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Light-Front Spin-1 Model: Parameters Dependence*

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Abstract We study the structure of the ρ -meson within a light-front model with constituent quark degrees of freedom. We calculate electroweak static observables: magnetic and quadrupole moments, decay constant and charge radius. The prescription used to compute the electroweak quantities is free of zero modes, which makes the calculation implicitly covariant. We compare the results of our model with other ones found in the literature. Our model parameters give a decay constant close to the experimental one.

Keywords ρ -meson · Light-Front model · Electromagnetic structure · Decay constant

1 Introduction

A central question today in physics is to understand the subatomic structure of matter in terms of fundamental degrees of freedom, i.e, quarks and gluons [1]. The standard model for the strong interaction, namely, quantum chromodynamics (QCD), with quarks and gluons [2; 3], exhibit confinement, beyond a perturbative expansion. Calculations with Lattice QCD and Schwinger-Dyson approaches are performed in Euclidian space, while it is still a challenge to extract information of the hadron in Minkowski space. In this sense, it is still useful to describe the composite states in constituent quark models (CQM), which are defined in Minkowski space, and driven towards experiments (see e.g. [4]).

A vast literature is devoted to investigate the hadronic electromagnetic structure (see e.g. [5; 6; 7; 8; 9; 10; 11; 12]) since it provides a laboratory to deeper our understanding of QCD. A natural framework to combine the quantum field theory (QFT) and CQM to describe bound states is the light-front quantization (LFQCM) [13; 14; 15]. One can resort to the usual covariant formulation of quantum field theory and the light-front quantisation to test the nonperturbative properties of QCD.

The aim of this contribution is to study the sensitivity of static electroweak properties of the ρ -meson with the model parameters of an ansatz of the covariant Bethe-Salpeter amplitude with constituent quarks proposed in Ref. [6]. The variables used to evaluate numerically the loop integrals is the light-front momentum, and the integrals are reduced to three-dimensional ones, by analytical integration on the minus component of the loop momentum, relating the calculations of the electroweak observables with the present model performed in a light-front framework considers only valence contributions. In the case of the electromagnetic observables, we use a prescription for computing the elastic form factors

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which was shown to be free of non-valence contributions in the Drell-Yan frame (see e.g. [30]). We also found that the vector meson decay constant in the present model is free of a zero-mode when computed with the plus component of the current operator. In our systematic study we address the dependence of the static electromagnetic observables and decay constant with the two model parameters, namely the constituent quark and the regulator masses. In addition, we compare our results with the ones obtained with different models.

2 Light-Front model, electromagnetic form factors and decay constant

The general framework for the light-front constituent quark model adopted here has been used to study several properties of hadronic states, like in the case of mesons and baryons in the vacuum [16; 17] or in nuclear matter [18]. Electromagnetic form factors of composite vector particles within the light-front framework have been addressed in many works [19; 20; 21; 22; 23; 24; 25; 26; 27], where a specific frame with momentum transfer $q^+ = q^0 + q^3 = 0$ (Drell-Yan condition) is chosen to compute the matrix elements of the plus component of the current operator. Here, we will use this frame and current component, with the choice of matrix elements suggested in [22], which has been shown to be free of zero modes [28; 29; 30]. In this case, the computation is performed only in the valence region. We will return to this point later in this section.

The $\rho - q\bar{q}$ vertex model for an on-mass-shell meson, is the same one proposed in reference [6]:

$$\Gamma^\mu = \left[\gamma^\mu - \frac{m_\rho}{2} \frac{2k^\mu - P^\mu}{(P^\mu k_\mu + m m_\rho) - i\epsilon} \right], \quad (1)$$

where the quark momentum is k^μ , the constituent quark mass is m , and the ρ meson mass is m_ρ .

The electromagnetic form factors of a spin-1 particle are computed from a linear combination of the matrix elements of the current, J_{ji}^μ with i and j indexing the polarization states. The general covariant expression of the current is [19]:

$$J_{\alpha\beta}^\mu = \left[F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m_\rho^2} \right] (p^\mu + p'^\mu) - F_3(q^2)(q_\alpha g_\beta^\mu - q_\beta g_\alpha^\mu), \quad (2)$$

where m_ρ is the rho meson mass, q^μ is the momentum transfer, and P^μ is the sum of the initial and final momentum. In the impulse approximation, the plus component of the electromagnetic current, J_{ji}^+ , is:

$$J_{ji}^+ = i \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon_j'^\nu \Gamma_\beta(k, k - p_f)(\not{k} - \not{p}_f + m)\gamma^+(\not{k} - \not{p}_i + m)\epsilon_i^\mu \Gamma_\alpha(k, k - p_i)(\not{k} + m)] \Lambda(k, p_f)\Lambda(k, p_i)}{(k^2 - m^2 + i\epsilon)((k - p_i)^2 - m^2 + i\epsilon)((k - p_f)^2 - m^2 + i\epsilon)}, \quad (3)$$

where ϵ_j' and ϵ_i are the polarization four-vectors of the final and initial states, respectively. In the cartesian instant form spin basis the initial polarization state is given by:

$$\epsilon_x^\mu = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \quad \epsilon_y^\mu = (0, 0, 1, 0), \quad \epsilon_z^\mu = (0, 0, 0, 1), \quad (4)$$

and for the final state is given by:

$$\epsilon_x^\mu = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \quad \epsilon_y^\mu = (0, 0, 1, 0), \quad \epsilon_z^\mu = (0, 0, 0, 1), \quad (5)$$

where $\eta = -q^2/4m_\rho^2$. The function $\Lambda(k, p) = N/((p - k)^2 - m_R^2 + i\epsilon)^2$ regularizes the loop integral in Eq.(3). The regulator mass is m_R and the normalisation factor N is fixed by the condition of unit charge.

The Breit-frame is used in the numerical calculations, with the choice $p^\mu = (p^0, -q_x/2, 0, 0)$ for the initial state and $p'^\mu = (p^0, q_x/2, 0, 0)$ for the final state, such that the momentum transfer $q^\mu = (p'^\mu - p^\mu)$ satisfies the the Drell-Yan condition, $q^+ = 0$. In this case four matrix elements are independent and only three electromagnetic form factors exists. The physical constraint is expressed by the angular condition, which in the light-front spin basis is written as:

$$\Delta(q^2) = (1 + 2\eta)I_{11}^+ + I_{1-1}^+ - \sqrt{8\eta}I_{10}^+ - I_{00}^+ = (1 + \eta)(J_{yy}^+ - J_{zz}^+) = 0. \quad (6)$$

This relation is violated in the present model by the presence of zero-modes not accounted by the naive integration in the loop momentum k^- , which leads only to the valence region contribution (see e.g. [30]). In principle, if the angular condition is satisfied, we have the freedom to extract in different ways the form factors from the matrix elements of the current [20; 21; 23; 24].

In the present work we calculate the electromagnetic observables, G_0 , G_1 and G_2 , without ambiguities due to zero-mode contributions by using the prescription proposed in [20; 22], which can be also computed in the instant form basis, as well as in the light-front spin basis:

$$\begin{aligned} G_0^{GK} &= \frac{1}{3}[(3 - 2\eta)I_{11}^+ + 2\sqrt{2\eta}I_{10}^+ + I_{1-1}^+] = \frac{1}{3}[J_{xx}^+ + (2 - \eta)J_{yy}^+ + \eta J_{zz}^+], \\ G_1^{GK} &= 2[I_{11}^+ - \frac{1}{\sqrt{2\eta}}I_{10}^+] = J_{yy}^+ - J_{zz}^+ - \frac{J_{zx}^+}{\sqrt{\eta}}, \\ G_2^{GK} &= \frac{2\sqrt{2}}{3}[-\eta I_{11}^+ + \sqrt{2\eta}I_{10}^+ - I_{1-1}^+] = \frac{\sqrt{2}}{3}[J_{xx}^+ - (1 + \eta)J_{yy}^+ + \eta J_{zz}^+]. \end{aligned} \quad (7)$$

The prescription [20; 22] amounts to eliminate the matrix element I_{00}^+ in the computation of the form factors. In [28; 29; 30], it was demonstrated that the zero-modes or non-valence contributions are present only in I_{00}^+ , and therefore the combinations expressed in Eq. (7) can be computed relying only on the valence region.

The low-energy electromagnetic observables considered here are the radius, $\langle r^2 \rangle$, the magnetic moment μ and the quadrupole moment Q_2 , which are given by [6; 25]:

$$\langle r^2 \rangle = \lim_{q^2 \rightarrow 0} \frac{6[G_0(q^2) - 1]}{q^2}, \quad \mu = \lim_{q^2 \rightarrow 0} G_1(q^2), \quad Q_2 = \lim_{q^2 \rightarrow 0} \frac{3\sqrt{2}G_2(q^2)}{q^2}. \quad (8)$$

In addition we calculate the ρ -meson decay constant f_ρ defined by the matrix element [31]:

$$\langle 0 | \bar{d} \Gamma^\mu u | \rho; i \rangle \frac{Q_u + Q_{\bar{d}}}{\sqrt{2}} = i\sqrt{2} f_\rho m_\rho \epsilon_i^\mu. \quad (9)$$

where $Q_u = 2/3$ and $Q_{\bar{d}} = 1/3$ are the charges of quark and antiquark. The matrix element of the electromagnetic current between the vacuum and ρ -meson states is computed from a loop integral, and we choose the plus momentum component and the polarisation ϵ_z defined in (4). After integrating in k^- the result is:

$$f_\rho = \frac{N_c N}{m_\rho} \int \frac{d^2 k_\perp}{(2\pi)^3} \int_0^1 dx \frac{4 \left(-x(p^+ + p^{+2}) + k_\perp^2 + m^2 \right) - \frac{m_\rho}{2} \frac{4mp^+(2x-1)(k_{on}^- - xp^+)}{[p^+(k_{on}^- + xp^+)/2 + m_\rho m]}}{x(1-x)^3(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2}, \quad (10)$$

where $p^\mu = (m_\rho, \mathbf{0})$, $N_c = 3$, is number of colors and $k_{on}^- = (k_\perp^2 + m^2)/x$ is the particle on-mass-shell relation. The other quantities in (10) are:

$$M_0^2 = \frac{\mathbf{k}_\perp^2 + m^2}{x} + \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2}{1-x} - p_\perp^2; \quad M_R^2 = \frac{\mathbf{k}_\perp^2 + m^2}{x} + \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m_R^2}{1-x} - \mathbf{p}_\perp^2, \quad (11)$$

In order to have bound state, the following conditions are necessary, $m > m_\rho/2$ e $m_R + m > m_\rho$, where m_R is the regulator mass. It is worthwhile to observe that the expression for the decay constant is free of zero-mode contributions, which are suppressed by our particular covariant choice of the vertex function.

3 Numerical Results

The input for the numerical calculations of the ρ meson electroweak observables are the quarks mass and regulator mass. The experimental decay constant $f_\rho^{exp} = 0.152 \pm 0.008$ GeV [1] is fitted with the Eq. (10) for $m = 0.430$ GeV and $m_R = 3$ GeV. In table I, we compare our results with other models [32; 33; 34]. The qualitative trend for the static electromagnetic observables seen in the table, indicates that for increasing charge radius the magnetic moment decreases, while the quadrupole

Model	f_ρ [GeV]	$\langle r^2 \rangle$ [fm^2]	μ [$e/2m_\rho$]	Q_2 [e/m_ρ^2]
our work	0.154	0.268	2.21	-0.882
[32]	0.134	0.296	2.10	-0.910
[33]	0.130	0.312	2.11	-0.850
[34]	0.207	0.540	2.01	-0.410

Table 1 ρ meson electromagnetic static observables. We compare our model with others ones from the literature [32; 33; 34]. In our model the quark mass is 0.430 GeV and the regulator mass is 3 GeV.

moment increases. It is suggestive that the contribution of the orbital motion to the magnetic and quadrupole moments is in part due to relativistic effects present in the quark spin coupling to form the composite state. It is natural that relativistic effects decrease as the charge radius increases and the quarks slow down. This general trend is further explored by computing the static observables for different regulator and quark masses in figures 1 and 2.

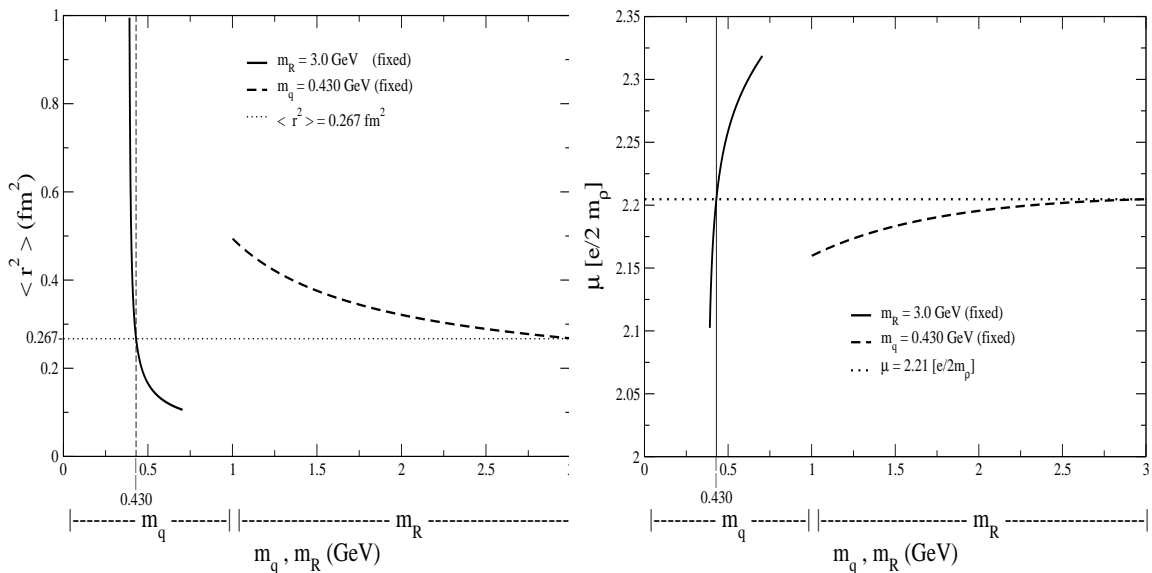


Fig. 1 ρ meson electromagnetic radius (left frame) and magnetic moment (right frame). Variation with the regulator mass with fixed quark mass (dashed line) and variation with the quark mass with fixed regulator mass (solid line).

In figure 1, we show the dependence of the charge radius and magnetic moment with the regulator and quark masses. We varied each mass independently while keeping the other one fixed. In the right frame of figure 1, we observe that the electromagnetic radius decreases by increasing any of the mass scales. The effect is sharp for variations in the quark mass, because in our model the quarks forms a bound state, with minimum quark mass of $m_\rho/2$, when the meson is dissolved in the continuum. Towards this limit the size grows to infinity and $\mu \rightarrow 2$, as clearly seen in the figure. The results for the quadrupole moment presented in figure 2 corroborate the general trend already discussed together with table I. As the mass scales decrease the quadrupole moment tends to have its magnitude decreased.

4 Summary

In the present work, we adopted the prescription proposed in Refs. [20; 22] to compute the ρ -meson electromagnetic form factors. The matrix element I_{00}^+ in the light-front spin basis is not used in the calculation of the form factors, and due to that the contribution of non-valence or zero-mode terms are

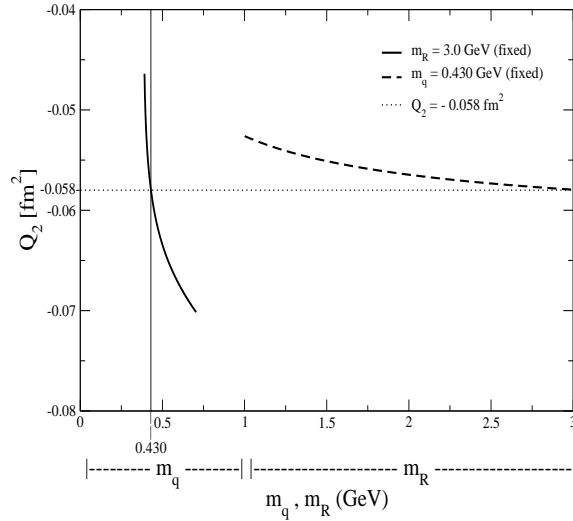


Fig. 2 Quadrupole momentum of the ρ meson. Variation with the regulator mass with fixed quark mass (dashed line) and variation with the quark mass with fixed regulator mass (solid line).

not present in these observables [29; 30]. Using the Bethe-Salpeter amplitude model for the ρ meson proposed in [6], we calculate the decay constant, charge radius, magnetic and quadrupole moments. We perform a quantitative analysis of the static electromagnetic observables with the variation of the model parameters, namely the constituent quark and regulator masses. The results obtained for the charge radius in the present model are in agreement with those expected for a composite state of nonrelativistic constituent quarks: the radius decreases by increasing the quark mass, the same behaviour is also found when the regulator mass increases.

The values of constituent quark mass $m = 0.430$ GeV and regulator mass $m_R = 3.0$ GeV give $f_\rho = 0.154$ GeV (see also table I), which is close to the experimental decay constant of $f_\rho^{exp} = 0.152 \pm 0.008$ GeV [1]. In this case we compare our results for the charge radius, magnetic and quadrupole moments to the outcome of other models found in the literature [32; 33; 34]. We note that the magnetic moment is pretty much close to 2 in all calculations, while a larger variation is found for the decay constant, charge radius and quadrupole moment. The magnetic moment is more robust to model variations, as it approaches the expected sum of individual quark magnetic moments, while the other observables depends crucially on the spatial distribution of the quarks, which is more sensitive to the details of the model.

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