Charged Fermion Masses and Mixing from a $SU(3)$ Family Symmetry Model

Albino Hernández-Galeana[1, *]

[1] *Departamento de Física, Escuela Superior de Física y Matemáticas, I.P.N., U. P. "Adolfo López Mateos". C. P. 07738, México, D.F., México.*

Within the framework of a Beyond Standard Model (BSM) with a local $SU(3)$ family symmetry, we report an updated fit of parameters which account for the known spectrum of quarks and charged lepton masses and the quark mixing in a 4×4 non-unitary V_{CKM}. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions, U, D, E, N, with N a sterile neutrino. The $N_{L,R}$ sterile neutrinos allow the implementation of a 8×8 general See-saw Majorana neutrino mass matrix with four massless eigenvalues at tree level. Hence, light fermions, including neutrinos, obtain masses from loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. $SU(3)$ family symmetry is broken spontaneously in two stages, whose hierarchy of scales yield an approximate $SU(2)$ global symmetry associated with the $Z_1, Y_1^{\pm}$ gauge boson masses of the order of 2 TeV. A global fit of parameters to include neutrino masses and lepton mixing is in progress.

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*Electronic address: albino@esfm.ipn.mx

I. INTRODUCTION

The origen of the hierarchy of fermion masses and mixing is one of the most important open problems in particle physics. Any attempt to account for this hierarchy introduce a mass generation mechanism which distinguish among the different Standard Model (SM) quarks and leptons.

After the discovery of the scalar Higgs boson on 2012, LHC has not found a conclusive evidence of new physics. However, there are theoretical motivations to look for new particles in order to answer some open questions like; neutrino oscillations, dark matter, stability of the Higgs mass against radiative corrections,etc.

In this report, we address the problem of charged fermion masses and quark mixing within the framework of an extension of the SM introduced by the author in [1]. This BSM proposal include a vector gauged $SU(3)$ family symmetry[1] commuting with the SM group and introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses of the top and bottom quarks and that of the tau lepton are generated at tree level from "Dirac See-saw"[3] mechanisms through the introduction of a new set of $SU(2)_L$ weak singlets U, D, E and N vector-like fermions, which do not couple to the W boson, such that the mixing of U and D vector-like quarks with the SM quarks gives rise to and extended 4×4 non-unitary CKM quark mixing matrix [4].

II. MODEL WITH $SU(3)$ FLAVOR SYMMETRY

A. Fermion content

Before "Electroweak Symmetry Breaking"(EWSB) all ordinary SM fermions remain massless, and the global symmetry in this limit, including R-handed neutrinos, is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$
$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3) \tag{1}$$

We define the gauge symmetry group

$$G \equiv SU(3) \otimes G_{SM} \tag{2}$$

where $SU(3)$ is the gauged family symmetry among families, eq.(1) , and $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group, with g_H, g_s, g and g' the corresponding coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

Ordinary Fermions: $q^o_{iL} = \begin{pmatrix} u^o_{iL} \\ d^o_{iL} \end{pmatrix}$, $l^o_{iL} = \begin{pmatrix} \nu^o_{iL} \\ e^o_{iL} \end{pmatrix}$, $Q = T_{3L} + \frac{1}{2}Y$

$$\Psi^o_q = (3, 3, 2, \frac{1}{3})_L = \begin{pmatrix} q^o_{1L} \\ q^o_{2L} \\ q^o_{3L} \end{pmatrix} \quad , \quad \Psi^o_l = (3, 1, 2, -1)_L = \begin{pmatrix} l^o_{1L} \\ l^o_{2L} \\ l^o_{3L} \end{pmatrix}$$

$$\Psi^o_u = (3, 3, 1, \frac{4}{3})_R = \begin{pmatrix} u^o_R \\ c^o_R \\ t^o_R \end{pmatrix} \quad , \quad \Psi^o_d = (3, 3, 1, -\frac{2}{3})_R = \begin{pmatrix} d^o_R \\ s^o_R \\ b^o_R \end{pmatrix}$$

[1] See [1, 2] and references therein for some other $SU(3)$ family symmetry model proposals.

$$\Psi_e^o = (3,1,1,-2)_R = \begin{pmatrix} e_R^o \\ \mu_R^o \\ \tau_R^o \end{pmatrix} \ ,$$

where the last entry corresponds to the hypercharge Y. The model also includes two types of extra $SU(2)_L$ weak singlet fermions:

Right Handed Neutrinos: $\Psi_{\nu_R}^o = (3,1,1,0)_R = \begin{pmatrix} \nu_{e_R} \\ \nu_{\mu_R} \\ \nu_{\tau_R} \end{pmatrix} \ ,$

and the vector-like fermions:

Sterile Neutrinos: $N_L^o, N_R^o = (1,1,1,0) \ ,$

The Vector Like quarks:

$$U_L^o, U_R^o = (1,3,1,\frac{4}{3}) \quad , \quad D_L^o, D_R^o = (1,3,1,-\frac{2}{3}) \tag{3}$$

and

The Vector Like electron: $E_L^o, E_R^o = (1,1,1,-2).$

The transformation of these vector-like fermions allows the gauge invariant mass terms

$$M_U \, \bar{U}_L^o \, U_R^o \, + \, M_D \, \bar{D}_L^o \, D_R^o \, + \, M_E \, \bar{E}_L^o \, E_R^o + h.c. \ , \tag{4}$$

and

$$m_D \, \bar{N}_L^o \, N_R^o \, + \, m_L \, \bar{N}_L^o \, (N_L^o)^c \, + \, m_R \, \bar{N}_R^o \, (N_R^o)^c \, + \, h.c \tag{5}$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the $SU(3)$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies [5]. The $SU(2)_L$ weak singlet vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions via Dirac See-saw mechanisms. These vector like fermions, together with the radiative corrections, play a crucial role to implement a hierarchical spectrum for ordinary quarks and charged lepton masses.

III. $SU(3)$ FAMILY SYMMETRY BREAKING

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)$, we introduce the flavon scalar fields: $\eta_i, \ i = 2,3,$

$$\eta_i = (3,1,1,0) = \begin{pmatrix} \eta_{i1}^o \\ \eta_{i2}^o \\ \eta_{i3}^o \end{pmatrix} \quad , \quad i = 2,3,$$

acquiring the "Vacuum ExpectationValues" (VEV's):

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) \ . \tag{6}$$

The corresponding $SU(3)$ gauge bosons are defined in Eq.(20) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(6) read

	Z_1	Z_2
Z_1	M_2^2	$-\frac{M_2^2}{\sqrt{3}}$
Z_2	$-\frac{M_2^2}{\sqrt{3}}$	$\frac{M_2^2+4M_3^2}{3}$

TABLE I: $Z_1 - Z_2$ mixing mass matrix

- η_2 : $\frac{g_{H_2}^2\Lambda_2^2}{2}(Y_1^+Y_1^- + Y_3^+Y_3^-) + \frac{g_{H_2}^2\Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1\frac{Z_2}{\sqrt{3}})$
- η_3 : $\frac{g_{H_3}^2\Lambda_3^2}{2}(Y_2^+Y_2^- + Y_3^+Y_3^-) + g_{H_3}^2\Lambda_3^2\frac{Z_2^2}{3}$

These two scalars in the fundamental representation is the minimal set of scalars to break down completely the $SU(3)$ family symmetry. Therefore, neglecting tiny contributions from electroweak symmetry breaking, Eq.(14), we obtain the gauge boson mass terms:

$$M_2^2\,Y_1^+Y_1^- + M_3^2\,Y_2^+Y_2^- + (M_2^2+M_3^2)\,Y_3^+Y_3^- + \frac{1}{2}M_2^2\,Z_1^2 + \frac{1}{2}\frac{M_2^2+4M_3^2}{3}\,Z_2^2 - \frac{1}{2}(M_2^2)\frac{2}{\sqrt{3}}\,Z_1\,Z_2 \tag{7}$$

$$M_2^2 = \frac{g_H^2\Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_H^2\Lambda_3^2}{2} \quad , \quad y \equiv \frac{M_3}{M_2} = \frac{\Lambda_3}{\Lambda_2} \tag{8}$$

Diagonalization of the $Z_1 - Z_2$ squared mass matrix yield the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_2^2 + M_3^2 - \sqrt{(M_3^2-M_2^2)^2 + M_2^2M_3^2}\right) = M_2^2\,y_- \tag{9}$$

$$M_+^2 = \frac{2}{3}\left(M_2^2 + M_3^2 + \sqrt{(M_3^2-M_2^2)^2 + M_2^2M_3^2}\right) = M_2^2\,y_+ \tag{10}$$

and the gauge boson mass eigenvalues

$$M_2^2\,Y_1^+Y_1^- + M_3^2\,Y_2^+Y_2^- + (M_2^2+M_3^2)\,Y_3^+Y_3^- + M_-^2\,\frac{Z_-^2}{2} + M_+^2\,\frac{Z_+^2}{2} \tag{11}$$

or

$$M_2^2\,Y_1^+Y_1^- + M_2^2\,y^2\,Y_2^+Y_2^- + M_2^2(1+y^2)\,Y_3^+Y_3^- + M_2^2\,y_-\,\frac{Z_-^2}{2} + M_2^2\,y_+\,\frac{Z_+^2}{2}\,, \tag{12}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{13}$$

$$\cos\phi\,\sin\phi = \frac{\sqrt{3}}{4}\,\frac{M_2^2}{\sqrt{M_2^4 + M_3^2(M_3^2-M_2^2)}}$$

Notice that in the limit $y = \frac{M_3}{M_2} \gg 1$, $\sin\phi \to 0$, $\cos\phi \to 1$, and we get an approximate $SU(2)$ global symmetry for the $Z_1, Y_1^\pm$ almost degenerated gauge boson masses of order M_2. Thus, the hierarchy of scales in the SSB yields an approximate $SU(2)$ global symmetry in the spectrum of $SU(3)$ gauge boson masses. Actually this approximate $SU(2)$ symmetry may play the role of a custodial symmetry to suppress properly the tree level $\Delta F = 2$ "Flavour Changing Neutral Currents" (FCNC) processes mediated by the lower scale of horizontal gauge bosons with masses of few TeV's

IV. ELECTROWEAK SYMMETRY BREAKING

Recently ATLAS [6] and CMS [7] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The Electroweak Symmetry Breaking (EWSB) in the $SU(3)$ family symmetry model involves the introduction of two triplets of $SU(2)_L$ Higgs doublets, namely;

$$\Phi^u = (3,1,2,-1) = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}^u_1 \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}^u_2 \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}^u_3 \end{pmatrix} \quad , \quad \Phi^d = (3,1,2,+1) = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}^d_1 \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}^d_2 \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}^d_3 \end{pmatrix} ,$$

with the VEV?s

$$\Phi^u\rangle = \begin{pmatrix} \langle\Phi^u_1\rangle \\ \langle\Phi^u_2\rangle \\ \langle\Phi^u_3\rangle \end{pmatrix} \quad , \quad \langle\Phi^d\rangle = \begin{pmatrix} \langle\Phi^d_1\rangle \\ \langle\Phi^d_2\rangle \\ \langle\Phi^d_3\rangle \end{pmatrix} ,$$

where

$$\Phi^u_i\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle\Phi^d_i\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} .$$

The contributions from $\langle\Phi^u\rangle$ and $\langle\Phi^d\rangle$ yield the W and Z gauge boson masses and mixing with the $SU(3)$ gauge bosons

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+{g'}^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$

$$+\,\frac{1}{4}\sqrt{g^2+{g'}^2}\,g_H\,Z_o\,\Big[\,(v_{1u}^2-v_{2u}^2-v_{1d}^2+v_{2d}^2)\,Z_1+(v_{1u}^2+v_{2u}^2-2v_{3u}^2-v_{1d}^2-v_{2d}^2+2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}$$

$$+2\,(v_{1u}v_{2u}-v_{1d}v_{2d})\,\frac{Y_1^++Y_1^-}{\sqrt{2}}+2\,(v_{1u}v_{3u}-v_{1d}v_{3d})\,\frac{Y_2^++Y_2^-}{\sqrt{2}}+2\,(v_{2u}v_{3u}-v_{2d}v_{3d})\,\frac{Y_3^++Y_3^-}{\sqrt{2}}\Big]$$

$$+\,\frac{g_H^2}{4}\,\Big\{\,\frac{1}{2}\,(v_{1u}^2+v_{2u}^2+v_{1d}^2+v_{2d}^2)\,Z_1^2+\frac{1}{2}\,(v_{1u}^2+v_{2u}^2+4v_{3u}^2+v_{1d}^2+v_{2d}^2+4v_{3d}^2)\,\frac{Z_2^2}{3}$$

$$+\,(v_{1u}^2+v_{2u}^2+v_{1d}^2+v_{2d}^2)\,Y_1^+Y_1^-+(v_{1u}^2+v_{3u}^2+v_{1d}^2+v_{3d}^2)\,Y_2^+Y_2^-+(v_{2u}^2+v_{3u}^2+v_{2d}^2+v_{3d}^2)\,Y_3^+Y_3^-$$

$$+\,(v_{1u}^2-v_{2u}^2+v_{1d}^2-v_{2d}^2)\,Z_1\,\frac{Z_2}{\sqrt{3}}+(v_{2u}v_{3u}+v_{2d}v_{3d})\,(Y_1^+Y_2^-+Y_1^-Y_2^+)$$

$$+\,(v_{1u}v_{2u}+v_{1d}v_{2d})\,(Y_2^+Y_3^-+Y_2^-Y_3^+)+(v_{1u}v_{3u}+v_{1d}v_{3d})\,(Y_1^+Y_3^++Y_1^-Y_3^-)$$

$$+2\,(v_{1u}v_{2u}+v_{1d}v_{2d})\,\frac{Z_2}{\sqrt{3}}\,\frac{Y_1^++Y_1^-}{\sqrt{2}}+(v_{1u}v_{3u}+v_{1d}v_{3d})\,(Z_1-\frac{Z_2}{\sqrt{3}})\,\frac{Y_2^++Y_2^-}{\sqrt{2}}$$

$$-(v_{2u}v_{3u}+v_{2d}v_{3d})\,(Z_1+\frac{Z_2}{\sqrt{3}})\,\frac{Y_3^++Y_3^-}{\sqrt{2}}\Big\} \qquad (14)$$

$v_u^2=v_{1u}^2+v_{2u}^2+v_{3u}^2$, $v_d^2=v_{1d}^2+v_{2d}^2+v_{3d}^2$. Hence, if we define as usual $M_W=\frac{1}{2}gv$, we may write $v=\sqrt{v_u^2+v_d^2}\approx 246$ GeV.

$$Y_j^1=\frac{Y_j^++Y_j^-}{\sqrt{2}} \quad , \quad Y_j^\pm=\frac{Y_j^1\mp iY_j^2}{\sqrt{2}} \qquad (15)$$

The mixing of Z_o neutral gauge boson with the $SU(3)$ gauge bosons modify the couplings of the standard model Z boson with the ordinary quarks and leptons

V. FERMION MASSES

A. Dirac See-saw mechanisms

Now we describe briefly the procedure to get the masses for fermions. The analysis is presented explicitly for the charged lepton sector, with a completely analogous procedure for the u and d quarks and Dirac neutrinos. With the fields of particles introduced in the model, we may write the gauge invariant Yukawa couplings, as

$$h\,\bar{\psi}_l^o\,\Phi^d\,E_R^o\;+\;h_2\,\bar{\psi}_e^o\,\eta_2\,E_L^o\;+\;h_3\,\bar{\psi}_e^o\,\eta_3\,E_L^o\;+\;M\,\bar{E}_L^o\,E_R^o\;+h.c \qquad (16)$$

where M is a free mass parameter because its mass term is gauge invariant and h, h_2 and h_3 are Yukawa coupling constants. When the involved scalar fields acquire VEV's we get, in the gauge basis ${\psi_{L,R}^o}^T=(e^o,\mu^o,\tau^o,E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o\mathcal{M}^o\psi_R^o+h.c$, where

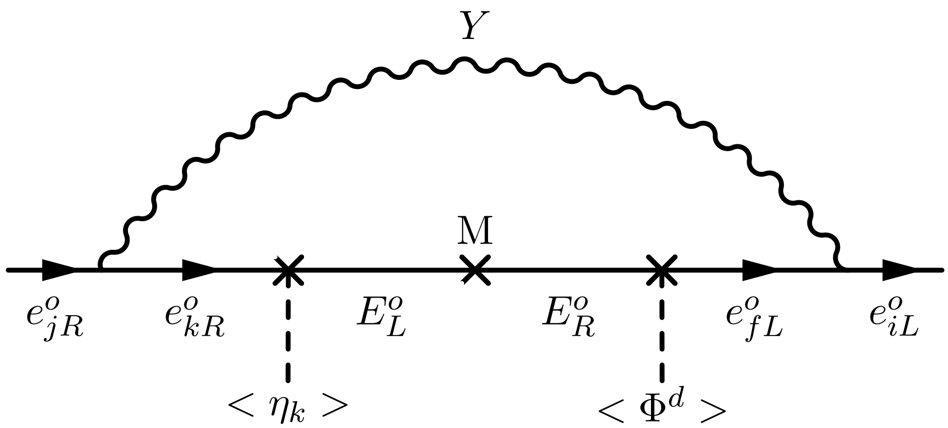

FIG. 1: Generic one loop diagram contribution to the mass term $m_{ij}\, \bar{e}^o_{iL} e^o_{jR}$

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\, v_1 \\ 0 & 0 & 0 & h\, v_2 \\ 0 & 0 & 0 & h\, v_3 \\ 0 & h_2\Lambda_2 & h_3\Lambda_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & M \end{pmatrix} . \tag{17}$$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi^o_{L,R} = V^o_{L,R}\, \chi_{L,R}$. The orthogonal matrices V^o_L and V^o_R are obtained explicitly in Appendix A. From V^o_L and V^o_R, and using the relationships defined there, one computes

$$V^{oT}_L \mathcal{M}^o\, V^o_R = Diag(0, 0, -\lambda_3, \lambda_4) \tag{18}$$

$$V^{oT}_L \mathcal{M}^o \mathcal{M}^{oT}\, V^o_L = V^{oT}_R \mathcal{M}^{oT} \mathcal{M}^o\, V^o_R = Diag(0, 0, \lambda_3^2, \lambda_4^2) . \tag{19}$$

where λ_3^2 and λ_4^2 are the nonzero eigenvalues defined in Eqs.(A4-A5), λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(18,19) that at tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions.

VI. ONE LOOP CONTRIBUTION TO FERMION MASSES

Subsequently, the masses for the light fermions arise through one loop radiative corrections. After the breakdown of the electroweak symmetry we can construct the generic one loop mass diagram of Fig. 1. Internal fermion line in this diagram represent the Dirac see-saw mechanism implemented by the couplings in Eq.(16). The vertices read from the $SU(3)$ flavor symmetry interaction Lagrangian

$$i\mathcal{L}_{int} = \frac{g_H}{2}\, (\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu \mu^o)\, Z_1^\mu + \frac{g_H}{2\sqrt{3}}\, (\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu \mu^o - 2\bar{\tau^o}\gamma_\mu \tau^o)\, Z_2^\mu + \frac{g_H}{\sqrt{2}} \left(\bar{e^o}\gamma_\mu \mu^o Y_1^+ + \bar{e^o}\gamma_\mu \tau^o Y_2^+ + \bar{\mu^o}\gamma_\mu \tau^o Y_3^+ + h.c.\right) , \tag{20}$$

where g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and Y_i^j , $i = 1, 2, 3$, $j = 1, 2$, are the eight gauge bosons. The crosses in the internal fermion line mean tree level mixing, and the mass M generated by the Yukawa couplings in Eq.(16) after the scalar fields get VEV's. The one loop diagram of Fig. 1 gives the generic contribution to the mass term $m_{ij}\, \bar{e}^o_{iL} e^o_{jR}$

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o\, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi} \tag{21}$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, Eq.(20), $m_3^o = -\sqrt{\lambda_3^2}$ and $m_4^o = \lambda_4$ are the See-saw mass eigenvalues, Eq.(18), and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$. Using the results of Appendix A, we compute

$$\sum_{k=3,4} m_k^o\, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i\, b_j\, M}{\lambda_4^2 - \lambda_3^2}\, F(M_Y)\,, \tag{22}$$

$i = 1,2,3$, $j = 2,3$, and $F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$. Adding up all the one loop $SU(3)$ gauge boson contributions, we get the mass terms $\bar{\psi}_L^o \mathcal{M}_1^o\, \psi_R^o + h.c.$,

$$\mathcal{M}_1^o = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi}\,, \tag{23}$$

$$\begin{aligned}
D_{11} &= \frac{1}{2}(\mu_{22} F_1 + \mu_{33} F_2) \\
D_{12} &= \mu_{12}(-\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12}) \\
D_{13} &= -\mu_{13}(\frac{F_{Z_2}}{6} + F_m) \\
D_{22} &= \mu_{22}(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} - F_m) + \frac{1}{2}\mu_{33} F_3 \\
D_{23} &= -\mu_{23}(\frac{F_{Z_2}}{6} - F_m) \\
D_{32} &= -\mu_{32}(\frac{F_{Z_2}}{6} - F_m) \\
D_{33} &= \mu_{33}\frac{F_{Z_2}}{3} + \frac{1}{2}\mu_{22} F_3\,,
\end{aligned}$$

$$F_1 \equiv F(M_{Y_1}) \quad , \quad F_2 \equiv F(M_{Y_2}) \quad , \quad F_3 \equiv F(M_{Y_3})$$

$$M_{Y_1}^2 = M_2^2 \quad , \quad M_{Y_2}^2 = M_3^2 \quad , \quad M_{Y_3}^2 = M_2^2 + M_3^2$$

$$F_m = \frac{\cos\phi \sin\phi}{2\sqrt{3}}\, [\, F(M_-) - F(M_+)\,]$$

with M_2, M_3, M_- and M_+ the boson masses defined in Eqs.(8-10).

Due to the $Z_1 - Z_2$ mixing, we diagonalize the propagators involving Z_1 and Z_2 gauge bosons according to Eq.(13)

$$Z_1 = \cos\phi\, Z_- - \sin\phi\, Z_+ \quad , \quad Z_2 = \sin\phi\, Z_- + \cos\phi\, Z_+$$

$$\langle Z_1 Z_1\rangle = \cos^2\phi\,\langle Z_- Z_-\rangle + \sin^2\phi\,\langle Z_+ Z_+\rangle$$

$$\langle Z_2 Z_2\rangle = \sin^2\phi\,\langle Z_- Z_-\rangle + \cos^2\phi\,\langle Z_+ Z_+\rangle$$

$$\langle Z_1 Z_2\rangle = \sin\phi\,\cos\phi\,(\langle Z_- Z_-\rangle - \langle Z_+ Z_+\rangle)$$

So, in the one loop diagram contributions:

$$F_{Z_1} = \cos^2\phi\,F(M_-) + \sin^2\phi\,F(M_+) \qquad , \qquad F_{Z_2} = \sin^2\phi\,F(M_-) + \cos^2\phi\,F(M_+)\,,$$

$$\mu_{ij} = \frac{a_i\,b_j\,M}{\lambda_4^2 - \lambda_3^2} = \frac{a_i\,b_j}{a\,b}\,\lambda_3\,c_\alpha\,c_\beta\,, \tag{24}$$

and $c_\alpha \equiv \cos\alpha\,,\ c_\beta \equiv \cos\beta\,,\ s_\alpha \equiv \sin\alpha\,,\ s_\beta \equiv \sin\beta$, as defined in the Appendix, Eq.(A6). Therefore, up to one loop corrections we obtain the fermion masses

$$\bar{\psi}^o_L \mathcal{M}^o\,\psi^o_R + \bar{\psi}^o_L \mathcal{M}^o_1\,\psi^o_R = \bar{\chi_L}\,\mathcal{M}\,\chi_R\,, \tag{25}$$

with $\mathcal{M} \equiv \left[Diag(0,0,-\lambda_3,\lambda_4) + V_L^{oT}\mathcal{M}^o_1\,V^o_R\right]$. Using V^o_L, V^o_R from Eqs.(A2-A3) we get the mass matrix

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\,m_{13} & s_\beta\,m_{13} \\ m_{21} & m_{22} & c_\beta\,m_{23} & s_\beta\,m_{23} \\ c_\alpha\,m_{31} & c_\alpha\,m_{32} & (-\lambda_3 + c_\alpha c_\beta\,m_{33}) & c_\alpha s_\beta\,m_{33} \\ s_\alpha\,m_{31} & s_\alpha\,m_{32} & s_\alpha c_\beta\,m_{33} & (\lambda_4 + s_\alpha s_\beta\,m_{33}) \end{pmatrix}\,, \tag{26}$$

where

$$m_{11} = \frac{1}{2}\frac{a_2}{a'}\Pi_1 \quad , \quad m_{12} = -\tfrac{1}{2}\tfrac{a_1 b_3}{a' b}(\Pi_2 - 6\mu_{22}F_m) \tag{27}$$

$$m_{21} = \frac{1}{2}\frac{a_1 a_3}{a' a}\Pi_1 \quad , \quad m_{31} = \tfrac{1}{2}\tfrac{a_1}{a}\Pi_1 \tag{28}$$

$$m_{13} = -\frac{1}{2}\frac{a_1 b_2}{a' b}[\Pi_2 + 2(2\frac{b_3^2}{b_2^2} - 1)\mu_{22}F_m] \tag{29}$$

$$m_{22} = \frac{1}{2}\frac{a_3 b_3}{a\,b}\left[\frac{a_2}{a'}(\Pi_2 - 6\mu_{22}F_m) + \frac{a' b_2}{a_3 b_3}(\Pi_3 + \Delta)\right] \tag{30}$$

$$m_{23} = \frac{1}{2}\frac{a_3 b_3}{a\,b}\left[\frac{a_2 b_2}{a' b_3}(\Pi_2 + 2(2\frac{b_3^2}{b_2^2} - 1)\mu_{22}F_m) - \frac{a'}{a_3}(\Pi_3 - \frac{b_2^2}{b_3^2}\Delta + 2\frac{b^2}{b_3^2}\mu_{33}F_m)\right] \tag{31}$$

$$m_{32} = \frac{1}{2}\frac{a_3 b_3}{a\,b}\left[\frac{a_2}{a_3}(\Pi_2 - 6\mu_{22}F_m) - \frac{b_2}{b_3}(\Pi_3 - \frac{a'^2}{a_3^2}\Delta - 2\frac{a^2}{a_3^2}\mu_{33}F_m)\right] \tag{32}$$

$$m_{33} = \frac{1}{2}\frac{a_3 b_3}{a\, b}\left[\frac{a_2 b_2}{a_3 b_3}(\Pi_2 - 2\mu_{22}F_m) + \Pi_3 + \frac{a'^2 b_2^2}{a_3^2 b_3^2}\Delta - \frac{1}{3}\frac{a^2 b^2}{a_3^2 b_3^2}\mu_{33}F_{Z_2} + 2(\frac{b_2^2}{b_3^2} + 2\frac{a_2^2}{a_3^2} - \frac{a'^2}{a_3^2})\mu_{33}F_m\right] \tag{33}$$

$$\Pi_1 = \mu_{22}F_1 + \mu_{33}F_2 \quad , \qquad \Pi_2 = \mu_{22}F_{Z_1} + \mu_{33}F_3$$

$$\Pi_3 = \mu_{22}F_3 + \mu_{33}F_{Z_2} \quad , \qquad \Delta = \tfrac{1}{2}\mu_{33}(F_{Z_2} - F_{Z_1}) \tag{34}$$

Notice that the m_{ij} mass terms depend just on the $\frac{a_i}{a_j}$ and $\frac{b_i}{b_j}$ ratios of the tree level parameters.

$$a' = \sqrt{a_1^2 + a_2^2}\ \ ,\ \ a = \sqrt{a'^2 + a_3^2}\ \ ,\ \ b = \sqrt{b_2^2 + b_3^2}\ , \tag{35}$$

The diagonalization of $\mathcal{M}$, Eq.(26) gives the physical masses for u, d, and e charged fermions. Using a new biunitary transformation $\chi_{L,R} = V_{L,R}^{(1)}\ \Psi_{L,R}$; $\bar{\chi}_L\ \mathcal{M}\ \chi_R = \bar{\Psi}_L\ {V_L^{(1)}}^T \mathcal{M}\ V_R^{(1)}\ \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$${V_L^{(1)}}^T \mathcal{M}\,\mathcal{M}^T\ V_L^{(1)} = {V_R^{(1)}}^T \mathcal{M}^T\ \mathcal{M}\ V_R^{(1)} = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\ , \tag{36}$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons.

A. Quark $(V_{CKM})_{4\times 4}$ mixing matrix

Within this $SU(3)$ family symmetry model, the transformations from massless to physical mass fermion eigenfields for quarks and charged leptons are

$$\psi_L^o = V_L^o\, V_L^{(1)}\ \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\ \Psi_R\,.$$

Recall that vector like quarks, Eq.(3), are $SU(2)_L$ weak singlets, and hence they do not couple to the W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson may be written as

$$\frac{g}{\sqrt{2}}\,\bar{f^o}_{uL}\gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}}\,\bar{\Psi}_{uL}\ [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\ (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4}\ \gamma_\mu \Psi_{dL}\ W^{+\mu}\ , \tag{37}$$

where g is the $SU(2)_L$ gauge coupling. Therefore, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times 4} = [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\ (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4} \tag{38}$$

VII. NUMERICAL RESULTS

To illustrate the spectrum of masses and mixing, let us consider the following fit of space parameters at the M_Z scale [8]

Taking the input values

$$M_2 = 2\,\text{TeV} \quad , \quad M_3 = 2000\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2$$

for the M_2, M_3 horizontal boson masses, Eq.(8), and the $SU(3)$ coupling constant, respectively, and the ratio of the electroweak VEV's: v_{iu} from Φ^u (v_{id} from Φ^d)

$$v_{1u} = 0 \quad , \quad \frac{v_{2u}}{v_{3u}} = 0.1 \quad , \quad \frac{v_{1d}}{v_{2d}} = 0.23257 \quad , \quad \frac{v_{2d}}{v_{3d}} = 0.08373\,,$$

we obtain the following mass and mixing matrices, and mass eigenvalues:

A. Quark masses and mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 0. \\ 0 & 0 & 0 & 29834. \\ 0 & 0 & 0 & 298340. \\ 0 & 1.49495\times 10^7 & -730572. & 1.58511\times 10^7 \end{pmatrix} \text{MeV}\,, \tag{39}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 1.38 & 0. & 0. & 0. \\ 0. & -532.587 & -2587.14 & -2442.42 \\ 0. & 7064.64 & -172017. & 31927.1 \\ 0. & 70.6499 & 338.204 & 2.18023\times 10^7 \end{pmatrix} \text{MeV}\,, \tag{40}$$

and the u-quark masses

$$(\, m_u \,,\ m_c \,,\ m_t \,,\ M_U \,) = (\, 1.38 \,,\ 638.22 \,,\ 172181 \,,\ 2.18023\times 10^7 \,)\,\text{MeV} \tag{41}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 13375.7 \\ 0 & 0 & 0 & 57510.3 \\ 0 & 0 & 0 & 686796. \\ 0 & 723708. & -37338.1 & 6.89219\times 10^7 \end{pmatrix} \text{MeV} \tag{42}$$

$$\mathcal{M}_d = \begin{pmatrix} 2.82461 & 0.0338487 & -0.656039 & -0.00689715 \\ 0.65453 & -25.1814 & -217.369 & -2.28527 \\ 0.0562685 & 423.166 & -2820.62 & 46.5371 \\ 0.000562713 & 4.23187 & 44.2671 & 6.89291\times 10^7 \end{pmatrix} \text{MeV} \tag{43}$$

$$(\, m_d \ , \ m_s \ , \ m_b \ , \ M_D \,) = (\, 2.82368 \ , \ 57.0005 \ , \ 2860 \ , \ 6.89291 \times 10^7 \,) \ \mathrm{MeV} \tag{44}$$

and the quark mixing

$$V_{CKM} = \begin{pmatrix} 0.97362 & 0.225277 & -0.0362485 & 0.000194044 \\ -0.226684 & 0.973105 & -0.040988 & -0.000310055 \\ 0.0260403 & 0.0481125 & 0.998387 & -0.00999333 \\ -0.000234396 & -0.000826552 & -0.011432 & 0.000114632 \end{pmatrix} \tag{45}$$

B. Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 37956.9 \\ 0 & 0 & 0 & 189784. \\ 0 & 0 & 0 & 1.93543 \times 10^6 \\ 0 & 548257. & -30307.4 & 1.94497 \times 10^8 \end{pmatrix} \mathrm{MeV} \tag{46}$$

$$\mathcal{M}_e = \begin{pmatrix} -0.486368 & -0.00536888 & 0.0971221 & 0.000274163 \\ -0.0967909 & -34.7536 & -250.305 & -0.706579 \\ -0.0096786 & 485.768 & -1661.27 & 10.8107 \\ -0.0000967909 & 4.85792 & 38.2989 & 1.94507 \times 10^8 \end{pmatrix} \mathrm{MeV} \tag{47}$$

fit the charged lepton masses:

$$(m_e \, , \, m_\mu \, , \, m_\tau \, , \, M_E) = (0.486095 \, , \, 102.7 \, , \, 1746.17 \, , \, 3.15956 \times 10^8 \,) \, \mathrm{MeV}$$

and the charged lepton mixing

$$V_{e\,L}^o \, V_{e\,L}^{(1)} = \begin{pmatrix} 0.973942 & 0.221206 & 0.050052 & 0.000194 \\ -0.226798 & 0.949931 & 0.214927 & 0.0008342 \\ -2.90427 \times 10^{-6} & -0.220675 & 0.975296 & 0.009963 \\ 2.62189 \times 10^{-7} & 0.0013632 & -0.009906 & 0.99995 \end{pmatrix} \tag{48}$$

VIII. CONCLUSIONS

We reported recent numerical analysis on charged fermion masses and mixing within a BSM with a local $SU(3)$ family symmetry, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical mass generation mechanism for quarks and charged leptons.
In section VII we show a parameter space region where this scenario account for the known hierarchical spectrum of ordinary quarks and charged lepton masses, and the quark mixing in a non-unitary $(V_{CKM})_{4\times 4}$ within allowed values[2] reported in PDG 2014 [9].

Let me point out here that the solutions for fermion masses and mixing reported in section VII suggest that the dominant contribution to EWSB comes from the weak doublets which couple to the third family.

It is also worth to comment that fermion content, scalar fields, and their transformation under the gauge group, Eq. (2)*, all together, forbid tree level Yukawa couplings between ordinary standard model fermions. Consequently, the flavon scalar fields introduced to break the symmetries:* Φ^u, Φ^d, η_2 *and* η_3*, couple only ordinary fermions to their corresponding vector like fermion at tree level. Thus, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in the quark mixing* $(V_{CKM})_{4\times 4}$*, Eq.(45), and the charged lepton mixing, Eq.* (48)*, may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.*

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[2] except $(V_{CKM})_{13}$ and $(V_{CKM})_{31}$

Appendix A: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \tag{A1}$$

Using the biunitary transformations $\psi_L^o = V_L^o \, \chi_L$ and $\psi_R^o = V_R^o \, \chi_R$ to diagonalize $\mathcal{M}^o$, the orthogonal matrices V_L^o and V_R^o may be written explicitly as

$$V_L^o = \begin{pmatrix} \frac{a_2}{a'} & \frac{a_1 a_3}{a' a} & \frac{a_1}{a}\cos\alpha & \frac{a_1}{a}\sin\alpha \\ -\frac{a_1}{a'} & \frac{a_2 a_3}{a' a} & \frac{a_2}{a}\cos\alpha & \frac{a_2}{a}\sin\alpha \\ 0 & -\frac{a'}{a} & \frac{a_3}{a}\cos\alpha & \frac{a_3}{a}\sin\alpha \\ 0 & 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \tag{A2}$$

$$V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{b_3}{b} & \frac{b_2}{b}\cos\beta & \frac{b_2}{b}\sin\beta \\ 0 & -\frac{b_2}{b} & \frac{b_3}{b}\cos\beta & \frac{b_3}{b}\sin\beta \\ 0 & 0 & -\sin\beta & \cos\beta \end{pmatrix} \tag{A3}$$

where

$$\lambda_3^2 = \frac{1}{2}\left(B - \sqrt{B^2 - 4D}\right) \quad , \quad \lambda_4^2 = \frac{1}{2}\left(B + \sqrt{B^2 - 4D}\right) \tag{A4}$$

are the nonzero eigenvalues of $\mathcal{M}^o \mathcal{M}^{oT}$ ($\mathcal{M}^{oT}\mathcal{M}^o$), and

$$B = a^2 + b^2 + c^2 = \lambda_3^2 + \lambda_4^2 \quad , \quad D = a^2 b^2 = \lambda_3^2 \lambda_4^2 \, , \tag{A5}$$

$$\cos\alpha = \sqrt{\frac{\lambda_4^2 - a^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \cos\beta = \sqrt{\frac{\lambda_4^2 - b^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \tag{A6}$$