

S-WAVE NONLEPTONIC HYPERON DECAYS AND $\Xi_b^- \rightarrow \pi^- \Lambda_b$

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The decay $\Xi_b^- \rightarrow \pi^- \Lambda_b$ has recently been observed by the LHCb Collaboration at CERN. In contrast to most weak decays of b -flavored baryons, this process involves the decay of the strange quark in Ξ_b , and thus has features in common with nonleptonic weak decays of hyperons. Thanks to the expected pure S-wave nature of the decay in question in the heavy b quark limit, we find that its amplitude may be related to those for S-wave nonleptonic decays of Λ , Σ , and Ξ in a picture inspired by duality. The calculated branching fraction $\mathcal{B}(\Xi_b^- \rightarrow \pi^- \Lambda_b) = (6.3 \pm 4.2) \times 10^{-3}$ is consistent with the range allowed in the LHCb analysis. The error is dominated by an assumed 30% uncertainty in the amplitude due to possible U(3) violation. A more optimistic view based on sum rules involving nonleptonic hyperon decay S-wave amplitudes reduces the error on the branching fraction to 2.0×10^{-3} .

PACS codes: 14.20.Mr, 14.20.Jn, 12.40.Nn, 13.30.Eg

I INTRODUCTION

Most decays of b -flavored baryons observed up to now occur with the b quark decaying to c or u and a virtual W^- , or via a $b \rightarrow s$ or $b \rightarrow d$ penguin amplitude. However, it has long been noted that strange b -flavored baryons are heavy enough that their s quarks can decay instead via the subprocess $s \rightarrow \pi^- u$ or $su \rightarrow ud$, with the b quark acting as a spectator [1–6]. Such processes enable the decays $\Xi_b^{(-,0)} \rightarrow \pi^{(-,0)} \Lambda_b$. The strange quark in a charmed-strange baryon can also decay to a nonstrange one. Here the additional subprocess $cs \rightarrow dc$ can contribute. Thus the decays $\Xi_c^{(+,0)} \rightarrow \pi^{(+,0)} \Lambda_c$ are permitted.

The LHCb Collaboration at CERN has now observed the decay $\Xi^- \rightarrow \pi^- \Lambda_b$ [7]. The quantity measured is the product of the ratio of fragmentation functions $f_{\Xi_b^-}^f/f_{\Lambda_b}$ and the branching fraction $\mathcal{B}(\Xi_b^- \rightarrow \pi^- \Lambda_b)$:

$$\frac{f_{\Xi_b^-}^f}{f_{\Lambda_b}} \mathcal{B}(\Xi_b^- \rightarrow \pi^- \Lambda_b) = (5.7 \pm 1.8_{-0.9}^{+0.8}) \times 10^{-4} . \quad (1)$$

Assuming a range $0.1 \leq f_{\Xi_b^-}/f_{\Lambda_b} \leq 0.3$ based on measured production rates of other strange particles relative to their non-strange counterparts, the branching ratio is then expected to lie between $(0.57 \pm 0.21)\%$ and $(0.19 \pm 0.07)\%$ [7].

This decay is expected to proceed purely via S-wave. As a result, we find in a framework inspired by duality that its amplitude may be related to those for S-wave nonleptonic decays of Λ , Σ , and Ξ . The calculated branching fraction $\mathcal{B}(\Xi_b^- \rightarrow \pi^- \Lambda_b) = (6.3 \pm 4.2) \times 10^{-3}$ is consistent with the range allowed in the LHCb analysis.

Earlier studies of $\Xi_b^- \rightarrow \pi^- \Lambda_b$ (and other heavy-flavor-conserving heavy baryon decays) [1–6] have applied current algebra and a soft pion limit, thereby relating this amplitude to a matrix element of a strangeness-changing four-fermion operator between the initial and final heavy baryon states. Calculations of this matrix element are model-dependent and involve large theoretical uncertainties.

The S-wave nature of the decay $\Xi_b^- \rightarrow \pi^- \Lambda_b$ is discussed in Sec. II. A duality-inspired description of the S-wave decays of the strange hyperons Λ , Σ , and Ξ is recalled and updated in Sec. III. As the bottom quark acts as a spectator in $\Xi_b^- \rightarrow \pi^- \Lambda_b$, we relate the amplitude for decay of its strange quark to S-wave amplitudes of the strange hyperons in Sec. IV. The parameters extracted from an updated analysis of hyperon S-wave amplitudes are then applied to calculate the rate for $\Xi_b^- \rightarrow \pi^- \Lambda_b$ in Sec. V. A more optimistic view of possible uncertainties is presented in Sec. VI. Section VII concludes.

II S-WAVE NATURE OF THE DECAY $\Xi_b^- \rightarrow \pi^- \Lambda_b$

In the $\Lambda_b = b[ud]$, the light quarks u and d are in an S-wave state with $I = S = 0$ (denoted by the square brackets). To the extent that $|m_s - m_d|$ can be neglected in comparison with m_b [8–10], the light quarks s and d in Ξ_b^- also are in an S-wave state antisymmetric in flavor with $S = 0$. In the decay $\Xi_b^- \rightarrow \pi^- \Lambda_b$, the b quark acts as a spectator. The transition among light quarks is thus one with $J^P = 0^+ \rightarrow \pi 0^+$, and hence is purely a parity-violating S wave. We shall see that it thus may be related to parity-violating S-wave amplitudes in the nonleptonic decays of Λ , Σ , and Ξ .

III S-WAVE NONLEPTONIC HYPERON DECAYS

Many attempts have been made to systematize nonleptonic weak decays of the hyperons belonging to the lowest SU(3) octet. For discussions of both parity-violating S-wave and parity-conserving P-wave decays, see, e.g., Refs. [11–13] and numerous references therein. The P-wave decays are sensitive to delicate cancellations among pole contributions. In contrast, there is a compact parametrization of the S-wave decays [14] based on Dolen-Horn-Schmid duality [15, 16] which does a very good job in describing the parity-violating amplitudes. We will adopt this approach, which has been shown to be equivalent to earlier studies of these S-wave amplitudes using current algebra and partial conservation of the axial current (PCAC) and assuming octet dominance [17, 18].

We recall the notation of Refs. [13, 19]. With the effective Lagrangian for the decay given

by

$$\mathcal{L}_{\text{eff}} = G_F m_\pi^2 [\bar{\psi}_2 (A + B \gamma_5) \psi_1] \phi_\pi , \quad (2)$$

the partial width for $B_1 \rightarrow \pi B_2$ is

$$\Gamma(B_1 \rightarrow \pi B_2) = \frac{(G_F m_\pi^2)^2}{8\pi m_1^2} q [(m_1 + m_2)^2 - m_\pi^2] |A|^2 + [(m_1 - m_2)^2 - m_\pi^2] |B|^2 . \quad (3)$$

Here $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi decay constant, q is the magnitude of the final three-momentum of either particle in the B_1 rest frame, and A and B are S-wave and P-wave decay amplitudes, respectively.

The nonleptonic hyperon decays are observed to follow an approximate $|\Delta I| = 1/2$ rule, so a decay $B_1 \rightarrow \pi B_2$ may be regarded as a scattering process $\sigma B_1 \rightarrow \pi B_2$, where the spurion σ transforms as a K^0 and has $J^P = 0^-$ for parity-violating S-wave amplitudes and 0^+ for parity-conserving P-wave amplitudes. In Ref. [14] it was noted that one could regard the S-wave amplitudes as if the spurion and pion coupled to a t -channel K^* or K^{**} Regge trajectory, with relatively small [O(15%)] contributions from octet pole terms. Such a Regge-dominance picture would not hold for the pole-dominated P-wave amplitudes. With the assumption that all S-wave decays can be regarded as if a $K^0\pi$ pair were exchanging a Regge trajectory with the $\bar{B}_1 B_2$ pair, one can calculate all S-wave $B_1 \rightarrow \pi B_2$ amplitudes in terms of an overall scale x and a parameter F expressing the ratio of antisymmetric to symmetric three-octet coupling at the baryon vertex. In our convention, $F + D = 1$. A typical graph illustrating the coupling in $\Sigma^- \rightarrow \pi^- n$ is shown in Fig. 1(a).

An exchanged trajectory E and the baryons B_1 and B_2 may be represented by 3×3 matrices. For π^- emission, E represents the product of $\bar{\sigma}$ (transforming as a \bar{K}^0) and the π^- matrix, or

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (4)$$

For octet baryons,

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{bmatrix} , \quad \bar{B} = B^T . \quad (5)$$

The couplings at the baryon vertex may be represented by traces $\langle \dots \rangle$ of 3×3 matrices, where we use the notation of Ref. [20]:

$$g_{\bar{B}_1 E B_2} = \gamma \left[(1 - F) \langle \bar{B}_1 \{E, B_2\} \rangle + F \langle \bar{B}_1 [E, B_2] \rangle - (1 - 2F) \langle \bar{B}_1 B_2 \rangle \langle E \rangle \right] . \quad (6)$$

The last term enforces nonet symmetry, decoupling $s\bar{s}$ trajectories from the nucleon. One then obtains the nonleptonic S-wave decay amplitudes (correcting a sign in [14])

$$\Lambda_- \equiv A(\Lambda \rightarrow \pi^- p) = -(2F + 1)x/\sqrt{6} , \quad (7)$$

$$\Sigma_+^+ \equiv A(\Sigma^+ \rightarrow \pi^+ n) = 0 , \quad (8)$$

$$\Sigma_-^- \equiv A(\Sigma^- \rightarrow \pi^- n) = -(2F - 1)x , \quad (9)$$

$$\Xi_-^- \equiv A(\Xi^- \rightarrow \pi^- \Lambda) = (4F - 1)x/\sqrt{6} . \quad (10)$$

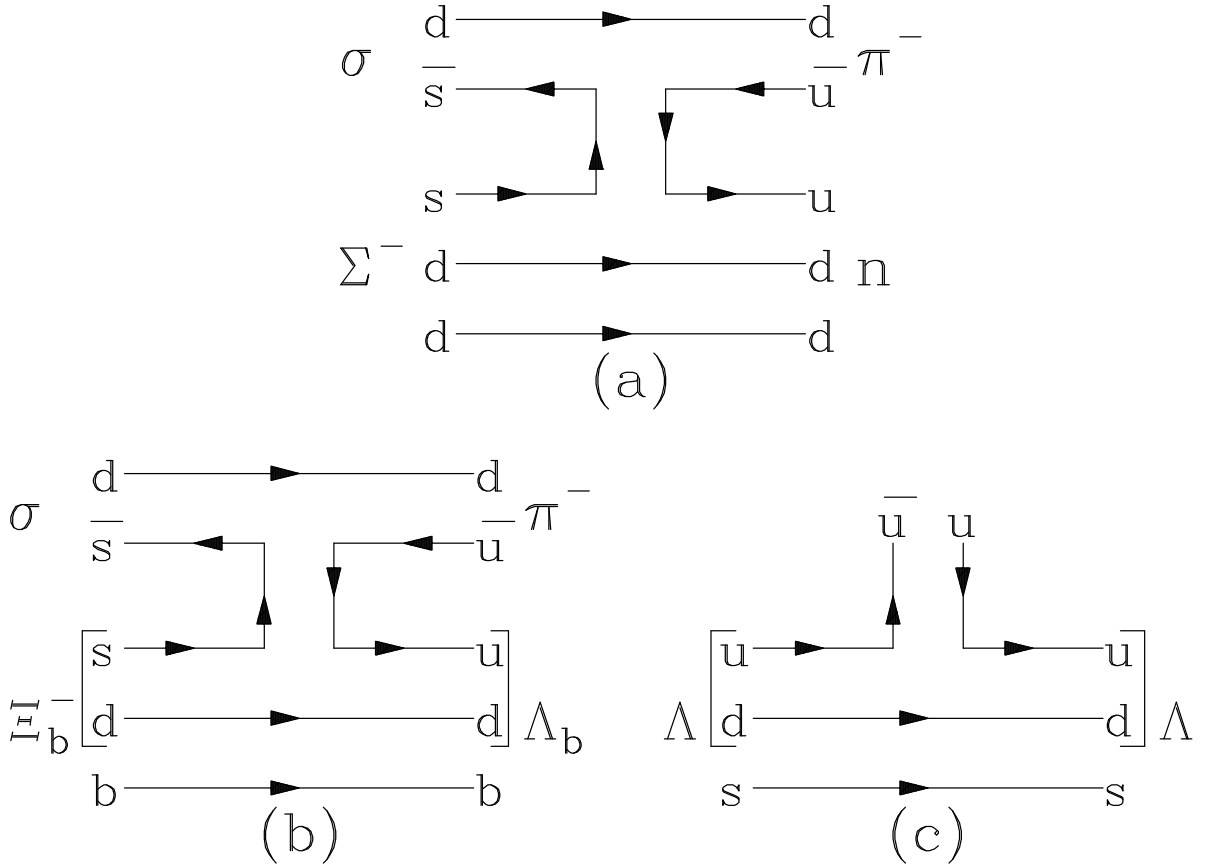


Figure 1: Illustration of Reggeon couplings. (a) Coupling contributing to the decay $\Sigma^- \rightarrow \pi^- n$ induced by a spurion σ transforming as a K^0 . (b) Corresponding coupling contributing to the heavy-flavor-conserving process $\Xi_b^- \rightarrow \pi^- \Lambda_b$. The brackets around the initial sd and final ud pair denote flavor and spin antisymmetry, so the pair are in a 3^* of flavor $SU(3)$ and a state of spin 0. (c) Corresponding coupling at the baryon vertex to a quark in hyperons belonging to a flavor- and spin-antisymmetric state.

Here x is an arbitrary scale factor. Two relations involving factors $\mp\sqrt{2}$, between Λ_- and Λ_0 and between Ξ_- and Ξ_0^0 , follow from the $|\Delta I| = 1/2$ nature of the transition. A similar relation between Σ_- and Σ_0^+ follows from isospin and the suppression of $A(\Sigma^+ \rightarrow \pi^+ n)$, thus providing a test of octet dominance or our duality assumption. The full set of S-wave amplitudes is summarized in Table I, along with observed values [19]. Also shown are values predicted from a best fit, which selects $F = 1.652$, $x = 0.8605$.

These relations were also obtained in Refs. [17, 18] within a current-algebra framework. They imply the triangle relation [21, 22]

$$2A(\Xi^- \rightarrow \pi^- \Lambda) + A(\Lambda \rightarrow \pi^- p) = -(3/2)^{1/2} A(\Sigma^- \rightarrow \pi^- n). \quad (11)$$

Table I: Predicted and observed S-wave amplitudes A for nonleptonic hyperon decays. Predicted values are for best-fit parameters $F = 1.652$, $x = 0.8605$

Decay	Predicted A amplitude	Observed value	Predicted value
$\Lambda \rightarrow \pi^- p$	$-(2F + 1)x/\sqrt{6}$	-1.47 ± 0.01	-1.51
$\Lambda \rightarrow \pi^0 n$	$(2F + 1)x/(2\sqrt{3})$	1.07 ± 0.01	1.07
$\Sigma^+ \rightarrow \pi^+ n$	0	0.06 ± 0.01	0
$\Sigma^+ \rightarrow \pi^0 p$	$-(2F - 1)x/\sqrt{2}$	-1.48 ± 0.05	-1.40
$\Sigma^- \rightarrow \pi^- n$	$-(2F - 1)x$	-1.93 ± 0.01	-1.98
$\Xi^0 \rightarrow \pi^0 \Lambda$	$(4F - 1)x/(2\sqrt{3})$	1.55 ± 0.03	1.39
$\Xi^- \rightarrow \pi^- \Lambda$	$(4F - 1)x/\sqrt{6}$	2.04 ± 0.01	1.97

IV RELATION TO Ξ_b DECAY

The decay $\Xi_b^- \rightarrow \pi^- \Lambda_b$ involves the transformation of a strange quark in the $\Xi_b^- \simeq b[sd]$ into a u quark in the $\Lambda_b = b[ud]$ with emission of a pion. (We use the symbol \simeq to recall that the light diquark in the Ξ_b^- is nearly, but not totally, spinless.) The spectator b quark and the d member of the light diquark are untouched. This transition is illustrated in Fig. 1(b) as a scattering process involving the spurion σ transforming as a K^0 .

Note that the upper vertex is the same in Figs. 1(a) and 1(b). We seek a transition at the light baryon vertex in which the exchanged light-quark trajectory couples to a spinless 3^* of flavor SU(3), as the $u\bar{s}$ trajectory couples at the heavy baryon vertex in Fig. 1(b). Such a coupling is possessed by the Λ when coupling to a $u\bar{u}$ trajectory, as illustrated in Fig. 1(c). Here we make use of nonet symmetry, in which the coupling of an octet strange trajectory to a pair of SU(3) antitriplets is related to the coupling of an octet-singlet mixture (the $u\bar{u}$ trajectory) to the antitriplet pair. Such symmetry is familiar from the case of the vector mesons, in which a quark triplet and an antiquark antitriplet form a single nonet with octet and singlet properties related to one another.

The 3×3 matrix for the $u\bar{u}$ exchanged trajectory E' is then $E' = \text{Diag}(1, 0, 0)$. We calculate $g_{\bar{\Lambda}E'\Lambda}$ using the methods of the previous section and find

$$A(\Xi_b^- \rightarrow \pi^- \Lambda_b) = (5F - 2)x/3 = 1.796 \pm 0.269 \pm 0.539 = 1.796 \pm 0.602 \quad (12)$$

for the best-fit values $F = 1.652$, $x = 0.8605$. The first error of 15% has been assigned in accord with the estimate associated with Reggeon dominance [14]. (The quality of the fit in Table I is better than that.) The second error is a conservative 30% associated with possible U(3) breaking in comparing exchange of strange and nonstrange Regge trajectories. (A more optimistic view of uncertainties is presented in Sec. VI.) Implicit in this relation is the assumption that the properties of the light diquark are not greatly affected by the nature of the spectator quark (b in Fig. 1(b), s in Fig. 1(c)). A certain enhancement of the decay amplitude of the spatially more compact Ξ_b^- relative to the hyperon amplitude has been suggested in Ref. [4] due to an enhanced short-distance correlation inside the scalar diquark

in the heavy baryon. The fact that our calculated rate [see Eq. (14) below] turns out to be compatible with experiment [7] provides an *a posteriori* validation of our assumptions.

V CALCULATION OF DECAY RATE

We now apply Eq. (12) to the rate calculation in Eq. (3), based on the A amplitude alone, using masses from Ref. [23]: $m_{\Xi_b^-} = 5.7944$ GeV, $m_{\Lambda_b} = 5.6195$ GeV, $m_{\pi^-} = 0.13957$ GeV (from which we derive $q = 0.1038$ GeV), obtaining

$$\Gamma(\Xi_b^- \rightarrow \pi^- \Lambda_b) = 8.27|A|^2 \times 10^{-16} \text{ GeV} = (2.67 \pm 1.79) \times 10^{-15} \text{ GeV} , \quad (13)$$

where the error is dominated by the assumed 30% uncertainty in the amplitude due to U(3) breaking. With a Ξ_b^- lifetime of 1.56 ± 0.04 ps [23], this corresponds to a branching fraction

$$\mathcal{B}(\Xi_b^- \rightarrow \pi^- \Lambda_b) = (6.32 \pm 4.24 \pm 0.16) \times 10^{-3} = (6.3 \pm 4.2) \times 10^{-3} , \quad (14)$$

where the errors correspond to those of A and the Ξ_b^- lifetime. Neglecting the effect of small mass differences, the branching fraction for $\Xi_b^0 \rightarrow \pi^0 \Lambda_b$ is then expected to be half of this.

VI ALTERNATIVE VIEW OF UNCERTAINTIES

One can express relations for couplings of the strangeness-changing spurion and π^- purely in terms of strangeness-changing observables. Then one needs an estimate of the accuracy of U(3) symmetry in relating couplings of the *same* trajectory to different particles. For SU(3), this was done long ago by Barger and Olsson [24], with no deviation found to an accuracy of about 25%. However, one can do better if restricting attention to S-wave nonleptonic hyperon decays.

As $A(\Xi_b^- \rightarrow \pi^- \Lambda_b)$ is predicted by our assumption to depend just on the two parameters F and x , one can express it as a linear combination of any two A amplitudes for nonleptonic hyperon decays. Specifically, one has

$$A(\Xi_b^- \rightarrow \pi^- \Lambda_b) = -\frac{1}{2\sqrt{6}}A(\Lambda \rightarrow \pi^- p) - \frac{3}{4}A(\Sigma^- \rightarrow \pi^- n) = 1.75 , \quad (15)$$

$$= \frac{1}{\sqrt{6}}[A(\Lambda \rightarrow \pi^- p) + 3A(\Xi^- \rightarrow \pi^- \Lambda)] = 1.90 , \quad (16)$$

$$= -\frac{1}{2}A(\Sigma^- \rightarrow \pi^- n) + \frac{1}{\sqrt{6}}A(\Xi^- \rightarrow \pi^- \Lambda) = 1.80 , \quad (17)$$

inserting the experimental values from Table I. One finds a maximum deviation of about 6% from the value using fitted F, x . This precision provides a test for SU(3) as the above amplitudes involve different members of the baryon octet in the initial and final states. It is well within the 15% associated with the expected error from assuming Reggeon dominance. Adding amplitude uncertainties of 6% and 15% in quadrature and taking account of the small uncertainty in the Ξ_b^- lifetime, the total uncertainty in the predicted branching fraction is reduced from 4.2×10^{-3} to 2.0×10^{-3} , still consistent with the LHCb range but now favoring its upper end.

VII CONCLUSIONS

We have shown that an application of Dolen-Horn-Schmid duality to the recently observed decay $\Xi_b^- \rightarrow \pi^- \Lambda_b$ leads to a calculated branching fraction $\mathcal{B} = (6.3 \pm 4.2) \times 10^{-3}$, consistent with the LHCb range [7]. If a more optimistic view of the accuracy of U(3) is adopted, our prediction would favor the upper end of this range. It will be interesting to see if this estimate is borne out when the suggestion of Ref. [25] or reducing the uncertainty on $f_{\Xi_b^-} / f_{\Lambda_b}$ is applied.

ACKNOWLEDGMENTS

The work of J.L.R. was supported in part by the United States Department of Energy through Grant No. DE-FG02-13ER41598.

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