Semileptonic decays of $B(B_s)$ to light tensor mesons

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Abstract

The semileptonic $B_s(B) \to K_2^*(a_2, f_2)\ell\nu$, $\ell = \tau, \mu$ transitions are investigated in the frame work of the three-point QCD sum rules. Considering the quark condensate contributions, the relevant form factors of these transitions are estimated. The branching ratios of these channel modes are also calculated at different values of the continuum thresholds of the tensor mesons and compared with the obtained data for other approaches.

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I. INTRODUCTION

Investigation of the B meson decays into tensor mesons are useful in several aspects such as CP asymmetries, isospin symmetries and the longitudinal and transverse polarization fractions. A large isospin violation has already been experimentally detected in $B \rightarrow \omega K_2^*(1430)$ mode [1]. Also, the decay mode $B \rightarrow \phi K_2^*(1430)$ is mainly dominated by the longitudinal polarization [2, 3], in contrast with the $B \rightarrow \phi K^*$ where the transverse polarization is comparable with the longitudinal one [4]. Therefore, nonleptonic and semileptonic decays of B meson can play an important role in the study of the particle physics.

In the flavor $SU(3)$ symmetry, the light p-wave tensor mesons with $J^P = 2^+$ containing iso-vector mesons $a_2(1320)$, iso-doublet states $K_2^*(1430)$, and two iso-singlet mesons $f_2(1270)$ and $f_2'(1525)$, are building the ground state nonet which have been experimentally established [5, 6]. The quark content $q\bar{q}$ for the iso-vector and iso-doublet tensor resonances are obvious. The iso-scalar tensor states, $f_2(1270)$ and $f_2'(1525)$ have a mixing wave functions where mixing angle should be small [7, 8]. Therefore, $f_2(1270)$ is primarily a $(u\bar{u} + d\bar{d})/\sqrt{2}$ state, while $f_2'(1525)$ is dominantly $s\bar{s}$ [9].

As a nonperturbative method, the QCD sum rules is a well established technique in the hadron physics since it is based on the fundamental QCD Lagrangian. The semileptonic decays of B to the light mesons involving π, $K(K^*, K_0^*)$, and a_1 have been studied via the three-point QCD sum rules (3PSR), for instance $B \rightarrow \pi \ell \nu$ [10], $B \rightarrow K \ell^+ \ell^-$, $B \rightarrow K^* \ell^+ \ell^-$ [11, 12], $B_s \rightarrow K_0^* \ell \nu$ [13], $B_s \rightarrow (K_0^*, f_0) \ell^+ \ell^-$ [14] and $B \rightarrow a_1 \ell^+ \ell^-$ [15]. The determination of the form factor value $T_1(0) = 0.35 \pm 0.05$ relevant for the $B \rightarrow K^* \gamma$ and $B \rightarrow K^* \ell^+ \ell^-$ [12, 16] decays allowed to predict the ratio $\Gamma(B \rightarrow K^* \gamma)/\Gamma(b \rightarrow s\gamma) = 0.17 \pm 0.05$, which agrees with the experimental measurements [17–19]. The obtained results of the decay $B \rightarrow \pi \ell \nu$ [10], and simulations on the lattice [20–22] are in a reasonable agreement.

However, considering the structure of the 3PSR for heavy-light transitions shows a difficulty which is due to the dependence of the various terms of the short-distance expansion to mass of the b-quark [23]. Therefore, some authors claimed that the 3PSR is not a well-established tool for heavy-light transition, and is ill-behaved for large m_b mass which has been discussed in [24].

It should be noted that the treatment of the 3PSR and light-cone QCD sum rules

(LCSR) are different for the light hadron in the final state. The 3PSR, without considering wave functions, introduces a three-point correlation function with appropriate interpolating currents and extract the perturbative and nonperturbative contributions of the transition form factors. In the limit of $m_b \to \infty$, the coefficients of the nonperturbative effects such as quark-quark and quark-gluon condensate are increased with m_b faster than the perturbative coefficient. In the light-cone sum rules (LCSR) method, this problem does not appear since the nonperturbative effects are included in the hadron distribution amplitudes [25].

This problem could be irrelevant for the actual value of the b-quark mass, and also for particular processes and final states. For instance, the obtained results based on the 3PSR for the $B \to \pi$, and $B \to K^*$ transitions are in good agreement with the experimental values, as it was already mentioned. On the other hand, using the Borel transformation exponentially suppresses the contributions of the highest-order operators. However, it is necessary to compare the results obtained for heavy-light transition from the 3PSR with other methods, especially the LCSR and experimental data if exist.

It should be emphasized that a suitable choice of the Borel parameter interval keeps the convergence of the condensate expansion under control. Therefore, neglecting the higher-dimensional terms do not introduce a large error. In Ref. [24], authors have obtained $T_1^{B \to K^*}(0) \simeq 0.5 - 0.6$ and 0.32 via the 3PSR and the LCSR, respectively, where the 3PSR value is larger than that for the LCSR. Also, this quantity has been calculated 0.38 based on the 3PSR in [26], which has a reasonable agreement with the LCSR value. The main reason for the difference between results based on the 3PSR in the two papers is due to the difference in selection of the Borel parameter interval [24].

In this work, we investigate the $B(B_s) \to K_2^*(a_2, f_2)\ell\nu$ decays within the 3PSR method. For analysis of these decays, the form factors and their branching ratio values are calculated. So far, the form factors of the semileptonic decays $B(B_s) \to K_2^*(a_2, f_2)\ell\nu$ have been studied via different approaches such as the LCSR [27], the perturbative QCD (PQCD) [5], the large energy effective theory (LEET) [28–30], and the ISGW II model [31]. A comparison of our results for the form factor values in $q^2 = 0$ and branching ratio data with predictions obtained from other approaches, especially the LCSR, is also made.

The plan of the present paper is as follows: The 3PSR approach for calculation of the relevant form factors of the $B(B_s) \to K_2^*(a_2, f_2)\ell\nu$ decays presented in Section II. In the

final section, the value of the form factors in $q^2 = 0$ and the branching ratio of the considered decays are reported. For a better analysis, the form factors and differential branching ratios related to these semileptonic decays are plotted with respect to the momentum transfer squared q^2.

II.   THEORETICAL FRAMEWORK

In order to study of $B(B_s) \to K_2^*(a_2, f_2)\ell\nu$ decays, we focus on the exclusive decay $B_s \to K_2^*$ via the 3PSR. The $B_s \to K_2^*\ell\nu$ decay governed by the tree level $b \to u$ transition (see Fig. 1). In the framework of the 3PSR, the first step is appropriate definition of
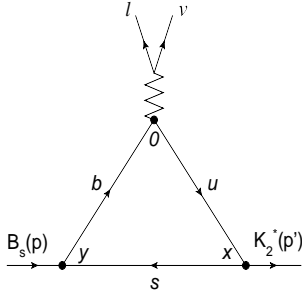

FIG. 1: Schematic picture of the spectator mechanism for the $B_s \to K_2^*\ell\nu$ decay.

correlation function. In this work, the correlation function should be taken as

$$\Pi_{\alpha\beta\mu}(p^2, p'^2, q^2) = i \int \int d^4x \, d^4y \, e^{i(p'x-py)} \left\langle 0 \left| \mathcal{T} \left\{ j_{\alpha\beta}^{K_2^*}(x) j_\mu(0) j^{B_s}(y) \right\} \right| 0 \right\rangle, \qquad (1)$$

where p and p' are four-momentum of the initial and final mesons, respectively. q^2 is the squared momentum transfer and $\mathcal{T}$ is the time ordering operator. $j_\mu = \bar{u}\gamma_\mu(1 - \gamma_5)b$ is the transition current. j^{B_s} and $j_{\alpha\beta}^{K_2^*}$ are also the interpolating currents of the B_s and the tensor meson K_2^*, respectively. With considering all quantum numbers, their interpolating currents can be written as

$$
\begin{aligned}
j^{B_s}(y) &= \bar{b}(y)\gamma_5 s(y), \\
j_{\alpha\beta}^{K_2^*}(x) &= \frac{i}{2} \left[\bar{s}(x)\gamma_\alpha \overleftrightarrow{D}_\beta(x) u(x) + \bar{s}(x)\gamma_\beta \overleftrightarrow{D}_\alpha(x) u(x) \right],
\end{aligned}
\qquad (2)
$$

where $\overleftrightarrow{D}_\mu(x)$ is the four-derivative with respect to x acting at the same time on the left and right. It is given as

$$\overleftrightarrow{D}_\mu(x) = \frac{1}{2} \left[\overrightarrow{D}_\mu(x) - \overleftarrow{D}_\mu(x) \right],$$

$$\vec{D}_\mu(x) = \vec{\partial}_\mu(x) - i\frac{g}{2}\lambda^a \boldsymbol{A}_\mu^a(x),$$
$$\overleftarrow{D}_\mu(x) = \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a \boldsymbol{A}_\mu^a(x),$$

where λ^a and $\boldsymbol{A}_\mu^a(x)$ are the Gell-Mann matrices and the external gluon fields, respectively.

The correlation function is a complex function of which the imaginary part comprises the computations of the phenomenology and real part comprises the computations of the theoretical part (QCD). By linking these two parts via the dispersion relation, the physical quantities are calculated. In the phenomenological part of the QCD sum rules approach, the correlation function in Eq. (1) is calculated by inserting two complete sets of intermediate states with the same quantum numbers as B_s and K_2^*. After performing four-integrals over x and y, it will be:

$$\Pi_{\alpha\beta\mu} = -\frac{\langle 0|j_{\alpha\beta}^{K_2^*}|K_2^*(p')\rangle\langle K_2^*(p')|j_\mu|B_s(p)\rangle\langle B_s(p)|j^{B_s}|0\rangle}{(p^2 - m_{B_s}^2)(p'^2 - m_{K_2^*}^2)} + \text{higher states.} \qquad (3)$$

In Eq. (3), the vacuum to initial and final meson state matrix elements are defined as

$$\langle 0|j_{\alpha\beta}^{K_2^*}|K_2^*(p',\varepsilon)\rangle = f_{K_2^*}m_{K_2^*}^2\varepsilon_{\alpha\beta}, \qquad \langle 0|j^{B_s}|B_s(p)\rangle = -i\frac{f_{B_s}m_{B_s}^2}{(m_b + m_s)}, \qquad (4)$$

where $f_{K_2^*}$ and f_{B_s} are the leptonic decay constants of K_2^* and B_s mesons, respectively. $\varepsilon_{\alpha\beta}$ is polarization tensor of K_2^*. The transition current give a contribution to these matrix elements and it can be parametrized in terms of some form factors using the Lorentz invariance and parity conservation. The correspondence between a vector meson and a tensor meson allows us to get these parametrization in a comparative way (for more information see [5]). The parametrization of $B \to T$ form factors is analogous to the $B \to V$ case except that the ε is replaced by ε_T, as follows:

$$c_V\langle K_2^*(p',\varepsilon)|\bar{u}\gamma_\mu(1-\gamma_5)b|B_s(p)\rangle = -i\varepsilon_{T\mu}^*(m_{B_s} + m_{K_2^*})A_1(q^2) + i(p+p')_\mu(\varepsilon_T^*.q)\frac{A_2(q^2)}{m_{B_s} + m_{K_2^*}}$$
$$+ iq_\mu(\varepsilon_T^*.q)\frac{2m_{K_2^*}}{q^2}\left(A_3(q^2) - A_0(q^2)\right) + \epsilon_{\mu\nu\rho\sigma}\varepsilon_T^{*\nu}p^\rho p'^\sigma\frac{2V(q^2)}{m_{B_s} + m_{K_2^*}} \qquad (5)$$

with $A_3(q^2) = \frac{m_{B_s} + m_{K_2^*}}{2m_{K_2^*}}A_1(q^2) - \frac{m_{B_s} - m_{K_2^*}}{2m_{K_2^*}}A_2(q^2)$ and $A_0(0) = A_3(0)$, $\qquad (6)$

where $q = p - p'$, $P = p + p'$, and $\varepsilon_{T\mu}^* = \frac{p_\lambda}{m_{B_s}}\varepsilon_{\mu\lambda}^*$. The factor c_V accounts for the flavor content of particles: $c_V = \sqrt{2}$ for a_2, f_2 and $c_V = 1$ for K_2^* [32]. Inserting Eqs. (4) and (5) in Eq. (3) and performing summation over the polarization tensor as

$$\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta} = \frac{1}{2}T_{\mu\alpha}T_{\nu\beta} + \frac{1}{2}T_{\mu\beta}T_{\nu\alpha} - \frac{1}{3}T_{\mu\nu}T_{\alpha\beta},$$

5

where $T_{\mu\nu} = -g_{\mu\nu} + \frac{p'_\mu p'_\nu}{m^2_{K^*_2}}$, the final representation of the physical side is obtained as

$$
\begin{aligned}
\Pi_{\alpha\beta\mu} &= \frac{f_{B_s} m_{B_s}}{(m_b + m_s)} \frac{f_{K^*_2} m^2_{K^*_2}}{(p^2 - m^2_{B_s})(p'^2 - m^2_{K^*_2})} \left\{ V'(q^2) i\epsilon_{\beta\mu\rho\sigma} p_\alpha p^\rho p'^\sigma + A'_0(q^2) p_\alpha p_\beta p'_\mu \right. \\
&+ \left. A'_1(q^2) g_{\beta\mu} p_\alpha + A'_2(q^2) p_\alpha p_\beta p_\mu \right\} + \text{higher states.} \qquad (7)
\end{aligned}
$$

For simplicity in calculations, the following redefinitions have been used in Eq. (7):

$$
\begin{aligned}
V'(q^2) &= \frac{V(q^2)}{m_{B_s} + m_{K^*_2}}, & A'_0(q^2) &= -\frac{m_{K^*_2}(A_3(q^2) - A_0(q^2))}{q^2}, \\
A'_1(q^2) &= -\frac{(m_{B_s} + m_{K^*_2})}{2} A_1(q^2), & A'_2(q^2) &= \frac{A_2(q^2)}{2(m_{B_s} + m_{K^*_2})}.
\end{aligned}
$$

Now, the QCD part of the correlation function is calculated by expanding it in terms of the OPE at large negative value of q^2:

$$
\Pi_{\alpha\beta\mu} = C^{(0)}_{\alpha\beta\mu} I + C^{(3)}_{\alpha\beta\mu} \langle 0|\bar{\Psi}\Psi|0\rangle + C^{(4)}_{\alpha\beta\mu} \langle 0|G^a_{\rho\nu} G^{\rho\nu}_a|0\rangle + C^{(5)}_{\alpha\beta\mu} \langle 0|\bar{\Psi}\sigma_{\rho\nu} T^a G^{\rho\nu}_a \Psi|0\rangle + ..., \quad (8)
$$

where $C^{(i)}_{\alpha\beta\mu}$ are the Wilson coefficients, I is the unit operator, $\bar{\Psi}$ is the local fermion field operator and $G^a_{\rho\nu}$ is the gluon strength tensor. In Eq. (8) the first term is contribution of the perturbative and the other terms are contribution of the non-perturbative part.

To compute the portion of the perturbative part (Fig. 1), using the Feynman rules for the bare loop, we obtain:

$$
\begin{aligned}
C^{(0)}_{\alpha\beta\mu} &= -\frac{i}{4} \int\int d^4x\, d^4y\, e^{i(p'x - py)} \left\{ \text{Tr}\left[S_s(x-y)\gamma_\alpha \overset{\leftrightarrow}{D}_\beta (x) S_u(-x)\gamma_\mu(1 - \gamma_5) S_b(y)\gamma_5 \right] \right. \\
&+ \left. \text{Tr}[\alpha \leftrightarrow \beta] \right\}, \qquad (9)
\end{aligned}
$$

taking the partial derivative with respect to x of the quark free propagators, and performing the Fourier transformation and using the Cutkosky rules, i.e., $\frac{1}{p^2 - m^2} \to -2i\pi\delta(p^2 - m^2)$, imaginary part of the $C^{(0)}_{\alpha\beta\mu}$ is calculated as

$$
\begin{aligned}
\text{Im}\left[C^{(0)}_{\alpha\beta\mu} \right] &= \frac{1}{8\pi} \int d^4k \delta(k^2 - m^2_s)\delta((p+k)^2 - m^2_b)\delta((p'+k)^2 - m^2_u)(2k + p')_\beta \\
&\times \text{Tr}\left[(\slashed{k} + m_s)\gamma_\alpha(\slashed{p}' + \slashed{k} + m_u)\gamma_\mu(1 - \gamma_5)(\slashed{p} + \slashed{k} + m_b)\gamma_5 \right] + \{\alpha \leftrightarrow \beta\}, (10)
\end{aligned}
$$

where k is four-momentum of the spectator quark s. To solve the integral in Eq. (10), we will have to deal with the integrals such as I_0, I_α, $I_{\alpha\beta}$ and $I_{\alpha\beta\mu}$ with respect to k. For example $I_{\alpha\beta\mu}$ can be as:

$$
I_{\alpha\beta\mu}(s, s', q^2) = \int d^4k\, [k_\alpha k_\beta k_\mu]\delta(k^2 - m^2_s)\delta((p+k)^2 - m^2_b)\delta((p'+k)^2 - m^2_u).
$$

where $s = p^2$ and $s' = p'^2$. I_0, I_α, $I_{\alpha\beta}$ and $I_{\alpha\beta\mu}$ can be taken as an appropriate tensor structure as follows:

$$
\begin{aligned}
I_0 &= \frac{1}{4\sqrt{\lambda(s, s', q^2)}}, \\
I_\alpha &= B_1[p_\alpha] + B_2[p'_\alpha], \\
I_{\alpha\beta} &= D_1[g_{\alpha\beta}] + D_2[p_\alpha p_\beta] + D_3[p_\alpha p'_\beta + p'_\alpha p_\beta] + D_4[p'_\alpha p'_\beta], \\
I_{\alpha\beta\mu} &= E_1[g_{\alpha\beta}p_\mu + g_{\alpha\mu}p_\beta + g_{\beta\mu}p_\alpha] + E_2[g_{\alpha\beta}p'_\mu + g_{\alpha\mu}p'_\beta + g_{\beta\mu}p'_\alpha] + E_3[p_\alpha p_\beta p_\mu] \\
&\quad + E_4[p'_\alpha p_\beta p_\mu + p_\alpha p'_\beta p_\mu + p_\alpha p_\beta p'_\mu] + E_5[p'_\alpha p'_\beta p_\mu + p'_\alpha p_\beta p'_\mu + p_\alpha p'_\beta p'_\mu] \\
&\quad + E_6[p'_\alpha p'_\beta p'_\mu],
\end{aligned}
\tag{11}
$$

The quantities $\lambda(s, s', q^2)$, B_l $(l = 1, 2)$, D_j $(j = 1, ..., 4)$, and E_r $(r = 1, ..., 6)$, are indicated in the Appendix. Using the relations in Eq. (11), $\mathrm{Im}[C_{\alpha\beta\mu}^{(0)}]$ can be calculated for the each structure corresponding to Eq. (7) as follows:

$$
\mathrm{Im}\left[C_{\alpha\beta\mu}^{(0)}\right] = \rho_V(i\epsilon_{\beta\mu\rho\sigma}p_\alpha p^\rho p'^\sigma) + \rho_0(p_\alpha p_\beta p'_\mu) + \rho_1(g_{\beta\mu}p_\alpha) + \rho_2(p_\alpha p_\beta p_\mu).
\tag{12}
$$

where the spectral densities ρ_i, $(i = V, 0, 1, 2)$ are found as

$$
\begin{aligned}
\rho_V(s, s', q^2) &= 24B_1\sqrt{\lambda}\left[B_1(m_s - m_b) + B_2(m_s - m_u) + m_s I_0\right], \\
\rho_0(s, s', q^2) &= 12[D_2(m_s - m_b) + D_3(m_s - m_u) + 2B_1 m_s - 2E_4(m_b - m_s)], \\
\rho_1(s, s', q^2) &= 3B_1[2m_s^2(m_b + m_u - m_s) - m_s(2m_b m_u + u) + \Delta(m_s - m_u) + \Delta'(m_s - m_b)] \\
&\quad + 6D_1(m_s - m_u) - 24E_1(m_b - m_s), \\
\rho_2(s, s', q^2) &= 24[D_2 m_s + E_3(m_s - m_b)].
\end{aligned}
$$

Using the dispersion relation, the perturbative part contribution of the correlation function can be calculated as follows:

$$
C_i^{(0)} = \int ds' \int ds \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)}.
\tag{13}
$$

For calculation of the non-perturbative contributions (condensate terms), we consider the condensate terms of dimension $3, 4$ and 5 related to the contributions of the quark-quark, gluon-gluon and quark-gluon condensate, respectively. They are more important than the other terms in the OPE. In the 3PSR, when the light quark is a spectator, the gluon-gluon condensate contributions can be easily ignored [23]. On the other hand, the

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quark condensate contributions of the light quark which is a non spectator, are zero after applying the double Borel transformation with respect to the both variables p^2 and p'^2, because only one variable appears in the denominator. Therefore, only two important diagrams of dimension 3, 4 and 5 remain from the non-perturbative part contributions. The diagrams of these contributions corresponding to $C_{\alpha\beta\mu}^{(3)}$ and $C_{\alpha\beta\mu}^{(5)}$ are depicted in Fig. 2. After some calculations, the non-perturbative part of the correlation function are obtained
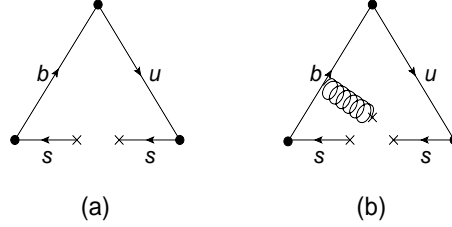

FIG. 2: The diagrams of the effective contributions of the condensate terms.

as follows:

$$
\begin{aligned}
C_V^{(3)} + C_V^{(5)} &= -\frac{2\kappa}{(p^2 - m_b^2)^2 (p'^2 - m_u^2)}, \\
C_0^{(3)} + C_0^{(5)} &= -\frac{4\kappa}{(p^2 - m_b^2)^2 (p'^2 - m_u^2)}, \\
C_1^{(3)} + C_1^{(5)} &= \frac{\kappa}{(p^2 - m_b^2)(p'^2 - m_u^2)} + \frac{\kappa[(m_b + m_u)^2 - q^2]}{(p^2 - m_b^2)^2 (p'^2 - m_u^2)}, \\
C_2^{(3)} + C_2^{(5)} &= -\frac{4\kappa}{(p^2 - m_b^2)^2 (p'^2 - m_u^2)},
\end{aligned}
\tag{14}
$$

where $\kappa = \frac{(m_s^2 - \frac{m_0^2}{2})}{48}\langle 0|s\bar{s}|0\rangle$, $m_0^2 = (0.8 \pm 0.2)\text{GeV}^2$ [33], and $\langle 0|\bar{s}s|0\rangle = (0.8 \pm 0.2)\langle 0|\bar{u}u|0\rangle$, $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -(0.240 \pm 0.010 \text{ GeV})^3$ that we choose the value of the condensates at a fixed renormalization scale of about 1 GeV.

The next step is to apply the Borel transformations with respect to the $p^2(p^2 \rightarrow M_1^2)$ and $p'^2(p'^2 \rightarrow M_2^2)$ on the phenomenological as well as the perturbative and non-perturbative parts of the correlation functions and equate these two representations of the correlations. The following sum rules for the form factors are derived:

$$
\begin{aligned}
V'(q^2) &= \frac{(m_b + m_s)e^{m_{B_s}^2/M_1^2} e^{m_{K_2^*}^2/M_2^2}}{f_{B_s} m_{B_s} f_{K_2^*} m_{K_2^*}^2} \left\{ \frac{-1}{(2\pi)^2} \int_{m_s^2}^{s_0'} ds' \int_{s_L}^{s_0} ds\, \rho_V(s, s', q^2)e^{-s/M_1^2} e^{-s'/M_2^2} \right. \\
&+ \left. \widetilde{B}\left[C_V^{(3)} + C_V^{(5)}\right] \right\},
\end{aligned}
$$

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$$A'_n(q^2) = \frac{(m_b + m_s)e^{m_{B_s}^2/M_1^2}e^{m_{K_2^*}^2/M_2^2}}{f_{B_s}m_{B_s}f_{K_2^*}m_{K_2^*}^2}\left\{\frac{-1}{(2\pi)^2}\int_{m_s^2}^{s'_0}ds'\int_{s_L}^{s_0}ds\,\rho_n(s,s',q^2)e^{-s/M_1^2}e^{-s'/M_2^2}\right.$$
$$\left. + \widetilde{B}\left[C_n^{(3)} + C_n^{(5)}\right]\right\}, \tag{15}$$

where $n = 0,...,2$, s_0 and s'_0 are the continuum thresholds in the initial and final channels, respectively. The lower limit in the integration over s is: $s_L = m_b^2 + \frac{m_b^2}{m_b^2-q^2}s'$. Also $\widetilde{B}$ transformation is defined as follows:

$$\widetilde{B}\left[\frac{1}{(p^2 - m_b^2)^m(p'^2 - m_u^2)^n}\right] = \frac{(-1)^{m+n}}{\Gamma(n)\Gamma(m)}\frac{e^{-m_b^2/M_1^2}e^{-m_u^2/M_2^2}}{(M_1^2)^{m-1}(M_2^2)^{n-1}}, \tag{16}$$

where M_1^2 and M_2^2 are Borel mass parameters.

We would like to provide the same results for the $B \to a_2\ell\nu$, and $B \to f_2\ell\nu$ decays. With a little bit of change in the above expressions such as $s \leftrightarrow d(u)$ and $m_{K_2^*} \leftrightarrow m_{a_2}(m_{f_2})$, we can easily find similar results in Eq. (15) for the form factors of the new transitions.

III.   NUMERICAL ANALYSIS

In this section, we numerically analyze the sum rules for the form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ as well as branching ratio values of the transitions $B(B_s) \to T$, where T can be one of the tensor mesons K_2^*, a_2, or f_2. The values of the meson masses and leptonic decay constants are chosen as presented in Table I. Also $m_b = 4.820$ GeV, $m_s = 0.150$ GeV

TABLE I: The values of the meson masses [35] and decay constants [5] in GeV.

meson	B_s	B	K_2^*	a_2	f_2
Mass	5.366	5.279	1.425	1.318	1.275
Decay Constant	0.230	0.190	0.118	0.107	0.102

[34], $m_\tau = 1.776$ GeV, and $m_\mu = 0.105$ GeV [35].

From the 3PSR, it is clear that the form factors also contain the continuum thresholds s_0 and s'_0 and the Borel parameters M_1^2 and M_2^2 as the main input. These are not physical quantities, hence the form factors, should be independent of these parameters. The continuum thresholds, s_0 and s'_0 are not completely arbitrary, but these are in correlation with
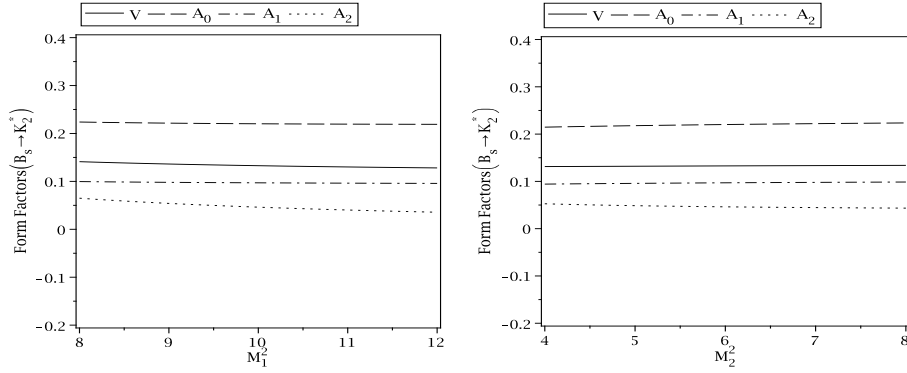
FIG. 3: The form factors of $B_s \to K_2^*$ on M_1^2 and M_2^2.

the energy of the first exited state with the same quantum numbers as the considered inter-polating currents. The value of the continuum threshold $s_0^{B(B_s)} = 35$ GeV^2 [36] calculated from the 3PSR. The values of the continuum threshold s_0' for the tensor mesons K_2^*, a_2 and f_2 are taken to be $s_0^{K_2^*} = 3.13$ GeV^2, $s_0^{a_2} = 2.70$ GeV^2 and $s_0^{f_2} = 2.53$ GeV^2, respectively [9].

We search for the intervals of the Borel parameters so that our results are almost insen-sitive to their variations. One more condition for the intervals of these parameters is the fact that the aforementioned intervals must suppress the higher states, continuum and con-tributions of the highest-order operators. In other words, the sum rules for the form factors must converge. As a result, we get 8 GeV$^2 \leq M_1^2 \leq 12$ GeV^2 and 4 GeV$^2 \leq M_2^2 \leq 8$ GeV^2. To show how the form factors depend on the Borel mass parameters, as examples, we depict the variations of the form factors V, A_0, A_1 and A_2 for $B_s \to K_2^* \ell \nu$ at $q^2 = 0$ with respect to the variations of the M_1^2 and M_2^2 parameters in their working regions in Fig. 3. From these figures, it revealed that the form factors weakly depend on these parameters in their working regions.

The sum rules for the form factors are truncated at about $0 \leq q^2 \leq 11$ GeV^2. The dependence of the form factors V, A_0, A_1 and A_2 on q^2 for $B \to T$ transitions are shown in Fig. 4. However, it is necessary to obtain the behavior of the form factors with respect to q^2 in the full physical region, $0 \leq q^2 \leq (m_{B(B_s)} - m_T)^2$, in order to calculate the decay width of the $B \to T$ transitions. So to extend our results, we look for a parametrization of the form factors in such a way that in the region $0 \leq q^2 \leq (m_{B(B_s)} - m_T)^2$, this parametrization coincides with the sum rules predictions. Our numerical calculations show that the sufficient
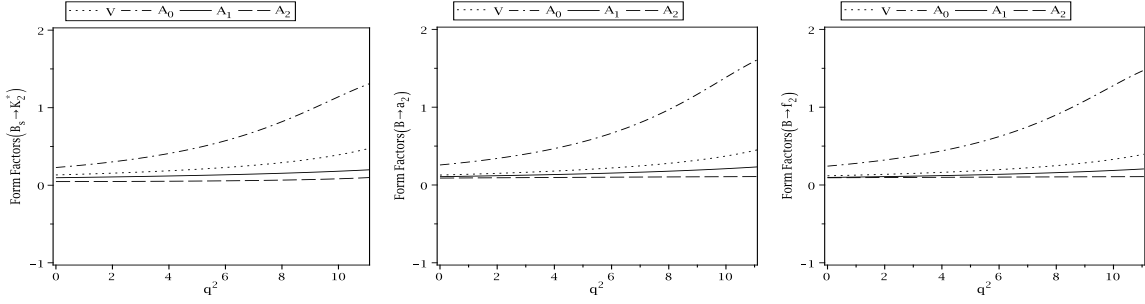
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FIG. 4: The SR predictions for the form factors of the $B(B_s) \to T\ell\nu$ transitions on q^2.

parametrization of the form factors with respect to q^2 is as follows:

$$f(q^2) = \frac{f(0)}{1 - a(\frac{q^2}{m^2_{B(B_s)}}) + b(\frac{q^2}{m^2_{B(B_s)}})^2}.$$ (17)

The values of the parameters $f(0)$, a, and b for the transition form factors of the $B \to T$ are given in the Table II.

TABLE II: Parameter values appearing in the fit functions of the $B \to T\ell\nu$ decays.

Form Factor	$f(0)$	a	b	Form Factor	$f(0)$	a	b
$V^{B_s \to K_2^*}$	0.13	2.19	0.83	$A_0^{B_s \to K_2^*}$	0.23	3.77	4.21
$A_1^{B_s \to K_2^*}$	0.10	1.36	0.09	$A_2^{B_s \to K_2^*}$	0.05	0.21	-2.99
$V^{B \to a_2}$	0.13	2.10	0.75	$A_0^{B \to a_2}$	0.26	3.71	4.03
$A_1^{B \to a_2}$	0.11	1.45	0.23	$A_2^{B \to a_2}$	0.09	0.63	0.46
$V^{B \to f_2}$	0.12	2.01	0.60	$A_0^{B \to f_2}$	0.24	3.70	4.02
$A_1^{B \to f_2}$	0.10	1.40	0.16	$A_2^{B \to f_2}$	0.09	0.46	0.29

In Table III, our results for the form factors of $B \to T\ell\nu$ decays in $q^2 = 0$ is compared with those of other approaches such as the LCSR, the PQCD, the LEET, and the ISGW II model. Our results are in good agreement with those of the LCSR, PQCD and LEET in all cases.

At the end of this section, we would like to present the differential decay widths of the process under consideration. Using the parametrization of these transitions in terms of the form factors, the differential decay width for $B \to T\ell\nu$ transition is obtained as:

$$\frac{d\Gamma(B \to T\ell\nu)}{dq^2} = \frac{|G_F V_{ub}|^2 \sqrt{\lambda(m_B^2, m_T^2, q^2)}}{256\ m_B^3\ \pi^3 q^2}(1 - \frac{m_\ell^2}{q^2})^2(X_L + X_+ + X_-),$$ (18)

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TABLE III: Comparison of the form factor values of the $B \to T\ell\nu$ decays in $q^2 = 0$ in different approaches.

Form Factor	This Work	LCSR[27]	PQCD[5]	LEET[28–30]	ISGW II[31]
$V^{B_s \to K_2^*}$	0.13 ± 0.03	0.15 ± 0.02	$0.18^{+0.05}_{-0.04}$	–	–
$A_0^{B_s \to K_2^*}$	0.23 ± 0.06	0.22 ± 0.04	$0.15^{+0.04}_{-0.03}$	–	–
$A_1^{B_s \to K_2^*}$	0.10 ± 0.02	0.12 ± 0.02	$0.11^{+0.03}_{-0.02}$	–	–
$A_2^{B_s \to K_2^*}$	0.05 ± 0.01	0.05 ± 0.02	$0.07^{+0.02}_{-0.02}$	–	–
$V^{B \to a_2}$	0.13 ± 0.03	0.18 ± 0.02	$0.18^{+0.05}_{-0.04}$	0.18 ± 0.03	0.32
$A_0^{B \to a_2}$	0.26 ± 0.07	0.21 ± 0.04	$0.18^{+0.06}_{-0.04}$	0.14 ± 0.02	0.20
$A_1^{B \to a_2}$	0.11 ± 0.04	0.14 ± 0.02	$0.11^{+0.03}_{-0.03}$	0.13 ± 0.02	0.16
$A_2^{B \to a_2}$	0.09 ± 0.02	0.09 ± 0.02	$0.06^{+0.02}_{-0.01}$	0.13 ± 0.02	0.14
$V^{B \to f_2}$	0.12 ± 0.04	0.18 ± 0.02	$0.12^{+0.03}_{-0.03}$	0.18 ± 0.02	0.32
$A_0^{B \to f_2}$	0.24 ± 0.06	0.20 ± 0.04	$0.13^{+0.04}_{-0.03}$	0.13 ± 0.02	0.20
$A_1^{B \to f_2}$	0.10 ± 0.02	0.14 ± 0.02	$0.08^{+0.02}_{-0.02}$	0.12 ± 0.02	0.16
$A_2^{B \to f_2}$	0.09 ± 0.02	0.10 ± 0.02	$0.04^{+0.01}_{-0.01}$	0.13 ± 0.02	0.14

m_ℓ represents the mess of the charged lepton. The other parameters are defined as

$$X_L = \frac{1}{9} \frac{\lambda}{m_T^2 \, m_B^2} [(2q^2 + m_\ell^2)h_0^2(q^2) + 3\lambda m_\ell^2 A_0^2(q^2)],$$

$$X_\pm = \frac{2q^2}{3}(2q^2 + m_\ell^2) \frac{\lambda}{8m_T^2 \, m_B^2} [(m_B + m_T)A_1(q^2) \mp \frac{\sqrt{\lambda}}{m_B + m_T} V(q^2)]^2,$$

$$h_0(q^2) = \frac{1}{2m_T}[(m_B^2 - m_T^2 - q^2)(m_B + m_T)A_1(q^2) - \frac{\lambda}{m_B + m_T} A_2(q^2)].$$

Integrating Eq. (18) over q^2 in the whole physical region, and using $V_{ub} = (3.89 \pm 0.44) \times 10^{-3}$ [35], the branching ratios of the $B \to T\ell\nu$ are obtained. The differential branching ratios of the $B \to T\ell\nu$ decays on q^2 are shown in Fig. 5. The branching ratio values of these decays are also obtained as presented in Table IV. Furthermore, this table contains the results estimated via the PQCD. Considering the uncertainties, our estimations for the branching ratio values of the $B \to T\ell\nu$ decays are in consistent agreement with those of the PQCD.

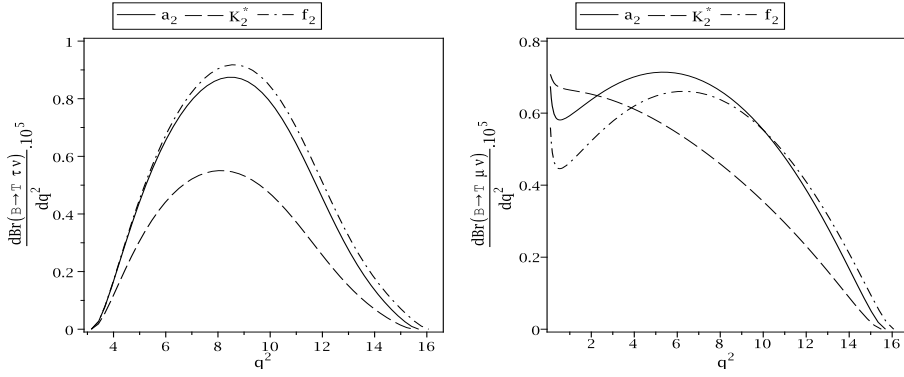In summary, we considered the $B_s(B) \to K_2^*(a_2, f_2)\ell\nu$ channels and computed the rel-

FIG. 5: The differential branching ratios of the semileptonic $B \to T\ell\nu$ decays on q^2.

TABLE IV: Comparison of the branching ratio values of the $B \to T\ell\nu$ decays with those of the PQCD (in units of 10^{-4}).

	This Work	PQCD[5]
$\mathrm{Br}(B \to a_2\mu\nu)$	0.82 ± 0.25	$1.16^{+0.81}_{-0.57}$
$\mathrm{Br}(B_s \to K_2^*\mu\nu)$	0.65 ± 0.20	$0.73^{+0.48}_{-0.33}$
$\mathrm{Br}(B \to f_2\mu\nu)$	0.77 ± 0.23	$0.69^{+0.48}_{-0.34}$
$\mathrm{Br}(B \to a_2\tau\nu)$	0.51 ± 0.17	$0.41^{+0.29}_{-0.20}$
$\mathrm{Br}(B_s \to K_2^*\tau\nu)$	0.35 ± 0.11	$0.25^{+0.17}_{-0.12}$
$\mathrm{Br}(B \to f_2\tau\nu)$	0.53 ± 0.18	$0.25^{+0.18}_{-0.13}$

evant form factors considering the contribution of the quark condensate corrections. Our results are in good agreement with those of the LCSR, PQCD and LEET in all cases. We also evaluated the total decays widthes and the branching ratios of these decays. Our branching ratio values of these decays are in consistent agreement with those of the PQCD.

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Appendix

In this appendix, the explicit expressions of the coefficients $\lambda(s, s', q^2)$, B_l $(l = 1, 2)$, D_j $(j = 1, ..., 4)$, and E_r $(r = 1, ..., 6)$, are given.

$$\lambda(s, s', q^2) = s^2 + s'^2 + (q^2)^2 - 2sq^2 - 2s'q^2 - 2ss',$$

$$B_1 = \frac{I_0}{\lambda(s, s', q^2)} \left[2s'\Delta - \Delta'u\right],$$

$$B_2 = \frac{I_0}{\lambda(s, s', q^2)} \left[2s\Delta' - \Delta u\right],$$

$$D_1 = -\frac{I_0}{2\lambda(s, s', q^2)}[4ss'm_s^2 - s\Delta'^2 - s'\Delta^2 - u^2 m_s^2 + u\Delta\Delta'],$$

$$D_2 = -\frac{I_0}{\lambda^2(s, s', q^2)}[8ss'^2 m_s^2 - 2ss'\Delta' - 6s'^2\Delta^2 - 2u^2 s'm_s^2 + 6s'u\Delta\Delta' - u^2\Delta'^2],$$

$$D_3 = \frac{I_0}{\lambda^2(s, s', q^2)} \left[4ss'um_s^2 + 4ss'\Delta\Delta' - 3su\Delta'^2 - 3u\Delta^2 s' - u^3 m_3^2 + 2u^2\Delta\Delta'\right],$$

$$D_4 = \frac{I_0}{\lambda^2(s, s', q^2)}[-6s'u\Delta\Delta' + 6s^2\Delta'^2 - 8s^2 s'm_s^2 + 2u^2 s\, m_s^2 + u^2\Delta^2 + 2ss'\Delta^2],$$

$$\begin{aligned}
E_1 = \frac{I_0}{2\lambda^2(s, s', q^2)} & \left[8s'^2 m_s^2\Delta s - 2s'm_s^2\Delta u^2 - 4um_s^2\Delta' ss' + u^3 m_s^2\Delta' - 2s'^2\Delta^3 \right. \\
& \left. + \; 3s'u\Delta^2\Delta' - 2\Delta'^2\Delta ss' - \Delta'^2\Delta u^2 + us\Delta^3\right],
\end{aligned}$$

$$\begin{aligned}
E_2 = \frac{I_0}{2\lambda^2(s, s', q^2)} & \left[8s^2 m_s^2\Delta' s' - 2s^2\Delta'^3 - 4um_s^2\Delta ss' - 2\Delta^2\Delta' ss' + 3us\Delta'^2\Delta \right. \\
& \left. - \; 2sm_s^2\Delta'u^2 + s'u\Delta^3 + u^3 m_s^2\Delta - \Delta^2\Delta'u^2\right],
\end{aligned}$$

$$\begin{aligned}
E_3 = -\frac{I_0}{\lambda^3(s, s', q^2)} & \left[48sm_s^2\Delta s'^3 - 24ss'^2 um_s^2\Delta' - 12ss'^2\Delta'^2\Delta + 6su\Delta'^3 s' - 20s'^3\Delta^3 \right. \\
& \left. + \; 30s'^2 u\Delta^2\Delta' - 12s'^2 m_s^2\Delta u^2 - 12s'\Delta'^2\Delta u^2 + 6s'u^3 m_s^2\Delta' + u^3\Delta'^3\right],
\end{aligned}$$

$$\begin{aligned}
E_4 = -\frac{I_0}{\lambda^3(s, s', q^2)} & \left[16s^2 m_s^2\Delta' s'^2 - 4s^2\Delta'^3 s' - 12ss'^2\Delta^2\Delta' - 24ss'^2 um_s^2\Delta + 3u^3\Delta'^2\Delta \right. \\
& + \; 18su\Delta'^2\Delta s' - 4s\Delta'^3 u^2 + 10s'^2 u\Delta^3 + 6s'u^3 m_s^2\Delta - 12s'\Delta^2\Delta'u^2 - 2m_s^2\Delta'u^4 \\
& \left. + \; 4ss'u^2 m_s^2\Delta'\right],
\end{aligned}$$

$$\begin{aligned}
E_5 = -\frac{I_0}{\lambda^3(s, s', q^2)} & \left[16s^2 m_s^2\Delta s'^2 - 24s^2 s'um_s^2\Delta' - 12s^2 s'\Delta'^2\Delta + 10us^2\Delta'^3 - 4ss'^2\Delta^3 \right. \\
& + \; 4ss'u^2 m_s^2\Delta + 18su\Delta^2\Delta' s' + 6su^3 m_s^2\Delta' - 12s\Delta^2\Delta u^2 - 4s'\Delta^3 u^2 - 2m_s^2\Delta u^4 \\
& \left. + \; 3u^3\Delta^2\Delta'\right],
\end{aligned}$$

$$\begin{aligned}
E_6 = -\frac{I_0}{\lambda^3(s, s', q^2)} & \left[48s^3 m_s^2\Delta' s' - 20s^3\Delta'^3 - 12s^2\Delta^2\Delta' s' - 24s^2 s'um_s^2\Delta - 12s^2 m_s^2\Delta'u^2 \right. \\
& \left. + \; 30us^2\Delta'^2\Delta + 6su\Delta^3 s' - 12s\Delta^2\Delta'u^2 + 6su^3 m_s^2\Delta + u^3\Delta^3\right],
\end{aligned}$$

$$\Delta = s + m_s^2 - m_b^2, \qquad \Delta' = s' + m_s^2 - m_u^2, \qquad u = s + s' - q^2.$$

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