

# Nucleon Effective E-Mass in Neutron-Rich Matter from the Migdal-Luttinger Jump

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The well-known Migdal-Luttinger theorem states that the jump of the single-nucleon momentum distribution at the Fermi surface is equal to the inverse of the nucleon effective E-mass. Recent experiments studying short-range correlations (SRC) in nuclei using electron-nucleus scatterings at the Jefferson National Laboratory (JLAB) together with model calculations constrained significantly the Migdal-Luttinger jump at saturation density of nuclear matter. We show that the corresponding nucleon effective E-mass is consequently constrained to  $M_0^{*,E}/M \approx 2.22 \pm 0.35$  in symmetric nuclear matter (SNM) and the E-mass of neutrons is smaller than that of protons in neutron-rich matter. Moreover, the average depletion of the nucleon Fermi sea increases (decreases) approximately linearly with the isospin asymmetry  $\delta$  according to  $\kappa_{p/n} \approx 0.21 \pm 0.06 \pm (0.19 \pm 0.08)\delta$  for protons (neutrons). These results will help improve our knowledge about the space-time non-locality of the single-nucleon potential in neutron-rich nucleonic matter useful in both nuclear physics and astrophysics.

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## I. INTRODUCTION

In the framework of Landau Fermi liquid theory [1–8], the (Landau) effective mass of a Fermion is a fundamental quantity describing to leading order effects related to the space-time nonlocality of the underlying interactions and the Pauli exclusion principle. The study of nucleon effective mass in finite nuclei and/or infinite nuclear matter has a long history because of the great challenges involved and its significance for both nuclear physics and astrophysics, see, e.g., refs. [9–11] for earlier reviews. Moreover, there are interesting new issues related to the isospin dependence of space-time nonlocality determining, such as the neutron-proton effective mass splitting and their interaction cross sections in neutron-rich nucleonic matter, see, e.g., ref. [12] for a recent review. Despite of the impressive progress made in this field, our current knowledge on the nucleon effective mass especially its isospin dependence is still rather poor. It is thus widely recognized that better knowledge on the nucleon effective mass is critical for us to make further progress in solving many other interesting problems in both nuclear physics and astrophysics. For example, the isospin dependence of space-time non-locality affects the symmetry energy of asymmetric nuclear matter (ANM) [13–26], the momentum dependence of both the isoscalar and isovector parts of the single-nucleon potential [27–35] and the in-medium nucleon-nucleon scattering cross sections [36–39] used in simulating heavy-ion collisions especially those induced by rare isotopes, the level densities and thermal properties of hot nuclei [40–45] as well as the cooling rate and transport properties of neutron stars [46, 47].

The effective mass of a nucleon  $J = (n,p)$  can be calculated from the derivative of its potential  $U_J$  with respect

to either its energy  $E$  or momentum  $\mathbf{k}$  [11]

$$\begin{aligned} \frac{M_J^*}{M} &= 1 - \frac{dU_J(\mathbf{k}(E), E, \rho, \delta)}{dE} \\ &= \left[ 1 + \frac{M}{\hbar^2 k_F^J} \frac{dU_J(\mathbf{k}, E(\mathbf{k}), \rho, \delta)}{d|\mathbf{k}|} \right]^{-1} \end{aligned} \quad (1)$$

where  $M$  is the average mass of nucleons in free-space. Moreover, we take  $|\mathbf{k}| = k_F^J$  in this work with  $k_F^J = (1 + \tau_3^J \delta)^{1/3} \cdot k_F$  and  $k_F = (3\pi^2 \rho/2)^{1/3}$  being the nucleon Fermi momentum in symmetric nuclear matter at density  $\rho$ ,  $\tau_3^J = +1$  or  $-1$  for neutrons or protons and  $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$  is the isospin asymmetry of the medium. The  $M_J^*/M$  at saturation density  $\rho_0$  can be extracted from the energy/momentum dependence of nucleon optical potentials using an on-shell energy-momentum dispersion relation [48–50]. It is known that the total nucleon effective mass can be decomposed into [9–11]

$$\frac{M_J^*}{M} = \frac{M_J^{*,E}}{M} \cdot \frac{M_J^{*,k}}{M} \quad (2)$$

with

$$\frac{M_J^{*,E}}{M} = 1 - \frac{\partial U_J}{\partial E} \quad \text{and} \quad \frac{M_J^{*,k}}{M} = \left[ 1 + \frac{M_J}{|\mathbf{k}|} \frac{\partial U_J}{\partial |\mathbf{k}|} \right]^{-1} \quad (3)$$

the nucleon E-effective mass (E-mass) and k-effective mass (k-mass) to characterize the energy and momentum dependence of the single-nucleon potential  $U_J$  due to the non-locality in time and space of the underlying interaction, respectively.

Most experiments and phenomenological models probe only the total effective mass  $M_J^*/M$  [9–12, 48–51]. From Eq. (2), it is seen that an independent determination of either the E-mass or k-mass together with the total effective mass will then allow us to know all three kinds of nucleon effective masses. Interestingly, the Migdal-Luttinger theorem [52, 53] connects the nucleon E-mass

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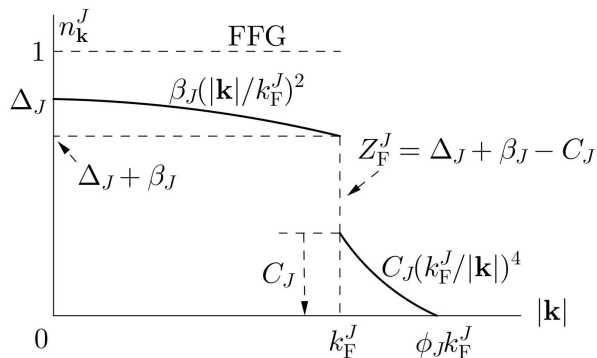


FIG. 1: A sketch of the single nucleon momentum distribution with a high momentum tail used in this work.

directly with the jump (discontinuity)  $Z_F^J \equiv n_{\mathbf{k}}^J(k_{F-0}^J) - n_{\mathbf{k}}^J(k_{F+0}^J)$  of the single-nucleon momentum distribution  $n_{\mathbf{k}}^J$  at the Fermi momentum  $k_F^J$  illustrated in Fig. 1 via

$$M_J^{*,E}/M = 1/Z_F^J. \quad (4)$$

The nuclear physics community has devoted much efforts to probing the depletion of the nucleon Fermi sphere by using transfer, pickup and (e,e'p) reactions. Results of these studies normally given in terms of the nucleon spectroscopic factors can constrain the  $n_{\mathbf{k}}^J(k_{F-0}^J)$  [9–11]. On the other hand, quantitative information about both the shape and magnitude of the high-momentum tail (HMT) above the Fermi surface have been extracted recently from analyzing cross sections of both inclusive and exclusive electron-nucleus scatterings [54–58] as well as medium-energy photonuclear absorptions [59, 60], providing a constraint on the  $n_{\mathbf{k}}^J(k_{F+0}^J)$ . These experimental results together with model analyses provide a significant empirical constraint on the Migdal-Luttinger jump. In this work, we show that the corresponding nucleon E-mass is consequently constrained to  $M_J^{*,E}/M \approx 2.22 \pm 0.35$  in symmetric nuclear matter and the E-mass of neutrons is smaller than that of protons in neutron-rich matter. Moreover, the average depletion of the nucleon Fermi sea increases (decreases) approximately linearly with the isospin asymmetry  $\delta$  according to  $\kappa_{p/n} \approx 0.21 \pm 0.06 \pm (0.19 \pm 0.08)\delta$  for protons (neutrons).

## II. THE SINGLE-NUCLEON MOMENTUM DISTRIBUTION FUNCTION IN COLD NEUTRON-RICH NUCLEONIC MATTER

We briefly recall here the main features of  $n_{\mathbf{k}}^J$  used in the present work [61]. It is well known that the SRC due to tensor components and/or the repulsive core of nuclear forces leads to a high (low) momentum tail (depletion) in the single-nucleon momentum distribution above (below) the nucleon Fermi momentum in cold nucleonic matter, see refs. [62–65] for comprehensive reviews. It

has been found from analyzing electron-nucleus scattering data that the percentage of nucleons in the HMT is about 25% in SNM but decreases gradually to about only 1% in pure neutron matter (PNM) [54, 55].

We parameterize the single-nucleon momentum distribution in cold ANM with

$$n_{\mathbf{k}}^J(\rho, \delta) = \begin{cases} \Delta_J + \beta_J I(|\mathbf{k}|/k_F^J), & 0 < |\mathbf{k}| < k_F^J, \\ C_J (k_F^J/|\mathbf{k}|)^4, & k_F^J < |\mathbf{k}| < \phi_J k_F^J. \end{cases} \quad (5)$$

As sketched in Fig. 1, the  $\Delta_J$  measures the depletion of the Fermi sphere at zero momentum with respect to the free Fermi gas (FFG) model prediction while the  $\beta_J$  is the strength of the momentum dependence  $I(\mathbf{k}/k_F^J)$  of the depletion near the Fermi surface. The parameters  $\Delta_J$ ,  $C_J$ ,  $\phi_J$  and  $\beta_J$  depend linearly on the isospin asymmetry according to  $Y_J = Y_0(1 + Y_1\tau_3^J\delta)$  [61]. The amplitude  $C_J$  and the cutoff coefficient  $\phi_J$  determine the fraction of nucleons in the HMT via

$$x_J^{\text{HMT}} = 3C_J \left(1 - \frac{1}{\phi_J}\right). \quad (6)$$

The normalization condition  $[2/(2\pi)^3] \int_0^\infty n_{\mathbf{k}}^J(\rho, \delta) d\mathbf{k} = \rho_J = (k_F^J)^3/3\pi^2$  requires that only three of the four parameters, i.e.,  $C_J$ ,  $\phi_J$ ,  $\beta_J$  and  $\Delta_J$ , are independent. Here we choose the first three as independent and determine the  $\Delta_J$  by [61]

$$\Delta_J = 1 - \frac{3\beta_J}{(k_F^J)^3} \int_0^{k_F^J} I\left(\frac{k}{k_F^J}\right) k^2 dk - 3C_J \left(1 - \frac{1}{\phi_J}\right). \quad (7)$$

The  $C/|\mathbf{k}|^4$  shape of the HMT both for SNM and PNM is strongly supported by recent findings theoretically and experimentally. Combining results of analyzing the d(e,e'p) cross sections [55] with an evaluation of medium-energy photonuclear absorption cross sections [59] leads to a value of  $C_0 \approx 0.161 \pm 0.015$ . With this  $C_0$  and the value of  $x_{\text{SNM}}^{\text{HMT}} = 28\% \pm 4\%$  [54, 55, 66] obtained from systematic analyses of inclusive (e,e') reactions and data from exclusive two-nucleon knockout reactions, the HMT cutoff parameter in SNM is determined to be  $\phi_0 = (1 - x_{\text{SNM}}^{\text{HMT}}/3C_0)^{-1} = 2.38 \pm 0.56$  [61]. The value of  $C_n^{\text{PNM}} = C_0(1 + C_1)$  is extracted by applying the adiabatic sweep theorem [67] to the EOS of PNM predicted by microscopic many-body theories [68–72] as well as that from the EOS of Fermi systems under unitary condition [67, 73]. More quantitatively, we obtained  $C_n^{\text{PNM}} \approx 0.12$  and subsequently  $C_1 = -0.25 \pm 0.07$  [61]. By inserting the value of  $x_{\text{PNM}}^{\text{HMT}} = 1.5\% \pm 0.5\%$  [54, 55, 66] and the  $C_n^{\text{PNM}}$  into Eq. (6), the high momentum cutoff parameter for PNM is determined to be  $\phi_n^{\text{PNM}} \equiv \phi_0(1 + \phi_1) = (1 - x_{\text{PNM}}^{\text{HMT}}/3C_n^{\text{PNM}})^{-1} = 1.04 \pm 0.02$  [61]. Consequently, we get  $\phi_1 = -0.56 \pm 0.10$  [61] by using the  $\phi_0$  determined earlier. Moreover, a quadratic momentum-dependence  $I(k/k_F^J) = (k/k_F^J)^2$  is adopted [61] from predictions of some nuclear many-body theories [74], then Eq. (7) gives us  $\Delta_J = 1 - 3\beta_J/5 - 3C_J(1 - 1/\phi_J)$ .

Specifically, we have  $\beta_0 = (5/3)[1 - \Delta_0 - 3C_0(1 - \phi_0^{-1})] = (5/3)[1 - \Delta_0 - x_{\text{SNM}}^{\text{HMT}}]$  for SNM. Then, using the predicted value of  $\Delta_0 \approx 0.88 \pm 0.03$  [75–77] and the experimental value of  $x_{\text{SNM}}^{\text{HMT}} \approx 0.28 \pm 0.04$ , the value of  $\beta_0$  is estimated to be about  $-0.27 \pm 0.08$ . Similarly, the condition  $\beta_J = \beta_0(1 + \beta_1 \tau_3^J \delta) < 0$ , i.e.,  $n_{\mathbf{k}}^J$  is a decreasing function of momentum towards  $k_{\text{F}}^J$ , indicates generally that  $|\beta_1| \leq 1$ . For more details of these parameters, see ref. [61].

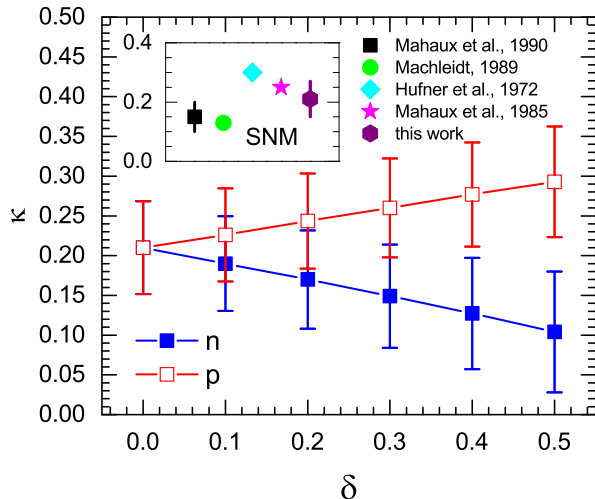


FIG. 2: (Color Online) The average depletion of the neutron and proton Fermi surface as a function of isospin asymmetry in neutron-rich matter. The inset shows the average nucleon depletion in symmetric matter from this work and earlier studies [10, 80–82].

The average depletion of the Fermi sphere in asymmetric nuclear matter

$$\begin{aligned} \kappa_J &= 1 - \Delta_J - \frac{1}{k_{\text{F}}^J} \int_0^{k_{\text{F}}^J} \beta_J \left( \frac{|\mathbf{k}|}{k_{\text{F}}^J} \right)^2 dk \\ &= \frac{4}{15} \beta_J + 3C_J \left( 1 - \frac{1}{\phi_J} \right) \end{aligned} \quad (8)$$

depends strongly on the tensor part of the nucleon-nucleon interaction [77, 78]. It provides a quantitative measure of the validity of the Hugenholtz-Van Hove (HVH) theorem [79] and more generally independent particle models. A deeper depletion indicates a more serious violation of the HVH theorem [80–83]. Experimentally, it can be measured by using the nucleon spectroscopic factor from transfer, pickup and  $(e, e'p)$  reactions [80]. A well-known example is the finding that mean-field models overpredict the occupation of low-momentum nucleon orbitals compared to data of electron scatterings on nuclei from  ${}^7\text{Li}$  to  ${}^{208}\text{Pb}$  by about 30-40% due to the neglect of correlations [84]. The  $\kappa_J$  is also believed to determine the rate of convergence of the hole-line expansion of the nuclear potential [10, 62, 80, 82]. In Fig. 2, the average depletion of the neutron and proton Fermi surface is shown separately as a function of isospin asymme-

try in neutron-rich matter. It is interesting to see that the neutron/proton depletion decreases/increases with  $\delta$  approximately linearly, indicating that protons with energies near the Fermi surface experience larger correlations with increasing asymmetry in qualitative agreement with findings from both analyses using microscopic many-body theories [76, 78] and phenomenological models [87]. This is also consistent with experimental findings from earlier studies of nucleon spectroscopic factors [85], dispersive optical model analyses of proton-nucleus scatterings [86] and the neutron-proton dominance model analyses of electron-nucleus scattering experiments [54]. More quantitatively, the neutron-proton splitting of the  $\kappa_J$  is approximately  $\kappa_n - \kappa_p \approx [8\beta_0\beta_1/15 + 6C_0\phi_1/\phi_0 + 6C_0C_1(1 - \phi_0^{-1})]\delta \approx (-0.37 \pm 0.16)\delta$ . For symmetric nuclear matter, we have  $\kappa = 4\beta_0/15 + x_{\text{SNM}}^{\text{HMT}} \approx 0.21 \pm 0.06$  comparable with the results obtained earlier from other studies [10, 80–82], as shown in the inset of Fig. 2.

Several consequences of the SRC modified nucleon momentum distribution have been studied recently. In particular, it was found that the nucleon kinetic symmetry energy is reduced compared to the FFG model prediction [61, 66, 88–95]. This has important consequences on isovector observables in heavy-ion collisions [66, 96–98] and on the critical densities for forming different charge states of  $\Delta(1232)$  resonances in neutron stars [99, 100]. Moreover, the SRC was also found to enhance the isospin-quartic term in the kinetic energy of ANM within both non-relativistic [61] and relativistic models [95]. Very recently, it was shown that the SRC-induced depletion of the nucleon Fermi surface affects significantly the neutrino emissivity, heat capacity and neutron superfluidity in neutron stars [101].

Before going further, it is necessary to address a possible drawback of our parameterization for the single-nucleon momentum distribution in Eq. 5. To our best knowledge, the original derivations [52, 53] of the Migdal-Luttinger theorem do not explicitly require the derivative of the momentum distribution  $[dn_{\mathbf{k}}/dk]_{k=k_{\text{F}} \pm 0}$  at the Fermi momentum  $k_{\text{F}}$  to be  $-\infty$ . Of course, one can associate mathematically loosely the finite drop in  $n_{\mathbf{k}}$  over zero increase in momentum at  $k_{\text{F}}$  to a slope of  $-\infty$ . Some later derivations using various approximation schemes, such as the “derivative expansion” in which the momentum distribution is expressed in terms of energy derivatives of the mass operator by Mahaux and Saror [102], have shown that the slope of the momentum distribution should have the asymptotic behavior of  $[dn_{\mathbf{k}}/dk]_{k=k_{\text{F}} \pm 0} = -\infty$ . Similar to many other calculations including some examples given in refs. [10, 102], our parameterization of Eq. 5 does not have such behavior from neither side of the discontinuity. However, similar to what has been done in ref. [103], in parameterizing the  $n_{\mathbf{k}}$  both above and below the  $k_{\text{F}}$  one can add a term that is vanishingly small in magnitude but asymptotically singular in slope at  $k_{\text{F}}$ , such as  $\eta \cdot \left( \frac{k-k_{\text{F}}}{\Lambda} \right) \cdot \ln\left( \frac{k-k_{\text{F}}}{\Lambda} \right)$  where  $\eta$  is a constant much smaller than  $\Delta_0$  and  $C_0$ . Of course, one then has to determine the totally 4 additional

parameters ( $\eta$  and  $\Lambda$  for neutrons and protons above and below their respective Fermi momenta) and readjust the other parameters already used in Eq. 5. This has the potential of reducing the error bars of the quantities we extract but requires more experimental information. Unfortunately, the analyses of existing experimental data we mentioned above have so far not considered such corrections. While we do not expect the corrections will affect significantly the size of the Migdal-Luttinger jump since they have vanishingly small magnitudes at  $k_F$ , our description about the discontinuity of  $n_{\mathbf{k}}$  at  $k_F$  certainly should be investigated further and possibly improved in the future. For the present exploratory study using information from phenomenological model analyses of limited experimental data, we feel that the parameterization of Eq. 5 is good enough.

### III. NUCLEON E-EFFECTIVE MASS AND ITS ISOSPIN SPLITTING IN NEUTRON-RICH NUCLEONIC MATTER

We now turn to the nucleon E-mass obtained through the Migdal-Luttinger theorem of Eq. (4). In terms of the parameters describing the single-nucleon momentum distribution  $n_{\mathbf{k}}^J$ , we have  $Z_F^J = \Delta_J + \beta_J - C_J = 1 + 2\beta_J/5 - 4C_J + 3C_J\phi_J^{-1}$ . For SNM, it is given by  $Z_F^0 = 1 + 2\beta_0/5 - 4C_0 + 3C_0\phi_0^{-1} = 1 + 2\beta_0/5 - C_0 - x_{\text{SNM}}^{\text{HMT}}$ , then using the values for  $\beta_0$ ,  $\phi_0$ ,  $C_0$  and  $x_{\text{SNM}}^{\text{HMT}}$  given above, we obtain a value of  $M_0^{*,E}/M \approx 2.22 \pm 0.35$ . Shown in Fig. 3 with the filled squares are the extracted nucleon E-mass in SNM within the uncertainty range of the  $\beta_0$  parameter. It is seen that the variation of  $M_0^{*,E}/M$  with  $\beta_0$  is rather small. For comparisons, also shown are earlier predictions based on (1) a semi-realistic parametrization through a relative s-wave exponential nucleon-nucleon interaction potential (red dash line) [104], (2) a Green's function method considering collective effects due to the coupling of nucleons with the low-lying particle-hole excitations of the medium (green solid line) [105], (3) a correlated basis function (CBF) method using the Reid and Bethe-Johnson potentials (black and magenta solid lines) [106, 107], (4) two non-relativistic models with the Paris nuclear potential (purple and red solid line) [103, 108], (5) a low density expansion of the optical potential (orange solid line) [109] and (6) a relativistic Dirac-Brueckner approach (dash black line) [83]. While we are unable to comment on possible origins of the different model predictions and their differences from the empirical values presented here, to our best knowledge, it is the first time that the nucleon E-mass is extracted using the Migdal-Luttinger theorem from the single-nucleon momentum distribution constrained by experiments phenomenologically.

In neutron-rich nucleonic matter, an interesting quantity is the neutron-proton E-mass splitting generally ex-

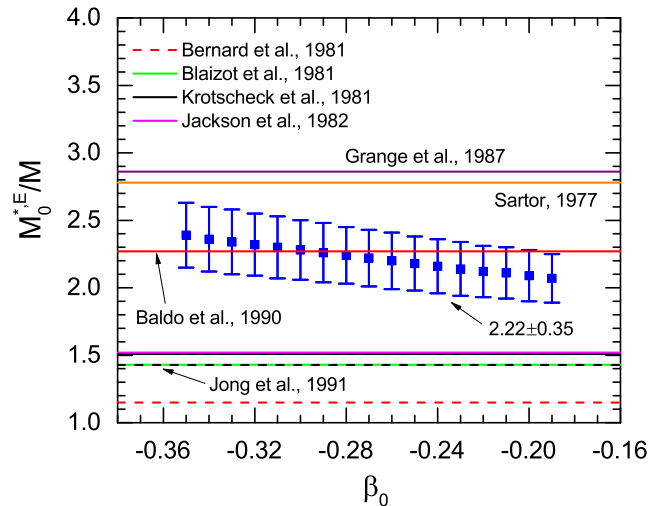


FIG. 3: (Color Online) The nucleon effective E-mass in symmetric nuclear matter (blue lines with error bars) at normal density within the uncertainty range of the shape parameter  $\beta_0$  of the nucleon momentum distribution extracted using the Migdal-Luttinger theorem in this work in comparison with predictions of earlier studies [83, 103–109], see detailed descriptions in the text.

pressed as

$$\frac{M_n^{*,E} - M_p^{*,E}}{M} = s_E \delta + t_E \delta^3 + \mathcal{O}(\delta^5) \quad (9)$$

where  $s_E$  and  $t_E$  are the linear and cubic splitting functions, respectively. The latter generally depend on the nucleon momentum and the density of the medium. Shown in Fig. 4 are the values of  $s_E$  and  $t_E$  at the nucleon Fermi momentum in nuclear matter at  $\rho_0$  within the uncertainty range of the  $\beta_1$ -parameter. More quantitatively, at the lower limit, mid-value and upper limit of the  $\beta_1$ -parameter, we have  $s_E(\beta_1 = -1) \approx -3.29 \pm 1.23$ ,  $t_E(\beta_1 = -1) \approx -1.49 \pm 1.47$ ,  $s_E(\beta_1 = 0) \approx -2.22 \pm 0.84$ ,  $t_E(\beta_1 = 0) \approx -0.41 \pm 0.42$ ,  $s_E(\beta_1 = 1) \approx -1.16 \pm 0.64$  and  $t_E(\beta_1 = 1) \approx -0.09 \pm 0.05$ , respectively. We note that the cubic splitting function  $t_E$  generally can not be neglected (e.g., for  $\beta_1 = 0$ ,  $t_E/s_E \approx 18\%$ ), and it may have sizable effects on the cooling and thermal properties of neutron stars [46, 101].

An important feature shown in Fig. 4 is that in neutron-rich nucleonic matter, the E-mass of a neutron is smaller than that of a proton, i.e.,  $M_n^{*,E} < M_p^{*,E}$ . However, the neutron-proton E-mass splitting in ANM has an appreciable dependence on the largely uncertain  $\beta_1$ -parameter characterizing the isospin-dependence of the nucleon momentum distribution near the Fermi surface. Unfortunately, currently there exists no reliable constraint on the parameter  $\beta_1$ . Thus, it is interesting to mention that there are experimental efforts to measure the isospin dependence of the nucleon spectroscopic factors using direct reactions with radioactive beams [110] and the isospin-dependence of SRC with both electrons

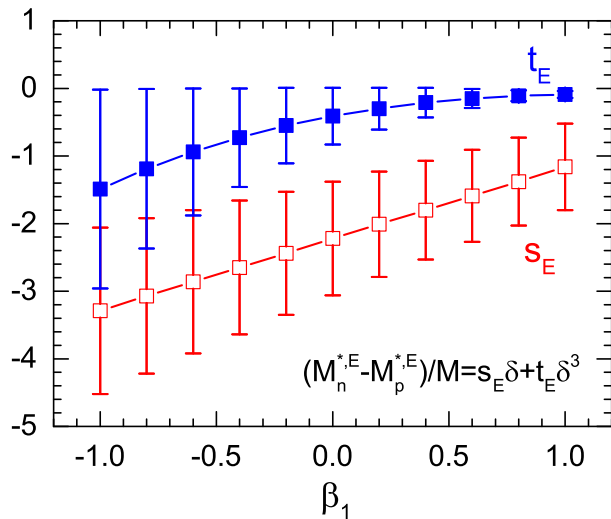


FIG. 4: (Color Online) The linear and cubic splitting functions  $s_E$  and  $t_E$  at normal density within the uncertain range of the  $\beta_1$ -parameter characterizing the isospin-dependence of the nucleon momentum distribution near the Fermi surface.

and hadrons [111]. These experiments have the potential to constrain the  $\beta_1$  and thus the neutron-proton E-mass splitting in neutron-rich matter.

#### IV. SUMMARY

In summary, using the Migdal-Luttinger theorem relating the discontinuity of the single-nucleon momentum

distribution function at the Fermi surface with the nucleon E-mass, we have extracted the latter and its isospin splitting in neutron-rich nucleonic matter at normal density using the single-nucleon momentum distribution constrained by recent experiments at the JLAB. We found that the nucleon E-mass in SNM is  $M_0^{*,E}/M \approx 2.22 \pm 0.35$  while in neutron-rich matter the E-mass of neutrons is smaller than that of protons. Moreover, the average depletion of the nucleon Fermi sea increases (decreases) approximately linearly with the isospin asymmetry  $\delta$  according to  $\kappa_{p/n} \approx 0.21 \pm 0.06 \pm (0.19 \pm 0.08)\delta$  for protons (neutrons). These results provide useful references for microscopic nuclear many-body theories and will help improve our knowledge about the space-time non-locality of the single-nucleon potential in neutron-rich nucleonic matter.

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