Low Scale Higgs Inflation with Gauss-Bonnet Coupling

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Abstract

Recent LHC data provides precise values of coupling constants of the Higgs field, however, these measurements do not determine its coupling with gravity. We explore this freedom to see whether Higgs field non-minimally coupled to Gauss-Bonnet term in 4-dimensions can lead to inflation generating the observed density fluctuations. We obtain analytical solution for this model and that the exit of inflation (with a finite number of e-folding) demands that the energy scale of inflation is close to Electro-weak scale. We compare the scalar and tensor power spectrum of our model with PLANCK data and discuss its implications.

Keywords: Higgs Inflation, Gauss-Bonnet Inflation

1. Introduction

Cosmological Inflation [1, 2, 3, 4, 5] provides a causal mechanism to generate the primordial density perturbations that are responsible for the anisotropies in the cosmic microwave background (CMB) and the formation of the large scale structure (LSS). CMB and LSS data have been used to constrain the parameters of the inflationary model. In the case of canonical scalar field, the CMB and LSS data provide constraints on the height and the slope of the potential ref [6, 7, 8].

The fact that the temperature fluctuations of the CMB is close to scale-invariance is a highly demanding requirement for inflation model building ref [9, 10, 11, 12] than providing approximately 60 e-foldings of inflation needed to solve the various initial conditions problems. More specifically, the near scale-invariance imposes a condition that the canonical scalar field potential should be almost flat — almost like cosmological constant — so that the quantum fluctuations that exit the horizon during inflation is nearly scale-invariant. While these flat potentials are phenomenologically successful, however, in the standard model of particle physics there is no candidate with such flat potentials that could sustain inflation [9, 10, 11, 12]. For instance, the renormalizability of the Higgs field in 4-dimensions puts a constraint on the scalar field potential ($V(\phi) = m^2\phi^2 + \lambda\phi^4$, where m is the mass and λ is the coupling parameter), however, inflationary models require potentials of the form $V(\phi) = \sum_{n=0}^{N} c_{2n}\phi^{2n}$ where c_{2n}'s are real numbers and $N > 2$.

To achieve the flatness of the potential, inflationary models using the standard model Higgs field as the inflaton, couples Higgs field non-minimally with gravity [13, 14, 15, 16, 17, 18]. In Higgs Inflation [13, 14, 15, 19, 20, 21, 22, 23, 24, 25], the flatness of the Higgs potential is achieved by large non-minimal coupling of the Higgs field to the Ricci scalar ($\sim \xi R\phi^2$ where ξ is the coupling constant and R is the Ricci scalar) i. e. $\xi \sim 10^4$.

One of the main assumption in the above models of Higgs inflation is that the standard model physics remains to hold until Planck energy. Which may be consistent with the current LHC measurements — since there are no evidence of new physics so far (e.g., supersymmetry or extra dimension(s), etc.) ref [26, 27, 28] — however, it also points to the fact that λ can be negative at high energies [29, 30, 31, 32, 33, 34, 35]. But a non-minimal Higgs Ricci coupling may prevent this up to inflationary scale [20, 23].

In this work, we ask the following question: Can Higgs field drive inflation without invoking any new Physics in the particle physics sector with exit at low-energies, order of 100 GeV to 1000 GeV? While the LHC measurements determine the coupling constants of the Higgs field precisely, it does not determine its coupling with Gravity. We use this freedom and assume that the Higgs field couples with the Gauss-Bonnet Gravity, instead of Ricci Scalar.

Gauss-Bonnet Gravity is a part of the general extension of Gravity theories referred to as Lovelock theories of gravity [36]. One important feature of Lovelock theories, as against the $f(R)$ gravity theories, is that the gravity equations of motion remain second order (and quasilinear in second order). They provide a natural arena for understanding many deep features of gravity and recently they have been a subject of study. (For a recent review see [37].) Some higher dimensional Lovelock theories arise also as a weak field limit of string theory [38, 39]. While a pure Gauss-Bonnet term is non-dynamical in 4-dimensions — topologically invariant in 4-dimensions — non-minimal coupling with the Higgs field makes it dynamical.

Since Gauss-Bonnet term is higher-derivative term, it may be natural to expect that non-minimal coupling of the scalar field may only lead to modifications at high-energies. However, in this work, we show explicitly that the non-minimal coupling of the Higgs scalar lead to exit of inflation at low-energies i.e. close to Electroweak scale. This an unique feature of our model. We also explicitly compute the power-spectrum and show that it is consistent with the recent PLANCK data [8]. There has been recent interest in coupling scalar field with Gauss-Bonnet gravity (see, for instance, [40, 18]). Our analysis differs from their analyses: In Ref. [40], authors have assumed that Einstein-Hilbert term is irrelevant and, hence, have ignored the linear term. In Ref. [18], the authors have coupled the scalar field to both the Ricci and Gauss-Bonnet gravity. Their analysis is based on slow-roll and makes predictions similar to [15]. It is important to note that they found the Gauss-Bonnet term to be significant only at late times where as the Higgs-Ricci coupling dominating the initial epoch and was responsible for the spectrum. As mentioned earlier, our model couples the Higgs scalar with Gauss-Bonnet gravity leading to a dynamical model of inflation.

The paper is organized as follows: In the next section, to get the physical picture of the dynamical equations, we obtain exact generalized power-law inflation for our model. We show that the generalized power-law solution exists only when the mass of the scalar field is identically zero. In Sec. 3, we show that the Higgs potential leads to dynamical model of inflation where the exit occurs close to the electro-weak scale. We show that the non-zero Higgs mass lead to the exit. In Sec. 4, we compute the power-spectrum of our model and compare the results with the recent PLANCK data. We discuss the key results and possible implications of our model in Sec. 5.

In this work, we consider $(-, +, +, +)$ metric signature. We use natural units $c = \hbar = 1$, $\kappa = 1/M_{\mathrm{Pl}}^2$, and $M_{\mathrm{Pl}}^2 = \frac{\hbar c}{8\pi G}$ is the reduced Planck mass. We denote dot as derivative with respect to cosmic time t and $H(t) \equiv \dot{a}(t)/a(t)$.

2. Generalized power-law inflation

Consider the following action where the scalar field ϕ is non-minimally coupled of Gauss-Bonnet ($\mathcal{L}_{\mathrm{GB}}$) term:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + f(\phi)\mathcal{L}_{\mathrm{GB}} - \frac{1}{2}\nabla_a\phi\nabla^a\phi - V(\phi) \right], \tag{1}$$

where R is the Ricci scalar, $V(\phi)$ is the scalar field potential, $f(\phi)$ is the coupling parameter and

$$\mathcal{L}_{GB} = R^2 - 4R^{ab}R_{ab} + R^{abcd}R_{abcd} \tag{2}$$

Varying the action (1) w.r.t the field ϕ and the metric leads to the following equations of motion [41]:

$$\Box\phi + \mathcal{L}_{GB}\, f_{,\phi}(\phi) - V_{,\phi}(\phi) = 0 \tag{3}$$

$$\frac{1}{\kappa}G_{pq} = \left(8G_{pq}g^{ab}\nabla_{ab}f(\phi) + 4R\nabla_{pq}f(\phi) - 8R_p^a\nabla_{aq}f(\phi) - 8R_q^a\nabla_{ap}f(\phi)\right.$$

$$8\nabla_{ab}f(\phi)R^{ab}g_{pq} - 8\nabla_{ab}f(\phi)R_{p\ q}^{\ a\ b} + \nabla_p\phi\nabla_q\phi - g_{pq}\left(\frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi + V(\phi)\right)\right) \tag{4}$$

It must be noted that the field equations being second order implies that this model doesn't have the problem of unitarity.

In this section, we are interested in obtaining exact solution for the above set of equations of motion for a spatially flat Friedmann-Robertson-Walker (FRW) background

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right) \tag{5}$$

where $a(t)$ is the scale factor. The equation of the field $\phi(t)$ and the scale factor $a(t)$ follows from Eqs. (3,4), respectively

$$-24H^2\frac{\ddot{a}}{a}f_{,\phi}(\phi) + \ddot{\phi} + V_{,\phi}(\phi) + 3H\dot{\phi} = 0 \tag{6a}$$

$$-3H^2\left(\frac{1}{\kappa} + 8H\dot{f}(\phi)\right) + \frac{1}{2}\dot{\phi}^2 + V(\phi) = 0 \tag{6b}$$

$$-H^2\left(\frac{1}{\kappa} + 8\ddot{f}(\phi)\right) - \frac{2\ddot{a}}{a}\left(\frac{1}{\kappa} + 8H\dot{f}(\phi)\right) + V(\phi) - \frac{1}{2}\dot{\phi}^2 = 0 \tag{6c}$$

It is important to note that the above differential equations are quasilinear i.e. they are linear with respect to all the highest order derivatives of $a(t)$ and $\phi(t)$. Rewriting Eqs. (6a, 6b), we get

$$-2H^2 + \kappa\dot{\phi}^2 - 24\,\kappa\,H^3\dot{f}(\phi) + 2\frac{\ddot{a}}{a} + 16\,\kappa H\frac{\ddot{a}}{a}\dot{f}(\phi)$$
$$+8\kappa H^2\ddot{f}(\phi) = 0 \tag{7}$$

In the rest of this section, we consider the solution of (6) for the following ansatz

$$a(t) = a_0\left(\frac{t}{t_0} + \Upsilon\right)^p \quad \text{and} \quad \phi = \phi_0\left(\frac{t}{t_0} + \Upsilon\right)^n \tag{8}$$

where $p > 1$ is a constant; n is a constant; a_0, t_0 are arbitrary constants whose values will not appear in any physical measured quantities and Υ is given by

$$\Upsilon = \left(\frac{\phi(t_0)}{\phi_0}\right)^{1/n} - 1. \tag{9}$$

Usually in cosmology, power-law inflation is given by $a(t) \propto t^p$. Ansatz (8) is a generalization. For real integer p, we have

$$a(t) = a_p t^p + a_{p-1}\,t^{p-1} + a_{p-2}\,t^{p-2} + \cdots + a_0$$

where in our case the coefficients $a_p, a_{p-1}, \cdots a_0$ are related. Since, $a(t)$ is a series, ϕ should also be a series like

$$\phi(t) = \phi_n t^n + \phi_{n-1}\,t^{n-1} + \phi_{n-2}\,t^{n-2} + \cdots + \phi_0$$

where, again, all the coefficients $\phi_n, \phi_{n-1}, \cdots \phi_0$. At $t = t_0$, $\phi(t_0) \neq \phi_0$ and ϕ_0 is an independent parameter. We refer to the above ansatz (8) for the scale factor as generalized power-law inflation.

Substituting the above ansatz (8) in Eq. (6), we get the following *exact relations*

$$V(\phi) = \tilde{\lambda}_1\,\phi^{-\frac{2}{n}} + \tilde{\lambda}_2\,\phi^{\frac{2(n-1)}{n}} + \tilde{\lambda}_3\,\phi^{\frac{p-1}{n}} \tag{10a}$$

$$f(\phi) = \tilde{\alpha}_1\,\phi^{\frac{2}{n}} + \tilde{\alpha}_2\,\phi^{\frac{2(n+1)}{n}} + \tilde{\alpha}_3\,\phi^{\frac{3+p}{n}} \tag{10b}$$

where

$$\tilde{\lambda}_1 = \frac{3(p-1)p^2}{\kappa(p+1)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^2 \qquad \tilde{\lambda}_2 = \frac{(5n^2p - n^2 + 2n^3)}{2(1 - 2n + p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^2 \qquad \tilde{\lambda}_3 = 24p^3C\left(\frac{\phi_0^{1/n}}{t_0}\right)^{1-p}$$

$$\tilde{\alpha}_1 = \frac{-1}{8\kappa p(1+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-2} \qquad \tilde{\alpha}_2 = \frac{n^2}{16p^2(1+n)(1-2n+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-2} \qquad \tilde{\alpha}_3 = \frac{C}{p+3}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-(p+3)} \tag{11}$$

and C is the constant of integration.

The following points are important to note regarding the above generalized power-law solution: (i) The ansatz (8) is the most general power-law *exact* solution satisfying Eqs. (6) for the potential and coupling (10) and *does not* depend on the constant of integration C. (ii) The above solutions are valid for any $p > 1$ and n. Imposing the condition that the potential be non-negative leads to $n > -2$ and $C \geq 0$. (iii) The coefficient of the first term in RHS of (10a) dominates the coefficients of the other two terms. Similarly, the coefficient of the first term in RHS of (10b) dominates the coefficients of the other two terms:

$$\left|\frac{\tilde{\alpha}_2}{\tilde{\alpha}_1}\right| = \left|\frac{1}{M_{\text{Pl}}^2} \frac{n^2(p+1)}{2(n+1)p^2(-2n+p+1)}\right| \qquad \left|\frac{\tilde{\lambda}_2}{\tilde{\lambda}_1}\right| = \left|\frac{1}{M_{\text{Pl}}^2} \frac{n^2(p+1)(2n+5p-1)}{6(p-1)p^2(-2n+p+1)}\right| \tag{12}$$

Hence for $\phi \ll M_{\text{Pl}}$; $\tilde{\lambda}_1\phi^{-2/n}$ and $\tilde{\alpha}_1\phi^{2/n}$ are the dominant terms in the potential and coupling respectively (iv) In the case of $C = 0$ and $\phi \ll M_{\text{Pl}}$, the general solution (10) leads to

$$V(\phi) = \tilde{\lambda}_1\phi^{\tilde{n}} \qquad f(\phi) = \tilde{\alpha}_1\phi^{-\tilde{n}} \qquad \text{where } \tilde{n} = -2/n$$

and is identical to the one studied by Guo and Schwarz [42, 43]. The authors [42] claimed that the solution has a graceful exit. However, our analysis clearly indicate that the above solution is a subset of the generalized power-law inflationary solution where there is no exit from inflation.

To make it clear, let us now repeat the analysis of [42] in the limit $\phi \ll M_{\text{Pl}}$. Rewriting their variables in terms of our variables, we have $\omega = 1$, $V_0 = \tilde{\lambda}_1$, $\xi_0 = -2\tilde{\alpha}_1$, $\alpha = -(8/3)\tilde{\alpha}_1\tilde{\lambda}_1$. Hence from Eq. (11), we get $\alpha = \frac{p(p-1)}{(p+1)^2}$. Now $0 < \alpha < 1$ implies $p > 1$, i.e at late times when our approximations turn valid the solution asymptotically approaches the power-law solution (8). For $\alpha = 0.25$ our analysis shows that the slow-roll parameter asymptotically takes the value $\epsilon = 0.4641$, which is also obtained numerically as the late time behaviour by Bruck and Longden [18] [see Fig. (1) in that reference].

In the rest of this section, we consider the special case $C = 0$, however, without imposing the condition $\phi \ll M_{\text{Pl}}$ and find an interesting scenario for Higgs inflation.

2.1. Special case: C = 0

Condition on n and C are that $n > -2$ and $C \geq 0$. We are interested in looking at the case where the scalar field potential has the form of power-law and consistent with standard model of particle physics. Hence, for the special case $C = 0$ and $n = -1/2$, scalar field potential and the coupling function take the following simple form:

$$V(\phi) = \lambda_4\phi^4 + \lambda_6\phi^6 \tag{13a}$$

$$f(\phi) = \frac{\alpha_2}{\phi^2} + \frac{\alpha_4}{\phi^4} \tag{13b}$$

where

$$\lambda_4 = \frac{3p^2(p-1)}{\phi_0^4\kappa(p+1)t_0^2} \quad \lambda_6 = \frac{1}{4}\frac{(5p-2)}{\phi_0^4t_0^2(4+2p)} \quad \alpha_2 = \frac{1}{32}\frac{\phi_0^4t_0^2}{p^2(2+p)} \quad \alpha_4 = -\frac{1}{8}\frac{\phi_0^4t_0^2}{\kappa p(p+1)} \tag{14}$$

This is one of the main results of this work regarding which we would like to stress the following points: (i) The dominant term in the potential and coupling function are λ_4 and α_4, respectively:

$$\left|\frac{\lambda_4}{\lambda_6}\right| \approx \left|5p^2M_{\text{Pl}}^2\right| \qquad \left|\frac{\alpha_4}{\alpha_2}\right| \approx \left|4pM_{\text{Pl}}^2\right|. \tag{15}$$

In other words, approximating

$$V(\phi) \simeq \lambda_4\phi^4 \qquad f(\phi) \simeq \frac{\alpha_4}{\phi^4} \tag{16}$$

lead to power-law expansion. While (8) is an exact solution of the background field equations (6) for the form of potential and coupling given by (13), including the sub-dominant terms. The ansatz (8) is an approximate solution for the form of potential and coupling given by (16). It is interesting to note that the above approximate scalar field potential is the potential for the chaotic inflation [44]. In the case of chaotic inflation, the scalar field is not coupled to the Gauss-Bonnet gravity, here, the non-minimal coupling catalyzes scalar field to accelerate and, hence, there is no exit with a pure ϕ^4 potential. (ii) For inflation to occur, Eq. (14) implies $\alpha_4 < 0$ and $\lambda_4 > 0$. (iii) Physical relevance

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of ϕ_0 can be seen from Eq. (14). The value of Gauss-Bonnet coupling parameter at the epoch of inflation depends on the ratio of ϕ_0 and $\phi(t_0)$, i. e.,

$$f(\phi(t_0)) \propto \left(\frac{\phi_0}{\phi(t_0)}\right)^4 \frac{1}{\kappa} \tag{17}$$

ϕ_0 decides the epoch of inflation in our model. Since, this can not be fixed *ab initio*, it can be fixed from observations. (iv) From (14), we obtain a relation between λ_4 and α_4

$$\lambda_4 |\alpha_4| = \frac{3p(p-1)}{8(p+1)^2 \kappa^2} \tag{18}$$

The above relation shows that for a given power-law inflation, λ_4 and α_4 are inversely related to each other. We show that the above relation is approximately satisfied also for Higgs inflation. This is interesting because λ_4 is measured precisely, and our model makes a precise prediction for α_4 and can lead to potential signatures at the LHC. We discuss this in the Conclusion. (v) The above relations also provide interesting connection between ϕ_0 (the initial value of the field) with the coupling constants. From (14), we get,

$$\phi_0^2 = \frac{1}{t_0}\left(24p^3(p-1)\frac{|\alpha_4|}{\lambda_4}\right)^{1/4} \tag{19}$$

Our model makes a precise prediction on the initial value of the inflaton field once we fix the inflation epoch of inflation and hence, setting the scale of inflation. Our model does not impose any restriction on the epoch of power-law inflation. As we show explicitly in the next section, including the mass term in the potential leads to exit from inflation, interestingly, the exit occurs at low-energies.

3. Higgs inflation

Following the discussion in the previous section, the tree-level SM Higgs Lagrangian is

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + f(\mathcal{H})\mathcal{L}_{GB} - |D_\mu \mathcal{H}|^2 - \lambda(|\mathcal{H}|^2 - v^2)^2\right], \tag{20}$$

where D_μ is the covariant derivative with respect to the SM gauge symmetry, $\mathcal{H}$ is the SM Higgs boson, v is its vacuum expectation value ($v = 246$ GeV), and λ is the self-coupling constant.

Taking the gauge ${}^t\mathcal{H} = (0, v + h)/\sqrt{2}$ (where h stands for the real, neutral component of the Higgs doublet $\mathcal{H}$) — the action (20) becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + f(h)\mathcal{L}_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu h \partial_\nu h - \lambda_4 \left(h^2 - v^2\right)^2\right], \tag{21}$$

$$\text{where } \lambda_4 = \lambda/4$$

In the limit $h \gg v$, this is identical to the action (1) — with the coupling constant and potential given by (16) — that lead to power-law inflation. While the condition $h \gg v$ may be valid at the initial epoch of inflation, it may not be a good approximation at the end of inflation.

As in the previous section, we set the Gauss-Bonnet coupling function to be

$$f(h) = \frac{\alpha_4}{h^4} \tag{22}$$

Before we proceed to obtain the background solutions, we would like to point the following: (i) As mentioned above, at the initial epoch of inflation, the value of the Higgs h is much larger than the vacuum expectation value. During this epoch, the background dynamics can be expressed as described in the previous section. (ii) However, as the Universe cools down during inflation, the value of the Higgs field is of the order of the vacuum expectation value and hence, the potential term will be sub-dominant. This suggests that the background dynamics may change dramatically.

Unlike the earlier case, the background equations of motion (6) can not be computed analytically especially in the region when the field value h is of the order of vacuum expectation value v. We solved the equations of motion (6) numerically for a time step of 10^{-16} secs and precision of the field h/h_0 (in dimensionless units) to be 10^{-16}. Using LHC results of $\lambda = 0.1291$ ($\lambda = \frac{M_H^2}{2v^2}$, where M_H is the Higgs mass and v its vacuum expectation value), we have $\lambda_4 = \lambda/4 = 0.032275$. (Appendix contains the details of the numerics.)

In Figures (1 and 2), we have plotted different physically relevant background quantities, from which we infer the following:
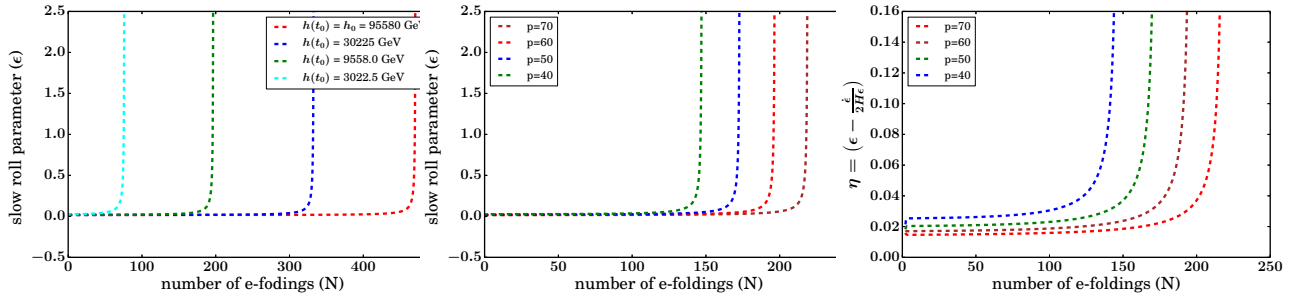

Figure 1: Slow roll parameters ϵ and η Vs number of e-foldings (i) ϵ for different values of Higgs field $h(t_0)$ (for p=60),(ii) ϵ for different values of $\alpha_4(p)$ and (iii) slow-roll parameter η for different $\alpha_4(p)$

1. In Fig. (1), we have plotted the slow-roll parameters ϵ and η

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = \epsilon - \frac{1}{2}\frac{\dot{\epsilon}}{H\epsilon} \qquad (23)$$

as a function of Number of e-foldings $N(t) = \ln[a(t_{end})/a(t)] = \int_t^{t_{end}} H dt$ for (i) for different initial values of the field $h(t_0)$, (ii) assuming that initial epoch as a power-law $p > 1$ and (iii) assuming that initial epoch as a power-law $p > 1$. This clearly indicates that our model leads to a minimum of 100 e-foldings of inflation for any initial field value $h(t_0)$, greater than 5 TeV. Fig. (1) also shows that larger the initial value of the field it leads to longer number of e-foldings and η is a constant for all through the inflation and varies rapidly at the exit of inflation.

2. In Fig. (2), we have plotted the evolution of the Higgs field $h(t)$ as a function of number of e-foldings for by assuming (i) different initial values of the field $h(t_0)$ and (ii) assuming that initial epoch as a power-law $p > 1$. It is important to note, in our model like any other model of inflation the field value remains almost a constant throughout the evolution, however close to the exit it changes rapidly. From Figs. (2), we infer that whatever be the initial value of the field or power-law p, the exit of inflation — when ϵ becomes greater than 1 — occurs around the Electroweak scale.

We have explicitly shown that the Higgs neutral scalar field, non-minimally coupled to Gauss-Bonnet gravity leads to a dynamical model of inflation and exit of Inflation occurs around Electroweak scale ~ 250GeV.
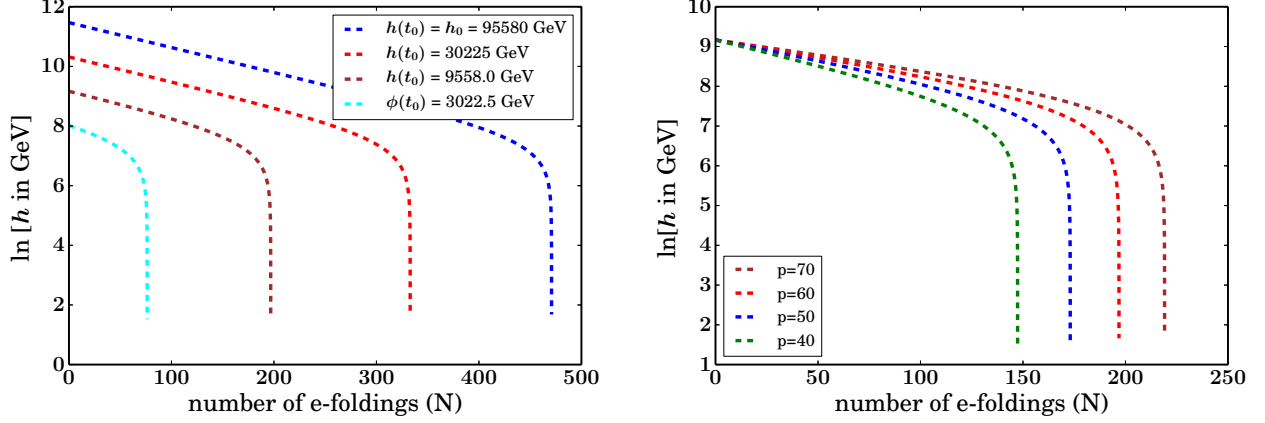
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Figure 2: The evolution of Higgs field $h(t)$ (i) for different values of initial field value $h(t_0)$ (for p=60) and (ii) for different values of the parameter $\alpha_4(p)$.

4. Power Spectrum

In this section, we compute the scalar and tensor power-spectrum for the Higgs inflation model, and compare it with the PLANCK data [8]. In this section we use the results obtained by Hwang and Noh [45] for our analysis. At linear order, the scalar perturbations can be simplified by writing in the uniform-field gauge. Mukhanov-Sasaki equation in the Fourier domain is given by, see Ref. [45] for details:

$$v_k'' + \left(c^2 k^2 - \frac{z''}{z}\right) v_k = 0 \tag{24}$$

where

$$z = a\sqrt{Q_{\mathcal{R}}} \tag{25a}$$

$$Q_{\mathcal{R}} = \frac{\dot{\phi}^2 + 12 H^2 \dot{f} \Gamma}{(H + \Gamma/2)^2} \qquad \Gamma = \frac{8 H^2 \dot{f}}{1/\kappa + 8 H \dot{f}} \tag{25b}$$

$$c^2 = 1 - 256 \dot{f} \Gamma^2 \frac{\ddot{f}/\dot{f} - H - 4\dot{H}/\Gamma}{\dot{\phi}^2 + 12 H^2 \dot{f} \Gamma} \tag{25c}$$

and prime denotes differentiation with respect to conformal time η. For the Higgs inflation, c turns out to be approximately constant with value slightly less than one and $Q_{\mathcal{R}}$ also is approximately a constant, see the equations below, for $p = 60$:

$$c^2 = \frac{1392 M_{\text{Pl}}^2 \left(60\sqrt{10797}t_0\sqrt{\frac{M_{\text{Pl}}^2}{\lambda t_0^2}} + 61 t \phi(t_0)^2 - 61 t_0 \phi(t_0)^2\right) + 3599\sqrt{10797}t_0\phi(t_0)^2\sqrt{\frac{M_{\text{Pl}}^2}{\lambda t_0^2}}}{1440 M_{\text{Pl}}^2 \left(60\sqrt{10797}t_0\sqrt{\frac{M_{\text{Pl}}^2}{\lambda t_0^2}} + 61 t \phi(t_0)^2 - 61 t_0 \phi(t_0)^2\right) + 3599\sqrt{10797}t_0\phi(t_0)^2\sqrt{\frac{M_{\text{Pl}}^2}{\lambda t_0^2}}} \tag{26}$$

$$Q_{\mathcal{R}} = 0.0017\left(M_{\text{Pl}}^2 + M_{\text{Pl}}^2\left(\frac{1}{0.235\sqrt{\lambda}(t-t_0)M_{\text{Pl}} + \frac{24 M_{\text{Pl}}^2}{\phi(t_0)^2}}\right)\right) \approx 0.0017\, M_{\text{Pl}}^2. \tag{27}$$

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Hence for our model Eq. (24) becomes :

$$v_k'' + \left(c^2 k^2 - \frac{\sigma^2 - 1/4}{\eta^2}\right)v = 0 \tag{28}$$

where

$$\sigma = \frac{3p-1}{2(p-1)} \tag{29}$$

and the general solution for Eq.(28) are a linear combination of Hankel functions.

$$v_k = \sqrt{|\eta|}\left(A H_\sigma^{(1)}(ck|\eta|) + B H_\sigma^{(2)}(ck|\eta|)\right) \tag{30}$$

Choosing the Bunch-Davies vacuum at past infinity $(ck|\eta| \to \infty)$, we get

$$A = \sqrt{\frac{\pi}{4}} e^{i(2\sigma+1)\pi/4} \quad \text{and} \quad B = 0\,. \tag{31}$$

The power spectrum of scalar curvature perturbations

$$P_\mathcal{R} = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2 \tag{32}$$

evaluated when the modes leave the Hubble radius leads to

$$P_\mathcal{R} = C\,k^{3-2\sigma}; \qquad C = 2^{4(\sigma-1)}c^{-2\sigma}\left(\frac{\Gamma(\sigma)}{\Gamma(3/2)}\right)^2 \frac{1}{4\pi^2}\left(\frac{a_0/t_0}{2\sigma-3}\right)^{2\sigma-1}\frac{1}{a_0^2 Q_\mathcal{R}}\,. \tag{33}$$

The scalar spectral index $n_\mathcal{R} - 1 = 3 - 2\sigma$.

The Fourier modes of the tensor perturbations satisfy, see Ref. [45] for details:

$$u'' + \left(c_\mathcal{T}^2 k^2 - \frac{z_\mathcal{T}''}{z_\mathcal{T}}\right)u = 0 \tag{34}$$

where

$$z_\mathcal{T} = a\sqrt{Q_g}; \quad Q_g = \frac{1}{\kappa} + 8H\dot{f}, \quad c_\mathcal{T}^2 = \frac{1 + 8\kappa\ddot{f}}{1 + 8\kappa H\dot{f}}\,. \tag{35}$$

Like in the case of scalar-perturbations, Q_g and $c_\mathcal{T}$ are approximately constants during inflation. So proceeding in the same way as like scalar perturbations, the tensor power-spectrum is given by:

$$P_\mathcal{T} = C_T k^{3-2\sigma_\mathcal{T}} \quad C_T = 8 \times 2^{4(\sigma_\mathcal{T}-1)}c_\mathcal{T}^{-2\sigma_\mathcal{T}}\left(\frac{\Gamma(\sigma_\mathcal{T})}{\Gamma(3/2)}\right)^2 \frac{1}{4\pi^2}\left(\frac{a_0/t_0}{2\sigma_\mathcal{T}-3}\right)^{2\sigma_\mathcal{T}-1}\frac{1}{a_0^2 Q_g} \tag{36}$$

The spectral index of scalar and tensor perturbations obey $n_\mathcal{R} - 1 = n_\mathcal{T}$. The tensor to scalar ratio, which is defined as

$$r \equiv \frac{P_\mathcal{T}}{P_\mathcal{R}} = 8 \times \left(\frac{c}{c_\mathcal{T}}\right)^{2\sigma} \frac{Q_\mathcal{R}}{Q_g} \tag{37}$$

4.1. Constraints from PLANCK

PLANCK [8] provides stringent constraints on the scalar spectral index $n_\mathcal{R} = 0.968 \pm 0.006$. Approximating inflation to be generalized power-law (for large part during inflation), Eq. (33) leads to $p \approx 60$. For $p \approx 60$, from Eq. (37), tensor to scalar ratio turns out to be $r = 0.012$. The above result is in agreement with PLANCK constraint of $r < 0.1$. It is important to note that as like other Higgs inflation model [15], our model predicts that the contribution from the tensor is significantly smaller than that of scalar perturbations.

8

Since our model does not have any free parameter, we can constraint all the parameters of the model by comparing the model's predicted power-spectrum and the observed power-spectrum values at the pivot scale $k_* = 0.05 \text{Mpc}^{-1}$. The observed power-spectrum at the pivot scale is $P_{\mathcal{R}}(k_*) = 2.2 \times 10^{-9}$ [8]. From Eq. (33), the time at which the perturbations exited the horizon radius during inflation is given as

$$t_* = 5.273 \times 10^{-36} \text{ secs} + t_0 - 60 \sqrt{8|f(h(t_0))|\kappa} \tag{38}$$

The field value at hubble crossing, $h_* \approx 10^{16}$ GeV. The above relations are the one of the main results of our work, regarding which we would like to stress the following: (i) Given that λ_4 is precisely measured leads to

$$h_0^2 t_0 \sim 5.7 \times 10^2 M_{\text{Pl}} . \tag{39}$$

The above expression clearly shows that while $h_0^2 t_0$ can be determined, however, h_0 and t_0 can not be determined independently. (ii) The time of exit of the perturbations depends on the value of the Gauss-Bonnet parameter at the epoch of inflation. As can be seen from above, t_* depends on t_0 and h_0. (iii) The scale at the time perturbations left the Hubble scale is $H_* \approx 10^{12}$ GeV.

5. Discussions

In this work, we presented a model of inflation in which the Higgs scalar is the inflaton. In our model, we have assumed Higgs is non-minimally coupled to Gauss-Bonnet Gravity in 4-dimensions. We have shown analytically that scalar field with ϕ^4 potential term leads to power-law inflation, however, adding a mass leads to the exit of inflation. We have explicitly shown that the exit is close to the Electroweak scale. Power-spectrum generated from our model is consistent with the PLANCK observations.

There have been earlier attempts to look for an inflationary model at electro-weak scales [46, 47, 48, 49, 50, 51]. Our model leads to inflation with exit at Electroweak scale due to non-minimal coupling of the Higgs to Gauss-Bonnet gravity term. In the model proposed by German et al Ref. [49], thermal effects lead to low-scale inflation in supersymmetric or large extra dimensions.

It is intriguing that Gauss-Bonnet gravity which are higher-derivative gravity corrections to Einstein gravity and hence are expected to have strong effects only in the early universe. However, we have explicitly shown that Gauss-Bonnet gravity leads a dynamical model at low-energies.

Our model is in spirit with Chaotic inflationary model of Linde [44]. Our model does not require an extension of standard model but is a natural phenomenon within standard model at the cost of a non minimal coupling of Higgs field with the Gauss-Bonnet coupling. Our analysis also indicate possible implications of the Gauss-Bonnet coupling at LHC.[1]

As mentioned in the previous section, PLANCK observations constrain all parameters of our model, however, it does not constrain the epoch of inflation. Using the non-Gaussianity constraints of PLANCK may help to break the degeneracy between h_0 and t_0. This is currently under investigation.

6. Acknowledgements

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[1] Inflation reheating requires coupling of the inflaton with the standard model particles. More precisely, to have efficient reheating the coupling has to be stronger so that the energy gets transferred from inflaton to standard model particles. If the exit happens close to EW scale, then this implies that the reheating process can provide some hint about the coupling of the inflaton with standard model.

7. Appendix: Numerical Evaluation

In order to solve the time evolution of the Higgs field $h(t)$, we obtain the expression for $\frac{d^2h}{dt^2}$ as a function only of h and $\frac{dh}{dt}$. We then use RK4 to numerically evaluate $h(t)$.

We solve the Hubble constant H from (6b). The cubic equation leads to three solutions for H, given by:

$$H1 = \frac{B^{\frac{1}{3}}}{24k\frac{df(t)}{dt}} + \frac{1}{24k\frac{df(t)}{dt}B^{\frac{1}{3}}} - \frac{1}{24k\frac{df(t)}{dt}} \tag{40a}$$

$$H2 = \frac{-B^{\frac{1}{3}}}{48k\frac{df(t)}{dt}} - \frac{1}{48k\frac{df(t)}{dt}B^{\frac{1}{3}}} - \frac{1}{24k\frac{df(t)}{dt}} + I\sqrt{3}\left(\frac{B^{\frac{1}{3}}}{48k\frac{df(t)}{dt}} - \frac{1}{48k\frac{df(t)}{dt}B^{\frac{1}{3}}}\right) \tag{40b}$$

$$H3 = \frac{-B^{\frac{1}{3}}}{48k\frac{df(t)}{dt}} - \frac{1}{48k\frac{df(t)}{dt}B^{\frac{1}{3}}} - \frac{1}{24k\frac{df(t)}{dt}} - I\sqrt{3}\left(\frac{B^{\frac{1}{3}}}{48k\frac{df(t)}{dt}} - \frac{1}{48k\frac{df(t)}{dt}B^{\frac{1}{3}}}\right) \tag{40c}$$

where

$$
\begin{aligned}
B &= 144\left(\frac{d}{dt}f(t)\right)^2 k^3\left(\frac{d}{dt}h(t)\right)^2 + 288\left(\frac{d}{dt}f(t)\right)^2 k^3 V(t) - 1 + \\
&\quad 12\sqrt{2}\left(\frac{d}{dt}f(t)\right)k^2\sqrt{A} \\
A &= \frac{1}{k}\left(72\left(\frac{d}{dt}f(t)\right)^2 k^3\left(\frac{d}{dt}h(t)\right)^4 + 288\left(\frac{d}{dt}f(t)\right)^2 k^3\left(\frac{d}{dt}h(t)\right)^2 V(t) - \right. \\
&\quad \left.\left(\frac{d}{dt}h(t)\right)^2 + 288\left(\frac{d}{dt}f(t)\right)^2 k^3(V(t))^2 - 2V(t)\right)
\end{aligned}
$$

Among the three solutions for H, *only* one solution satisfy the eqns. (6) all through the evolution and that is the physical solution. Substituting the corresponding solution in (6a), we get the second order differential equation in $h(t)$. From the fact that we can split $\frac{dH}{dt}$ as a sum of H_{t1} and $H_{t2}\frac{d^2h}{dt^2}$, where H_{t1} and H_{t2} are independent of $\frac{d^2h}{dt^2}$. we have:

$$\frac{d^2h}{dt^2} = \frac{24H^2\frac{df(h)}{dt}H_{t1} + 24H^4\frac{df(h)}{dt} - \frac{dV(h)}{dt} - 3\left(\frac{dh}{dt}\right)^2 H}{\frac{dh}{dt} - 24H^2\frac{df(h)}{dt}H_{t2}} \tag{41}$$

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