QCD description of backward vector meson hard electroproduction

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Abstract

We consider backward vector meson exclusive electroproduction off nucleons in the framework of collinear QCD factorization. Nucleon to vector meson transition distribution amplitudes arise as building blocks for the corresponding factorized amplitudes. In the near-backward kinematics, the suggested factorization mechanism results in the dominance of the transverse cross section of vector meson production ($\sigma_T \gg \sigma_L$) and in the characteristic $1/Q^8$-scaling behavior of the cross section. We evaluate nucleon to vector meson TDAs in the cross-channel nucleon exchange model and present estimates of the differential cross section for backward ρ^0, ω and ϕ meson production off protons. The resulting cross sections are shown to be measurable in the forthcoming JLab@12 GeV experiments.

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1. INTRODUCTION

The factorization of exclusive amplitudes into a short distance dominated part - the coefficient function - calculable in a perturbative way on the one hand, and universal hadronic matrix elements of non-local operators on the light-cone on the other hand, is a key feature of quantum chromodynamics (QCD). This allows one to extract information on the hadronic structure from measurements of specific exclusive processes in specific kinematics. The textbook examples of such factorization [1, 2] are the nearly forward deeply virtual Compton scattering (DVCS) and meson hard electroproduction, where generalized parton (quark and gluon) distributions (GPDs) are the relevant hadronic matrix elements. The extension of this strategy to other processes, such as backward meson hard electroproduction and the cross conjugated nucleon-antinucleon annihilation into a lepton pair in association with a light meson, has been advocated in [3–7] - although the corresponding factorization theorems are not yet rigorously proven. For this latter class of hard exclusive process, new hadronic matrix elements of three quark operators on the light cone, the baryon-to-meson transition distribution amplitudes (TDAs), appear. Baryon-to-meson TDAs share common features both with baryon DAs (that are defined as the baryon-to-vacuum matrix elements of the same three quark light-cone operators) and with GPDs, since the matrix element in question depends on the longitudinal momentum transfer between a baryon and a meson characterized by the skewness variable ξ. Also, similarly to GPDs [8–10], switching to the impact parameter space through the Fourier transform in Δ_T brings a novel transverse picture of the nucleon. It encodes new valuable complementary information on the hadronic 3-dimensional structure, whose detailed physical meaning still awaits its clarification.

A collinear factorized description for backward reactions requires the presence of a large scale Q, to ensure the perturbative expansion of the hard subprocess in the QCD coupling constant $\alpha_s(\mu)$ at the factorization scale $\mu = O(Q)$. The large scale Q can be taken either as the space-like virtuality of the electromagnetic probe in the case of electroproduction processes [6], or, respectively, time-like virtuality of a photon (or mass of heavy quarkonium) for the case of cross conjugated processes with lepton pair emission (or heavy quarkonium production) in association with a light meson in antinucleon nucleon annihilation [7, 11, 12].

Our previous studies [6, 7, 11, 13–15] were almost exclusively restricted to the case of πN TDAs. Thanks to the chiral properties of QCD, πN TDAs possess a well-understood soft pion limit, which can be related to the $\xi \to 1$ limit of the TDAs. This easily allows us to work out the physical normalization of πN TDAs and is helpful for practical model building.

The special πN TDA case however does not exhaust all interesting possibilities, and the vector meson sector should be experimentally accessible as well as the pseudoscalar meson sector [16–19]. In this paper we consider nucleon-to-vector meson (VN) TDAs and address the possibility of accessing them experimentally through backward hard electroproduction reactions. The spin-1 nature of the produced mesons gives rise to new structures for TDAs. This enables to define a set of the leading twist-3 VN TDAs, which in principle may be accessed separately through a rich variety of polarization dependent observables. In the present study we enlarge the scope of nucleon to meson distributions to the cases of ρ^0, ω and ϕ mesons. These three cases share the same J^{PC} quantum numbers, but each of them addresses a specific question. The ϕ-meson case deals with the issue of the strangeness content of the nucleon, which has been the subject of many experimental and theoretical studies (see *e.g.* [20] and Refs. therein), while the combined analysis of ρ and ω production allows one to disentangle the isotopic structure of VN TDAs in the non-strange sector.

The paper is organized as follows: in Section 2 we describe the kinematics of backward meson electroproduction. In Section 3 we propose a parametrization of nucleon-to-vector-meson TDAs. We calculate the hard amplitude in Section 4. In Section 5 we compute the unpolarized cross-section for backward hard meson electroproduction off nucleons and present estimates for the cross section of the backward $\omega(782)$, $\phi(1020)$ and $\rho^0(770)$ production within the cross-channel nucleon exchange model of the VN TDAs. Section 6 brings our conclusions. Appendix A explores the isospin constraints and the permutation properties of the VN TDAs; appendix B derives the cross channel nucleon exchange model for the VN TDAs.

2. KINEMATICS OF BACKWARD VECTOR MESON HARD ELECTROPRODUCTION

We consider the exclusive electroproduction of vector mesons off nucleons

$$e(k) + N(p_1, s_1) \to (\gamma^*(q, \lambda_\gamma) + N(p_1, s_1)) + e(k') \to e(k') + N(p_2, s_2) + V(p_V, \lambda_V), \quad (1)$$

within the generalized Bjorken limit, in which $Q^2 = -q^2$ and s are large[1]; the Bjorken variable $x_B \equiv \frac{Q^2}{2p_1 \cdot q}$ is fixed and the u-channel momentum transfer squared is small compared to Q^2 and s: $|u| \equiv |\Delta^2| \ll Q^2$, s. Within such kinematics, the amplitude of the hard subprocess of the reaction (1) is supposed to admit a collinear factorized description in terms of nucleon-to-vector-meson TDAs and nucleon DAs, as it is shown on Fig. 1. The small u corresponds to the vector meson produced in the near-backward direction in the $\gamma^* N$ center-of-mass system (CMS). Therefore in what follows, we refer the kinematical regime in question as the near-backward kinematics. We would like to emphasize that this kinematical regime is complementary to the more familiar generalized Bjorken limit (Q^2 and s - large; x_B - fixed; $|t| \ll Q^2$, s), known as the near-forward kinematics. In this latter kinematical regime the conventional collinear factorization theorem [1, 2] leading to the description of (1) in terms of GPDs and vector meson DAs is established (see Fig. 2).

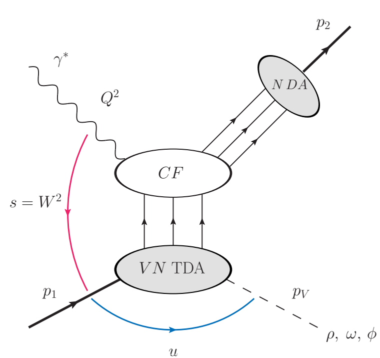


FIG. 1: Collinear factorization of $\gamma^* N \to N V$ in the near-backward kinematics regime (large Q^2, s; fixed x_B; $|u| \sim 0$); VN TDA stands for the transition distribution amplitude from a nucleon to a vector meson; N DA stands for the nucleon distribution amplitude; CF denotes hard subprocess amplitudes (coefficient functions).

We choose the z-axis along the colliding virtual-photon-nucleon and introduce the light-cone vectors p and n ($2p \cdot n = 1$). Keeping the first-order corrections in the masses and Δ_T^2, we establish the following Sudakov decomposition for the momenta of the reaction (1) in

[1] Throughout this paper we employ the usual Mandelstam variables for the hard subprocess of the reaction (1): $s = (p_1 + q)^2 \equiv W^2$; $t = (p_2 - p_1)^2$; $u = (p_V - p_1)^2 \equiv \Delta^2$.

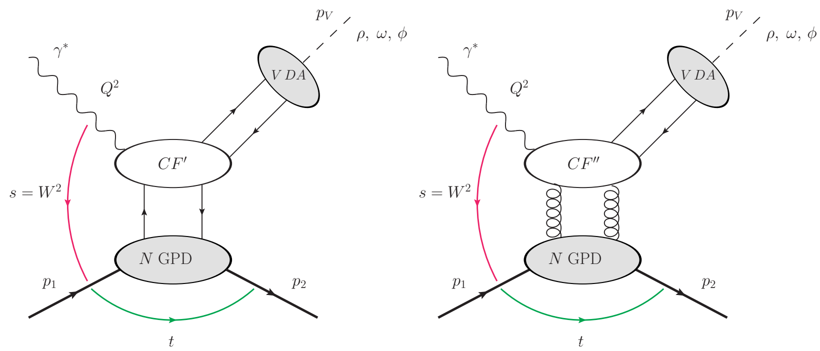


FIG. 2: Collinear factorization of $\gamma^* N \to NV$ in the near-forward kinematics regime (large Q^2, s; fixed x_B; $|t| \sim 0$); N GPD stand for, respectively, quark and gluon nucleon GPDs; V DA stands for the vector meson distribution amplitude; CF' and CF'' denote the corresponding hard subprocess amplitudes.

the near-backward kinematical regime (*cf.* [6]):

$$\begin{aligned} & p_1 = (1+\xi)p + \frac{M^2}{1+\xi} n\,; \\ & p_2 \simeq -2\xi \frac{(\Delta_T^2 - M^2)}{Q^2} p + \left[\frac{Q^2}{2\xi\Big(1 + \frac{(\Delta_T^2 - M^2)}{Q^2}\Big)} - \frac{m_V^2 - \Delta_T^2}{1-\xi} + \frac{M^2}{1+\xi} \right] n - \Delta_T; \\ & q \simeq -2\xi \Big(1 + \frac{(\Delta_T^2 - M^2)}{Q^2}\Big) p + \frac{Q^2}{2\xi\Big(1 + \frac{(\Delta_T^2 - M^2)}{Q^2}\Big)} n\,; \\ & p_V = (1-\xi)p + \frac{m_V^2 - \Delta_T^2}{1-\xi} n + \Delta_T\,; \\ & \Delta = -2\xi p + \Big[\frac{m_V^2 - \Delta_T^2}{1-\xi} - \frac{M^2}{1+\xi} \Big] n + \Delta_T\,. \end{aligned} \tag{2}$$

Here M and m_V denote respectively the nucleon and the vector meson masses and ξ stands for the u-channel skewness parameter introduced with respect to the u-channel momentum transfer

$$\xi = -\frac{(p_V - p_1)\cdot n}{(p_V + p_1)\cdot n}, \tag{3}$$

and

$$\Delta_T^2 = \frac{1-\xi}{1+\xi} \left(\Delta^2 - 2\xi \left[\frac{M^2}{1+\xi} - \frac{m_V^2}{1-\xi} \right] \right) \tag{4}$$

is the u-channel transverse momentum transfer squared. We introduce u_0 corresponding to

$\Delta_T^2 = 0$:

$$u_0 = -\frac{2\xi\left(m_V^2(1+\xi) - M^2(1-\xi)\right)}{1-\xi^2}. \tag{5}$$

It is the maximal possible value of u for given ξ. For $\Delta_T^2 = 0$ ($u = u_0$) the vector meson is produced exactly in the backward direction in the $\gamma^* N$ CMS ($\theta_V^* = \pi$).

In the initial nucleon rest frame (corresponding to the JLab laboratory frame, (LAB)) the light-cone vectors p and n read

$$p|_{\rm LAB} = \frac{M}{2(1+\xi)}\{1,0,0,-1\}; \quad n|_{\rm LAB} = \frac{1+\xi}{2M}\{1,0,0,1\}. \tag{6}$$

With the help of the appropriate boost we establish the expressions for the light-cone vectors in the $\gamma^* N$ CMS:

$$p|_{\gamma^* N\,{\rm CMS}} = \{\alpha,0,0,-\alpha\}; \quad p|_{\gamma^* N\,{\rm CMS}} = \{\beta,0,0,\beta\}, \tag{7}$$

with

$$\begin{aligned} \alpha &= \frac{M^2+Q^2+W^2+\Lambda\left(W^2,-Q^2,M^2\right)}{4(1+\xi)W}; \\ \beta &= \frac{(1+\xi)\left(M^2+Q^2+W^2-\Lambda\left(W^2,-Q^2,M^2\right)\right)}{4M^2W}, \end{aligned} \tag{8}$$

where Λ is the usual Mandelstam function

$$\Lambda(x,y,z) = \sqrt{x^2+y^2+z^2-2xy-2xz-2yz}\,. \tag{9}$$

The V-meson scattering angle in the $\gamma^* N$ CMS for the u-channel factorization regime then can be expressed as:

$$\cos\theta_V^* = \frac{-(1-\xi)\alpha + \frac{m_V^2-{\Delta_T}^2}{1-\xi}\beta}{\sqrt{(-(1-\xi)\alpha + \frac{m_V^2-{\Delta_T}^2}{1-\xi}\beta)^2 - {\Delta_T}^2}}\ . \tag{10}$$

One may check that for ${\Delta_T}^2 = 0$ indeed $\cos\theta_V^* = -1$, which means backward scattering.

For skewness variable ξ we employ the approximate expression neglecting order of mass and Δ_T^2 corrections

$$\xi \simeq \frac{x_B}{2-x_B} \simeq \frac{Q^2}{Q^2+2W^2}. \tag{11}$$

From the transversality of the polarization vector of the vector meson

$$\mathcal{E}^*(p_V, \lambda_V) \cdot p_V = 0 \tag{12}$$

we establish the following condition for the "−"-light-cone component of the polarization vector of the vector meson:

$$\mathcal{E}^*(p_V, \lambda_V) \cdot p = -\frac{m_V^2 - \Delta_T^2}{(1-\xi)^2}(\mathcal{E}^*(p_V, \lambda_V) \cdot n) - \frac{1}{1-\xi}(\mathcal{E}^*(p_V, \lambda_V) \cdot \Delta_T). \tag{13}$$

3. PARAMETRIZATION OF NUCLEON-TO-VECTOR MESON TRANSITION DISTRIBUTION AMPLITUDES

Nucleon-vector-meson TDAs are formally defined as the matrix elements of the light-cone three quark operator between a nucleon and a vector meson states. For simplicity we leave the discussion of isotopic spin properties of VN TDAs to the Appendix A and consider the transition matrix element of the uud light-cone operator[2]

$$\hat{O}^{uud}_{\rho\tau\chi}(\lambda_1 n, \lambda_2 n, \lambda_3 n) \equiv \varepsilon_{c_1 c_2 c_3} u^{c_1}_\rho(\lambda_1 n) u^{c_2}_\tau(\lambda_2 n) d^{c_3}_\chi(\lambda_3 n) \tag{14}$$

between the proton $|N_p\rangle$ state and a $I_3 = 0$, $J^{PC} = 1^{--}$ vector meson state (these could be *e.g.* $\langle\omega(782)|$, $\langle\rho^0(770)|$ or $\langle\phi(1020)|$ meson states).

The parametrization for the leading twist VN TDA involves 24 Dirac structures. Indeed, each of the three quarks and the nucleon have 2 helicity states, while the vector meson has 3. This leads to $3 \cdot 2^4 = 48$ helicity amplitudes. However, parity relates helicity amplitudes with all opposite helicities reducing the overall number of independent helicity amplitudes by the factor of 2. The procedure for building the corresponding leading twist Dirac structures was described in Ref. [21]. The revised version[3] of the leading twist-3 VN TDA parametrization

[2] We adopt the light-cone gauge $A^+ = 0$ and therefore the gauge link is not shown explicitly in the operator definition.

[3] The original parametrization of Ref. [21] erroneously lacked γ_5 factors for the Dirac structures. *C.f.* eq. (14) of Ref. [21] and eqs. (17)-(19) of the present paper.

reads

$$
\begin{aligned}
&4\mathcal{F}\langle V(p_V,\lambda_V)|\hat{O}^{uud}_{\rho\tau\chi}(\lambda_1 n,\lambda_2 n,\lambda_3 n)|N^p(p_1,s_1)\rangle\\
&=\delta(x_1+x_2+x_3-2\xi)\times M\Big[\sum_{\substack{\Upsilon=1\varepsilon,1T,1n,\\2\varepsilon,2T,2n}}(v^{VN}_{\Upsilon})_{\rho\tau,\chi}V^{VN}_{\Upsilon}(x_1,x_2,x_3,\xi,\Delta^2)\\
&+\sum_{\substack{\Upsilon=1\varepsilon,1T,1n,\\2\varepsilon,2T,2n}}(a^{VN}_{\Upsilon})_{\rho\tau,\chi}A^{VN}_{\Upsilon}(x_1,x_2,x_3,\xi,\Delta^2)+\sum_{\substack{\Upsilon=1\varepsilon,1T,1n,\varepsilon,2T,2n,\\3\varepsilon,3T,3n,4\varepsilon,4T,4n}}(t^{VN}_{\Upsilon})_{\rho\tau,\chi}T^{VN}_{\Upsilon}(x_1,x_2,x_3,\xi,\Delta^2)\Big],
\end{aligned}
\tag{15}
$$

where $\mathcal{F}$ stands for the conventional Fourier transform

$$
\mathcal{F}\equiv\mathcal{F}(x_1,x_2,x_3)(\ldots)=(P\cdot n)^3\int\left[\prod_{j=1}^{3}\frac{d\lambda_j}{2\pi}\right]e^{i\sum_{k=1}^{3}x_k\lambda_k(P\cdot n)}, \tag{16}
$$

and the leading twist Dirac structures are defined as

$$
\begin{aligned}
&(v^{VN}_{1\mathcal{E}})_{\rho\tau,\chi}=(\hat{p}C)_{\rho\tau}\big(\gamma^5\hat{\mathcal{E}}^*(p_V,\lambda_V)U^+(p_1,s_1)\big)_\chi;\\
&(v^{VN}_{1T})_{\rho\tau,\chi}=M^{-1}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\hat{p}C)_{\rho\tau}\big(\gamma^5U^+(p_1,s_1)\big)_\chi;\\
&(v^{VN}_{1n})_{\rho\tau,\chi}=M(\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\hat{p}C)_{\rho\tau}\big(\gamma^5U^+(p_1,s_1)\big)_\chi;\\
&(v^{VN}_{2\varepsilon})_{\rho\tau,\chi}=M^{-1}(\hat{p}C)_{\rho\tau}\big(\gamma^5\sigma^{\Delta_T\mathcal{E}^*}U^+(p_1,s_1)\big)_\chi;\\
&(v^{VN}_{2T})_{\rho\tau,\chi}=M^{-2}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\hat{p}C)_{\rho\tau}\big(\gamma^5\hat{\Delta}_TU^+(p_1,s_1)\big)_\chi;\\
&(v^{VN}_{2n})_{\rho\tau,\chi}=(\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\hat{p}C)_{\rho\tau}\big(\gamma^5\hat{\Delta}_TU^+(p_1,s_1)\big)_\chi;
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&(a^{VN}_{1\varepsilon})_{\rho\tau,\chi}=(\hat{p}\gamma^5C)_{\rho\tau}\big(\hat{\mathcal{E}}^*(p_V,\lambda_V)U^+(p_1,s_1)\big)_\chi;\\
&(a^{VN}_{1T})_{\rho\tau,\chi}=M^{-1}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\hat{p}\gamma^5C)_{\rho\tau}\big(U^+(p_1,s_1)\big)_\chi;\\
&(a^{VN}_{1n})_{\rho\tau,\chi}=M(\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\hat{p}\gamma^5C)_{\rho\tau}\big(U^+(p_1,s_1)\big)_\chi;\\
&(a^{VN}_{2\varepsilon})_{\rho\tau,\chi}=M^{-1}(\hat{p}\gamma^5C)_{\rho\tau}\big(\sigma^{\Delta_T\mathcal{E}^*}U^+(p_1,s_1)\big)_\chi;\\
&(a^{VN}_{2T})_{\rho\tau,\chi}=M^{-2}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\hat{p}\gamma^5C)_{\rho\tau}\big(\hat{\Delta}_TU^+(p_1,s_1)\big)_\chi;\\
&(a^{VN}_{2n})_{\rho\tau,\chi}=(\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\hat{p}\gamma^5C)_{\rho\tau}\big(\hat{\Delta}_TU^+(p_1,s_1)\big)_\chi;
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
(t^{VN}_{1\mathcal{E}})_{\rho\tau,\chi} &= (\sigma_{p\lambda}C)_{\rho\tau}(\gamma_5\sigma^{\lambda\mathcal{E}^*}U^+(p_1,s_1))_\chi;\\
(t^{VN}_{1T})_{\rho\tau,\chi} &= M^{-1}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\sigma_{p\lambda}C)_{\rho\tau}(\gamma_5\gamma^\lambda U^+(p_1,s_1))_\chi;\\
(t^{VN}_{1n})_{\rho\tau,\chi} &= M(\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\sigma_{p\lambda}C)_{\rho\tau}(\gamma_5\gamma^\lambda U^+(p_1,s_1))_\chi;\\
(t^{VN}_{2\mathcal{E}})_{\rho\tau,\chi} &= (\sigma_{p\mathcal{E}^*}C)_{\rho\tau}(\gamma_5 U^+(p_1,s_1))_\chi;\\
(t^{VN}_{2T})_{\rho\tau,\chi} &= M^{-2}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\sigma_{p\lambda}C)_{\rho\tau}(\gamma_5\sigma^{\lambda\Delta_T}U^+(p_1,s_1))_\chi;\\
(t^{VN}_{2n})_{\rho\tau,\chi} &= (\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\sigma_{p\lambda}C)_{\rho\tau}(\gamma_5\sigma^{\lambda\Delta_T}U^+(p_1,s_1))_\chi;\\
(t^{VN}_{3\mathcal{E}})_{\rho\tau,\chi} &= M^{-1}(\sigma_{p\Delta_T}C)_{\rho\tau}(\gamma_5\hat{\mathcal{E}}^*(p_V,\lambda_V)U^+(p_1,s_1))_\chi;\\
(t^{VN}_{3T})_{\rho\tau,\chi} &= M^{-2}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\sigma_{p\Delta_T}C)_{\rho\tau}(\gamma_5 U^+(p_1,s_1))_\chi;\\
(t^{VN}_{3n})_{\rho\tau,\chi} &= (\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\sigma_{p\Delta_T}C)_{\rho\tau}(\gamma_5 U^+(p_1,s_1))_\chi;\\
(t^{VN}_{4\mathcal{E}})_{\rho\tau,\chi} &= M^{-1}(\sigma_{p\mathcal{E}^*}C)_{\rho\tau}(\gamma_5\hat{\Delta}_T U^+)_\chi;\\
(t^{VN}_{4T})_{\rho\tau,\chi} &= M^{-3}(\mathcal{E}^*(p_V,\lambda_V)\cdot\Delta_T)(\sigma_{p\Delta_T}C)_{\rho\tau}(\gamma_5\hat{\Delta}_T U^+(p_1,s_1))_\chi;\\
(t^{VN}_{4n})_{\rho\tau,\chi} &= M^{-1}(\mathcal{E}^*(p_V,\lambda_V)\cdot n)(\sigma_{p\Delta_T}C)_{\rho\tau}(\gamma_5\hat{\Delta}_T U^+(p_1,s_1))_\chi. \qquad (19)
\end{aligned}
$$

Throughout this paper we employ Dirac's "hat" notation: $\hat{a} \equiv \gamma_\mu a^\mu$ and adopt the usual conventions: $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$; $\sigma^{v\nu} \equiv v_\mu \sigma^{\mu\nu}$, where v_μ is an arbitrary 4-vector. The large and small components of the nucleon Dirac spinor $U(p_1)$ are introduced as $U^+(p_1,s_1) = \hat{p}\hat{n}U(p_1,s_1)$ and $U^-(p_1,s_1) = \hat{n}\hat{p}U(p_1,s_1)$.

Each of the 24 VN TDAs defined in (15) are functions of three longitudinal momentum fractions x_1, x_2, x_3, skewness parameter ξ, u-channel momentum transfer squared Δ^2 and of factorization scale μ^2. TDAs $V^{VN}_\Upsilon(x_1,x_2,x_3,\xi,\Delta^2)$ and $T^{VN}_\Upsilon(x_1,x_2,x_3,\xi,\Delta^2)$ are defined symmetric under the interchange $x_1 \leftrightarrow x_2$, while $A^{VN}_\Upsilon(x_1,x_2,x_3,\xi,\Delta^2)$ are antisymmetric under the interchange $x_1 \leftrightarrow x_2$.

Note that with the use of the parametrization (15) the corresponding TDAs do not satisfy the polynomiality property in its simple form. Indeed, as explained in Ref. [14], since the light-cone kinematics implies the choice of a preferred z-direction, the parametrization (15) involves non-covariant kinematical quantities (such as Δ_T and light-cone vectors p and n). This results in the presence of kinematical singularities for the corresponding invariant amplitudes (TDAs). In principle, one can define the alternative set of the Dirac structures for VN involving only fully covariant kinematical quantities such as four-vectors $P = \frac{1}{2}(p_1+p_V)$ and Δ. Thus, for the price of the controllable admixture of higher twist contributions the corresponding set of VN TDAs turns to be free of kinematical singularities and satisfies the polynomiality condition in its simple form. Therefore, for this set of TDAs one can introduce the spectral representation [22] in terms of quadruple distributions. The relation between the free-of-kinematical-singularities-set of VN TDAs and those introduced in eq. (15) is given by the set of relations similar to eq. (C11) of Ref. [14].

In the present study we, nevertheless, prefer to stay with the VN TDA parametrization (15), since it is well suited to keep eye on the $\Delta_T = 0$ limit. Namely, in the limit $\Delta_T = 0$ only 7 TDAs out of 24 turn to be relevant: $V_{1\mathcal{E}}^{VN}$, V_{1n}^{VN}, $A_{1\mathcal{E}}^{VN}$, A_{1n}^{VN}, $T_{1\mathcal{E}}^{VN}$, T_{1n}^{VN}, $T_{2\mathcal{E}}^{VN}$.

4. CALCULATION OF THE HARD AMPLITUDE

Within the suggested factorized approach, in the leading order (both in α_s and $1/Q$) the amplitude of[4]

$$\gamma^*(q, \lambda_\gamma) + N^p(p_1, s_1) \to N^p(p_2, s_2) + V(p_V, \lambda_V) \tag{20}$$

involves 6 independent tensor structures

$$\mathcal{M}_{s_1 s_2}^{\lambda_\gamma \lambda_V} = -i \frac{(4\pi\alpha_s)^2 \sqrt{4\pi\alpha_{em}} f_N M}{54 Q^4} \sum_{k=1}^{6} \mathcal{S}_{s_1 s_2}^{(k)\, \lambda_\gamma \lambda_V} \mathcal{I}^{(k)}(\xi, \, \Delta^2). \tag{21}$$

Here $\alpha_{em} = \frac{1}{137}$ is the electromagnetic fine structure constant and f_N is the nucleon light-cone wave function normalization constant. Throughout this study we employ the value from Ref. [23]: $f_N = 5.2 \cdot 10^{-3}$ GeV^2.

There turn to be two tensor structures independent of Δ_T:

$$\begin{aligned} \mathcal{S}_{s_1 s_2}^{(1)\, \lambda_\gamma \lambda_V} &= \bar{U}(p_2, s_2) \hat{\varepsilon}(q, \lambda_\gamma) \hat{\mathcal{E}}^*(p_V, \lambda_V) U(p_1, s_1); \\ \mathcal{S}_{s_1 s_2}^{(2)\, \lambda_\gamma \lambda_V} &= M(\mathcal{E}^*(p_V, \lambda_V) \cdot n) \bar{U}(p_2, s_2) \hat{\varepsilon}(q, \lambda_\gamma) U(p_1, s_1), \end{aligned} \tag{22}$$

and four Δ_T-dependent tensor structures:

$$\begin{aligned} \mathcal{S}_{s_1 s_2}^{(3)\, \lambda_\gamma \lambda_V} &= \frac{1}{M} (\mathcal{E}^*(p_V, \lambda_V) \cdot \Delta_T) \, \bar{U}(p_2, s_2) \hat{\varepsilon}(q, \lambda_\gamma) U(p_1, s_1); \\ \mathcal{S}_{s_1 s_2}^{(4)\, \lambda_\gamma \lambda_V} &= \frac{1}{M^2} (\mathcal{E}^*(p_V, \lambda_V) \cdot \Delta_T) \, \bar{U}(p_2, s_2) \hat{\varepsilon}(q, \lambda_\gamma) \hat{\Delta}_T U(p_1, s_1); \\ \mathcal{S}_{s_1 s_2}^{(5)\, \lambda_\gamma \lambda_V} &= \frac{1}{M} \, \bar{U}(p_2, s_2) \hat{\varepsilon}(q, \lambda_\gamma) \hat{\mathcal{E}}^*(p_V, \lambda_V) \hat{\Delta}_T U(p_1, s_1); \\ \mathcal{S}_{s_1 s_2}^{(6)\, \lambda_\gamma \lambda_V} &= (\mathcal{E}^*(p_V, \lambda_V) \cdot n) \, \bar{U}(p_2, s_2) \hat{\varepsilon}(q, \lambda_\gamma) \hat{\mathcal{E}}^*(p_V, \lambda_V) \hat{\Delta}_T U(p_1, s_1). \end{aligned} \tag{23}$$

[4] For definiteness we take V to be a vector meson with $I_3 = 0$: ω, ρ^0 or ϕ.

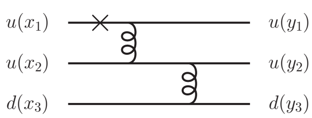
Here $\varepsilon(q, \lambda_\gamma)$ stands for the polarization vector of the incoming virtual photon and $\mathcal{E}^*(p_V, \lambda_V)$ is the polarization vector of the outgoing vector meson.

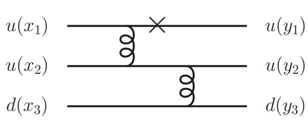
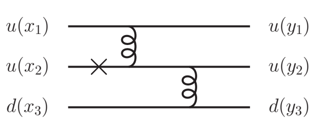
To the leading order in α_s, within the collinear factorized description in terms of VN TDAs, the amplitude of the reaction (20) can be computed from the same 21 diagrams listed in Table I of Ref. [6]. We adopt our common notations for the integral convolutions $\mathcal{I}^{(k)}$, $k = 1, \ldots, 6$:

$$\mathcal{I}^{(k)}(\xi, \Delta^2) \equiv \int_{-1+\xi}^{1+\xi} dx_1 \int_{-1+\xi}^{1+\xi} dx_2 \int_{-1+\xi}^{1+\xi} dx_3 \, \delta(x_1 + x_2 + x_3 - 2\xi)$$
$$\times \int_0^1 dy_1 \int_0^1 dy_2 \int_0^1 dy_3 \, \delta(y_1 + y_2 + y_3 - 1) \left(2 \sum_{\alpha=1}^{7} T_\alpha^{(k)} + \sum_{\alpha=8}^{14} T_\alpha^{(k)} \right). \quad (24)$$

The explicit expressions for the coefficients $T_\alpha^{(k)} \equiv D_\alpha \times N_\alpha^{(k)}$ (no summation over α assumed) are presented in Table I. Here D_α denote the singular kernels originating from the partonic propagators and $N_\alpha^{(k)} \equiv N_\alpha^{(k)}(x_1, x_2, \xi, \Delta^2; \, y_1, y_2, y_3)$ stand for the appropriate combinations of VN TDAs and nucleon DAs V^p, A^p and T^p arising in the numerator. The $\alpha = 1, \ldots, 21$ index refers for the diagram number and the index $k = 1, \ldots, 6$ runs for the contributions into 6 invariant amplitudes of eq. (21).

TABLE I: 14 of the 21 diagrams contributing to the hard-scattering amplitude with their associated coefficient $T_\alpha^{(k)} \equiv D_\alpha \times N_\alpha^{(k)}$ (no summation over α assumed). The seven first ones with u-quark lines inverted are not drawn. The crosses represent the virtual-photon vertex.

<table>
<tr><th>α</th><th>Diagram
D_α</th><th colspan="2">Numerators</th></tr>
<tr><td rowspan="6">1</td><td rowspan="6">

$\frac{Q_u(2\xi)^2}{(2\xi-x_1-i\epsilon)^2(x_3-i\epsilon)(1-y_1)^2 y_3}$</td><td>$N_\alpha^{(1)}$</td><td>$-\left(V^p - A^p\right)\left(V_{1\mathcal{E}}^{VN} + A_{1\mathcal{E}}^{VN}\right) + 2T^p\left(T_{1\mathcal{E}}^{VN} + T_{2\mathcal{E}}^{VN}\right)$</td></tr>
<tr><td>$N_\alpha^{(2)}$</td><td>$-\left(V^p - A^p\right)\left(V_{1n}^{VN} + A_{1n}^{VN}\right) + 4T^p\left(T_{1n}^{VN} + \frac{\Delta_T^2}{2M^2}T_{4n}^{VN}\right)$</td></tr>
<tr><td>$N_\alpha^{(3)}$</td><td>$-\left(V^p - A^p\right)\left(V_{1T}^{VN} + A_{1T}^{VN} + V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN}\right)$
$+4T^p\left(T_{1T}^{VN} + T_{3\mathcal{E}}^{VN} + \frac{\Delta_T^2}{2M^2}T_{4T}^{VN}\right)$</td></tr>
<tr><td>$N_\alpha^{(4)}$</td><td>$-\left(V^p - A^p\right)\left(V_{2T}^{VN} + A_{2T}^{VN}\right) + 2T^p\left(T_{2T}^{VN} + T_{3T}^{VN}\right)$</td></tr>
<tr><td>$N_\alpha^{(5)}$</td><td>$\left(V^p - A^p\right)\left(V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN}\right) - 2T^p\left(T_{3\mathcal{E}}^{VN} - T_{4\mathcal{E}}^{VN}\right)$</td></tr>
<tr><td>$N_\alpha^{(6)}$</td><td>$-\left(V^p - A^p\right)\left(V_{2n}^{VN} + A_{2n}^{VN}\right) + 2T^p\left(T_{2n}^{VN} + T_{3n}^{VN}\right)$</td></tr>
<tr><td rowspan="6">2</td><td rowspan="6">
</td><td>$N_\alpha^{(1)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(2)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(3)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(4)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(5)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(6)}$</td><td>0</td></tr>
<tr><td rowspan="6">3</td><td rowspan="6">

$\frac{Q_u(2\xi)^2)}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$</td><td>$N_\alpha^{(1)}$</td><td>$-2T^p\left(T_{1\mathcal{E}}^{VN} + T_{2\mathcal{E}}^{VN}\right)$</td></tr>
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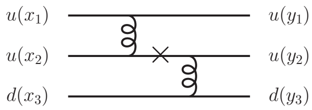
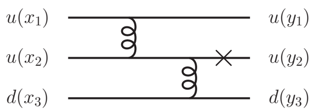
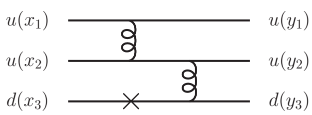
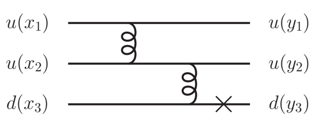
4	 $\frac{Q_u(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$	$N_\alpha^{(1)}$	$-\left(V^p - A^p\right)\left(V_{1\mathcal{E}}^{VN} + A_{1\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(2)}$	$-\left(V^p - A^p\right)\left(V_{1n}^{VN} + A_{1n}^{VN}\right)$
		$N_\alpha^{(3)}$	$-\left(V^p - A^p\right)\left(V_{1T}^{VN} + A_{1T}^{VN} + V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(4)}$	$-\left(V^p - A^p\right)\left(V_{2T}^{VN} + A_{2T}^{VN}\right)$
		$N_\alpha^{(5)}$	$\left(V^p - A^p\right)\left(V_{2\mathcal{E}}^{VN} + A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(6)}$	$\left(V^p - A^p\right)\left(V_{2n}^{VN} + A_{2n}^{VN}\right)$
5	 $\frac{Q_u(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_2)y_3}$	$N_\alpha^{(1)}$	$\left(V^p + A^p\right)\left(V_{1\mathcal{E}}^{VN} - A_{1\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(2)}$	$\left(V^p + A^p\right)\left(V_{1n}^{VN} - A_{1n}^{VN}\right)$
		$N_\alpha^{(3)}$	$\left(V^p + A^p\right)\left(V_{1T}^{VN} - A_{1T} + V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(4)}$	$\left(V^p + A^p\right)\left(V_{2T}^{VN} - A_{2T}\right)$
		$N_\alpha^{(5)}$	$-\left(V^p + A^p\right)\left(V_{2\mathcal{E}}^{VN} - A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(6)}$	$-\left(V^p + A^p\right)\left(V_{2n}^{VN} - A_{2n}^{VN}\right)$
6		$N_\alpha^{(1)}$	0
		$N_\alpha^{(2)}$	0
		$N_\alpha^{(3)}$	0
		$N_\alpha^{(4)}$	0
		$N_\alpha^{(5)}$	0
		$N_\alpha^{(6)}$	0
7	 $\frac{Q_d(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)^2 y_1(1-y_3)^2}$	$N_\alpha^{(1)}$	$-2\left(V^p V_{1\mathcal{E}}^{VN} - A^p A_{1\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(2)}$	$-2\left(V^p V_{1n}^{VN} - A^p A_{1n}^{VN}\right)$
		$N_\alpha^{(3)}$	$-2\left(V^p(V_{1T}^{VN} + V_{2\mathcal{E}}^{VN}) - A^p(A_{1T}^{VN} + A_{2\mathcal{E}}^{VN})\right)$
		$N_\alpha^{(4)}$	$-2\left(V^p V_{2T}^{VN} - A^p A_{2T}^{VN}\right)$
		$N_\alpha^{(5)}$	$2\left(V^p V_{2\mathcal{E}}^{VN} - A^p A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(6)}$	$2\left(V^p V_{2n}^{VN} - A^p A_{2n}^{VN}\right)$

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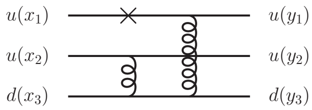
<tr><td rowspan="6">8</td><td rowspan="6"></td><td>$N_\alpha^{(1)}$</td><td>0</td></tr>
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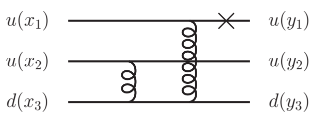
<tr><td rowspan="6">9</td><td rowspan="6">
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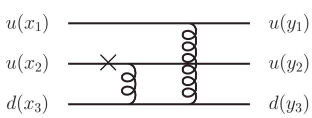
<tr><td rowspan="6">10</td><td rowspan="6">
$\frac{Q_u(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)^2y_1(1-y_2)^2}$</td><td>$N_\alpha^{(1)}$</td><td>$-\left(V^p-A^p\right)\left(V_{1\mathcal{E}}^{VN}+A_{1\mathcal{E}}^{VN}\right)+2T^p\left(T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN}\right)$</td></tr>
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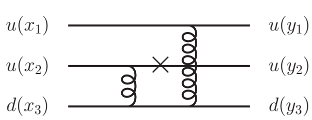
<tr><td rowspan="6">11</td><td rowspan="6"></td><td>$N_\alpha^{(1)}$</td><td>0</td></tr>
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<tr><td>$N_\alpha^{(3)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(4)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(5)}$</td><td>0</td></tr>
<tr><td>$N_\alpha^{(6)}$</td><td>0</td></tr>
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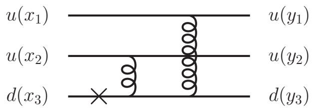
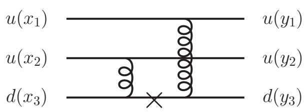
12	 $\frac{Q_d(2\xi)^2}{(x_1-i\epsilon)(x_2-i\epsilon)(2\xi-x_3-i\epsilon)y_1(1-y_2)y_2}$	$N_\alpha^{(1)}$	$(V^p+A^p)\left(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(2)}$	$(V^p+A^p)\left(V_{1n}^{VN}-A_{1n}^{VN}\right)$
		$N_\alpha^{(3)}$	$(V^p+A^p)\left(V_{1T}^{VN}-A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(4)}$	$(V^p+A^p)\left(V_{2T}^{VN}-A_{2T}^{VN}\right)$
		$N_\alpha^{(5)}$	$-(V^p+A^p)\left(V_{2\mathcal{E}}^{VN}-A_{2\mathcal{E}}^{VN}\right)$
		$N_\alpha^{(6)}$	$-(V^p+A^p)\left(V_{2n}^{VN}-A_{2n}^{VN}\right)$
13	 $\frac{Q_d(2\xi)^2)}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1(1-y_2)y_2}$	$N_\alpha^{(1)}$	$2T^p\left(T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN}\right)$
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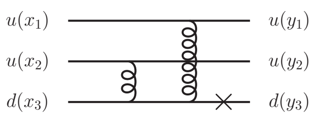
14	 $\frac{Q_d(2\xi)^2}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1y_2(1-y_3)}$	$N_\alpha^{(1)}$	$(V^p-A^p)\left(V_{1\mathcal{E}}^{VN}+A_{1\mathcal{E}}^{VN}\right)$
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		$N_\alpha^{(3)}$	$(V^p-A^p)\left(V_{1T}^{VN}+A_{1T}^{VN}+V_{2\mathcal{E}}^{VN}+A_{2\mathcal{E}}^{VN}\right)$
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		$N_\alpha^{(6)}$	$-(V^p-A^p)\left(V_{2n}^{VN}+A_{2n}^{VN}\right)$

5. CALCULATION OF THE UNPOLARIZED CROSS SECTION

First of all, we need to specify our conventions for the backward vector meson electroproduction cross section. Within the suggested factorization mechanism only the transverse virtual photoproduction cross section σ_T receives contribution at the leading twist. Analogously to how this was done in Ref. [14], using the explicit expression (eq. (2.12) of [24]) relating scattering amplitudes of vector meson electroproduction within one photon approximation and the amplitudes for the virtual photoproduction, we express the unpolarized cross section of hard electroproduction of a V meson off a nucleon through the helicity amplitudes of $\gamma^* N \to NV$ defined in (21) within the collinear factorized description framework:

$$\frac{d^5\sigma}{dE' d\Omega_{e'} d\Omega_V} = \Gamma \times \frac{\Lambda(s, m_V^2, M^2)}{128\pi^2 s(s - M^2)} \left\{ \frac{1}{2} \sum_{\lambda_{\gamma T}, \lambda_V, s_1, s_2} |\mathcal{M}_{s_1 s_2}^{\lambda_\gamma \lambda_V}|^2 + \ldots \right\} = \Gamma \times \left\{ \frac{d^2\sigma_T}{d\Omega_V} + \ldots \right\}. \quad (25)$$

Here $\Omega_{e'}$ is the differential solid angle for the scattered electron in the LAB frame; Ω_V is the differential solid angle of the produced vector meson in $N'V$ CMS frame; and Λ is the Mandelstam function (9).

By dots in the r.h.s. of eq. (25) we denote the subleading twist terms supressed by powers of $1/Q$. Γ stands for the virtual photon flux factor in Hand's convention

$$\Gamma = \frac{\alpha_{\rm em}}{2\pi^2} \frac{k'^L_0}{k^L_0} \frac{s - M^2}{2MQ^2} \frac{1}{1 - \epsilon}. \quad (26)$$

Here k_0^L and k'^L_0 denote the initial state and final state electron energies in the LAB frame and

$$\epsilon = \left[1 + 2 \frac{\left(k_0^L - k'^L_0\right)^2 + Q^2}{Q^2} \tan^2 \frac{\theta_e^L}{2} \right]^{-1}, \quad (27)$$

is the polarization parameter of the virtual photon, where θ_e^L denotes the electron scattering angle in the LAB frame.

We employ the following relation for the sum over photon's transverse polarizations

$$\sum_{\lambda_{\gamma T}} \varepsilon^\nu(q, \lambda_\gamma) \varepsilon^{\mu *}(q, \lambda_\gamma) = -g^{\mu\nu} + \frac{1}{(p \cdot n)} (p^\mu n^\nu + p^\nu n^\mu), \quad (28)$$

and the V-meson polarization sum reads

$$\sum_{\lambda_V} \mathcal{E}^\rho(p_V, \lambda_V) \mathcal{E}^\sigma(p_V, \lambda_V) = -g^{\rho\sigma} + \frac{p_V^\rho p_V^\sigma}{m_V^2}. \quad (29)$$

Let us introduce the notation

$$\mathcal{D} = -i \left(\frac{(4\pi\alpha_s)^2 \sqrt{4\pi\alpha_{em}} f_N M}{54} \right), \quad (30)$$

and define

$$\begin{aligned} \Gamma_H = {} & \hat{\varepsilon}(q, \lambda_\gamma) \hat{\mathcal{E}}(p_V, \lambda_V) \mathcal{I}^{(1)}(\xi, \Delta^2) + M(\mathcal{E}(p_V, \lambda_V) \cdot n) \hat{\varepsilon}(q, \lambda_\gamma) \mathcal{I}^{(2)}(\xi, \Delta^2) \\ & + \frac{(\mathcal{E}(p_V, \lambda_V) \cdot \Delta_T)}{M} \hat{\varepsilon}(q, \lambda_\gamma) \mathcal{I}^{(3)}(\xi, \Delta^2) + \frac{(\mathcal{E}(p_V, \lambda_V) \cdot \Delta_T)}{M^2} \hat{\varepsilon}(q, \lambda_\gamma) \hat{\Delta}_T \mathcal{I}^{(4)}(\xi, \Delta^2) \\ & + \frac{1}{M} \hat{\varepsilon}(q, \lambda_\gamma) \hat{\mathcal{E}}(p_V, \lambda_V) \hat{\Delta}_T \mathcal{I}^{(5)}(\xi, \Delta^2) + (\mathcal{E}(p_V, \lambda_V) \cdot n) \hat{\mathcal{E}}(p_V, \lambda_V) \hat{\Delta}_T \mathcal{I}^{(6)}(\xi, \Delta^2). \quad (31) \end{aligned}$$

We now square the amplitude and sum over the transverse polarization of the virtual photon, over the spin of outgoing nucleon and over the polarizations of the V-meson and sum over the spin of the initial nucleon. We make use of kinematic relations summarized in (2) and keep only the leading twist terms. The resulting expression then reads:

$$
\begin{aligned}
|\mathcal{M}_T|^2 &= \sum_{\lambda_{\gamma T},\,\lambda_V,\,s_1,\,s_2} \mathcal{M}^{\lambda_\gamma \lambda_V}_{s_1 s_2} \mathcal{M}^{\lambda_\gamma \lambda_V\,*}_{s_1 s_2} = |\mathcal{D}|^2 \frac{1}{Q^8} \sum_{\lambda_{\gamma T},\,\lambda_\omega} \mathrm{Tr}\left\{ (\hat{p}_2 + M)\Gamma_H (\hat{p}_1 + M)\gamma_0 \Gamma_H^\dagger \gamma_0 \right\} \\
&= |\mathcal{D}|^2 \frac{1}{Q^6} \frac{2(1+\xi)}{\xi} \Bigg[|\mathcal{I}^{(1)}|^2 \left(1 + \frac{M^2(1-\xi)^2 + (1+\xi)^2(m_V^2 - \Delta_T^2)}{m_V^2(1+\xi)^2} \right) \\
&+ |\mathcal{I}^{(2)}|^2 \frac{M^2(1-\xi)^2}{4m_V^2} + \left(\mathcal{I}^{(1)} \mathcal{I}^{(2)*} + \mathcal{I}^{(1)*} \mathcal{I}^{(2)} \right) \frac{M^2(1-\xi)^2}{2m_V^2(1+\xi)} \\
&+ \frac{\Delta_T^2}{M^2} \Bigg\{ -|\mathcal{I}^{(3)}|^2 \frac{(m_V^2 - \Delta_T^2)}{m_V^2} + \left(\mathcal{I}^{(1)} \mathcal{I}^{(3)*} + \mathcal{I}^{(1)*} \mathcal{I}^{(3)} \right) \frac{M^2(1-\xi)}{m_V^2(1+\xi)} \\
&+ \left(\mathcal{I}^{(2)} \mathcal{I}^{(3)*} + \mathcal{I}^{(2)*} \mathcal{I}^{(3)} \right) \frac{M^2(1-\xi)}{2m_V^2} + |\mathcal{I}^{(4)}|^2 \frac{\Delta_T^2 (m_V^2 - \Delta_T^2)}{M^2 m_V^2} \\
&+ \left(\mathcal{I}^{(1)} \mathcal{I}^{(4)*} + \mathcal{I}^{(1)*} \mathcal{I}^{(4)} \right) \frac{(m_V^2 - \Delta_T^2)}{m_V^2} - |\mathcal{I}^{(5)}|^2 \left(1 + \frac{M^2(1-\xi)^2 + (1+\xi)^2(m_V^2 - \Delta_T^2)}{m_V^2(1+\xi)^2} \right) \\
&+ \left(\mathcal{I}^{(1)} \mathcal{I}^{(5)*} + \mathcal{I}^{(1)*} \mathcal{I}^{(5)} \right) \frac{2M^2(1-\xi)}{m_V^2(1+\xi)} + \left(\mathcal{I}^{(2)} \mathcal{I}^{(5)*} + \mathcal{I}^{(2)*} \mathcal{I}^{(5)} \right) \frac{M^2(1-\xi)}{2m_V^2} \\
&- \left(\mathcal{I}^{(3)} \mathcal{I}^{(5)*} + \mathcal{I}^{(3)*} \mathcal{I}^{(5)} \right) \frac{m_V^2 - \Delta_T^2}{m_V^2} + \left(\mathcal{I}^{(4)} \mathcal{I}^{(5)*} + \mathcal{I}^{(4)*} \mathcal{I}^{(5)} \right) \frac{\Delta_T^2(1-\xi)}{m_V^2(1+\xi)} \\
&- |\mathcal{I}^{(6)}|^2 \frac{M^2(1-\xi)^2}{4m_V^2} - \left(\mathcal{I}^{(1)} \mathcal{I}^{(6)*} + \mathcal{I}^{(1)*} \mathcal{I}^{(6)} \right) \frac{M^2(1-\xi)}{2m_V^2} \\
&- \left(\mathcal{I}^{(4)} \mathcal{I}^{(6)*} + \mathcal{I}^{(4)*} \mathcal{I}^{(6)} \right) \frac{\Delta_T^2(1-\xi)}{2m_V^2} + \left(\mathcal{I}^{(5)} \mathcal{I}^{(6)*} + \mathcal{I}^{(5)*} \mathcal{I}^{(6)} \right) \frac{M^2(1-\xi)^2}{2m_V^2(1+\xi)} \Bigg\} \Bigg]. \qquad (32)
\end{aligned}
$$

We end up with the following expression for the LO unpolarized cross section of hard photoproduction of backward vector mesons off nucleons:

$$
\frac{d^2\sigma_T}{d\Omega_V} = \frac{\Lambda(s, m_V^2, M^2)}{128\pi^2 s(s - M^2)} \frac{1}{2} |\mathcal{M}_T|^2, \qquad (33)
$$

where σ_T refers to the transverse polarization of the virtual photon and $\frac{1}{2}$ stands for averaging over the initial nucleon spin.

Thus we conclude that there are two essential marking signs of the onset of the suggested factorization regime for hard vector meson production in the near-backward kinematics, which can be tested experimentally.

- The dominance of the transverse polarization of the virtual photon resulting in the suppression of the σ_L cross section by at least $1/Q^2$. In fact, the preliminary analysis

[25] of backward ω-meson production JLab Hall C 6 GeV data hints at $\sigma_T > \sigma_L$ already for $Q^2 \simeq 2.4$ GeV^2 and $W \simeq 2.2$ GeV.

- The characteristic $1/Q^8$-scaling behavior of the transverse cross section (33) for fixed x_B.

In what follows we present our estimates for the LO unpolarized cross section of hard photoproduction of backward $\omega(782)$, $\phi(1020)$ and $\rho^0(770)$ mesons off protons within the u-channel nucleon exchange model for VN TDAs presented in the Appendix B. This is a simple TDA model which populates VN TDAs only within the ERBL-like support region. However, it can be seen as a reliable estimate of the VN TDAs magnitude for the intermediate values of skewness parameter $\xi = 0.1 \div 0.4$. As inputs this model requires the values of the vector and tensor $G^{V,T}_{VNN}$ couplings (B1) and the phenomenological solutions for the leading twist nucleon DAs.

On the choice of phenomenological parametrization for the nucleon DA

The choice of the phenomenological solution for the leading twist nucleon DA and the corresponding value of the strong coupling represents a complicated problem (see *e.g.* discussion in Ref. [26]). Roughly speaking, there exist two distinct classes of the leading twist nucleon DA parametrizations.

- The models with the shape of nucleon DA close to the asymptotic form [27]

$$V^p(y_1, y_2, y_3) = T^p(y_1, y_2, y_3) = 120 y_1 y_2 y_3; \quad A^p(y_1, y_2, y_3) = 0 \tag{34}$$

  already at a low normalization scale. Prominent examples are the Bolz-Kroll (BK) [28] and Braun-Lenz-Wittmann (BLW) LO and NLO models [29, 30]. Also, the advanced lattice calculations of the nucleon DA [31] favor such nucleon DAs.

- The Chernyak-Zhitnitsky (CZ)-type models with a shape of nucleon DA considerably different from the the asymptotic limit at a low normalization scale. The examples of this type of nucleon DA models are the CZ [23], King-Sachrajda (KS) [32], Chernyak-Ogloblin-Zhitnitsky (COZ) [33] and Gari-Stefanis (GS) [34] solutions.

Both types of nucleon DA models were employed to provide a description of the nucleon electromagnetic form factors. As it is well known, the asymptotic form of the nucleon DA (34) results in a vanishing pQCD contribution for the proton form factor. Therefore, using the nucleon DA with a shape close to the asymptotic form implies that the standard pQCD contribution must be be complemented by the so-called soft or end-point corrections (see *e.g.* discussion in Refs. [28, 29, 35, 36]).

However, the nucleon electromagnetic form factor appears as a building block of the backward amplitude within our u-channel nucleon exchange model for VN TDAs. Therefore, within our simple model for VN TDAs, we need to assure that the pQCD contribution into the nucleon electromagnetic form factor is close to the experimental value. This implies that we are forced to use the CZ-type solutions for nucleon DA.

In the following phenomenological estimates we have chosen to employ the COZ [33] and KS [32] solutions for the leading twist nucleon DAs and set a compromise value of the strong coupling $\alpha_s = 0.3$.

Backward ω -meson hard photoproduction

As the first example we consider backward $\omega(782)$ meson hard photoproduction off proton

$$\gamma^*(q, \lambda_\gamma) + p(p_1, s_1) \to \omega(p_\omega, \lambda_\omega) + p(p_2, s_2). \tag{35}$$

We take the `Grein'80` estimates for the ω-meson couplings to nucleons [37] (see also Table 9.2 of Ref. [38])

$$G^V_{\omega NN} = 10.1; \quad G^T_{\omega NN} = 1.42. \tag{36}$$

We take the kinematical point from the foreseen Jlab@12 GeV setup for Hall C (`Fpi-3 (E06-12-101)` u-channel kinematics) [39] $W = 3.20$ GeV. On Fig. 3 we plot the resulting unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_\omega}$ (33) within the u-channel nucleon exchange model for ωN TDAs as a function of Q^2 for $\Delta_T^2 = 0$ (*i.e.* for the ω-meson produced exactly in the backward direction in the $\gamma^* N$ CMS: $\theta^*_\omega = \pi$).

On Fig. 4 we show the $\frac{d^2\sigma_T}{d\Omega_\omega}$ cross section as a function of Δ_T^2 for several values of W and Q^2 (*i.e.* for $u \le u_0$ or, equivalently, $\theta^*_\omega \le \pi$).

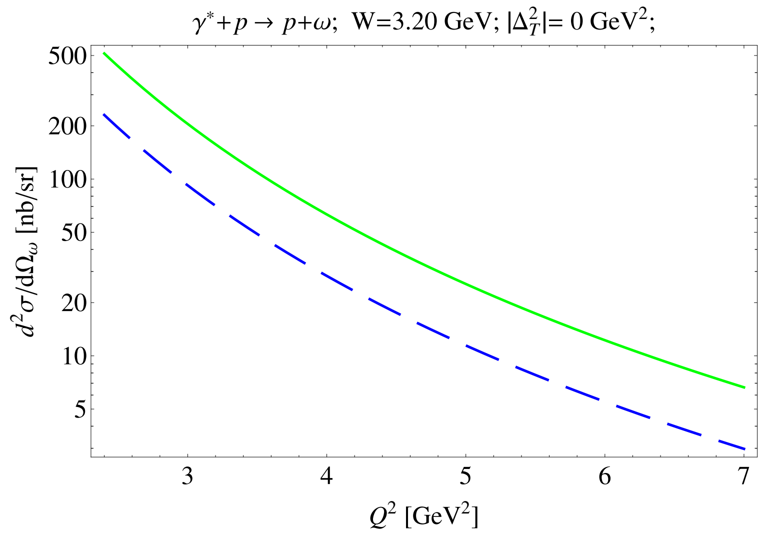

FIG. 3: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_V}$ (in nb/sr) for backward $\gamma^* + p \to p + \omega$ for fixed $W = 3.20$ GeV as a function of Q^2 in the u-channel nucleon exchange model for ωN TDAs. COZ [33] (long-dashed lines) and KS [32] (solid) solutions for the leading twist nucleon DA are used as the phenomenological input.

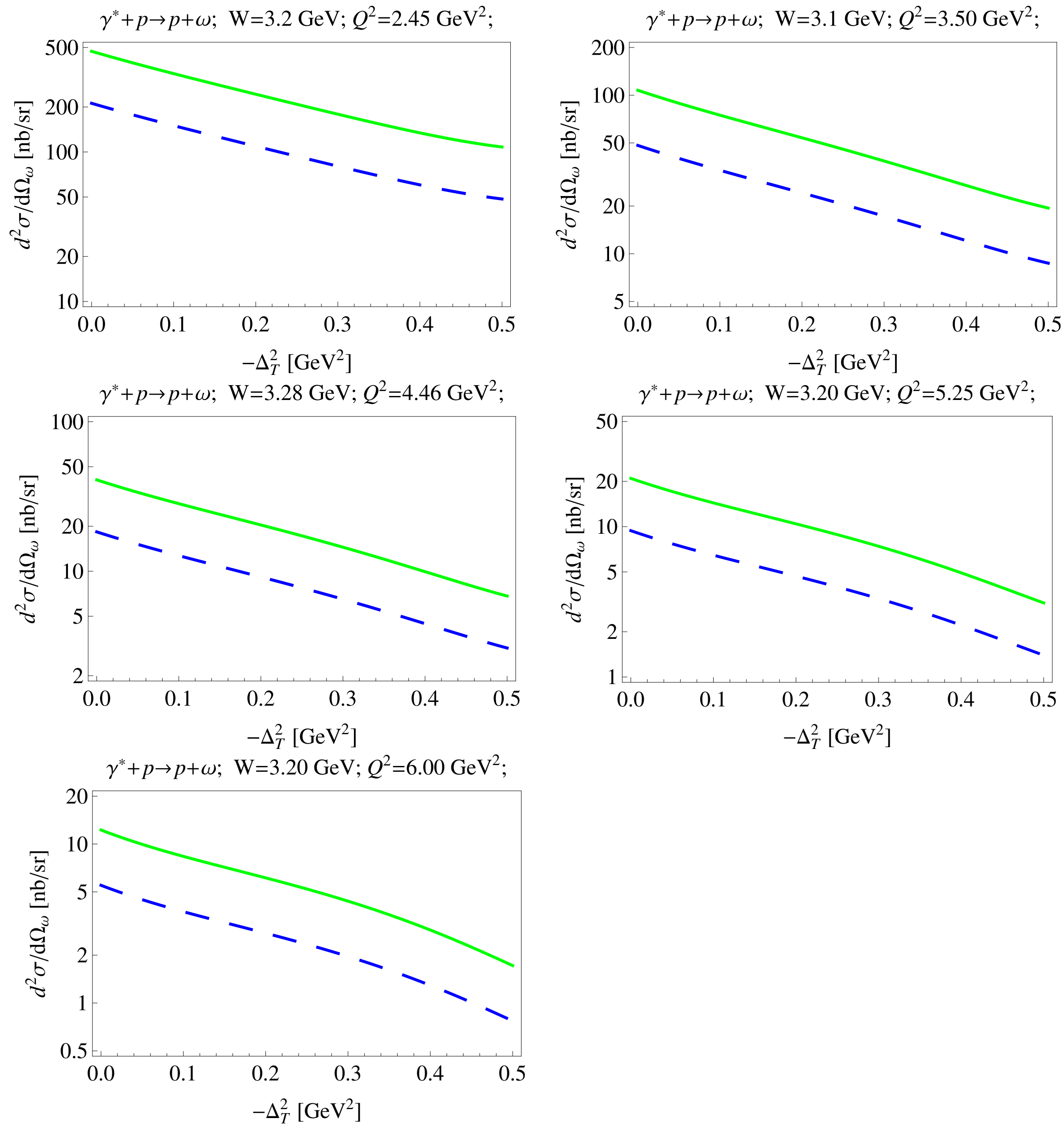

FIG. 4: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_\omega}$ (in nb/sr) for backward $\gamma^* + p \to p + \omega$ for several values of W and Q^2 as a function of Δ_T^2 in the u-channel nucleon exchange model for ωN TDAs. COZ [33] (long-dashed lines) and KS [32] (solid) solutions for the leading twist nucleon DA are used as the phenomenological input.

We may conclude that qualitatively the expected cross sections are similar to the backward π-electroproduction case. Depending on the phenomenological input, and kinematics the cross section turns to be about $\sim 10 \div 100$ [nb/sr] which lies within the reach of future

JLab Hall A, B and C experiments.

Backward ϕ -meson hard photoproduction

The second example is the $\phi(1020)$ meson hard photoproduction

$$\gamma^*(q,\lambda_\gamma)+p(p_1,s_1)\to p(p_2,s_2)+\phi(p_\phi,\lambda_\phi). \tag{37}$$

Some controversy exists in the literature for the values of the phenomenological ϕ-meson to nucleon vector and tensor couplings. As the numerical input for the u-channel nucleon exchange model for ϕN TDAs we employ the phenomenological ϕ-meson to nucleon vector and tensor coupling presented in Ref. [40] (see also Table 2 of Ref. [41]):

$$G^V_{\phi NN}=9.18; \quad G^T_{\phi NN}=-2.02, \tag{38}$$

that are roughly consistent with the estimates of Ref. [42].

On Fig. 5 we plot the backward ϕ-meson hard photoproduction cross section $\frac{d^2\sigma_T}{d\Omega_\phi}$ for $\Delta_T^2=0$ (*i.e.* corresponding to exactly backward scattering $\theta^*_\phi=-\pi$) as a function of Q^2 for the Q^2 range corresponding to `Fpi-3 (E06-12-101)` u-channel kinematics [39].

On Fig. 6 we show the $\frac{d^2\sigma_T}{d\Omega_\phi}$ cross section as a function of Δ_T^2 for several values of W and Q^2 (*i.e.* for $u\le u_0$ or, equivalently, $\theta^*_\phi\le\pi$).

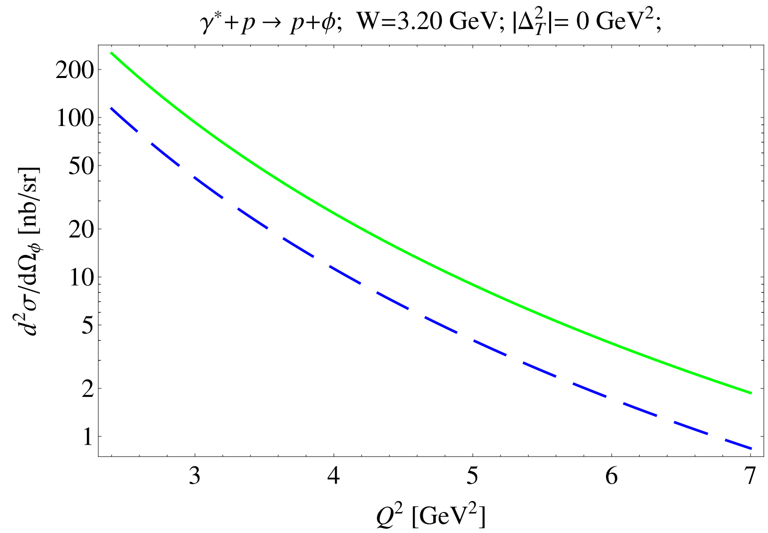

FIG. 5: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_\phi}$ (in nb/sr) for backward $\gamma^* + p \to p + \phi$ for fixed $W = 3.20$ GeV as a function of Q^2 in the u-channel nucleon exchange model for ϕN TDAs. COZ [33] (long-dashed lines) and KS [32] (solid) solutions for the leading twist nucleon DA are used as the phenomenological input.

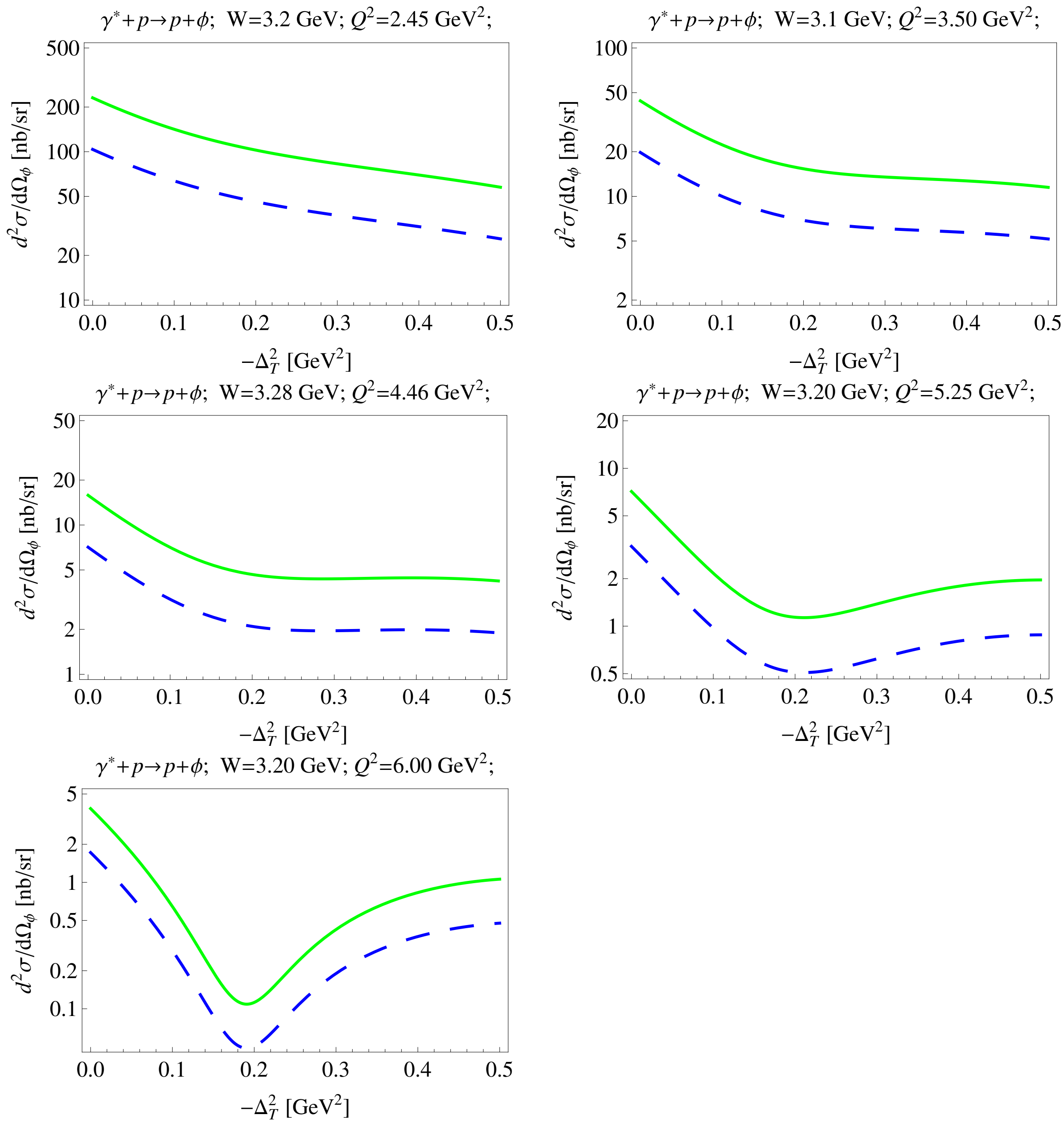

FIG. 6: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_\phi}$ (in nb/sr) for backward $\gamma^* + p \to p + \phi$ for several values of W and Q^2 as a function of Δ_T^2 in the u-channel nucleon exchange model for ϕN TDAs. COZ [33] (long-dashed lines) and KS [32] (solid) solutions for the leading twist nucleon DA are used as the phenomenological input.

Backward ρ^0 -meson hard photoproduction

Finally, we consider the $\rho^0(770)$ meson hard photoproduction

$$\gamma^*(q,\lambda_\gamma)+p(p_1,s_1)\to p(p_2,s_2)+\rho^0(p_\phi,\lambda_\phi). \tag{39}$$

As the numerical input for the u-channel nucleon exchange model for ρN TDAs we employ the `Pietarinen'77` phenomenological ρ-meson to nucleon vector and tensor couplings presented in Table 9.2 of Ref. [38]:

$$G^V_{\rho NN}=2.6; \quad G^T_{\rho NN}=16.1. \tag{40}$$

On Fig. 7 we plot the backward ϕ-meson hard photoproduction cross section $\frac{d^2\sigma_T}{d\Omega_\rho}$ for $\Delta_T^2=0$ (*i.e.* corresponding to exactly backward scattering $\theta^*_\rho=-\pi$) as a function of Q^2 for the Q^2 range corresponding to `Fpi-3 (E06-12-101)` u-channel kinematics [39].

On Fig. 8 we show the $\frac{d^2\sigma_T}{d\Omega_\rho}$ cross section as a function of Δ_T^2 for several values of W and Q^2 (*i.e.* for $u\le u_0$ or, equivalently, $\theta^*_\rho\le\pi$).

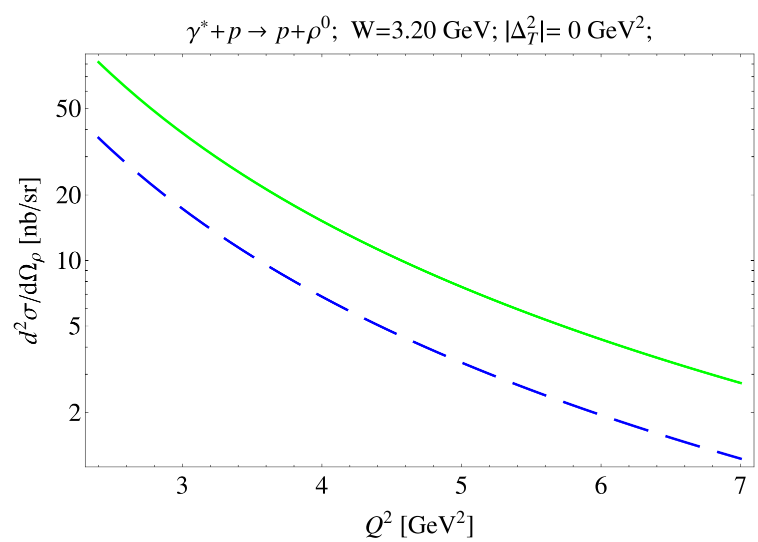

FIG. 7: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_\rho}$ (in nb/sr) for backward $\gamma^*+p\to p+\rho^0$ for fixed $W=3.20$ GeV as a function of Q^2 in the u-channel nucleon exchange model for ρN TDAs. COZ [33] (long-dashed lines) and KS [32] (solid) solutions for the leading twist nucleon DA are used as the phenomenological input.

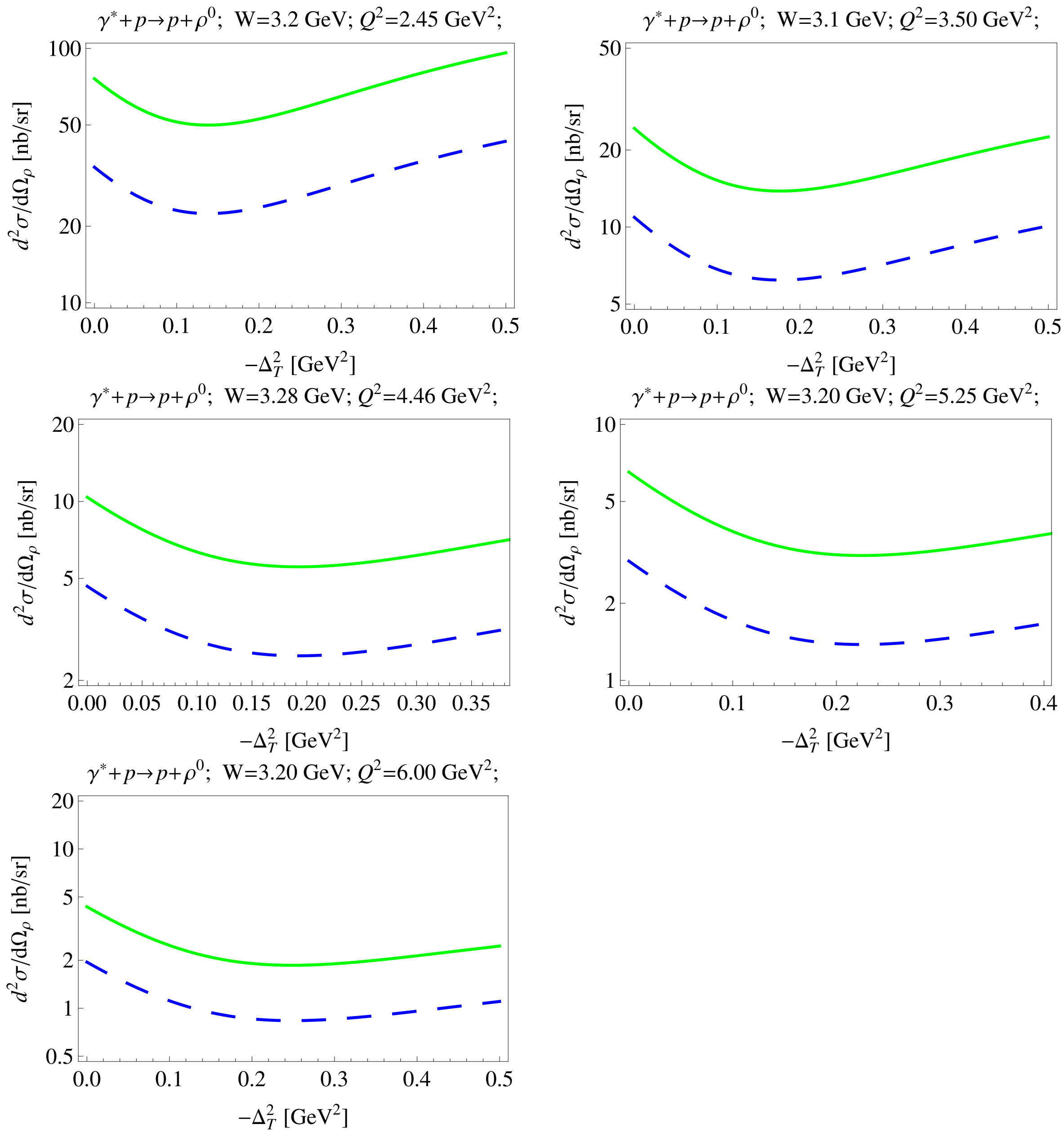

FIG. 8: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_\rho}$ (in nb/sr) for backward $\gamma^* + p \to p + \rho^0$ for several values of W and Q^2 as a function of Δ_T^2 in the u-channel nucleon exchange model for ρN TDAs. COZ [33] (long-dashed lines) and KS [32] (solid) solutions for the leading twist nucleon DA are used as the phenomenological input.

It is interesting to note that as the ρ-meson is mostly coupled to nucleon through the tensor coupling the cross section turns out to be suppressed at least by a factor 3 as compared to the ω meson case.

6. CONCLUSIONS AND OUTLOOK

In this paper we applied the nucleon-to-meson TDA approach to the case of near-backward leptoproduction of light vector mesons off nucleons. We defined 24 leading twist-3 VN TDAs and computed the corresponding leading order hard amplitude. We estimated the differential cross section of backward ρ^0, ω and ϕ production within a simple cross-channel nucleon exchange model for VN TDAs. The cross sections were found to be sizable enough to be studied at future JLab experiments. The study of hard leptoproduction of vector meson in the backward direction can also be seen as an opportunity for COMPASS at CERN and future EIC. Bringing experimental evidences for the suggested scaling behavior will improve our understanding of the on-set of perturbative QCD description of hard reactions.

The same VN TDAs can also be addressed in nucleon-antinucleon annihilation into a lepton pair in association with a vector meson to be studied at P̄ANDA, thus checking the universality of TDAs. Therefore, a feasibility study for $p\bar{p} \to \gamma^* V$ and $p\bar{p} \to J/\psi\, V$ reactions for the P̄ANDA conditions is highly needed.

Our formalism can be naturally generalized to the case of nucleon-to-photon TDAs [43], which can be accessed in backward DVCS. Potentially, this is the cleanest process involving TDAs that could bring new information on hadronic structure. However, the experimental feasibility of backward DVCS requires further studies.

Acknowledgements

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A. ISOSPIN AND PERMUTATION SYMMETRY PROPERTIES OF VN TDAS

In this Appendix we present a brief overview of the isospin properties of VN TDAs. Throughout this analysis we literally follow the system of notations and conventions adopted in Ref. [13].

- Letters from the beginning of the Greek alphabet are reserved for the $SU(2)$ isospin indices α, β, γ, ι, $\kappa = 1,\, 2$.

- We have to distinguish between upper (contravariant) and lower (covariant) $SU(2)$ isospin indices. We introduce the totally antisymmetric tensor $\varepsilon_{\alpha\beta}$ for lowering indices and $\varepsilon^{\alpha\beta}$ for rising indices ($\varepsilon_{12} = \varepsilon^{12} = 1$): $\Psi^\alpha \varepsilon_{\alpha\beta} = \Psi_\beta$, $\Psi_\alpha \varepsilon^{\alpha\beta} = \Psi^\beta$ and $\delta^\alpha_\beta = -\varepsilon^\alpha_{\ \beta} = \varepsilon_\beta^{\ \alpha}$.

- Letters from the beginning of the Latin alphabet $a, b, c = 1,\, 2,\, 3$ are reserved for indices of the adjoint representation of the $SU(2)$ isospin group. σ_a stand for the usual Pauli matrices.

- Letters from the second half of the Greek alphabet ρ, τ, χ are reserved for the Dirac indices.

- Letters c_1, c_2, c_3 stand for $SU(3)$ color indices.

We consider the VN matrix element of light-cone three-quark operators

$$\hat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(z_1, z_2, z_3) \equiv \hat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(1,2,3) = \varepsilon_{c_1c_2c_3} \Psi^{c_1\alpha}_\rho(z_1) \Psi^{c_2\beta}_\tau(z_2) \Psi^{c_3\alpha}_\chi(z_3). \tag{A1}$$

For the case of $I = 0$ vector meson ($\omega(782)$ and $\phi(1020)$ being the examples) the isotopic structure of VN TDA coincides with that of the leading twist nucleon DA. Therefore, the invariant isospin parametrization reads (for definiteness we consider the ωN TDA case)

$$4\langle \omega(p_\omega, \lambda_\omega)| \hat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(1,2,3) |N_\iota(p_1, s_1)\rangle = \varepsilon^{\alpha\beta} \delta^\gamma_\iota M^{\omega N\{12\}}_{\rho\chi\tau}(1,3,2) + \varepsilon^{\alpha\gamma} \delta^\gamma_\iota M^{\omega N\{12\}}_{\rho\tau\chi}(1,2,3), \tag{A2}$$

where the invariant amplitude $M^{\omega N \{12\}}_{\rho\tau\chi}(1,2,3)$ is symmetric with respect to interchange of first two variables:

$$M^{\omega N \{12\}}_{\rho\tau\chi}(1,2,3) = M^{\omega N \{12\}}_{\tau\rho\chi}(2,1,3), \tag{A3}$$

and satisfies the isospin identity (*c.f.* eq. (32) of Ref. [13])

$$M^{\omega N \{12\}}_{\rho\tau\chi}(1,2,3) + M^{\omega N \{12\}}_{\rho\chi\tau}(1,3,2) + M^{\omega N \{12\}}_{\tau\chi\rho}(2,3,1) = 0. \tag{A4}$$

The Dirac structure of $M^{\omega N\, \{12\}}_{\rho\tau\chi}(1,2,3)$ is that of eq. (15). It is straightforward to check that

$$4\langle \omega(p_\omega,\lambda_\omega)|\hat{O}^{uud}_{\rho\tau\chi}(1,2,3)|N_p(p_1,s_1)\rangle = -4\langle \omega(p_\omega,\lambda_\omega)|\hat{O}^{ddu}_{\rho\tau\chi}(1,2,3)|N_n(p_1,s_1)\rangle$$
$$= M^{\omega N\, \{12\}}_{\rho\tau\chi}(1,2,3). \tag{A5}$$

To work out the consequences of the isospin identity (A4) for particular TDAs of (15) one has to employ the set of the Fierz identities (A11)–(A17) for the relevant Dirac structures (17), (18), (19).

For the case of $I=1$ vector meson ($\rho(770)$ being the obvious example) the isotopic structure of VN TDA coincides with that of the leading twist nucleon πN TDA. Therefore we can write down the following isospin decomposition

$$4\langle \rho_a|\widehat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(z_1,\, z_2,\, z_3)|N_\iota\rangle = (f_a)^{\{\alpha\beta\gamma\}}_{\ \ \ \iota} M^{(\rho N)_{3/2}}_{\rho\tau\chi}(1,2,3)$$
$$+\varepsilon^{\alpha\beta}(\sigma_a)^{\gamma}_{\ \iota} M^{(\rho N)_{1/2}\,\{12\}}_{\rho\chi\tau}(1,3,2) + \varepsilon^{\alpha\gamma}(\sigma_a)^{\beta}_{\ \iota} M^{(\rho N)_{1/2}\,\{12\}}_{\rho\tau\chi}(1,2,3)\,, \tag{A6}$$

where the totally symmetric tensor $(f_a)^{\{\alpha\beta\gamma\}}_{\ \ \ \iota}$ reads

$$(f_a)^{\{\alpha\beta\gamma\}}_{\ \ \ \iota} = \frac{1}{3}\left((\sigma_a)^{\alpha}_{\ \delta}\varepsilon^{\delta\beta}\delta^{\gamma}_{\ \iota} + (\sigma_a)^{\alpha}_{\ \delta}\varepsilon^{\delta\gamma}\delta^{\beta}_{\ \iota} + (\sigma_a)^{\beta}_{\ \delta}\varepsilon^{\delta\gamma}\delta^{\alpha}_{\ \iota}\right). \tag{A7}$$

The properties of the u-channel isospin-$\frac{1}{2}$ invariant amplitude $M^{(\rho N)_{1/2}\,\{12\}}_{\rho\tau\chi}$ are fully analogous to that of the corresponding invariant amplitude of eq. (A2). It is symmetric under the simultaneous interchange of two first arguments and the Dirac indices:

$$M^{(\rho N)_{1/2}\,\{12\}}_{\rho\tau\chi}(1,2,3) = M^{(\rho N)_{1/2}\,\{12\}}_{\tau\rho\chi}(2,1,3), \tag{A8}$$

and satisfies the isospin identity (*c.f.* eq. (32) of Ref. [13])

$$M^{(\rho N)_{1/2}\,\{12\}}_{\rho\tau\chi}(1,2,3) + M^{(\rho N)_{1/2}\,\{12\}}_{\rho\chi\tau}(1,3,2) + M^{(\rho N)_{1/2}\,\{12\}}_{\tau\chi\rho}(2,3,1) = 0. \tag{A9}$$

The Dirac structure of $M^{(\rho N)_{1/2}\,\{12\}}_{\rho\tau\chi}$ is again that of eq. (15).

As a consequence of the permutation and isotopic symmetry, the u-channel isospin-$\frac{3}{2}$ invariant amplitude $M^{(\rho N)_{3/2}\,\{12\}}_{\rho\tau\chi}$ is completely symmetric under simultaneous permutations of the arguments and the Dirac indices:

$$\begin{aligned}
&M^{(\rho N)_{3/2}}_{\rho\tau\chi}(1,2,3) = M^{(\rho N)_{3/2}}_{\rho\chi\tau}(1,3,2) = M^{(\rho N)_{3/2}}_{\tau\rho\chi}(2,1,3) \\
&= M^{(\rho N)_{3/2}}_{\tau\chi\rho}(2,3,1) = M^{(\rho N)_{3/2}}_{\chi\tau\rho}(3,2,1) = M^{(\rho N)_{3/2}}_{\chi\rho\tau}(3,1,2)\,.
\end{aligned} \tag{A10}$$

The Dirac structure of $M^{(\rho N)_{3/2}\,\{12\}}_{\rho\tau\chi}$ is also that of eq. (15).

Below, to the leading twist-3 accuracy, we present the set of Fierz identities for the relevant Dirac structures (17), (18), (19) needed to establish the consequences of the isotopic and permutation symmetry (A10) for VN TDAs.

$$\begin{aligned}
(v^{VN}_{1\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{1\mathcal{E}} - a^{VN}_{1\mathcal{E}} + t^{VN}_{1\mathcal{E}} + t^{VN}_{2\mathcal{E}}\right)_{\chi\tau,\,\rho}; \\
(a^{VN}_{1\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(-v^{VN}_{1\mathcal{E}} + a^{VN}_{1\mathcal{E}} + t^{VN}_{1\mathcal{E}} + t^{VN}_{2\mathcal{E}}\right)_{\chi\tau,\,\rho}; \\
(t^{VN}_{1\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{1\mathcal{E}} + a^{VN}_{1\mathcal{E}} + t^{VN}_{1\mathcal{E}} - t^{VN}_{2\mathcal{E}}\right)_{\chi\tau,\,\rho} \\
(t^{VN}_{2\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{1\mathcal{E}} + a^{VN}_{1\mathcal{E}} - t^{VN}_{1\mathcal{E}} + t^{VN}_{2\mathcal{E}}\right)_{\chi\tau,\,\rho};
\end{aligned} \tag{A11}$$

$$\begin{aligned}
(v^{VN}_{1T})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{1T} - a^{VN}_{1T} + t^{VN}_{1T}\right)_{\chi\tau,\,\rho}; \\
(a^{VN}_{1T})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(-v^{VN}_{1T} + a^{VN}_{1T} + t^{VN}_{1T}\right)_{\chi\tau,\,\rho}; \\
(t^{VN}_{1T})_{\rho\tau,\,\chi} &= \left(v^{VN}_{1T} + (a^{VN}_{1T}\right)_{\chi\tau,\,\rho};
\end{aligned} \tag{A12}$$

$$\begin{aligned}
(v^{VN}_{1n})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{1n} - a^{VN}_{1n} + t^{VN}_{1n}\right)_{\chi\tau,\,\rho}; \\
(a_{1n})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(-v^{VN}_{1n} + a^{VN}_{1n} + t^{VN}_{1n}\right)_{\chi\tau,\,\rho}; \\
(t^{VN}_{1n})_{\rho\tau,\,\chi} &= \left(v^{VN}_{1n} + a^{VN}_{1n}\right)_{\chi\tau,\,\rho};
\end{aligned} \tag{A13}$$

$$\begin{aligned}
(v^{VN}_{2\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{2\mathcal{E}} - a^{VN}_{2\mathcal{E}} + t^{VN}_{3\mathcal{E}} - t^{VN}_{4\mathcal{E}}\right)_{\chi\tau,\,\rho}; \\
(a^{VN}_{2\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(-v^{VN}_{2\mathcal{E}} + a^{VN}_{2\mathcal{E}} + t^{VN}_{3\mathcal{E}} - t^{VN}_{4\mathcal{E}}\right)_{\chi\tau,\,\rho}; \\
(t^{VN}_{3\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(v^{VN}_{1T} + v^{VN}_{2\mathcal{E}} + a^{VN}_{1T} + a^{VN}_{2\varepsilon} + t^{VN}_{3\mathcal{E}} - t^{VN}_{1T} + t^{VN}_{4\mathcal{E}}\right)_{\chi\tau,\,\rho}; \\
(t^{VN}_{4\mathcal{E}})_{\rho\tau,\,\chi} &= \frac{1}{2}\left(-v^{VN}_{2\mathcal{E}} + v^{VN}_{1T} - a^{VN}_{2\mathcal{E}} + a^{VN}_{1T} + t^{VN}_{3\mathcal{E}} - t^{VN}_{1T} + t^{VN}_{4\mathcal{E}}\right)_{\chi\tau,\,\rho};
\end{aligned} \tag{A14}$$

$$(v_{2T}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(v_{2T}^{VN} - a_{2T}^{VN} + t_{2T}^{VN} + t_{3T}^{VN}\right)_{\chi\tau,\rho};$$
$$(a_{2T}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(-v_{2T}^{VN} + a_{2T}^{VN} + t_{2T}^{VN} + t_{3T}^{VN}\right)_{\chi\tau,\rho};$$
$$(t_{2T}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(v_{2T}^{VN} + a_{2T}^{VN} + t_{2T}^{VN} - t_{3T}^{VN}\right)_{\chi\tau,\rho};$$
$$(t_{3T}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(v_{2T}^{VN} + a_{2T}^{VN} - t_{2T}^{VN} + t_{3T}^{VN}\right)_{\chi\tau,\rho}; \quad \text{(A15)}$$

$$(v_{2n}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(v_{2n}^{VN} - a_{2n}^{VN} + t_{2n}^{VN} + t_{3n}^{VN}\right)_{\chi\tau,\rho};$$
$$(a_{2n}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(-v_{2n}^{VN} + a_{2n}^{VN} + t_{2n}^{VN} + t_{3n}^{VN}\right)_{\chi\tau,\rho};$$
$$(t_{2n}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(v_{2n}^{VN} + a_{2n}^{VN} + t_{2n}^{VN} - t_{3n}^{VN}\right)_{\chi\tau,\rho};$$
$$(t_{3n}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\left(v_{2n}^{VN} + a_{2n}^{VN} - t_{2n}^{VN} + t_{3n}^{VN}\right)_{\chi\tau,\rho}; \quad \text{(A16)}$$

$$(t_{4T}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\frac{\Delta_T^2}{M^2}\left(v_{1T}^{VN} - a_{1T}^{VN} - t_{1T}^{VN}\right)_{\chi\tau,\rho} + (t_{4T}^{VN})_{\chi\tau,\rho};$$
$$(t_{4n}^{VN})_{\rho\tau,\chi} = \frac{1}{2}\frac{\Delta_T^2}{M^2}\left(v_{1n}^{VN} - a_{1n}^{VN} - t_{1n}^{VN}\right)_{\chi\tau,\rho} + (t_{4n}^{VN})_{\chi\tau,\rho}. \quad \text{(A17)}$$

B. NUCLEON POLE EXCHANGE MODEL FOR VN TDA

In this Appendix we construct a simple u-channel nucleon exchange model for VN TDAs. This model populates only the Efremeov-Radyushkin-Brodsky-Lepage (ERBL)-like region of the VN TDA support domain and represents an analogue of the D-term contribution supplementary to the spectral representation [22] in terms of quadruple distributions.

The effective VNN vertex [38]

$$V_{\mathit{eff}}\big(N(p_1,s_1) \to V(p_V,\lambda_V)N(-\Delta,s_p)\big)$$
$$= \bar{U}(-\Delta,s_p)\left[G_{VNN}^V \hat{\mathcal{E}}^*(p_V,\lambda_V) + G_{VNN}^T \frac{\sigma_{\mu\nu}}{2M}(-\Delta)^\nu \mathcal{E}^{\mu *}(p_V,\lambda_V)\right] U(p_1,s_1) \quad \text{(B1)}$$

involves two dimensionless phenomenological couplings G_{VNN}^V and G_{VNN}^T.

The u-channel nucleon exchange contribution into the light-cone three-quark-operator

VN matrix element occurring in the VN TDA definition (15) reads:

$$\begin{aligned}
&\langle V(p_V,\lambda_V)|\hat{O}^{uud}_{\rho\tau\chi}(\lambda_1 n,\ \lambda_2 n,\ \lambda_3 n)|N(p_1,s_1)\rangle\Big|_{N(940)} \\
&= \sum_{s_p} \langle 0|\hat{O}^{uud}_{\rho\tau\chi}(\lambda_1 n,\ \lambda_2 n,\ \lambda_3 n)|N(-\Delta,s_p)\rangle \bar{U}(-\Delta,s_p)\frac{1}{\Delta^2-M^2}\Big[G^V_{VNN}\hat{\mathcal{E}}^*(p_V,\lambda_V) \\
&+G^T_{NN\omega}\frac{\sigma_{\mu\nu}}{2M}(-\Delta)^\nu \mathcal{E}^{\mu *}(p_V,\lambda_V)\Big]\,U(p_1,s_1).
\end{aligned} \tag{B2}$$

The $\langle 0|\hat{O}^{uud}_{\rho\tau\chi}(\lambda_1 n,\ \lambda_2 n,\ \lambda_3 n)|N(-\Delta,s_p)\rangle$ matrix element is then expressed through the leading twist-3 nucleon DA. Performing the Fourier transform (16) this yields:

$$\begin{aligned}
&4\mathcal{F}(x_1,x_2,x_3)\ \langle V(p_V,\lambda_V)|\hat{O}^{uud}_{\rho\tau\chi}(\lambda_1 n,\ \lambda_2 n,\ \lambda_3 n)|N_p(p_1,s_1)\rangle\Big|_{N(940)} = \delta(x_1+x_2+x_3-2\xi) \\
&\times M\ \Theta_{\rm ERBL}(x_1,x_2,x_3) f_N \frac{1}{M} \times \frac{1}{(2\xi)^2}\sum_{s_p}\Big\{V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)(-\hat{\Delta}C)_{\rho\tau}(\gamma^5 U(-\Delta,s_p))_\chi \\
&+A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)(-\hat{\Delta}\gamma_5 C)_{\rho\tau}U(-\Delta,s_p)_\chi \\
&+T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)(\sigma_{-\Delta\lambda}C)_{\rho\tau}(\gamma^\lambda\gamma^5 U(-\Delta,s_p))_\chi\Big\}\bar{U}(-\Delta,s_p)\frac{1}{\Delta^2-M^2}\Big[G^V_{VNN}\hat{\mathcal{E}}^*(p_V,\lambda_V) \\
&+G^T_{VNN}\frac{\sigma_{\mu\nu}}{2M}(-\Delta)^\nu\mathcal{E}^{\mu *}(p_V,\lambda_V)\Big]\,U(p_1,s_1).
\end{aligned} \tag{B3}$$

Here we employ the notation

$$\Theta_{\rm ERBL}(x_1,x_2,x_3) \equiv \left[\prod_{k=1}^{3}\theta(0\le x_k\le 2\xi)\right]. \tag{B4}$$

To work out from (B3) the nucleon pole exchange contributions to particular TDAs one has to expand it over the set of 24 basic Dirac structures (17), (18) and (19) which is a straightforward (though tedious) calculation. It turns out that the u-channel nucleon exchange model populates 22 out of the 24 VN TDAs (T^{VN}_{4T} and T^{VN}_{4n} vanish in this model).

It is convenient to show the results for the groups of VN TDAs interlinked through the set of the isospin relations (see Appendix A).

- $V_{1\mathcal{E}}$, $A_{1\mathcal{E}}$, $T_{1\mathcal{E}}$, $T_{2\mathcal{E}}$ satisfy the isospin symmetry relations based on the Fierz transformation set (A11).

$$
\begin{aligned}
V_{1\mathcal{E}}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1\mathcal{E}}^{VN}(\xi,\Delta^2);\\
A_{1\mathcal{E}}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1\mathcal{E}}^{VN}(\xi,\Delta^2);\\
T_{1\mathcal{E}}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1\mathcal{E}}^{VN}(\xi,\Delta^2);\\
T_{2\mathcal{E}}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1\mathcal{E}}^{VN}(\xi,\Delta^2),
\end{aligned}
\tag{B5}
$$

where

$$
K_{1\mathcal{E}}^{VN}(\xi,\Delta^2)=\frac{f_N}{\Delta^2-M^2}\left(G_{VNN}^V\frac{2\xi(1-\xi)}{1+\xi}+G_{VNN}^T\,\xi\left(\frac{2\xi}{1+\xi}-\frac{\Delta^2}{M^2}\right)\right). \tag{B6}
$$

- V_{1T}, A_{1T}, T_{1T} satisfy the isospin symmetry relations based on the Fierz transformation set (A12)

$$
\begin{aligned}
V_{1T}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1T}^{VN}(\xi,\Delta^2);\\
A_{1T}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1T}^{VN}(\xi,\Delta^2);\\
T_{1T}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1T}^{VN}(\xi,\Delta^2),
\end{aligned}
\tag{B7}
$$

where

$$
K_{1T}(\xi,\Delta^2)=\frac{f_N}{\Delta^2-M^2}\left(-G_{VNN}^V\frac{2\xi(1+3\xi)}{1-\xi}\right). \tag{B8}
$$

- V_{1n}, A_{1n}, T_{1n} satisfy the isospin symmetry relations based on the Fierz transformation set (A13)

$$
\begin{aligned}
V_{1n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1n}^{VN}(\xi,\Delta^2);\\
A_{1n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1n}^{VN}(\xi,\Delta^2);\\
T_{1n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{1n}^{VN}(\xi,\Delta^2),
\end{aligned}
\tag{B9}
$$

where

$$K_{1n}(\xi,\Delta^2) \\ = \frac{f_N}{\Delta^2-M^2}\left(\frac{(1+\xi)\,(m^2-\Delta_T^2)}{M^2(1-\xi)^2}-\frac{1}{1+\xi}\right)\left(G^V_{VNN}(-4\xi)+G^T_{VNN}\frac{\xi(1-\xi)}{1+\xi}\right). \tag{B10}$$

- $V_{2\mathcal{E}}$, $A_{2\mathcal{E}}$, $T_{3\mathcal{E}}$, $T_{4\mathcal{E}}$ satisfy the isospin symmetry relations based on the Fierz transformation set (A14). Within the nucleon u-channel exchange model this set decouples from the V_{1T}, A_{1T}, T_{1T} set.

$$\begin{aligned}
V^{VN}_{2\mathcal{E}}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2\mathcal{E}}(\xi,\Delta^2);\\
A^{VN}_{2\mathcal{E}}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2\mathcal{E}}(\xi,\Delta^2);\\
T^{VN}_{3\mathcal{E}}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2\mathcal{E}}(\xi,\Delta^2);\\
T^{VN}_{4\mathcal{E}}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2\mathcal{E}}(\xi,\Delta^2),
\end{aligned} \tag{B11}$$

where

$$K^{VN}_{2\mathcal{E}}(\xi,\Delta^2)=\frac{f_N}{\Delta^2-M^2}\left(G^V_{VNN}(-2\xi)+G^T_{VNN}\xi\right). \tag{B12}$$

- V_{2T}, A_{2T}, T_{2T}, T_{3T} satisfy the isospin symmetry relations based on the Fierz transformation set (A15).

$$\begin{aligned}
V^{VN}_{2T}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2T}(\xi,\Delta^2);\\
A^{VN}_{2T}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2T}(\xi,\Delta^2);\\
T^{VN}_{2T}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2T}(\xi,\Delta^2);\\
T^{VN}_{3T}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K^{VN}_{2T}(\xi,\Delta^2),
\end{aligned} \tag{B13}$$

where

$$K_{2T}(\xi,\Delta^2)=\frac{f_N}{\Delta^2-M^2}\left(G^T_{VNN}\frac{\xi(1+\xi)}{1-\xi}\right). \tag{B14}$$

- V_{2n}, A_{2n}, T_{2n}, T_{3n} satisfy the isospin symmetry relations based on the Fierz transformation set (A16).

$$
\begin{aligned}
V_{2n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}V^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{2n}^{VN}(\xi,\Delta^2);\\
A_{2n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= \Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}A^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{2n}^{VN}(\xi,\Delta^2);\\
T_{2n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{2n}^{VN}(\xi,\Delta^2);\\
T_{3n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)} &= -\Theta_{\rm ERBL}(x_1,x_2,x_3)\frac{1}{(2\xi)^2}T^p\left(\frac{x_1}{2\xi},\frac{x_2}{2\xi},\frac{x_3}{2\xi}\right)K_{2n}^{VN}(\xi,\Delta^2),
\end{aligned}
\tag{B15}
$$

where

$$
K_{2n}^{VN}(\xi,\Delta^2)=\frac{f_N}{\Delta^2-M^2}\xi\left(\frac{1+\xi}{(1-\xi)^2}\frac{(m_V^2-\Delta_T^2)}{M^2}-\frac{1}{1+\xi}\right)G_{VNN}^T. \tag{B16}
$$

- Finally,

$$
\begin{aligned}
&T_{4T}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)}=0;\\
&T_{4n}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)\big|_{N(940)}=0;
\end{aligned}
\tag{B17}
$$

The above formulas can be employed both for $I=0$ and $I=1$ vector-meson-to-nucleon TDAs. In the latter case the u-channel nucleon pole exchange contributes only to the u-channel isospin-$\frac{1}{2}$ invariant amplitude $M_{\rho\tau\chi}^{(\rho N)_{1/2}\,\{12\}}$, thus populating only u-channel isospin-$\frac{1}{2}$ TDAs.

It is straightforward to check that the $I=0$ and $I=1$ vector-meson-to-nucleon TDAs computed within the u-channel nucleon pole exchange model satisfy the set of isospin identities following from the appropriate isospin symmetry relations (A4) and (A9). The explicit form of these isospin identities can be established with the use of the set of the Fierz identities worked out in App. A. For example for the case of $V_{1\mathcal{E}}^{VN}$ TDA it reads

$$
\begin{aligned}
&V_{1\mathcal{E}}^{VN}(x_1,x_2,x_3,\xi,\Delta^2)+\frac{1}{2}\left(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN}+T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN}\right)(x_3,x_1,x_2,\xi,\Delta^2)\\
&+\frac{1}{2}\left(V_{1\mathcal{E}}^{VN}-A_{1\mathcal{E}}^{VN}+T_{1\mathcal{E}}^{VN}+T_{2\mathcal{E}}^{VN}\right)(x_3,x_2,x_1,\xi,\Delta^2)=0.
\end{aligned}
\tag{B18}
$$

The validity of this and all subsequent isospin identities for VN TDAs within the u-channel nucleon exchange model turns out to be the consequence of the familiar isospin identity for the leading twist nucleon DAs

$$2T^p(y_1, y_2, y_3) = (V^p - A^p)(y_1, y_3, y_2) - (V^p - A^p)(y_2, y_3, y_1). \tag{B19}$$

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