arXiv:1503.05727v3 [hep-ph] 14 May 2015

Light Quarkonium - Glueball Mixing from a Holographic QCD

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(Dated: October 9, 2018)

We study the mixing structure of isospin-singlet scalars, the light quarkonium ($\bar{q}q$) and glueball (gg) in two-flavor QCD, based on a holographic model of bottom-up hard-wall type. In the model the pure quarkonium and glueball states are unambiguously defined in terms of the different $U(1)_A$ charges in the restoration limit of the chiral $U(2)_L \times U(2)_R$ symmetry, in which the quarkonium gets massless as the chiral partner of the pion. Hence the $\bar{q}q$-gg mixing arises in the presence of the nonzero chiral condensate or pion decay constant. At the realistic point where the pion decay constant and other hadron masses reach the observed amount, we predict the tiny mixing between the lightest quarkonia and glueball: The smallness of the mixing is understood by the slightly small ratio of the chiral and gluon condensate scales. The low-lying two scalar masses are calculated to be $\simeq 1.25$ GeV and $\simeq 1.77$ GeV, which are compared with masses of $f_0(1370)$ and $f_0(1710)$. Our result implies that $f_0(1710)$ predominantly consists of glueball.

I. INTRODUCTION

The spectrum structure of the low-lying scalars around the mass 1 GeV still remains unsolved in the low-energy QCD, and has currently been pursued extensively by several approaches. Of particular interest are the isospin-singlet scalars, denoted as f_0's in the quark model [1], since they can arise as mixtures having the same quantum number $J^{PC} = 0^{++}$, such as the light two-flavor (u, d) quarkonium ($\bar{q}q$), glueball (gg). Understanding such a rich low-lying isoscalar spectrum is therefore of great importance, which is tied to revealing some part of the dynamics of low-energy QCD.

The straightforward investigation for the mixing structure has so far been performed on the full lattice QCD simulations [2], which focuses on the mixing $\bar{q}q$ and gg states, and also has been made by using QCD sum rules (Ref. [3] for a recent review). The quantitative estimate on the mixing angle is, however, still less accurate so that one cannot say anything about the constituent structure. On the one hand, other recent approaches based on phenomenological hadron models have attempted to give some insight on the $\bar{q}q$-gg mixing. By performing global fit to model parameters with use of the phenomenological inputs such as currently observed scalar-decay properties, it has been indicated that $f_0(1710)$ is almost constructed from the pure glueball state [4]. Still, however, the issue is controversial so it needs more indication from some different approaches.

In this paper, we study the mixing between the quarkonium and glueball states based on the gauge-gravity duality [5, 6], so-called holographic QCD. We employ a holographic model proposed in Ref. [7], an improved version of bottom-up hard-wall type proposed in Refs. [8, 9]. The model has succeeded in reproducing the characteristic features of QCD, such as the observed meson interaction properties as well as the ultraviolet-asymptotic behavior of QCD Green functions. What should be remarked in the model of [7] is, in particular, simultaneous incorporation of the five-dimensional bulk fields dual to the gluon and the chiral condensate operators, $G^2_{\mu\nu}$ and $\bar{q}q$. As emphasized in [7], one can thereby reproduce the ultraviolet scaling of QCD current correlators including terms along with the condensate of $G^2_{\mu\nu}$ as well as the leading logarithmic term and the chiral condensate term. The isospin-singlet scalars, arising as fluctuation modes around the condensates in the holographic bulk, thus include both the quarkonium and glueball states, hence the mixing among them can be evaluated straightforwardly.

We analyze the two-flavor QCD case, in which the chiral $U(2)_L \times U(2)_R$ symmetry is spontaneously broken down to the vectorial one $U(2)_V$ via the nonzero vacuum expectation value of a bulk-scalar dual to the $\bar{q}q$, accordingly due to the nonzero pion decay constant f_π. In the model the pure quarkonium and glueball states are unambiguously defined in terms of the different $U(1)_A$ charges in the restoration limit of the chiral $U(2)_L \times U(2)_R$ symmetry ($\langle\bar{q}q\rangle$ and $f_\pi \to 0$), in which the quarkonium gets massless, reflecting the chiral partner of the pion. Once the chiral condensate develops from zero, the $\bar{q}q$-gg mixing is turned on and grows monotonically with the chiral condensate. At the realistic point where f_π and masses of other mesons such as the vector/axialvector mesons reach the desired amount, we find the tiny mixing between the lightest quarkonium and glueball, which is consistent with the large N_c picture on that of basis the holographic model has been established. This is the definite prediction obtained without any phenomenological inputs such as observed isospin-singlet scalar decay properties, in contrast to other effective-hadron model approaches [4, 10–12]. We further provide a new insight on the small mixing in terms of generic quantities in QCD: the smallness of the mixing is understood by the somewhat small ratio of the chiral and gluon condensate scales at the realistic point.

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At the realistic point the lowest-lying two scalar masses are calculated to be $\simeq 1.25$ GeV and $\simeq 1.77$ GeV, which are compared with masses of $f_0(1370)$ and $f_0(1710)$. The lightest isospin-singlet scalar is almost completely degenerate with the isospin-triplet scalar, a_0 meson, which is due to the extremely-small mass split induced from the vanishingly small mixing among the isospin-singlet scalars. These two lower masses $\simeq 1.2$ GeV can actually be lifted up to the desired value around $\simeq 1.3-1.4$ GeV, when the mixing with a four-quark state would be taken into account [13].

Our result thus implies that $f_0(1710)$ predominantly consists of glueball with the mass around $\simeq 1.7-1.8$ GeV, in accord with the pure glueball mass estimate by lattice simulations [14], a recent study based on a holographic QCD of top-down type [15] and a different approach based on a phenomenological model [4].

This paper is organized as follows: In Sec. II we give a brief review of the holographic model in Ref. [7] and summarize things necessary for analysis of the isospin-singlet scalar mixing. In Sec. III we define the pure quarkonium and glueball states and introduce effective interactions describing the mixing of them deduced from the present holographic model. The effective mass matrix, obtained by keeping a few low-lying scalars, is then evaluated in Sec. IV. Summary of this paper is given in Sec. V.

II. MODEL: PRELIMINARIES

We begin by reviewing the holographic model proposed in Ref. [7] and list some equations and formulas necessary for the later discussions.

The model in Ref. [7] is based on deformations of a bottom-up approach for successful holographic dual of QCD [8, 9]. The model we shall employ is described as $U(2)_L \times U(2)_R$ gauge theory which is defined on the five-dimensional anti-de-Sitter (AdS) space-time. The five-dimensional space-time is characterized by the metric $ds^2 = g_{MN}dx^M dx^N = (L/z)^2 \left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$ with $\eta_{\mu\nu} = \mathrm{diag}[1,-1,-1,-1]$. Here, M and N (μ and ν) represent five-dimensional (four-dimensional) Lorentz indices, and L denotes the curvature radius of the AdS background. The fifth direction, denoted as z, is compactified on an interval extended from the ultraviolet (UV) brane located at $z=\epsilon$ to the infrared (IR) brane at $z=z_m$, i.e., $\epsilon \le z \le z_m$. The UV cutoff ϵ will be taken to be 0 after all calculations are done.

Besides the bulk left- (L_M) and right- (R_M) gauge fields, we introduce two bulk scalars Φ_{qq} and Φ_{gg}. The Φ_{qq} transforms as a bifundamental representation field under the $U(2)_L \times U(2)_R$ gauge symmetry, and therefore is dual to the quark bilinear operator $\bar{q}q$ having the $U(1)_A$ charge equal to 2. The Φ_{gg} is, on the other hand, chiral and $U(1)_A$-singlet, dual to the gluon condensate operator $G^2_{\mu\nu}$. The mass-parameter for these two bulk scalars $M_{\Phi_{qq}}$ and $M_{\Phi_{gg}}$ are then holographically given as $M^2_{\Phi_{qq}} = -3/L^2$ and $M^2_{\Phi_{gg}} = 0$, which reflect the scaling dimensions for $\bar{q}q$ and $G^2_{\mu\nu}$, respectively.

The action of the model is thus written as [7]

$$S_5 = S_{\rm bulk} + S_{\rm UV} + S_{\rm IR}\,, \tag{1}$$

where

$$\begin{aligned} S_{\rm bulk} = \int d^4x \int_\epsilon^{z_m} dz \sqrt{g}\frac{1}{g_5^2}\, e^{cg_5^2\Phi_{gg}} \Bigg[&\frac{1}{2}\partial_M \Phi_{gg}\partial^M\Phi_{gg} \\ &+ {\rm Tr}[D_M\Phi_{qq}^\dagger D^M\Phi_{qq} - M^2_{\Phi_{qq}}\Phi^\dagger_{qq}\Phi_{qq}] \\ &-\frac{1}{4}{\rm Tr}[L_{MN}L^{MN} + R_{MN}R^{MN}]\Bigg]\,, \end{aligned} \tag{2}$$

$$S_{\rm IR} = \int d^4x \int_\epsilon^{z_m} dz\, \delta(z-z_m)\sqrt{-\tilde{g}}\,\mathcal{L}_{\rm IR}\,, \tag{3}$$

with the boundary-induced metric $\tilde{g}_{\mu\nu} = (L/z)^2\eta_{\mu\nu}$. The covariant derivative acting on Φ_{qq} in Eq.(2) is defined as $D_M\Phi_{qq} = \partial_M\Phi_{qq} + iL_M\Phi_{qq} - i\Phi_{qq}R_M$, where $L_M(R_M) \equiv L^a_M(R^a_M)T^a$ with $T^a = \sigma^a/\sqrt{2}$ ($a = 0,1,2,3$) being the generators of $SU(2)$ and $\sigma^0 = 1_{2\times 2}$ normalized as ${\rm Tr}[T^aT^b] = \delta^{ab}$. $L(R)_{MN}$ is the five-dimensional field strength which is defined as $L(R)_{MN} = \partial_M L(R)_N - \partial_N L(R)_M - i[L(R)_M, L(R)_N]$, and g is defined as $g = \det[g_{MN}] = (L/z)^{10}$. The UV boundary action $S_{\rm UV}$ in Eq.(1) plays a role of the UV regulator to absorb the UV-divergent ϵ terms arising from the five-dimensional bulk dynamics, which we will not specify. The IR boundary action $S_{\rm IR}$ is introduced so as to realize minimization of the bulk potential by nonzero chiral condensate [7] with the IR Lagrangian:

$$\mathcal{L}_{\rm IR} = -e^{cg_5^2\langle\Phi_{gg}\rangle}\left(-m_b^2{\rm Tr}[|\Phi_{qq}|^2] + \lambda{\rm Tr}[|\Phi_{qq}|^2]^2\right)\,. \tag{4}$$

The gauge coupling g_5 and a parameter c appearing in the action are fixed by matching with the UV asymptotic behavior of vector/axialvector current correlators as [7]

$$\frac{L}{g_5^2} = \frac{N_c}{12\pi^2}, \quad c = -\frac{L}{16\pi g_5^2} = -\frac{N_c}{192\pi^3}\,, \tag{5}$$

where the latter has been determined from terms damping along with the gluon condensate.

A. Vacuum expectation values of scalars

The bulk scalar fields Φ_{qq} and Φ_{gg} are parametrized as

$$\begin{aligned}\Phi_{qq}(x,z) = &\frac{1}{\sqrt{2}}\left(v_{qq}(z) + \sigma_{qq}(x,z) + a_0^i(x,z)\sigma^i\right) \\ &\times e^{i\pi^a(x,z)\sigma^a/v_{qq}(z)}, \qquad (i=1,2,3)\end{aligned} \tag{6}$$

$$\chi(x,z) \equiv e^{\frac{cg_5^2}{2}\Phi_{gg}} = v_\chi(z)\, e^{\sigma_{gg}(x,z)}, \tag{7}$$

with the vacuum expectation values, $v_{qq} = \sqrt{2}\langle\Phi_{qq}\rangle$ and $v_\chi = \langle\chi\rangle = e^{\frac{cg_5^2}{2}\langle\Phi_{gg}\rangle}$. We hereafter disregard pion fields

π^a which will not be relevant for the present study. Plugging these expansion forms into the bulk action $S_{\rm bulk}$ Eq.(2) we find the coupled equations of motion for the vacuum expectation values v_{qq} and v_χ,

$$v_\chi^{-2}\partial_z\left(\frac{1}{z^3}v_\chi^2\partial_z v_{qq}\right)+\frac{3}{z^5}v_{qq}=0\,,$$
$$v_\chi^{-1}\partial_z\left(\frac{1}{z^3}\partial_z v_\chi\right)+2s^2\,L^2\left(\frac{(\partial_z v_{qq})^2}{z^3}+\frac{3v_{qq}^2}{z^5}\right)=0 \quad (8)$$

where

$$s\equiv\frac{cg_5^2}{2L}=-\frac{1}{32\pi}\simeq-0.01\,. \quad (9)$$

The last equality of the second line in Eq.(9) follows from Eq.(5). The boundary conditions for v_{qq} and v_χ are chosen (in the limit where the sources for $\bar{q}q$ and $G_{\mu\nu}^2$ are turned off) as [7]

$$v_{qq}(\epsilon)=0\,,\qquad v_{qq}(z_m)=\frac{\xi}{L}\,,$$
$$v_\chi(\epsilon)=1\,,\qquad v_\chi(z_m)=1+G\,, \quad (10)$$

with the IR boundary values ξ and G which are holographically related to the chiral and gluon condensates as shown in Ref. [7]. Looking at the numerical value of s in Eq.(9), as done in Ref. [7] one may neglect the second term of $\mathcal{O}(s^2)=\mathcal{O}(10^{-4})$ in the equation of motion for v_χ in Eq.(8) to decouple the v_{qq} and v_χ. Then the approximate equations of motion can be analytically solved to be [7]

$$v_{qq}\simeq\frac{\xi}{L}(1+G)\cdot\frac{(z/z_m)^3}{1+G(z/z_m)^4}\,,$$
$$v_\chi\simeq1+G(z/z_m)^4\,, \quad (11)$$

with the boundary conditions in Eq.(10) incorporated. [We have checked that, by numerically solving the coupled equation in Eq.(8) keeping the s^2 term, this approximation is satisfied within a few percent level in a wide range of the parameter space we will consider later.]

As noted above, the nonzero value of the chiral condensate parameter ξ can be realized by adjusting the IR potential in $S_{\rm IR}$ of Eq.(1): putting the solutions in Eq.(11) into the action S_5 in Eq.(1) and performing the minimization of the potential energy, one finds the stationary condition [7]

$$\xi^2=\frac{1}{\lambda}\left[m_b^2L-\frac{N_c}{12\pi^2}\left(3-\frac{4G}{1+G}\right)\right]\,. \quad (12)$$

B. Scalar sector

The five-dimensional bulk scalar fields $\sigma_{qq}(x,z)$, $\sigma_{gg}(x,z)$ and $a_0^i(x,z)$ in Eqs.(6) and (7) can be expanded in terms of normalizable modes, corresponding to QCD scalar mesons:

$$\sigma_{qq}(x,z)=\sum_n\sigma_{qq}^{(n)}(x)f_{\sigma_{qq}}^{(n)}(z)\,,$$
$$\sigma_{gg}(x,z)=\sum_n\sigma_{gg}^{(n)}(x)f_{\sigma_{gg}}^{(n)}(z)\,,$$
$$a_0^i(x,z)=\sum_n a_0^{i(n)}(x)f_{a_0}^{(n)}(z)\,, \quad (13)$$

with the wavefunctions $f_{\sigma_{qq}}^{(n)},f_{\sigma_{gg}}^{(n)},f_{a_0}^{(n)}$ having the normalizable UV boundary conditions,

$$f_{\sigma_{qq}}^{(n)}(\epsilon)=0\,,\qquad f_{a_0}^{(n)}(\epsilon)=0\,,\qquad f_{\sigma_{gg}}^{(n)}(\epsilon)=0\,. \quad (14)$$

Using Eq.(13) we expand the bulk scalar sector of the action $S_{\rm bulk}$ in Eq.(1) to find the induced effective Lagrangian in four-dimension:

$$\mathcal{L}_{\rm scalar}=\mathcal{L}_{\sigma_{qq}^2}+\mathcal{L}_{a_0^2}+\mathcal{L}_{\sigma_{gg}^2}+\mathcal{L}_{\sigma_{qq}\sigma_{gg}}\,, \quad (15)$$

where

$$\mathcal{L}_{\sigma_{qq}^2}=\sum_n\frac{1}{2}A_{\sigma_{qq}}^n\left(\partial_\mu\sigma_{qq}^{(n)}\right)^2-\frac{1}{2}B_{\sigma_{qq}}^n(\sigma_{qq}^{(n)})^2\,,$$
$$\mathcal{L}_{a_0^2}=\sum_n\frac{1}{2}A_{a_0}^n\left(\partial_\mu a_0^{i(n)}\right)^2-\frac{1}{2}B_{a_0}^n(a_0^{i(n)})^2\,,$$
$$\mathcal{L}_{\sigma_{gg}^2}=\sum_n\frac{1}{2}A_{\sigma_{gg}}^n\left(\partial_\mu\sigma_{gg}^{(n)}\right)^2-\frac{1}{2}B_{\sigma_{gg}}^n(\sigma_{gg}^{(n)})^2\,,$$
$$\mathcal{L}_{\sigma_{qq}\sigma_{gg}}=\sum_{m,n}C_{\sigma_{qq}\sigma_{gg}}^{(m,n)}\sigma_{qq}^{(m)}\sigma_{gg}^{(n)}\,, \quad (16)$$

with

$$A_{\sigma_{qq}}^n=\frac{N_c}{12\pi^2}\int\frac{dz}{z}2\left(\frac{L}{z}\right)^2v_\chi^2(f_{\sigma_{qq}}^{(n)})^2\,,$$
$$A_{a_0}^n=\frac{N_c}{12\pi^2}\int\frac{dz}{z}2\left(\frac{L}{z}\right)^2v_\chi^2(f_{a_0}^{(n)})^2\,,$$
$$A_{\sigma_{gg}}^n=\frac{N_c}{12\pi^2}\cdot\frac{1}{s^2}\cdot\int\frac{dz}{z}\left(\frac{L}{z}\right)^2v_\chi^2(f_{\sigma_{gg}}^{(n)})^2\,,$$
$$B_{\sigma_{qq}}^n=\frac{N_c}{12\pi^2}\int\frac{dz}{z}2\left(\frac{L}{z}\right)^2v_\chi^2\times\left[(\partial_z f_{\sigma_{qq}}^{(n)})^2-\frac{3}{z^2}(f_{\sigma_{qq}}^{(n)})^2\right]\,,$$
$$B_{a_0}^n=\frac{N_c}{12\pi^2}\int\frac{dz}{z}2\left(\frac{L}{z}\right)^2v_\chi^2\times\left[(\partial_z f_{a_0}^{(n)})^2-\frac{3}{z^2}(f_{a_0}^{(n)})^2\right]\,,$$
$$B_{\sigma_{gg}}^n=\frac{N_c}{12\pi^2}\cdot\frac{1}{s^2}\cdot\int\frac{dz}{z}\left(\frac{L}{z}\right)^2v_\chi^2(\partial_z f_{\sigma_{gg}}^{(n)})^2\,,$$
$$C_{\sigma_{qq}\sigma_{gg}}^{(m,n)}=\frac{N_c}{12\pi^2}\int dz\,4\left(\frac{L}{z}\right)^3v_\chi^2f_{\sigma_{gg}}^{(n)}\times\left[-(\partial_z v_{qq})(\partial_z f_{\sigma_{qq}}^{(m)})+\frac{3}{z^2}v_{qq}f_{\sigma_{qq}}^{(m)}\right]\,. \quad (17)$$

In deriving the effective Lagrangian we have used the equation of motion for v_{qq} and v_χ in Eq.(8) and imposed the IR boundary conditions for $f_{\sigma_{qq}}^{(n)}$, $f_{a_0}^{(n)}$ and $f_{\sigma_{gg}}^{(n)}$ so as to eliminate the IR boundary terms in quadratic order of fields as follows:

$$\partial_z f^{(n)}_{\sigma_{qq}}(z)|_{z=z_m} = \left[-\frac{24\pi^2}{N_c}\lambda\xi^2 + 3 - \frac{4G}{1+G}\right] f^{(n)}_{\sigma_{qq}}(z_m)\,,$$
$$\partial_z f^{(n)}_{a_0}(z)|_{z=z_m} = \left[-\frac{24\pi^2}{N_c}\lambda\xi^2 + 3 - \frac{4G}{1+G}\right] f^{(n)}_{a_0}(z_m)\,,$$
$$f^{(n)}_{\sigma_{gg}}(z_m) = 0\,, \qquad (18)$$

where use has been made of the stationary condition for v_{qq} in Eq.(12). It should be noted that the $\bar{q}q$-gg mixing strength $C^{(m,n)}_{\sigma_{qq}\sigma_{gg}}$ in Eq.(17) is proportional to the chiral condensate $\sim v_{qq}$. This implies that the $\bar{q}q$-gg mixing is turned off when the chiral symmetry is restored, where one can realize the definitely pure $\bar{q}q$ and gg-isospin singlet-scalar states, as will be discussed in the next section.

C. Vector and axialvector sectors

The five-dimensional vector and axial-vector gauge fields V_M and A_M are defined as

$$V_M = \frac{L_M + R_M}{\sqrt{2}}\,, \qquad A_M = \frac{L_M - R_M}{\sqrt{2}}\,. \qquad (19)$$

It is convenient to work with the gauge-fixing $V_z = A_z \equiv 0$ and take the boundary conditions $V_\mu(x,\epsilon) = v_\mu(x)$, $A_\mu(x,\epsilon) = a_\mu(x)$ and $\partial_z V_\mu(x,z)|_{z=z_m} = \partial_z A_\mu(x,z)|_{z=z_m} = 0$, where $v_\mu(x)$ and $a_\mu(x)$ correspond to sources for the vector and axial-vector currents, respectively. We then solve the equations of motion for (the transversely polarized components of) $V_\mu(x,z)$ and $A_\mu(x,z)$ and substitute the solutions back into the action in Eq.(1), to obtain the generating functional $W[v_\mu, a_\mu]$ holographically dual to QCD. Then we obtain the vector and the axial-vector current correlators, which are defined as

$$i\int d^4x e^{iqx} \langle 0|\, \mathrm{T}\, J^{a\mu}_{V,A}(x)\, J^{b\nu}_{V,A}(0)\, |0\rangle$$
$$= \delta^{ab}\left(\frac{q^\mu q^\nu}{q^2} - \eta^{\mu\nu}\right)\Pi_{V,A}(-q^2)\,, \qquad (20)$$

with the currents

$$J^{a\mu}_V = \bar{q}\left(\frac{T^a}{\sqrt{2}}\right)\gamma^\mu q,$$
$$J^{a\mu}_A = \bar{q}\left(\frac{T^a}{\sqrt{2}}\right)\gamma^\mu\gamma_5 q. \qquad (21)$$

$\Pi_V(Q^2)$ and $\Pi_A(Q^2)$ (where $Q \equiv \sqrt{-q^2}$ is the Euclidean momentum) are expressed as

$$\Pi_V(Q^2) = \frac{N_c}{12\pi^2}\frac{\partial_z V(Q^2,z)}{z}\Bigg|_{z=\epsilon\to 0}\,,$$
$$\Pi_A(Q^2) = \frac{N_c}{12\pi^2}\frac{\partial_z A(Q^2,z)}{z}\Bigg|_{z=\epsilon\to 0}\,, \qquad (22)$$

where the vector and axial-vector profile functions $V(Q^2,z)$ and $A(Q^2,z)$ are defined as $V_\mu(q,z) = v_\mu(q)V(q^2)$ and $A_\mu(q,z) = a_\mu(q)A(q^2)$ with the Fourier transforms of $v_\mu(x)$ and $a_\mu(x)$. These profile functions satisfy the following equations:

$$\left[-Q^2 + \omega^{-1}\partial_z\omega\partial_z\right]V(Q^2,z) = 0, \qquad (23)$$

$$\left[-Q^2 + \omega^{-1}\partial_z\omega\partial_z - 2\left(\frac{L}{z}\right)^2 v_{qq}^2\right]A(Q^2,z) = 0, \quad (24)$$

$$\omega \equiv \frac{L}{z}v_\chi^2\,, \qquad (25)$$

with the boundary conditions $V(Q^2,z)|_{z=\epsilon\to 0} = A(Q^2,z)|_{z=\epsilon\to 0} = 1$ and $\partial_z V(Q^2,z)|_{z=z_m} = \partial_z A(Q^2,z)|_{z=z_m} = 0$.

The vector and axial-vector current correlators, Π_V and Π_A, can be expanded in terms of towers of the vector and axial-vector resonances. We then identify poles for $\Pi_{V,A}$ as the infinite towers of ρ and a_1 mesons. Their masses, m_{ρ_n} and $m_{(a_1)_n}$, are calculated by solving the eigenvalue equations for the vector and axial-vector profile functions [7]:

$$\left[m^2_{\rho_n} + \omega^{-1}\partial_z\omega\partial_z\right]V_n(z) = 0, \qquad (26)$$

$$\left[m^2_{(a_1)_n} + \omega^{-1}\partial_z\omega\partial_z - 2\left(\frac{L}{z}\right)^2 v_{qq}^2\right]A_n(z) = 0, \quad (27)$$

with the same boundary conditions $V_n(\epsilon) = A_n(\epsilon) = 0$ and $\partial_z V_n(z)|_{z=z_m} = \partial_z A_n(z)|_{z=z_m} = 0$. Note that the ρ_n and $(a_1)_n$ meson masses get degenerate when the chiral condensate $\sim v_{qq} \sim \xi$ is sent to zero.

The pion decay constant f_π is expressed in terms of Π_V and Π_A as $f_\pi^2 = \Pi_V(0) - \Pi_A(0)$. We can express this f_π by using Eq.(22) as [7]:

$$f_\pi^2 = -\frac{N_c}{12\pi^2}\frac{\partial_z A(Q^2,z)}{z}\Bigg|_{z=\epsilon\to 0}\,. \qquad (28)$$

The vector and axialvector sectors are completely fixed once the parameters ξ, G and z_m are chosen to be certain values. In Refs. [7] and [16], the optimal values are found so as to reproduce the experimental values of f_π, masses of the lowest vector and axialvector mesons $m_{\rho_1} \equiv m_\rho$ and $m_{(a_1)_n} \equiv m_{a_1}$: the optimal numbers leading to the realistic point are

$$\xi \simeq 3.1\,, \qquad G \simeq 0.25\,, \qquad z_m^{-1} \simeq 347\,\mathrm{MeV}\,. \quad (29)$$

This parameter choice yields $f_\pi \simeq 92$ MeV, $m_\rho \simeq 775$ MeV, $m_{a_1} \simeq 1264$ MeV [1] and predicts

$$\langle -\bar{q}q\rangle \simeq (277\,\mathrm{MeV})^3\,, \qquad \langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle^{1/4} \simeq (331\,\mathrm{MeV})^4\,, \qquad (30)$$

in agreement with values estimated in Refs. [17, 18] [1].

[1] Our estimate on the chiral and gluon condensates is also consistent with values obtained based on other QCD calculations [19]

III. REALIZATION OF PURE $\bar{q}q$ AND gg STATES AND THEIR MIXING

As noted above, in the present model the $\bar{q}q$-gg mixing term is proportional to the chiral condensate $\sim v_{qq}$ [See Eq.(17)]. When one considers the ideal limit where the chiral symmetry is restored ($\langle\bar{q}q\rangle \to 0$, $f_\pi \to 0$), therefore, the pure $\bar{q}q$ and gg-states can be unambiguously defined. In the present model this limit can actually be achieved by sending the chiral-condensate parameter ξ to zero [See Eq.(11)].

A. Meson mass spectra in the restoration limit of the chiral symmetry

In the chiral-restoration limit, the masses of the pure $\bar{q}q$ and gg isospin-singlet scalars are determined by the following decoupled eigenvalue equations derived from $\mathcal{L}_{\rm scalar}$ in Eq.(15) with $\xi = 0$:

$$\left[v_\chi^{-2}\partial_z\left(\frac{1}{z^3}v_\chi^2\partial_z\right) + \frac{(m^0_{qq(n)})^2}{z^3} + \frac{3}{z^5}\right][f^{(n)}_{\sigma_{qq}}]^0 = 0\,,$$
$$\left[v_\chi^{-2}\partial_z\left(\frac{1}{z^3}v_\chi^2\partial_z\right) + \frac{(m^0_{gg(n)})^2}{z^3}\right][f^{(n)}_{\sigma_{gg}}]^0 = 0\,, \qquad (31)$$

where the superscript 0 denotes $\xi = 0$ ($\langle\bar{q}q\rangle = f_\pi = 0$). Note that, in the chiral-restoration limit the wavefunction for the isospin-triplet scalar $a_0^{(n)}$ obeys the same mass eigenvalue equation as that of the isospin-singlet scalar $\sigma_{qq}^{(n)}$. Since the same boundary condition is imposed on $[f^{(n)}_{\sigma_{qq}}]^0$ and $[f^{(n)}_{a_0}]^0$ as in Eqs.(14) and (18), the isospin-triplet and -singlet quarkonia are completely degenerate in the limit $\xi \to 0$ ($\langle\bar{q}q\rangle \to 0$ and $f_\pi \to 0$).

In addition, the vector and axialvector mesons satisfy the same eigenvalue equations in the limit $\xi \to 0$ [See Eqs.(26) and (27)], so their masses are degenerate as well. Note the explicit independence of ξ on the eigenvalue equation for ρ_n in Eq.(26). We may therefore fix the parameters G and z_m to the optimal values in Eq.(29) so that the ρ meson mass already reaches the realistic value in the chiral-restoration limit $\xi \to 0$, i.e.,

$$m^0_\rho = m^0_{a_1} = m_\rho \simeq 775\,\text{MeV}\,, \qquad (32)$$

where, again, the superscript 0 stands for $\xi = 0$.

Taking $\xi = 0$, $G \simeq 0.25$ and $z_m^{-1} \simeq 347$ MeV, we thus calculate the scalar meson masses in the chiral restoration limit. [One should note that the dependence of the potential parameter λ in the IR boundary condition Eq.(18) goes away when $\xi = 0$.] The lowest-three scalar masses below 2 GeV are found to be

$$m^0_{qq(1)} = 0\,,$$
$$m^0_{qq(2)} \simeq 1782\,\text{MeV}\,,$$
$$m^0_{gg(1)} \simeq 1782\,\text{MeV}\,. \qquad (33)$$

The lightest quarkonia, for both isospin-singlet and -triplet scalars, thus become massless in the chiral-restoration limit, reflecting the chiral partner of pions.

B. Defining mixing between pure $\bar{q}q$ and gg states

Once the chiral condensate develops from zero, the isospin-singlet $\bar{q}q$ and gg states start to mix according to the mixing form given in $\mathcal{L}_{\sigma_{qq}\sigma_{gg}}$ in Eq.(16). After the scalar fields are canonically normalized, the scalar mass terms in Eq.(15) take the form

$$\mathcal{L}^m_{\rm scalar} = -\sum_n \frac{1}{2}\left[(m^{(n)}_{\sigma_{qq}})^2(\sigma^{(n)}_{qq})^2 + (m^{(n)}_{a_0})^2(a_0^{i(n)})^2\right] - \sum_n \frac{1}{2}(m^{(n)}_{\sigma_{gg}})^2(\sigma^{(n)}_{gg})^2 - \sum_{k,n}(m^{(k,n)}_{\sigma_{qg}})^2\sigma^{(k)}_{qq}\sigma^{(n)}_{gg}\,, \qquad (34)$$

where

$$(m^{(n)}_{\sigma_{qq}})^2 = \frac{B^n_{\sigma_{qq}}}{A^n_{\sigma_{qq}}}\,,$$
$$(m^{(n)}_{a_0})^2 = \frac{B^n_{a_0}}{A^n_{a_0}}\,,$$
$$(m^{(n)}_{\sigma_{gg}})^2 = \frac{B^n_{\sigma_{gg}}}{A^n_{\sigma_{gg}}}\,,$$
$$(m^{(k,n)}_{\sigma_{qg}})^2 = \frac{C^{(k,n)}_{\sigma_{qq}\sigma_{gg}}}{\sqrt{A^k_{\sigma_{qq}}}\sqrt{A^n_{\sigma_{gg}}}}\,. \qquad (35)$$

Note, however, that the scalar fields $\sigma^{(n)}_{qq}$ and $\sigma^{(n)}_{gg}$ are no longer purely $\bar{q}q$ and gg-states due to the nonzero mixing triggered by the nonzero ξ.

For the purpose of addressing the mixing between the pure $\bar{q}q$ and gg-states, we shall now propose an effective mixing term inspired by the present holographic QCD. We first replace the mixed wavefunctions $f^{(n)}_{\sigma_{qq}}$ and $f^{(n)}_{\sigma_{gg}}$ in the off-diagonal mass-squared element $(m^{(k,n)}_{\sigma_{qg}})^2$ of Eq.(35) with the pure $\bar{q}q$ and gg wavefunctions in the chiral-restoration limit ($\xi \to 0$), $[f^{(n)}_{\sigma_{qq}}]^0$ and $[f^{(n)}_{\sigma_{gg}}]^0$ in the decoupled equations in (31). Then, the off-diagonal mass-squared element $(m^{(k,n)}_{\sigma_{qg}})^2$ is modified as

$$(m^{(k,n)}_{\sigma_{qg}})^2 \to \left.\frac{C^{(k,n)}_{\sigma_{qq}\sigma_{gg}}}{\sqrt{A^k_{\sigma_{qq}}}\sqrt{A^n_{\sigma_{gg}}}}\right|_{[f^{(n)}_{\sigma_{qq}}]^0,[f^{(n)}_{\sigma_{gg}}]^0} = 2\sqrt{2}\,s\,\frac{\langle[f^{(n)}_{\sigma_{gg}}]^0\left(-\dot{\bar{v}}_{qq}[f^{(k)}_{\sigma_{qq}}]^0 + \frac{3\bar{v}_{qq}[f^{(k)}_{\sigma_{qq}}]^0}{z^2}\right)\rangle}{\sqrt{\langle[(f^{(k)}_{\sigma_{qq}}]^0)^2\rangle\langle[(f^{(n)}_{\sigma_{gg}}]^0)^2\rangle}}\,, \qquad (36)$$

where $\bar{v}_{qq} \equiv L\cdot v_{qq}$ and $\langle A\rangle \equiv \int dz(L/z)^3 v_\chi^2(z)A(z)$ for an arbitrary function $A(z)$. Thus the redefined $(m^{(k,n)}_{\sigma_{qg}})^2$

as in Eq.(36) is evaluated as the mixing amplitude of the pure $\bar{q}q$ state overlapped with the pure gg state, which are properly defined in the chiral-restoration limit, to be developed by nonzero ξ only through the nonzero v_{qq}.

On the other hand, we keep the diagonal elements $(m^{(n)}_{\sigma_{qq}})^2$ and $(m^{(n)}_{\sigma_{gg}})^2$ unchanged and assume that the wavefunctions in these elements satisfy the decoupled equations similar to Eq.(31) in the chiral-restoration limit, but with the different IR boundary condition for $\sigma^{(n)}_{qq}$ having nonzero ξ as in Eq.(18).

We have also computed the mass eigenvalues directly by solving the mixed equations, and checked that these drastic deformations do not make significant difference in the scalar mass spectrum. This implies that the mixed wavefunctions $f^{(n)}_{\sigma_{qq}}$ and $f^{(n)}_{\sigma_{gg}}$ are well saturated by the pure wavefunctions $[f^{(n)}_{\sigma_{qq}}]^0$ and $[f^{(n)}_{\sigma_{gg}}]^0$ in the scalar mass spectrum.

Even when the chiral condensate grows from zero, the isospin-triplet scalars $a_0^{i(n)}$ are still isolated as well as in the case of the chiral-restoration limit, except the different IR boundary condition with nonzero term $\lambda\xi^2$ in Eq.(18). We may fix the IR potential parameter λ so that the lowest a_0 mass is set to the desired value at the realistic point achieved by the parameter choice in Eq.(29). As done in Ref. [7], for the reference value of the a_0 mass we may choose $m_{a_0} \simeq 1.24$ GeV by taking into account that the mass can be lifted up to that of $a_0(1450)$ when the mixing with a four-quark state is incorporated [13]. Then, the optimal value of λ is fixed to be [7]

$$\lambda \equiv \frac{N_c}{(4\pi)^2} \cdot \kappa \,, \qquad \text{with} \qquad \kappa \simeq 1 \,. \tag{37}$$

IV. ANALYSIS OF THE MIXING STRUCTURE

We are now ready to discuss the mixing strength between the pure $\bar{q}q$ and gg states, which evolves from zero at the chiral-restoration limit ($\xi = 0$) to the realistic point ($\xi \simeq 3.1$) with the parameter choice in Eqs.(29) and (37). In Fig. 1 we first show the evolution of the diagonal mass-squared elements $(m^{(n)}_{\sigma_{qq}})^2$ and $(m^{(n)}_{\sigma_{gg}})^2$ with respect to f_π, in place of ξ. Here, we have only picked up the low-lying three mass values, $m^{(1)}_{\sigma_{qq}} (= m_{a_0})$, $m^{(2)}_{\sigma_{qq}}$ and $m^{(1)}_{\sigma_{gg}}$. The figure tells us that the lowest-$\bar{q}q$ scalar mass dramatically develops from zero at the chiral restoration point ($f_\pi = 0$) to the fixed $m_{a_0} \simeq 1.24$ GeV at the realistic point ($f_\pi \simeq 92\,\mathrm{MeV}$). The growth of the mass will actually be saturated around the value $\simeq 1.3$ GeV, even if one extremely takes the large f_π. The lowest-gg scalar mass $m^{(1)}_{\sigma_{gg}}$ is completely independent of f_π, while the mass of the first-excited $\bar{q}q$-state, $m^{(2)}_{\sigma_{qq}}$, goes beyond that of the ground state of the gg-scalar, to be above 2 GeV at the realistic point, but not to be above 3 GeV even when $f_\pi \to \infty$ limit.

In Fig. 2 we display the mixing strengths $(m^{(1,1)}_{\sigma_{qg}})^2$ and $(m^{(2,1)}_{\sigma_{qg}})^2$ as a function of f_π. We see from the figure that the mixing strengths are still small enough, even after reaching the realistic point, compared to the diagonal mass-squared values: the mass-squared matrix in unit of GeV^2 looks like

$$\mathbf{m}^2 \simeq \begin{pmatrix} (1.25)^2 & (0.14)^2 & 0 \\ (0.14)^2 & (1.77)^2 & -(0.12)^2 \\ 0 & -(0.12)^2 & (2.26)^2 \end{pmatrix} \,, \tag{38}$$

which acts on the vector $(\sigma^{(1)}_{qq}, \sigma^{(1)}_{gg}, \sigma^{(2)}_{qq})^T$. By diagonalizing this matrix, one obtains the masses of the low-lying two isospin-singlet scalars below 2 GeV,

$$m_1 \simeq 1.25\,\mathrm{GeV}\,, \qquad m_2 \simeq 1.77\,\mathrm{GeV}\,. \tag{39}$$

The lowest mass can be lifted up to the amount of the $f_0(1370)$ mass $\simeq 1.3 - 1.4$ GeV in a way similar to the $a_0(1450)$ case through mixing with a four-quark state [13]. Thus, compared to the diagonal element in Eq.(38), we find that the mixing strengths are negligibly small, in accord with the assumption made in Ref. [7] and the large N_c picture on which the holographic model has been based.

The smallness of the mixing can be understood by somewhat smaller chiral-condensate scale than the gluon condensate scale: these condensates are actually related involving the parameters ξ, G and z_m as [7]

$$\xi = \frac{32}{3\sqrt{3}} \frac{G}{1+G} \frac{\langle -\bar{q}q\rangle / z_m}{\langle \frac{\alpha_s}{\pi} G^2_{\mu\nu}\rangle} \,. \tag{40}$$

Taking $G \simeq 0.25$, $\langle \frac{\alpha_s}{\pi} G^2_{\mu\nu}\rangle \simeq (331\,\mathrm{MeV})^4$ and $z_m^{-1} \simeq 347$ MeV, we numerically evaluate ξ to write

$$\xi \simeq 5 \times \frac{\langle -\bar{q}q\rangle}{(331\,\mathrm{MeV})^3} \simeq 5 \times \left(\frac{\Lambda_{\bar{q}q}}{\Lambda_{gg}}\right)^3 \,, \tag{41}$$

with the chiral and gluon condensate scales $\Lambda_{\bar{q}q} \equiv \langle -\bar{q}q\rangle^{1/3}$ and $\Lambda_{gg} \equiv \langle \frac{\alpha_s}{\pi} G^2_{\mu\nu}\rangle^{1/4}$. At the realistic point, from Eq.(30) we have $(\Lambda_{\bar{q}q}/\Lambda_{gg})^3 \simeq (0.84)^3 \simeq 0.6$, in accordance with the optimal value of ξ in Eq.(29). If $(\Lambda_{\bar{q}q}/\Lambda_{gg})^3$ was as much as/more than of $\mathcal{O}(1)$, ξ would get larger to enhance the mixing strengths to be comparable with the diagonal element in Eq.(38), as expected from Fig. 2.

Our result thus implies that $f_0(1710)$ is predominantly constructed from the glueball state with the mass around $\simeq 1.7 - 1.8$ GeV, in agreement with the pure glueball mass estimate by lattice simulations [14], a recent study based on a holographic QCD of top-down type [15] and a different approach based on a phenomenological model [4].

V. SUMMARY AND DISCUSSION

In this paper, we studied the mixing structure of isospin-singlet scalars, the light quarkonium ($\bar{q}q$) and glueball (gg) in two-flavor QCD, based on a holographic

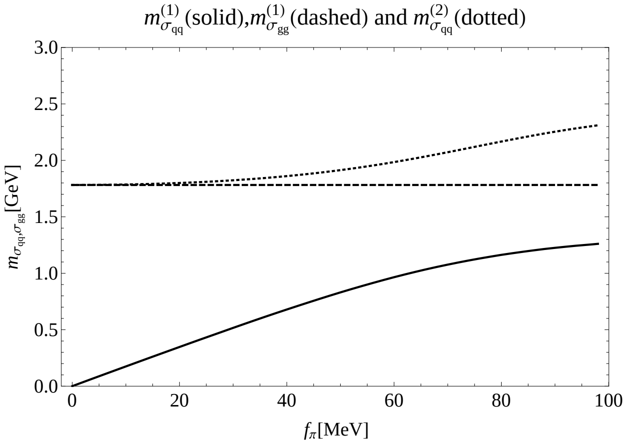

FIG. 1. The diagonal mass-squared elements $m^{(1)}_{\sigma_{qq}} (= m_{a_0})$ (solid curve), $m^{(2)}_{\sigma_{qq}}$ (dotted curve) and $m^{(1)}_{\sigma_{gg}}$ (dashed curve) versus the pion decay constant f_π with the parameter G, z_m^{-1} and λ fixed to be the optimal values, $G = 0.25$, $z_m^{-1} = 347$ MeV and $\lambda = 3/(4\pi)^2$ (See Eqs.(29) and (37)).

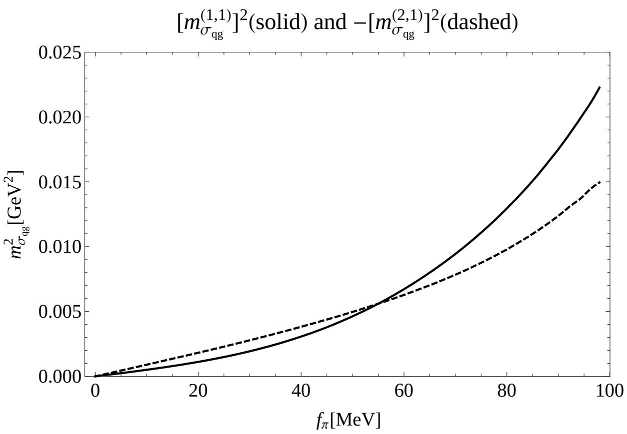

FIG. 2. The same for the mixing strengths $(m^{(1,1)}_{\sigma_{qg}})^2$ (solid curve) and $[-(m^{(2,1)}_{\sigma_{qg}})^2]$ (solid curve) as Fig. 1.

model of bottom-up hard-wall type. Based on that model, we have demonstrated that the $\bar{q}q$-gg mixing takes place due to the nonzero chiral condensate. In the model, the pure quarkonium and glueball states are unambiguously defined in terms of the different $U(1)_A$ charges in the restoration limit of the chiral $U(2)_L \times U(2)_R$ symmetry, in which the quarkonium gets massless as the chiral partner of the pion. We showed that, at the realistic point where the pion decay constant and other hadron masses reach the observed amount, the lightest quarkonia and glueball are hardly mixed at all, without any phenomenological inputs such as the currently observed isospin-singlet scalar-decay properties. This is consistent with the large N_c picture on that of basis the holographic model has been established. The smallness of the mixing strength can actually be understood by the slightly smaller ratio of the chiral and gluon condensates (See Eq.(41)). The low-lying two scalar masses are calculated to be $\simeq 1.25$ GeV and $\simeq 1.77$ GeV, which are compared with masses of $f_0(1370)$ and $f_0(1710)$. Our result implies that $f_0(1710)$ predominantly consists of glueball.

Several comments are in order:

In addressing the scalar meson masses, we have incorporated not only the lowest masses from each of $\bar{q}q$ and gg states, but also the next-to-lowest one from the $\bar{q}q$ state. As seen from the mass matrix element in Eq.(38), the mixing between the two lightest scalars from the $\bar{q}q$ and gg could be comparable with that between the lightest scalar from the gg and the next-to-lightest one from the $\bar{q}q$. This would imply some new possibility that incorporating the next-to-lowest $\bar{q}q$ scalar could be relevant in investigating the low-lying scalar mixing structure.

Also, we have neglected the higher level more than the third in the quarkonium state and the second in the glueball state. Actually, we have computed the higher masses to be $\gtrsim 3$ GeV, which has not yet been established in experiments on the scalar resonances, but could be accessible in the future, together with the slightly lighter $\bar{q}q$-like scalar with the mass $\simeq 2$ GeV predicted in the present model (See Eq.(38)).

Similar analyses as done in the present paper can be performed in a more realistic situation, where mixings with a four-quark state and a pure strange state with the sizable strange quark mass are incorporated by extending to the three-flavor case, which will be pursued in another publication.

ACKNOWLEDGMENTS

We are grateful to M. Harada and H. Nishihara for useful comments and discussions. S.M was supported in by the JSPS Grant-in-Aid for Scientific Research (S) No. 22224003.

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