

Chiral power counting of one- and two-body currents in direct detection of dark matter

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Abstract

We present a common chiral power-counting scheme for vector, axial-vector, scalar, and pseudoscalar WIMP–nucleon interactions, and derive all one- and two-body currents up to third order in the chiral expansion. Matching our amplitudes to non-relativistic effective field theory, we find that chiral symmetry predicts a hierarchy amongst the non-relativistic operators. Moreover, we identify interaction channels where two-body currents that previously have not been accounted for become relevant.

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1. Introduction

Elucidating the nature of dark matter is one of the most pressing challenges in contemporary particle physics and astrophysics. Still, one of the dominant paradigms rests on a weakly-interacting massive particle (WIMP), such as the neutralino in supersymmetric extensions of the standard model (SM). A WIMP can be searched for at colliders, in annihilation signals, or in direct-detection experiments, where the recoil energy deposited when the WIMP scatters off nuclei is measured. Recent years have witnessed an impressive increase in sensitivity, e.g., from XENON100 [1], LUX [2], and SuperCDMS [3], which will further improve dramatically with the advent of ton-scale detectors, XENON1T [4] and LZ [5]. In the absence of a signal, direct-detection experiments provide more and more stringent constraints on the parameter space of WIMP candidates. To derive these constraints and to interpret a future signal, it is mandatory that the nucleon matrix elements and the nuclear structure factors, which are required when transitioning from the SM to the nucleon to the nucleus level, be calculated systematically and incorporate what we know about QCD.

Effects at the level of the nucleus can be described by an effective field theory (EFT) whose degrees of freedom are non-relativistic (NR) nucleon and WIMP fields [6, 7]. This NREFT has been recently used in an analysis of direct-detection experiments [8]. In this approach, scales related to the spontaneous breaking of chiral symmetry of QCD are integrated out, with the corresponding effects subsumed into the coefficients of the EFT. In the context of nuclear forces, such an EFT is called pionless EFT. To derive limits on the WIMP parameter space, information from QCD has then to be included in the analysis in a second step.

Alternatively, one can start directly from chiral EFT (ChEFT) to incorporate the QCD constraints from chiral symmetry [9–16], which makes predictions for the hierarchy among one- and two-body currents. Based on ChEFT, scalar and axial-vector two-body currents were recently considered in [10]

and [11, 12], respectively. Moreover, lattice QCD can be used to constrain the couplings of two-body currents [17].

The goal of this Letter is to combine vector, axial-vector, scalar, and pseudoscalar interactions in a common chiral power counting, collect all relevant one- and two-body matrix elements, and match the result onto NREFT. This combines our knowledge of QCD at low energies: the one-body matrix elements correspond to the standard decomposition into form factors, while the two-body scalar [9, 10], vector [18–20], and axial-vector [15, 21] currents have been calculated as well, the vector current even at one-loop order. Here, we combine these results for their application in direct detection, extending the axial-vector two-body currents to finite momentum transfer and generalizing to the three-flavor case where appropriate. By matching to the NREFT, we find that the chiral symmetry of QCD predicts a hierarchy among the different operators and that two-body currents can be as important as one-body currents in some channels.

2. Effective Lagrangian and kinematics

We start from the following dimension-6 and -7 effective Lagrangian for the interaction of the WIMP χ , assumed to be a SM singlet, with the SM fields [22]

$$\begin{aligned} \mathcal{L}_\chi = & \frac{1}{\Lambda^3} \sum_q \left[C_q^{SS} \bar{\chi} \chi m_q \bar{q} q + C_q^{PS} \bar{\chi} i \gamma_5 \chi m_q \bar{q} q \right. \\ & \left. + C_q^{SP} \bar{\chi} \chi m_q \bar{q} i \gamma_5 q + C_q^{PP} \bar{\chi} i \gamma_5 \chi m_q \bar{q} i \gamma_5 q \right] \\ & + \frac{1}{\Lambda^2} \sum_q \left[C_q^{VV} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q + C_q^{AV} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu q \right. \\ & \left. + C_q^{VA} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu \gamma_5 q + C_q^{AA} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu \gamma_5 q \right] \\ & + \frac{1}{\Lambda^2} \sum_q \left[C_q^{TT} \bar{\chi} \sigma^{\mu\nu} \chi \bar{q} \sigma_{\mu\nu} q + \tilde{C}_q^{TT} \bar{\chi} \sigma^{\mu\nu} i \gamma_5 \chi \bar{q} \sigma_{\mu\nu} q \right] \end{aligned}$$

$$+ \frac{1}{\Lambda^3} \left[C_g^S \bar{\chi} \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} + C_g^P \bar{\chi} i \gamma_5 \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} + \tilde{C}_g^S \bar{\chi} \chi \alpha_s G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} + \tilde{C}_g^P \bar{\chi} i \gamma_5 \chi \alpha_s G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \right], \quad (1)$$

where the Wilson coefficients C_i parameterize the effect of new physics associated with the scale Λ (organizing the interactions in this way assumes Λ to be much larger than the typical QCD scale of 1 GeV). To render the scalar and pseudoscalar matrix elements renormalization-scale invariant we included explicitly the quark masses m_q in the definition of the respective operators. We further assumed χ to be a Dirac fermion (in the Majorana case, $C_q^{VV} = C_q^{VA} = C_q^{TT} = 0$), and defined the dual field strength tensor as

$$\tilde{G}_a^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} G_{\lambda\sigma}^a, \quad (2)$$

with sign convention $\epsilon^{0123} = +1$. Compared to the operator basis used in [23] we do not include the dimension-8 operators related to the traceless part of the QCD energy-momentum tensor. As shown in [23], these operators become relevant for heavy WIMPs and contribute to spin-independent interactions, decreasing significantly the single-nucleon contribution. Finally, we will ignore the tensor operators in (1) and concentrate on the chiral predictions for the V, A, S, P channels.

The kinematics for the WIMP–nucleon scattering process are taken as

$$N(p) + \chi(k) \rightarrow N(p') + \chi(k'), \quad (3)$$

the momentum transfer is defined as

$$q = k' - k = p - p', \quad q^2 = t, \quad (4)$$

and the pion, η , nucleon, nucleus, and WIMP masses will be denoted by M_π, M_η, m_N, m_A , and m_χ , respectively (Dirac spinors are normalized to 1). We will also need

$$P = p + p', \quad K = k + k'. \quad (5)$$

The cross section differential with respect to momentum transfer for the elastic WIMP–nucleus scattering process in the laboratory frame can be expressed as

$$\frac{d\sigma}{d\mathbf{q}^2} = \frac{1}{8\pi v^2 (2J+1)} \sum_{\text{spins}} |\mathcal{M}_{\text{NR}}|^2 + \mathcal{O}(\mathbf{q}^0), \quad (6)$$

with nucleus spin J , WIMP velocity v , and NR amplitude \mathcal{M}_{NR} defined as

$$\mathcal{M} = 2m_A 2m_\chi \mathcal{M}_{\text{NR}} + \mathcal{O}(\mathbf{q}^2), \quad (7)$$

where \mathcal{M} is the relativistic scattering amplitude. In the Majorana case, (6) receives an additional factor of 4.

3. Chiral power counting

We use the standard chiral power counting [24, 25]

$$\partial = \mathcal{O}(p), \quad m_q = \mathcal{O}(p^2), \quad a_\mu, v_\mu = \mathcal{O}(p), \quad (8)$$

with axial-vector and vector sources a_μ and v_μ . The velocity distribution in dark matter halo models indeed suggests to count

the momentum transfer $q \lesssim M_\pi$ as $\mathcal{O}(p)$ [10]. In the baryon sector we depart from the standard counting in chiral perturbation theory (ChPT) and adopt the more conventional ChEFT assumption (see, e.g., [26–28]) for the scaling of relativistic corrections

$$\frac{\partial}{m_N} = \mathcal{O}(p^2). \quad (9)$$

This counting is appropriate for a break-down scale around 500 MeV. As far as the WIMP is concerned, a chiral counting is only required for the NR expansion of the spinors. We assume the same counting as in the nucleon case, but display the corresponding additional powers explicitly. If $m_\chi \gtrsim m_N$, the suppression will be more pronounced, for $M_\pi \lesssim m_\chi \lesssim m_N$ the counting should be adapted, and for even smaller m_χ the naive counting breaks down.

For most of the channels it suffices to consider the leading-order Lagrangian to determine at which chiral order a given contribution starts. For the one-body matrix elements higher orders are subsumed into the nucleon form factors, which are obtained by their chiral expansion or could be taken from phenomenology. In this work, we consider all contributions up to $\mathcal{O}(p^3)$. Since the leading two-body terms start at $\mathcal{O}(p^2)$, this leaves the possibility that the next-to-leading-order (NLO) pion–nucleon Lagrangian involving the low-energy constants c_i [29] could be required, and this is indeed the case for the spatial component of the axial-vector current [11, 12] (indicated by “2b NLO” in Table 1). In the same channel, NN contact terms d_i [30] enter. We define both c_i and d_i in the conventions of [21] (with dimensionless c_6 and c_7).

As a preview of our results, the leading chiral orders of one- and two-body currents for time and space components of the axial-vector and vector currents, as well as for the scalar and pseudoscalar operators, are listed in Table 1. The suppression by two powers (“+2”) originating from the WIMP spinors is displayed separately. In the following sections, we give results for all one- and two-body currents involved in Table 1.

4. Nuclear matrix elements

4.1. Scalar

At zero momentum transfer the scalar couplings of the heavy quarks $Q = c, b, t$ can be determined from the trace anomaly of the QCD energy-momentum tensor [31]

$$\theta_\mu^\mu = \sum_q m_q \bar{q} q + \frac{\beta_{\text{QCD}}}{2g_s} G_{\mu\nu}^a G_a^{\mu\nu}, \quad \langle N | \theta_\mu^\mu | N \rangle = m_N, \quad (10)$$

$$\frac{\beta_{\text{QCD}}}{2g_s} = - \left(11 - \frac{2N_f}{3} \right) \frac{\alpha_s}{8\pi} + \mathcal{O}(\alpha_s^2).$$

For $N_f = 3$ active flavors, one obtains

$$\langle N | m_Q \bar{Q} Q | N \rangle = - \frac{\alpha_s}{12\pi} \langle N | G_{\mu\nu}^a G_a^{\mu\nu} | N \rangle = m_N f_Q^N, \quad (11)$$

where

$$f_Q^N = \frac{2}{27} \left(1 - \sum_{q=u,d,s} f_q^N \right), \quad m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle. \quad (12)$$

Nucleon		V		A		Nucleon		S	P
WIMP		t	\mathbf{x}	t	\mathbf{x}	WIMP			
	1b	0	1 + 2	2	0 + 2		1b	2	1
V	2b	4	2 + 2	2	4 + 2	S	2b	3	5
	2b NLO	–	–	5	3 + 2		2b NLO	–	4
	1b	0 + 2	1	2 + 2	0		1b	2 + 2	1 + 2
A	2b	4 + 2	2	2 + 2	4	P	2b	3 + 2	5 + 2
	2b NLO	–	–	5 + 2	3		2b NLO	–	4 + 2

Table 1: Left: leading chiral order of time (t) and space (\mathbf{x}) components of the WIMP and nucleon currents for vector and axial-vector interactions, for one-body (1b) and two-body (2b) operators. For the axial-vector nucleon operator, terms involving vertices from the NLO chiral Lagrangian (indicated by “2b NLO”) need to be included (see main text for details). The second number (“+2”) refers to the additional suppression originating from the NR expansion of the WIMP spinors, if momentum over WIMP mass is counted in the same way as for the nucleon mass. Right: leading chiral order of the WIMP and nucleon currents for scalar and pseudoscalar interactions.

Therefore, at leading order in α_s the effect of integrating out the heavy quarks can be absorbed into a redefinition of C_g^S

$$C_g^{rS} = C_g^S - \frac{1}{12\pi} \sum_{Q=c,b,t} C_Q^{SS}. \quad (13)$$

For the u - and d -quarks the couplings are intimately related to the pion–nucleon σ -term $\sigma_{\pi N}$ [32]

$$f_u^N = \frac{\sigma_{\pi N}(1 - \xi)}{2m_N} + \Delta f_u^N, \quad f_d^N = \frac{\sigma_{\pi N}(1 + \xi)}{2m_N} + \Delta f_d^N, \quad (14)$$

with $\xi = \frac{m_d - m_u}{m_d + m_u} = 0.36 \pm 0.04$ [33] and corrections $\Delta f_{u,d}^N$ related to the strong proton–neutron mass difference via the low-energy constant c_5 . For the strange quark, the most accurate determination comes from lattice QCD [34]. The above $\mathcal{O}(\alpha_s)$ analysis may not be accurate enough for the charm quark, see [23, 35, 36] for a study of higher orders in α_s .

This analysis generalizes to finite t if one defines

$$m_N f_q^N(t) = \langle N(p') | m_q \bar{q} q | N(p) \rangle, \quad \theta_0^N(t) = \langle N(p') | \theta_\mu^\mu | N(p) \rangle, \\ f_Q^N(t) = \frac{2}{27} \left(\frac{\theta_0^N(t)}{m_N} - \sum_{q=u,d,s} f_q^N(t) \right), \quad (15)$$

and replaces $f_q^N \rightarrow f_q^N(t)$, $f_Q^N \rightarrow f_Q^N(t)$ accordingly.

The chiral expansion of $\sigma_{\pi N}$ starts with

$$\sigma_{\pi N} = -4c_1 M_\pi^2 + \mathcal{O}(p^3), \quad (16)$$

in line with the $\mathcal{O}(p^2)$ listed in Table 1 for the scalar one-body current. Note, however, that the power 2 does not imply a momentum-dependent coupling in this case, but a quark-mass suppression. As far as the t -dependence is concerned, the slope of the scalar form factors is dominated by $\pi\pi$ scattering, which is known to not be adequately described by ChPT, but to require a reconstruction based on dispersion relations [37–39]. The t -dependence generated by other sources but light-quark scalar form factors was shown to be higher order in the chiral expansion in [10].

Defining

$$f_N(t) = \frac{m_N}{\Lambda^3} \left(\sum_{q=u,d,s} C_q^{SS} f_q^N(t) - 12\pi f_Q^N(t) C_g^{rS} \right), \quad (17)$$

the NR one-body matrix element for the scalar channel becomes¹

$$\mathcal{M}_{1,\text{NR}}^{SS} = \chi_{r'}^\dagger \chi_r \chi_{s'}^\dagger f_N(t) \chi_s, \quad (18)$$

where $\chi_{r,s}$ ($\chi_{r',s'}$) are NR spinors for the incoming (outgoing) WIMP and nucleon, respectively. $\mathcal{M}_{1,\text{NR}}^{PS}$ is of higher chiral order since the NR reduction of γ_5 produces a term $-\boldsymbol{\sigma} \cdot \mathbf{q}/(2m_\chi)$, which we count as $\mathcal{O}(p^2)$ for $m_\chi \gg m_N$.

4.2. Vector

The decomposition of the vector current at the quark level reads

$$\langle N(p') | \bar{q} \gamma^\mu q | N(p) \rangle = \langle N' | \gamma^\mu F_1^{q,N}(t) - \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu F_2^{q,N}(t) | N \rangle, \quad (19)$$

where the sign of the Pauli term is due to the convention in (4). To obtain a flavor decomposition of the vector current, one usually assumes isospin symmetry (corrections can again be calculated in ChPT [41]):

$$F_i^{u,p}(t) = F_i^{d,n}(t), \quad F_i^{d,p}(t) = F_i^{u,n}(t), \quad F_i^{s,p}(t) = F_i^{s,n}(t). \quad (20)$$

In this way, one obtains

$$F_i^{u,p}(t) = F_i^{d,n}(t) = 2F_i^{\text{EM},p}(t) + F_i^{\text{EM},n}(t) + F_i^{s,N}(t), \\ F_i^{d,p}(t) = F_i^{u,n}(t) = F_i^{\text{EM},p}(t) + 2F_i^{\text{EM},n}(t) + F_i^{s,N}(t), \quad (21)$$

with electromagnetic form factors $F_i^{\text{EM},N}(t)$. At vanishing momentum transfer this defines the vector couplings

$$\langle N | \bar{q} \gamma^\mu q | N \rangle = f_{V_q}^N \langle N | \gamma^\mu | N \rangle, \quad f_{V_u}^p = f_{V_d}^n = 2f_{V_d}^p = 2f_{V_u}^n = 2. \quad (22)$$

¹The nucleon spinors include isospin indices according to $\chi_{s'}^\dagger f_N(t) \chi_s \equiv \frac{1}{2} \chi_{s'}^\dagger [(f_p(t) + f_n(t))\mathbb{1} + (f_p(t) - f_n(t))\tau^3] \chi_s$. The Wilson coefficients match onto the conventions of [40] by means of the identification $f_N(0) = \sqrt{2} G_F c_0$.

Corrections to (22) can be worked out in terms of magnetic moments $\mu_N = Q_N + \kappa_N$, electric radii $\langle r_E^2 \rangle^N$, as well as strangeness moments $\mu_N^s = \kappa_N^s$ and radii $\langle r_{E,s}^2 \rangle^N$, explicitly

$$\begin{aligned}
F_1^{u,p}(t) &= 2 + 2\left(\frac{\langle r_E^2 \rangle^p}{6} - \frac{\kappa_p}{4m_p^2}\right)t + \left(\frac{\langle r_E^2 \rangle^n}{6} - \frac{\kappa_n}{4m_n^2}\right)t \\
&\quad + \left(\frac{\langle r_{E,s}^2 \rangle^N}{6} - \frac{\kappa_s}{4m_N^2}\right)t + O(t^2), \\
F_1^{d,p}(t) &= 1 + \left(\frac{\langle r_E^2 \rangle^p}{6} - \frac{\kappa_p}{4m_p^2}\right)t + 2\left(\frac{\langle r_E^2 \rangle^n}{6} - \frac{\kappa_n}{4m_n^2}\right)t \\
&\quad + \left(\frac{\langle r_{E,s}^2 \rangle^N}{6} - \frac{\kappa_s}{4m_N^2}\right)t + O(t^2), \\
F_1^{s,N}(t) &= \left(\frac{\langle r_{E,s}^2 \rangle^N}{6} - \frac{\kappa_N^s}{4m_N^2}\right)t + O(t^2), \\
F_2^{u,N} &= \kappa_N + O(t), \quad F_2^{d,N} = -\kappa_N - \kappa_N^s + O(t), \\
F_2^{s,N} &= \kappa_N^s + O(t), \tag{23}
\end{aligned}$$

with the Sachs form factors

$$\begin{aligned}
G_E^N(t) &= F_1^N(t) + \frac{t}{4m_N^2} F_2^N(t) = Q_N + \frac{\langle r_E^2 \rangle^N}{6} t + O(t^2), \\
G_M^N(t) &= F_1^N(t) + F_2^N(t) = \mu_N \left(1 + \frac{\langle r_M^2 \rangle^N}{6} t\right) + O(t^2). \tag{24}
\end{aligned}$$

The NR one-body matrix elements involving a nucleon vector current are

$$\begin{aligned}
\mathcal{M}_{1,\text{NR}}^{VV} &= \chi_r^\dagger \chi_r \chi_{s'}^\dagger \left[f_1^{V,N}(t) - \frac{\mathbf{q}}{4m_N^2} \cdot (\mathbf{q} - i\boldsymbol{\sigma} \times \mathbf{P}) f_2^{V,N}(t) \right] \chi_s \\
&\quad + \frac{1}{2m_N} \chi_r^\dagger [\mathbf{K} + i\boldsymbol{\sigma} \times \mathbf{q}] \chi_r \cdot \frac{1}{2m_N} \chi_{s'}^\dagger i\boldsymbol{\sigma} \times \mathbf{q} f_2^{V,N}(t) \chi_s, \\
\mathcal{M}_{1,\text{NR}}^{AV} &= \frac{1}{2m_N} \chi_r^\dagger \boldsymbol{\sigma} \cdot \mathbf{K} \chi_r \chi_{s'}^\dagger f_1^{A,N}(t) \chi_s \\
&\quad - \chi_r^\dagger \boldsymbol{\sigma} \chi_r \cdot \frac{1}{2m_N} \chi_{s'}^\dagger \left[(\mathbf{P} - i\boldsymbol{\sigma} \times \mathbf{q}) f_1^{A,N}(t) - i\boldsymbol{\sigma} \times \mathbf{q} f_2^{A,N}(t) \right] \chi_s, \tag{25}
\end{aligned}$$

where

$$\begin{aligned}
f_i^{V,N}(t) &= \frac{1}{\Lambda^2} \sum_{q=u,d,s} C_q^{VV} F_i^{q,N}(t), \\
f_i^{A,N}(t) &= \frac{1}{\Lambda^2} \sum_{q=u,d,s} C_q^{AV} F_i^{q,N}(t). \tag{26}
\end{aligned}$$

4.3. Axial vector

The decomposition of the axial-vector current at the quark level reads (see, e.g., [42, 43])

$$\begin{aligned}
\langle N(p') | \bar{q} \gamma^\mu \gamma_5 q | N(p) \rangle &= \langle N' | \gamma^\mu \gamma_5 G_A^{q,N}(t) - \gamma_5 \frac{q^\mu}{2m_N} G_P^{q,N}(t) \\
&\quad - \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu \gamma_5 G_T^{q,N}(t) | N \rangle. \tag{27}
\end{aligned}$$

$G_T^{q,N}(t)$ corresponds to a second-class current [44], i.e., it violates G -parity, and will be ignored in the following. At vanishing momentum transfer only $G_A^{q,N}$ contributes. Its coefficients are conventionally defined as

$$\langle N(p) | \bar{q} \gamma^\mu \gamma_5 q | N(p) \rangle = \Delta q^N \langle N | \gamma_\mu \gamma_5 | N \rangle, \tag{28}$$

and isospin symmetry is assumed

$$\Delta u^p = \Delta d^n, \quad \Delta u^n = \Delta d^p, \quad \Delta s^p = \Delta s^n. \tag{29}$$

The combinations

$$\begin{aligned}
a_3^p &= -a_3^n = \Delta u^p - \Delta d^p = g_A, \\
a_8^N &= \Delta u^N + \Delta d^N - 2\Delta s^N = 3F - D, \tag{30}
\end{aligned}$$

are determined by the axial charge of the nucleon in the case of a_3 , or can be inferred from semileptonic hyperon decays for a_8 , yielding $D \approx 0.8$, $F \approx 0.46$. The third combination

$$\Delta \Sigma^N = \Delta u^N + \Delta d^N + \Delta s^N \tag{31}$$

is related to the spin structure function of the nucleon, it is not a scale-independent quantity. At $Q^2 = 5 \text{ GeV}^2$ and $O(\alpha_s^2)$ the following values were obtained in [45]

$$\begin{aligned}
\Delta u^p &= 0.842 \pm 0.012, & \Delta d^p &= -0.427 \pm 0.013, \\
\Delta s^p &= -0.085 \pm 0.018. \tag{32}
\end{aligned}$$

Besides the coefficients at zero also the momentum dependence of the flavor combinations

$$\begin{aligned}
A_\mu^3 &= \bar{Q} \gamma_\mu \gamma_5 \frac{\lambda^3}{2} Q = \frac{1}{2} (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d), \\
A_\mu^8 &= \bar{Q} \gamma_\mu \gamma_5 \frac{\lambda^8}{2} Q = \frac{1}{2\sqrt{3}} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2\bar{s} \gamma_\mu \gamma_5 s), \tag{33}
\end{aligned}$$

can be analyzed in $SU(N_f)$ ChPT, but due to the anomalously broken $U(1)_A$ current this is not the case for the isoscalar component. One obtains

$$\begin{aligned}
\langle N(p') | A_\mu^3 | N(p) \rangle &= \langle N' | \left(\gamma^\mu \gamma_5 G_A^3(t) - \gamma_5 \frac{q^\mu}{2m_N} G_P^3(t) \right) \frac{\tau^3}{2} | N \rangle, \\
\langle N(p') | A_\mu^8 | N(p) \rangle &= \langle N' | \left(\gamma^\mu \gamma_5 G_A^8(t) - \gamma_5 \frac{q^\mu}{2m_N} G_P^8(t) \right) \frac{1}{2} | N \rangle, \tag{34}
\end{aligned}$$

with leading-order results

$$\begin{aligned}
G_A^3(t) &= g_A, & G_A^8(t) &= \frac{3F - D}{\sqrt{3}} \equiv g_A^8, \\
G_P^3(t) &= -\frac{4m_N^2 g_A}{t - M_\pi^2}, & G_P^8(t) &= -\frac{4m_N^2 g_A^8}{t - M_\eta^2}. \tag{35}
\end{aligned}$$

Empirically, the momentum dependence of $G_A^3(t)$, extracted from neutrino scattering off nucleons and charged-pion electroproduction, follows a dipole fit

$$G_A^3(t) = \frac{g_A}{(1 - t/M_A^2)^2}, \tag{36}$$

with mass parameter M_A around 1 GeV [42, 43]. Since for general t the flavor structure cannot be inverted without additional input for the singlet component, we decompose the quark sum according to²

$$\sum_q C_q^{AA} G_{A,P}^{q,N}(t) = C_0^{AA} G_{A,P}^0(t) + C_3^{AA} G_{A,P}^3(t)\tau^3 + C_8^{AA} G_{A,P}^8(t), \quad (37)$$

with

$$C_0^{AA} = \frac{1}{3}[C_u^{AA} + C_d^{AA} + C_s^{AA}], \quad C_3^{AA} = \frac{1}{2}[C_u^{AA} - C_d^{AA}],$$

$$C_8^{AA} = \frac{\sqrt{3}}{6}[C_u^{AA} + C_d^{AA} - 2C_s^{AA}], \quad (38)$$

and define

$$g_{A,P}^N(t) = \frac{1}{\Lambda^2}[C_0^{AA} G_{A,P}^0(t) + C_3^{AA} G_{A,P}^3(t)\tau^3 + C_8^{AA} G_{A,P}^8(t)]. \quad (39)$$

In terms of these quantities, the NR amplitude reads

$$\mathcal{M}_{1,\text{NR}}^{AA} = -\chi_r^\dagger \chi_r \cdot \chi_{s'}^\dagger \left[\sigma g_A^N(t) - \frac{\mathbf{q}}{4m_N^2} \sigma \cdot \mathbf{q} g_P^N(t) \right] \chi_s. \quad (40)$$

Similarly, for the VA channel we define

$$h_{A,P}^N(t) = \frac{1}{\Lambda^2}[C_0^{VA} G_{A,P}^0(t) + C_3^{VA} G_{A,P}^3(t)\tau^3 + C_8^{VA} G_{A,P}^8(t)], \quad (41)$$

to obtain

$$\mathcal{M}_{1,\text{NR}}^{VA} = \chi_r^\dagger \chi_r \frac{1}{2m_N} \chi_{s'}^\dagger \left[\sigma \cdot \mathbf{P} h_A^N(t) - \frac{\mathbf{q} \cdot \mathbf{P}}{4m_N^2} \sigma \cdot \mathbf{q} h_P^N(t) \right] \chi_s$$

$$- \frac{1}{2m_\chi} \chi_r^\dagger \left[\mathbf{K} + i\sigma \times \mathbf{q} \right] \chi_r \cdot \chi_{s'}^\dagger \left[\sigma h_A^N(t) - \frac{\mathbf{q}}{4m_N^2} \sigma \cdot \mathbf{q} h_P^N(t) \right] \chi_s.$$

4.4. Pseudoscalar

The pseudoscalar matrix element is usually parameterized as

$$\langle N(p') | m_q \bar{q} i\gamma_5 q | N(p) \rangle = \langle N' | m_N G_5^{q,N}(t) i\gamma_5 | N \rangle. \quad (43)$$

By means of the Ward identity

$$\sum_q \partial_\mu \bar{q} \gamma^\mu \gamma_5 q = \sum_q 2im_q \bar{q} \gamma_5 q - \frac{\alpha_s N_f}{4\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu}, \quad (44)$$

the corresponding form factor $G_5^{q,N}(t)$ follows from $G_A^{q,N}(t)$ and $G_P^{q,N}(t)$, except for the singlet component, where the anomaly does not drop out,

$$G_5^i(t) = G_A^i(t) + \frac{t}{4m_N^2} G_P^i(t), \quad i = 3, 8. \quad (45)$$

Accordingly, we have

$$\mathcal{M}_{1,\text{NR}}^{SP} = \chi_r^\dagger \chi_r \frac{i}{2} \chi_{s'}^\dagger \sigma \cdot \mathbf{q} g_5^N(t) \chi_s,$$

$$\mathcal{M}_{1,\text{NR}}^{PP} = \frac{1}{2m_\chi} \chi_r^\dagger \sigma \cdot \mathbf{q} \chi_r \frac{1}{2} \chi_{s'}^\dagger \sigma \cdot \mathbf{q} h_5^N(t) \chi_s, \quad (46)$$

²At vanishing momentum transfer this equation maps onto the notation of [40] by means of $\sum_q C_q^{AA} \Delta q^N = \sqrt{2} G_F \Lambda^2 \frac{1}{2} (a_0 + a_1 \tau^3)$.

where

$$g_5^N(t) = \frac{1}{\Lambda^2} [C_3^{SP} G_5^3(t)\tau^3 + C_8^{SP} G_5^8(t)],$$

$$h_5^N(t) = \frac{1}{\Lambda^2} [C_3^{PP} G_5^3(t)\tau^3 + C_8^{PP} G_5^8(t)]. \quad (47)$$

5. Two-body currents

5.1. Scalar

The scalar meson-exchange currents, involving both pion and η contributions, have been considered before in [9, 10]. The full expression reads

$$\mathcal{M}_{2,\text{NR}}^{SS} = -\chi_r^\dagger \chi_r \left(\frac{g_A}{2F_\pi} \right)^2 f_\pi M_\pi^2 \chi_{s_1}^\dagger \chi_{s_2}^\dagger \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 X_{12}^\pi \chi_{s_1} \chi_{s_2}$$

$$- \chi_r^\dagger \chi_r \left(\frac{g_A}{2F_\pi} \right)^2 \left(\frac{4\alpha - 1}{\sqrt{3}} \right)^2 f_\eta M_\eta^2 \chi_{s_1}^\dagger \chi_{s_2}^\dagger X_{12}^\eta \chi_{s_1} \chi_{s_2}, \quad (48)$$

where

$$X_{12}^i = \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{q}_2}{(\mathbf{q}_1^2 + M_i^2)(\mathbf{q}_2^2 + M_i^2)}, \quad i = \pi, \eta, \quad (49)$$

pion decay constant $F_\pi = 92.2$ MeV [46], χ_{s_i} (χ_{s_i}') denote NR spinors for the incoming (outgoing) nucleons, with momenta p_i (p_i'), $q_i = p_i' - p_i$, $\alpha = F/(D + F)$, and the Wilson coefficients are collected in

$$f_\pi = \frac{1}{\Lambda^3} \sum_{q=u,d} C_q^{SS} f_q^\pi, \quad f_\eta = \frac{1}{\Lambda^3} \sum_{q=u,d,s} C_q^{SS} f_q^\eta, \quad (50)$$

with scalar meson couplings

$$f_u^\pi = \frac{m_u}{m_u + m_d} = 0.32 \pm 0.03, \quad f_d^\pi = \frac{m_d}{m_u + m_d} = 0.68 \pm 0.03, \quad (51)$$

and

$$f_u^\eta = \frac{1}{3} \frac{m_u}{m_u + m_d} \frac{M_\pi^2}{M_\eta^2} = (6.9 \pm 0.4) \times 10^{-3},$$

$$f_d^\eta = \frac{1}{3} \frac{m_d}{m_u + m_d} \frac{M_\pi^2}{M_\eta^2} = (14.7 \pm 0.4) \times 10^{-3},$$

$$f_s^\eta = \frac{2}{3} \frac{M_{K^0}^2 + M_{K^+}^2 - M_\pi^2}{M_\eta^2} = 1.05. \quad (52)$$

One particular feature of the scalar two-body currents is that they cannot be written as a correction to the one-body coupling f_N , since the scalar couplings of pions and η mesons probe a different combination of Wilson coefficients [10]. For this reason, even in the isospin limit they cannot be parameterized in terms of a single coupling c_0 as conventionally done for the one-body currents, see e.g. [40].

5.2. Vector

The only two-body vector current up to $\mathcal{O}(p^3)$ appears in the AV channel

$$\mathcal{M}_{2,\text{NR}}^{AV} = -\frac{2}{\Lambda^2} C_3^{AV} \left(\frac{g_A}{2F_\pi} \right)^2 \chi_r^\dagger \sigma \chi_r \cdot \chi_{s_1}^\dagger \chi_{s_2}^\dagger i[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3 \left[\frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}_1 \boldsymbol{\sigma}_2}{\mathbf{q}_1^2 + M_\pi^2} \right.$$

$$\left. - \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2 \boldsymbol{\sigma}_1}{\mathbf{q}_2^2 + M_\pi^2} + (\mathbf{q}_1 - \mathbf{q}_2) X_{12}^\pi \right] \chi_{s_1} \chi_{s_2}. \quad (53)$$

While the nucleon vector current itself has been studied in detail before [18–20], the present application to direct detection is new.

In fact, there are neither terms with $i = 8$ nor η contributions to $i = 3$. The reason for this can be traced back to the operator structure of the chiral Lagrangian: the coupling to the vector current occurs via a commutator $[v_\mu, \phi]$ of vector source and meson matrix. Expanded in Gell-Mann matrices, this leaves SU(3) structure factors f^{3ij} and f^{8ij} , and the only non-trivial ones, apart from the direct couplings to the nucleon that led to (35), reduce to the SU(2) subset ϵ^{ijk} .

5.3. Axial vector

The axial-vector two-body currents are

$$\begin{aligned} \mathcal{M}_{2,\text{NR}}^{\text{AA}} = & \frac{1}{\Lambda^2} C_3^{\text{AA}} \chi_r^\dagger \sigma \chi_r \cdot \chi_{s_1'}^\dagger \chi_{s_2'}^\dagger \left\{ \left[\frac{g_A}{F_\pi^2} [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3 \left[\frac{c_6}{4} \boldsymbol{\sigma}_1 \times \mathbf{q} \right. \right. \right. \\ & + c_4 \left(1 - \frac{\mathbf{q}}{\mathbf{q}^2 + M_\pi^2} \mathbf{q} \cdot \right) \boldsymbol{\sigma}_1 \times \mathbf{q}_2 \left. \left. \left. \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + M_\pi^2} \right] \right. \right. \\ & + \frac{2g_A}{F_\pi^2} \tau_2^3 \left[2c_1 M_\pi^2 \frac{\mathbf{q}}{\mathbf{q}^2 + M_\pi^2} \right. \\ & + c_3 \left(\mathbf{q}_2 - \frac{\mathbf{q}}{\mathbf{q}^2 + M_\pi^2} \mathbf{q} \cdot \mathbf{q}_2 \right) \left. \left. \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + M_\pi^2} \right] \right. \\ & + 2d_1 \tau_1^3 \left(\boldsymbol{\sigma}_1 - \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \mathbf{q}}{\mathbf{q}^2 + M_\pi^2} \right) \left. \left. \right] + (1 \leftrightarrow 2) \right. \\ & \left. + 2d_2 [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3 (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \left(1 - \frac{\mathbf{q} \cdot \mathbf{q}}{\mathbf{q}^2 + M_\pi^2} \right) \right\} \chi_{s_1} \chi_{s_2}, \end{aligned} \quad (54)$$

where the terms that do not contain an explicit \mathbf{q} -dependence ($\mathbf{q} = -\mathbf{q}_1 - \mathbf{q}_2$) and the c_6 -term are taken from [21], while the finite- \mathbf{q} pion-pole corrections were derived in [15]. The AA two-body current as in [21] has been applied in the calculation of structure factors for spin-dependent scattering in [11, 12], whereas the two-body current in the VA channel,

$$\begin{aligned} \mathcal{M}_{2,\text{NR}}^{\text{VA}} = & -\frac{1}{\Lambda^2} C_3^{\text{VA}} \frac{g_A}{2F_\pi^2} \chi_r^\dagger \chi_r \chi_{s_1'}^\dagger \chi_{s_2'}^\dagger \left\{ i [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3 \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + M_\pi^2} \right. \\ & \left. + (1 \leftrightarrow 2) \right\} \chi_{s_1} \chi_{s_2}, \end{aligned} \quad (55)$$

has not been considered before.

For similar reasons as in the vector case there are no $i = 8$ or η contributions from the leading-order Lagrangian. In principle, one could calculate corrections from the NLO SU(3) Lagrangian, in analogy to the SU(2) result for $\mathcal{M}_{2,\text{NR}}^{\text{AA}}$. However, there is a large number of poorly-known low-energy constants (see [47] or [48] for the matching to SU(2)), which would severely limit the predictive power.

Finally, due to the derivative in the Ward identity (44), there are no pseudoscalar two-body currents at $\mathcal{O}(p^3)$.

6. Matching to NREFT

Next, we express our results in terms of the operator basis from [7]

$$\begin{aligned} \mathcal{O}_1 = \mathbb{1}, & \quad \mathcal{O}_2 = (\mathbf{v}^\perp)^2, & \quad \mathcal{O}_3 = i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp), \\ \mathcal{O}_4 = \mathbf{S}_\chi \cdot \mathbf{S}_N, & \quad \mathcal{O}_5 = i\mathbf{S}_\chi \cdot (\mathbf{q} \times \mathbf{v}^\perp), & \quad \mathcal{O}_6 = \mathbf{S}_\chi \cdot \mathbf{q} \mathbf{S}_N \cdot \mathbf{q}, \\ \mathcal{O}_7 = \mathbf{S}_N \cdot \mathbf{v}^\perp, & \quad \mathcal{O}_8 = \mathbf{S}_\chi \cdot \mathbf{v}^\perp, & \quad \mathcal{O}_9 = i\mathbf{S}_\chi \cdot (\mathbf{S}_N \times \mathbf{q}), \\ \mathcal{O}_{10} = i\mathbf{S}_N \cdot \mathbf{q}, & \quad \mathcal{O}_{11} = i\mathbf{S}_\chi \cdot \mathbf{q}, \end{aligned} \quad (56)$$

where $\mathbf{S} = \boldsymbol{\sigma}/2$ and the velocity is defined as

$$\mathbf{v}^\perp = \frac{\mathbf{K}}{2m_\chi} - \frac{\mathbf{P}}{2m_N}. \quad (57)$$

We find the relations

$$\begin{aligned} \mathcal{M}_{1,\text{NR}}^{\text{SS}} &= \chi_r^\dagger \chi_{s'}^\dagger \mathcal{O}_1 f_N(t) \chi_r \chi_s, \\ \mathcal{M}_{1,\text{NR}}^{\text{SP}} &= \chi_r^\dagger \chi_{s'}^\dagger \mathcal{O}_{10} g_5^N(t) \chi_r \chi_s, \\ \mathcal{M}_{1,\text{NR}}^{\text{PP}} &= \frac{1}{m_\chi} \chi_r^\dagger \chi_{s'}^\dagger \mathcal{O}_6 h_5^N(t) \chi_r \chi_s, \\ \mathcal{M}_{1,\text{NR}}^{\text{VV}} &= \chi_r^\dagger \chi_{s'}^\dagger \left[\mathcal{O}_1 (f_1^{\text{V},N}(t) + \frac{t}{4m_N^2} f_2^{\text{V},N}(t)) + \frac{1}{m_N} \mathcal{O}_3 f_2^{\text{V},N}(t) \right. \\ & \quad \left. + \frac{1}{m_N m_\chi} (t \mathcal{O}_4 + \mathcal{O}_6) f_2^{\text{V},N}(t) \right] \chi_r \chi_s, \\ \mathcal{M}_{1,\text{NR}}^{\text{AV}} &= \chi_r^\dagger \chi_{s'}^\dagger \left[2\mathcal{O}_8 f_1^{\text{A},N}(t) + \frac{2}{m_N} \mathcal{O}_9 (f_1^{\text{A},N}(t) + f_2^{\text{A},N}(t)) \right] \chi_r \chi_s, \\ \mathcal{M}_{1,\text{NR}}^{\text{AA}} &= \chi_r^\dagger \chi_{s'}^\dagger \left[-4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2} \mathcal{O}_6 g_P^N(t) \right] \chi_r \chi_s, \\ \mathcal{M}_{1,\text{NR}}^{\text{VA}} &= \chi_r^\dagger \chi_{s'}^\dagger \left[-2\mathcal{O}_7 + \frac{2}{m_\chi} \mathcal{O}_9 \right] h_A^N(t) \chi_r \chi_s. \end{aligned} \quad (58)$$

This shows that as a result of QCD effects, the operators in the NREFT are not independent. For example, both axial and pseudoscalar operators combine in the nuclear matrix element $\mathcal{M}_{1,\text{NR}}^{\text{AA}}$. In addition, up to $\mathcal{O}(p^3)$ only 8 of the 11 operators of (56) are present. However, because $\mathcal{M}_{1,\text{NR}}^{\text{PS}}$ itself enters only at $\mathcal{O}(p^4)$, they are mapped onto 7 amplitudes, so that the relations cannot be inverted. This is because $\mathcal{M}_{1,\text{NR}}^{\text{AV}}$ and $\mathcal{M}_{1,\text{NR}}^{\text{VA}}$ involve the three operators \mathcal{O}_{7-9} . This implies that some operators, e.g. \mathcal{O}_6 , can be isolated by having a particular quark-level interaction, but this is not possible in general, as demonstrated by the example of \mathcal{O}_{7-9} . If we retain subleading corrections in the NR expansion of the spinors, the missing operators appear, accompanied by additional combinations: \mathcal{O}_{11} in terms of $\mathcal{M}_{1,\text{NR}}^{\text{PS}}$, \mathcal{O}_2 and \mathcal{O}_5 in $\mathcal{M}_{1,\text{NR}}^{\text{VV}}$, $\mathcal{O}_3 \mathcal{O}_8$ in $\mathcal{M}_{1,\text{NR}}^{\text{AV}}$, and $\mathcal{O}_7 \mathcal{O}_8$ in $\mathcal{M}_{1,\text{NR}}^{\text{AA}}$.

In the limit where m_χ becomes (significantly) larger than the nucleon mass also $\mathcal{M}_{1,\text{NR}}^{\text{PP}}$ should be dropped, as well as the $1/m_\chi$ suppressed terms in $\mathcal{M}_{1,\text{NR}}^{\text{VV}}$ and $\mathcal{M}_{1,\text{NR}}^{\text{VA}}$. In contrast, all two-body currents up to $\mathcal{O}(p^3)$ are independent of m_χ . They appear in the SS, AV, AA, and VA channels.

We stress that the above discussion merely pertains to the mapping of operator structures, it does not take into account the evolution of the scale dependence that is required when matching the coefficients of a pionless theory, valid for scales below

the pion mass, and ChEFT, defined at chiral scales. This involves also effects related to the limitations of the “Weinberg” counting scheme applied here [49], and would have to be taken into account in the matching relations required for translating NREFT coefficients to the QCD scale. In addition, there may be effects from operator mixing, originating from the interplay between the nucleon-spin dependence in the ChEFT WIMP–nucleon scattering operator and that in the high-momentum part of the ChEFT NN potential, which would also have to be considered when evolving NREFT operators to the QCD scale.

7. Summary and discussion

In this Letter, we have developed the constraints that chiral symmetry of QCD imposes on the nuclear matrix elements that can enter in dark matter direct detection. We provide explicit expressions for one- and two-body currents in WIMP–nucleus scattering for vector, axial-vector, scalar, and pseudoscalar interactions up to third order in the chiral expansion. The chiral power counting, summarized in Table 1, shows that at this order there are two-body currents that have not been considered and may be of similar or greater importance than some of the one-body operators, see (53) and (55). Moreover, the matching to NREFT shows that not all allowed one-body operators appear at this chiral order and that the operators in the NREFT are not independent.

The chiral power counting applies to the one- and two-nucleon level. In nuclei, the different interactions can lead to a coherent response that scales with the number of nucleons in the nucleus or to a single-particle-like response. In a next step, we will evaluate the nuclear structure factors, including the contributions from two-body currents, and provide a set of response functions for the analysis of direct-detection experiments. This will also allow us to assess how constructive or destructive the interference of operators based on the constraints provided by chiral symmetry proves to be.

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