, which would manifest themselves through the  $K_L \rightarrow invisible$  decay. As an example of such hidden sector we consider the one of the mirror matter models. The idea that along with the ordinary matter may exist its exact mirror copy, introduced for the parity conservation, is not new [\[22](#page-4-15)]. Accordingly, each ordinary particle of the SM has a corresponding mirror partner of exactly the same mass as the ordinary one. The mirror fields are all singlets under the SM  $SU_c(3) \otimes SU_L(2) \otimes U(1)$  gauge group. Mirror matter is dark in terms of the SM interactions, and could be a good candidate for dark matter, see, e.g., Refs.[\[23\]](#page-4-16), and recent [\[24\]](#page-5-3). In addition to gravity, the interaction between our and this type of dark matter could be transmitted by some gauge singlet particles interacting with both sectors. Any neutral, elementary or composite particle, in principle, can have mixing with its mirror duplicate. This results in several interesting phenomena, such, e.g. as Higgs [\[25\]](#page-5-4), positronium [\[26\]](#page-5-5), muonium [\[27\]](#page-5-6), or neutron [\[28](#page-5-7)] oscillations into their hidden partner, which have been or planned to be experimentally tested [\[29](#page-5-8)[–32](#page-5-9)].

In particular, the neutral  $K_L$ -meson can mix with it mirror  $(m)$  analog  $K_{L,m}$  due to effective four-fermion interaction

$$
L_{int} = \frac{1}{M_m^2} [\bar{d}\gamma^\mu (1 - \gamma_5) s \bar{s}_m \gamma_\mu (1 - \gamma_5) d_m]
$$
 (30)

The interaction (30) leads to conversion of ordinary  $K_L$ meson to mirror  $K_L$ -meson. The decays of mirror  $K_L$ meson are invisible in our world that leads to invisible  $K_L$  decay with the branching ratio

$$
Br(K_L \to invisible) = \frac{\delta^2}{2(\delta^2 + \Gamma_{tot}^2(K_L))},\qquad(31)
$$

where

$$
\delta = \frac{1}{M_{K_L}} < K_{L,m} |L_{int}| K_L > . \tag{32}
$$

For the interaction (27) in the vacuum insertion approximation we find that

$$
\delta \approx \frac{F_K^2 M_{K_L}}{M_m^2} \,. \tag{33}
$$

Numerically, for  $Br(K_L \to invisible) = 10^{-6}$  we can probe the value of  $M_m$  up to

$$
M_m \lesssim 8.4 \cdot 10^8 \; GeV \,. \tag{34}
$$

In our estimates we used nonrenormalizable effective four-fermion interaction (30). It is possible to obtain the effective interaction (27) from renormalizable mirror world model with the Higgs doublet extension of the SM model (see previous discussions) and with the additional interaction term between our and mirror world

$$
L_m = \lambda_m (H^+ H_2)(H_m H_{m,2}^+) + h.c. \tag{35}
$$

After electroweak symmetry breaking in our and mirror worlds  $(< H > = < H_m > \approx 174 \text{ GeV})$  we find an effective four-fermion interaction

$$
L_{eff} = \frac{1}{M_m^2} \bar{d}_L s_R \bar{s}_{R,m} d_{L,m} + h.c. ,
$$
 (36)

where

$$
\frac{1}{M_m^2} = \frac{h_{2d_Ls_R}^2}{M_{H_2}^2} \cdot \frac{\lambda_m |\langle H \rangle|^2}{M_{H_2}^2} \,. \tag{37}
$$

## V. CONCLUSION

In conclusion, the observation of the  $K_L \rightarrow invisible$ decay with the branching ratio  $Br(K_L \rightarrow invisible)$ 

 $10^{-10}$  would unambiguously signal the presence of BSM physics. We consider the  $K_L \rightarrow invisible$  decay in several natural extensions of the SM, such as the 2HDM, 2HDM and light neutral scalar field  $\phi$ , and mirror dark matter model. Using constraints from the experimental value for the  $Br(K^+ \to \pi^+ \nu \bar{\nu})$  we find that the  $K_L \rightarrow invisible$  decay branching ratio could be in the region  $Br(K_L \to invisible) \simeq 10^{-8} - 10^{-6}$ , which is experimentally accessible allowing to test new-physics scales well above 100 TeV. In some scenarios these bounds can be avoided, as in the model with the massive righthanded neutrino and scalar  $\phi$ -particle. This makes the  $K_L \rightarrow invisible$  decay a powerful clean probe of new physics, that is complementary to other rare  $K$  decay channels. Additionally, in the case of observation the  $K_L \rightarrow invisible$  decay could influence the precision of the Bell-Steinberger analysis of the  $K^0 - \overline{K}^0$  system. The results obtained provide a strong motivation for a sensitive search for this process in a near future  $K$  decay experiment proposed in [\[9](#page-4-4)]. It should be noted that in full analogy with the case of  $K_L$  invisible decay we can expect the existence of invisible decays of  $B_d$  and  $B_s$ mesons, see e.g. [\[33,](#page-5-10) [34](#page-5-11)], with the branchings similar to those discussed above.

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- <span id="page-5-1"></span>[36] The quark masses  $m_d, m_s$  and the effective mass  $M_X$  implicitly depend on the renormalization point  $\mu$  but their combination  $M_X^2(m_d + m_s)$  and hence the decay width (7) is renormalization groop invariant and does not depend on the renormalization point  $\mu$ .
- <span id="page-5-2"></span>[37] In our estimate (8) we used the values  $\tau(K_L) = 5.17$ .  $10^{-8}$  sec,  $m_{\nu_{\tau}} = 0$  and  $(m_d + m_s)(\mu = 1 \text{ GeV}) =$ 160 MeV