# Anomalous pion production induced by nontrivial topological structure of QCD vacuum

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A new mechanism for the pion production in high energy reactions is suggested. It is related to a possibility for the direct production of the pions induced by instantons, topologically nontrivial gluonic excitations of the QCD vacuum. This mechanism does not require any fragmentation functions for the production of pseudoscalar mesons in high energy reactions with hadrons. We calculate the contribution of the new mechanism to the inclusive  $\pi^0$ -meson production in high energy protonproton collisions. It is shown that it gives the dominant contribution to the inclusive cross-sections in the few GeV region for the transverse momentum of the final pion with large rapidity. We discuss the possible applications of the new mechanism to the phenomenon of large spin effects observed in numerous high energy reactions and to particle productions in the relativistic heavy ion collisions.

## I. INTRODUCTION

The inclusive production of mesons in high energy reactions is one of the powerful tools to investigate the structure of strong interaction. At the very large transverse momentum the leading behavior of the inclusive cross-section should be dominated by the perturbative t-channel one-gluon exchange which leads to the wellknown dependency  $d\sigma \sim 1/p_t^4$ . However, one cannot believe in the validity of the perturbative QCD (pQCD) approach in the small transverse momentum region where the nonperturbative QCD effects can play a crucial role. It is clear that the value of the transfer momentum for the applicability of the pQCD depends on the nonperturbative dynamics of QCD. This dynamics is deeply related to the complicated structure of the QCD vacuum. One of the powerful models to calculate the nonperturbative QCD effects in hadron physics, which are induced by the nontrivial topological structure of the QCD vacuum, is the instanton model (see reviews [1, 2]). The instantons describe the subbarrier transitions between the classical QCD vacua with the different topological charge. The existence of instantons is very important for hadron physics. For example, they provide a natural mechanism of the spontaneous chiral symmetry breaking (SCSB) in the strong interaction. As the result, large dynamical quark masses arise. One of the places where the SCSB effects might be important is the high energy reactions with hadrons. In particular, it was demonstrated in [3] that in the few GeV region for the momentum transfer the instanton effects give a significant contribution to the high energy quark-quark scattering cross section. These effects come from the anomalous chromomagnetic quark-gluon interaction induced by instantons [4]. Furthermore, this interaction leads to the quark chirality flip and might give an important contribution to the spindependent cross sections [5-8]. The problem with the pQCD description of the high energy inclusive pion production was discussed for the first time in [9]. It was mentioned that the cross section is well described by the pQCD only in the region of small  $x_F$  for the fixed target experiments. In [10], the large partonic intrinsic momentum in hadrons was introduced to describe the data. However, it is not easy to justify the existence of such a large intrinsic partonic momentum from the confinement dynamics. The possible violation of the pQCD factorization in inclusive production of hadrons induced by high twist contributions was discussed by Brodsky with collaborators (see [11, 12] and references therein), but the microscopic mechanism of such a violation was not presented.

In this Letter, we suggest a new mechanism of the inclusive production of pions in high energy reactions. We call it the anomalous pion production (APP) because the formation of the pions happens at short distances due to instantons which have a much smaller size in comparison with the confinement scale. Furthermore, in our approach it is not needed to include in the calculation any fragmentation functions which are related to the hadronization and, therefore, to the confinement dynamics. As the result, the value of the inclusive pion cross section is determined by the structure of the short range fluctuations of gluonic fields in the QCD vacuum. This mechanism breaks the pQCD factorization and might be a corner stone of various phenomena observed in high energy reactions in the few GeV range for the transfer momentum.

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### II. ANOMALOUS PRODUCTION OF THE PIONS INDUCED BY CHROMOMAGNETIC VERTEX

It was shown that instantons generate a new type of the quark-gluon chromomagnetic interaction [6]

$$\mathcal{L}_I = i \frac{g_s \mu_a}{4M_q} \bar{q} \sigma^{\mu\nu} t^a q G^a_{\mu\nu}, \qquad (1)$$

where  $\mu_a$  is the anomalous quark chromomagnetic moment (AQCM),  $M_q$  is the dynamical quark mass,  $g_s$ is the strong coupling constant, and  $G^a_{\mu\nu}$  is the gluon field strength. Within the instanton model the value of AQCM is (see for details [2, 13])

$$\mu_a = -\frac{3\pi (M_q \rho_c)^2}{4\alpha_s(\rho_c)}.$$
(2)

Therefore, the value of AQCM is determined by the dynamical mass of the quark in the instanton vacuum. For example, for the dynamical quark mass  $M_q = 170$  MeV from the mean field approximation [1] and  $\alpha_s(\rho_c) \approx 0.5$ at an average size of instantons in the QCD vacuum  $\rho_c = 1/600 \text{ MeV}^{-1}$  [2], we obtain

$$\mu_a = -0.378, \tag{3}$$

which is very large in comparison with the Schwingertype of the pQCD contribution to the AQCM

$$\mu_a^{pQCD} = -\frac{\alpha_s}{12\pi} \approx -1.3 \times 10^{-2}.$$
 (4)

It is evident that Lagrangian, Eq.1, violates chiral symmetry. Therefore, in [2, 14] the generalization of Eq.1 was suggested by inclusion of the pion field into consideration to preserve the explicit chiral invariance. The modified Lagrangian is

$$\mathcal{L}_{I} = i \frac{g_{s} \mu_{a}}{4M_{q}} \bar{q} \sigma^{\mu\nu} t^{a} e^{i\gamma_{5} \vec{\tau} \cdot \vec{\phi}_{\pi}/F_{\pi}} q G^{a}_{\mu\nu}, \qquad (5)$$

where  $F_{\pi} = 93$  MeV. The expansion of this Lagrangian



FIG. 1: The diagram a) presents the general quark-gluon vertex generated by instantons for  $N_f = 2 \text{ case}$ , b) corresponds to the case with one of the quark lines connected through the quark condensate, and c) describes the direct production of the pion from the instanton.

up to the first order in the pion field gives [28]

$$\mathcal{L}_{I} = i \frac{g_{s} \mu_{a}}{4M_{q}} \bar{q} \sigma^{\mu\nu} t^{a} q G^{a}_{\mu\nu} - \frac{g_{s} \mu_{a}}{4M_{q} F_{\pi}} \bar{q} \sigma^{\mu\nu} t^{a} \gamma_{5} \vec{\tau} \cdot \vec{\phi}_{\pi} q G^{a}_{\mu\nu}.$$
(6)

Within the instanton model the two terms in this equation can be represented by the diagrams b), c) in Fig.1. The diagrams which give the contribution to the anomalous production of the pion on the parton level are shown in Fig.2. Using the Sudakov parametrization of the four-



FIG. 2: The pion production induced by instanton a) in quark-quark scattering and b)in quark-gluon scattering.

momenta of particles [15]

$$p'_i = \alpha_i p_2 + z_i p_1 + p'_{i,t}, \quad p'_{i,t} \cdot p_{1,2} = 0, \quad p'^2_{i,t} = -\overline{p}'^2_i < 0,$$

we obtain the cross section of the  $\pi^0$  production in the quark-quark scattering by incoming *u*- or *d*-quark [29]

$$\frac{zd\sigma^{qq \to qq\pi^{0}}}{dzd^{2}\vec{k}} = g_{\pi^{0}qq}^{2} \frac{\rho_{c}^{4}}{2^{7}\pi} \int \frac{\alpha_{s}^{2}(q^{2})}{\alpha_{s}^{2}(\rho_{c})} \frac{\vec{q}^{2}}{(\vec{q}^{2} + m_{g}^{2})^{2}} F_{g}^{2}(\rho_{c}|\vec{q}|)d^{2}\vec{q}, \quad (7)$$

where  $F_g(t) = 4/t^2 - 2K_2(t)$  is the form factor of the instanton [13], z is a fraction of the initial quark momentum carried by the final pion,  $m_g$  is the dynamical gluon mass related to the infrared behavior of the gluon propagator, and

$$g_{\pi^0 qq} = \frac{M_q}{F_\pi}.$$
(8)

For the quark-gluon scattering (the diagram in Fig.2b), we obtained the cross section which is larger than quarkquark scattering case by a factor of 9/4. The instanton corresponds to the subbarrier transition between vacua with the different topological charge. Therefore, the single instanton approximation, which we are using, is correct when the invariant mass of the partonic system produced by the instanton does not exceed the height of the potential barrier between these vacua. This height is given by the energy of the so-called sphaleron  $E_{sph} = 3\pi/(4\alpha_s(\rho)\rho)$  (see, for example, [2]). For  $1/\rho_c = 0.6$ GeV and  $\alpha(\rho_c) = 0.5$  we obtain  $E_{sph} = 2.83$  GeV, which is rather a large value. By using the condition  $(k + p'_1)^2 = M_X^2 \leq E_{sph}^2$  and the relation

$$(z\vec{q} - \vec{k})^2 = z(1-z)M_X^2, \tag{9}$$

where  $\vec{k}$  is the transverse momentum of the pion, we finally obtain the APP cross section in the quark-quark scattering

$$\frac{zd\sigma^{qq \to qq\pi^0}}{dzd^2\vec{k}} = g_{\pi^0 qq}^2 \frac{\rho_c^4}{\alpha_s^2(\rho_c)2^8\pi} \int_0^{y_{max}} dy \int_0^{2\pi} d\varphi \frac{\alpha_s^2(x,y,\varphi)F(x,y,z,\varphi)F_g^2(\sqrt{F(x,y,z,\varphi)}/z)}{(F(x,y,z,\varphi) + z^2m_g^2\rho_c^2)^2} z(1-z), \tag{10}$$

where

$$F(x, y, z, \varphi) = z(1-z)y + x^2 + 2x\sqrt{y}\sqrt{z(1-z)}\cos\varphi$$
(11)

and  $x = \rho_c |\vec{k}|$ ,  $y = \rho_c^2 M_X^2$ . This cross section can be compared with the leading order pQCD cross section

$$\frac{zd\sigma^{qq \to qq\pi^0}}{dzd^2\vec{k}}(pQCD) = \frac{D_q^{\pi^0}(z)}{\pi z}\frac{d\sigma}{dq^2},\qquad(12)$$

where  $D_q^{\pi^0}(z)$  is the fragmentation function and  $d\sigma/dq^2 = 8\pi \alpha_s^2(q^2)/(q^2 - m_g^2)^2/9$ . We should mention that in the kinematic region of few GeV for the transverse momentum of the pion and at high energy one can neglect the change of the longitudinal momentum of the quark in partonic subprocess in the pQCD. In this case,

the meaning of 
$$z$$
 becomes the same for both pQCD and  
APP cases. In Fig.3, the result of calculation of the cross  
section of the APP, Eq.10, in the quark-quark scattering  
is presented [30]. We compare it with the pQCD cross  
section, Eq.12, calculated with the strong coupling con-

stant given by the analytical pQCD [16]

$$\alpha_s(Q^2) = \frac{1}{\beta_0} \left[ \frac{1}{\log(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right], \quad (13)$$

where  $Q^2 = -q^2$ ,  $\beta_0 = (33 - 2N_f)/12\pi$ ,  $\Lambda = 250$  MeV, and  $N_f = 3$  were used. The fragmentation function  $D_q^{\pi^0}(z)$  was taken from [17], set KKP-1. The dynamical gluon mass  $m_g \approx 0.65$  GeV was fixed according to the result of the lattice data in [18][31].



FIG. 3: The z dependency of the pQCD (black line) and the APP (blue line) cross sections, in the units  $10^2 \mu b/GeV^2$ , for the different values of the pion transverse momentum: the left panel  $k_t = 1$  GeV, the central panel  $k_t = 2$  GeV, and the right panel  $k_t = 3$  GeV. Here, the relation  $k_t = |\vec{k}| \approx \sqrt{-q^2}$  was used.

Let us discuss, in the beginning on a qualitative level, the features of kinematics of the pQCD and the APP contributions to  $\pi^0$  production in the fixed target experiments for a few GeV transverse momentum of the final pion where the big difference from the pQCD prediction was observed. In this case, the main contribution comes from the scattering of the valence quarks which carry about  $\langle x \rangle \approx 0.2$  momentum of the initial proton. Therefore, it follows from Fig.3 that at  $k_t = 1$  GeV the APP contribution starts to be dominant at  $x_F \approx 0.08$ [32], at  $k_t = 2$  GeV the APP contribution dominates at  $x_F \approx 0.11$ , and at  $k_t = 3$  GeV the APP above  $x_F \approx 0.15$ 

gives the main contribution to the cross section. So we come to the conclusion that the pQCD might give the main contribution only in the region of very small  $x_F$ . Furthermore, the APP contribution is very big even at large  $k_t$  in the large  $x_F$  region. To calculate the APP cross section at the hadron level, we use an approach similar to the pQCD approach (see [20]). In particular, in the high energy limit we will neglect all masses of the hadrons and partons in the kinematics of the inclusive pion production. In this way, for the APP contribution to the inclusive cross section of the  $\pi^0$  production in the proton-proton scattering, we have

$$\frac{E_k d\sigma}{d^3 k} = \sum_{a,b} \int_{x_a^{min}}^1 dx_a \int_{x_b^{min}}^1 G_{A \to a}(x_a, \mu^2) G_{B \to b}(x_b, \mu^2) \frac{z d\sigma^{ab}}{dz d^2 \vec{k}} (z = z_c, k_t),$$
(14)

where  $G_{A,B\to a,b}$  are parton distribution functions (PDFs) and the sum over the different types of partons is carried out. Additionally, we should use the scale  $\mu^2 \approx 1/\rho_c^2$  in our nonperturbative calculation.



In Eq.14

$$x_{a}^{min} = x_{1}/(1-x_{2}), \quad x_{b}^{min} = x_{2}x_{a}/(x_{a}-x_{1}),$$

$$x_{1} = \frac{1}{2}x_{T}/tan(\theta_{cm}/2), \quad x_{2} = \frac{1}{2}x_{T}tan(\theta_{cm}/2),$$

$$z_{c} = x_{2}/x_{b} + x_{1}/x_{a},$$
(15)

where  $x_T = 2k_t/\sqrt{s}$  and  $\theta_{cm}$  is the c.m. angle of the produced pion and the GRV98 NLO parametrization at  $\mu^2 = 0.4 \text{ GeV}^2$  was used [19]. This value of the scale is very close to the scale of our instanton based calculation  $1/\rho_c^2 = 0.36 \text{ GeV}^2$ . The results are represented in Fig.4 in comparison with the experimental data. As one can see, the APP mechanism provides a rather good description of both fixed target and collider data at  $x_F > 0.2$  and few GeV value for the transverse momentum of the pion. Some deviation from STAR data might be related to the PDF uncertainties. On the other hand, it is known that in this kinematic region for the fixed target experiments the pQCD contribution is very small and cannot explain the data without introducing large intrinsic parton momentum [9, 10] in the leading order calculation. The NLO pQCD predictions also fail to explain these data. We would like to emphasize that it is very difficult to uncerstand in the conventional approach why the pQCD works well at RHIC energy but fails to explain large  $x_F$ ISR data. In both the cases the energy is much larger than any other scales, i.e., hadron masses, transverse momentum of partons and hadrons, transfer momentum in a partonic subprocess, etc. Furthermore, the contribution from gluonic  $qq \rightarrow qq$ , which could be one of the candidates to explain this difference, is expected to be small in the forward rapidity region even for the RHIC energies [23]. It might be that a rather good agreement for the collider case [24] is related to the internal uncertainties in the pQCD calculation due to uncertainties in the value of factorization scale and in the shape of fragmentation functions. Indeed, the comparison with the STAR data shows rather large sensitivity of the NLO pQCD prediction to the choice of the fragmentation function [22] and to the factorization scale [25]. Therefore, we believe that the complete picture of the inclusive particle production in the strong interaction should include both pQCD and APP mechanisms.

### **III. CONCLUSION**

FIG. 4: The APP cross sections for inclusive  $\pi^0$  in comparison with the ISR (CERN Intersecting Storage Rings) fixed target [21] and the STAR [22] data for  $k_t \approx 1 \div 3$  GeV at large pseudorapidity.

We discussed the new mechanism for the inclusive production of the pions in the hadron-hadron interaction. It is related to the anomalous quark-gluon-pion coupling induced by the nontrivial topological structure of the QCD vacuum. It is shown that this APP mechanism gives the dominant contribution to the inclusive pion production for the  $x_F \gtrsim 0.1$  in the few GeV region for the transverse momentum of the pion. Furthermore, the APP violates factorization in the inclusive pion production and allows one to calculate the cross section without any fragmentation functions. The APP mechanism could be extended to the other pseudoscalar mesons. In the case of K- and  $\eta$ -meson production, it is needed to introduce the suppression for the chromomagnetic interaction due to the large mass of the s-quark. Additionally, one should include the effect from the  $\eta - \eta'$  mixing in the consideration of  $\eta$ -meson production. It is evident that the APP mechanism should give the contribution to other observables in the pseudoscalar meson production in high energy reactions with hadrons. We should emphasize that the anomalous quark-gluon-pion coupling, Eq.5, flips the quark spin. Therefore, it should contribute to the large single spin asymmetries (SSA) in the inclusive pseudoscalar meson production observed in the different high energy reactions with hadrons. The other longstanding problem is the large transverse polarization of  $\Lambda$  polarization observed in different high energy reactions (see discussion in [26, 27]). In our approach, such polarization might come from quark spin-flip in the K-meson production by the APP-type mechanism, e.g.  $u + q(g) \to K^+ + s + q(g)$ , and following fragmentation of the strange quark to  $\Lambda$ . In this case, the SSA in the K-meson production and the transverse polarization of  $\Lambda$  should be deeply related to each other. However, the calculation of SSA is a much more complicated task due to the loop diagrams contribution even in the leading order. The APP mechanism is completely different from the mechanism related to the first term in Eq.6 which was considered in [3, 6] at the partonic level. However, at the hadron level one should include the fragmentation function into consideration. It is clear that such a contribution to the pseudoscalar meson inclusive cross section should have approximately the same  $x_F$  dependency as the pQCD contribution which cannot solve the problem of the pion production at large  $x_F$ . Our separate task is to study the effects of the APP mechanism to the inclusive production of the pseudoscalar mesons in high energy heavy ion collision, in particular, to the so-called nuclear modification factor  $R_{AA}$ . In the case of pQCD production, the size of the pion is fixed by the fragmentation dynamics and should be rather large,  $R_{\pi}^{pQCD} \approx 1$ fm. For the APP mechanism the size of the pion is small,  $R_{\pi}^{APP} \approx \rho_c \approx 0.3$  fm. It follows that the final state interaction cross section of the pion in the nuclear matter for the APP process is about one order of magnitude smaller in comparison with the pQCD case. Therefore, the pQCD and the APP mechanisms should have different nuclear dependency. This difference, for example, might be responsible for the peak in  $R_{AA}$  observed at RHIC and LHC in the few GeV region for transverse momentum of the final mesons. The work in these directions is now in progress.

#### IV. ACKNOWLEDGMENTS

We would like to thank A.E. Dorokhov and S.B. Gerasimov for useful discussions. We are very grateful to Umberto D'Alesio for providing us with the tables of experimental data for the cross section of the inclusive  $\pi^0$  production in different high energy reactions. This work was partially supported by the National Natural Science Foundation of China (Grant No. 11035006 and 11175215), and by the Chinese Academy of Sciences visiting professorship for senior international scientists (Grant No. 2013T2J0011). This research was also supported in part by the Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education(2013R1A1A2009695)(HJL).

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- [27] M. Anselmino, D. Boer, U. D'Alesio and F. Murgia, Int. J. Mod. Phys. A 18, 1237 (2003)
- [28] We will neglect high order terms. Their contribution to the cross section is expected to be suppressed in a large  $N_c$  limit by a factor of  $1/N_c$ , because  $F_{\pi} \sim \sqrt{N_c}$ . Additionally, due to increasing of the number of the final particles, it should be suppressed at large  $x_F$  in compar-

ison with a leading contribution.

- [29] The production cross section of  $\pi^+$  ( $\pi^-$ ) in forward *u*-(*d*-) quark scattering is twice larger.
- [30] The pion production by our nonperturbative mechanism in the quark-gluon scattering is scaled by the same color factor 9/4 as in the pQCD case. Therefore, the inclusion into consideration of the quark-gluon scattering does not influence our conclusion. We also don't consider the pQCD contribution coming from the  $gg \rightarrow gg$  subprocess, because it gives the sizable contribution only at very small value of z. The pQCD contribution from the gluonic subprocess  $gg \rightarrow gg$  is expected to be small in the large  $x_F$  region even for the RHIC energies [23].
- [31] We do not consider the possible effect coming from the running dynamical gluon mass.
- [32] The  $x_F$  is part of the initial proton momentum carried by the final pion in the center of mass system in the longitudinal direction.