Influence on Starobinsky inflation by other fields with large amplitude

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The Starobinsky model is one of the inflation models consistent with the result of CMB observation by the Planck satellite. We consider the dynamics of the Starobinsky inflation in the presence of another scalar field with a large expectation value during inflation due to a negative Hubble-induced mass. We find that it would be affected if the other field has an amplitude close to the Planck scale. In this case, we may observe such effects on the Starobinsky model by future CMB experiments.

I. INTRODUCTION

Inflation is a very attractive cosmological paradigm. It solves the flatness and homogeneity problems of big bang cosmology, and could provide the seeds for the large-scale structure of the Universe as well. Inflation is usually considered to be driven by a scalar field called the inflaton which slow-rolls on its potential. The shape of the potential is carefully chosen in order to have both long enough inflation and density perturbations observed. Starobinsky inflation [\[1\]](#page-5-0) is one of the models consistent with observations of the cosmic microwave background by the Planck satellite [\[2\]](#page-5-1).

The inflaton potential may, however, be affected by other fields. In Ref. [\[3](#page-5-2)], it is considered that the potential of the chaotic inflation [\[4](#page-5-3)] could be suppressed by a scalar field with large amplitude due to a negative Hubble-induced mass term. In particular, they showed that this effect renders the chaotic inflation with quadratic potential consistent with Planck observations, otherwise marginally excluded [\[2\]](#page-5-1). It is then sensible to think the other way around: Is there any situation in which this mechanism could affect the observationally preferable inflation model considerably? Or we may think about any observational evidence for other fields that have large amplitudes during such inflation in the future CMB experiments.

In this article, we consider the influence on the Starobinsky inflation model by other fields with a large field amplitude due to a negative Hubble-induced mass term. As well known, it is reasonable for a scalar field to have a negative Hubble-induced mass term in supergravity [\[5](#page-5-4)]. Moreover, the Starobinsky inflation is realized in supergravity in many ways, such as in no-scale supergravity [\[6\]](#page-5-5) or superconformal theory [\[7\]](#page-5-6).

The structure of the article is as follows: In the next section, we review the Starobinsky model as a scalar-fielddriven inflation. In Sec. III, we introduce another scalar field whose coupling to the inflaton leads to a negative Hubble-induced mass term in the Starobinsky model, and we derive numerically the observables such as the spectral index and the tensor-to-scalar ratio, in addition to the inflaton dynamics in Sec. IV. In the same section, we obtain the constraints on model parameters by current CMB observations and consider the possibility of detecting such effects in future CMB experiments. In Sec. V, we present that the model can be derived naturally in supergravity. We give our conclusion in Sec. VI.

II. $R + R^2$ STAROBINSKY MODEL

The Starobinsky model [\[1\]](#page-5-0) is obtained by generalization of Einstein-Hilbert action which contains an R^2 term as in

$$
S = \frac{1}{2} \int d^4 x \sqrt{-g} \left(R + \frac{R^2}{6M^2} \right),
$$
 (1)

where M is a mass scale much less than the reduced Planck mass M_P (=2.4 × 10¹⁸ GeV), and $M_P = 1$ unit is adopted. It is equivalent to canonical gravity with a scalar field \tilde{I} by conformal transformation. Taking the transformation $\bar{g}_{\mu\nu} = (1 + \tilde{I}/3M^2)g_{\mu\nu}$ and the field redefinition $I = \sqrt{\frac{3}{2}} \log[1 + \tilde{I}/3M^2]$, one obtains the redefined action

$$
S = \frac{1}{2} \int d^4x \sqrt{-\bar{g}} \left[\bar{R} + (\partial_{\mu}I)^2 - \frac{3}{2}M^2 \left(1 - e^{-\sqrt{\frac{2}{3}}}I \right)^2 \right].
$$
 (2)

Thus, the potential of the inflaton I is

$$
V_{\rm S} = \frac{3M^2}{4} \left(1 - e^{-\sqrt{\frac{2}{3}}I} \right)^2.
$$
 (3)

The spectral index of the curvature perturbation n_s and the tensor-to-scalar ratio r are described, respectively, by slow-roll parameters as $n_s = 1 - 6\varepsilon_{\rm S} + 2\eta_{\rm S}$ and $r = 16\varepsilon_{\rm S}$, where $\varepsilon_{\rm S}$ and $\eta_{\rm S}$ are given by

$$
\varepsilon_{\rm S}(I) \equiv \frac{1}{2} \left(\frac{V'_{\rm S}}{V_{\rm S}} \right)^2 = \frac{4}{3} \frac{e^{-2\sqrt{\frac{2}{3}}I}}{\left(1 - e^{-\sqrt{\frac{2}{3}}I} \right)^2},\tag{4}
$$

$$
\eta_{\rm S}(I) \equiv \frac{V_{\rm S}^{\prime\prime}}{V_{\rm S}} = -\frac{4}{3} \frac{e^{-\sqrt{\frac{2}{3}}I} \left(1 - 2e^{-\sqrt{\frac{2}{3}}I}\right)}{\left(1 - e^{-\sqrt{\frac{2}{3}}I}\right)^2},\tag{5}
$$

respectively. The prime denotes a derivative with respect to the inflaton I. The number of e-folds of slow-roll inflation is calculated as

$$
N = \int_{I_{\text{end}}}^{I_N} dI \frac{V_{\text{S}}}{V'_{\text{S}}} = \frac{3}{4} \left(e^{\sqrt{\frac{2}{3}}I_N} - e^{\sqrt{\frac{2}{3}}I_{\text{end}}} \right) - \frac{\sqrt{6}}{4} (I_N - I_{\text{end}}),\tag{6}
$$

where I_N and I_{end} represent the amplitudes of the inflation at the time of N e-folds and at the end of the inflation $[\eta_{\rm S}(I_{\rm end}) = 1]$, respectively. In the Starobinsky model, $N=50$ (60) leads to $n_s = 0.961$ (0.968) and $r = 0.0042$ (0.0030), which are favored by the Planck result [\[2](#page-5-1)].

III. STAROBINSKY INFLATION IN THE PRESENCE OF ANOTHER SCALAR FIELD WITH LARGE AMPLITUDE

Let us now introduce another scalar field ϕ , which obtains large amplitude due to a negative Hubble-induced mass term. To illustrate how it may affect the inflaton dynamics, we assume the following coupling to the inflaton [\[3\]](#page-5-2):

$$
V(I, \phi) = V_{\rm S}(I) + V(\phi) = V_{\rm S} \left(1 - c\phi^2 \right) + \frac{\lambda^2}{2q} \phi^{2q}, \tag{7}
$$

where c and λ are positive coupling constants. As shown later, Eq. (7) is naturally obtained in supergravity. During inflation, ϕ stays at the minimum

$$
\phi_*(I) = \left[\frac{3c}{2\lambda^2}M^2\left(1 - e^{-\sqrt{\frac{2}{3}}I}\right)^2\right]^{\frac{1}{2(q-1)}},\tag{8}
$$

which changes as I moves during inflation. Therefore, the inflaton potential receives backreaction to be

$$
V(I) = V_{\rm S} \left(1 - \frac{q-1}{q} c \phi_*^2 \right),\tag{9}
$$

where the potential is suppressed at large I compared to V_S only. The slow-roll parameters are thus obtained as

$$
\varepsilon(I) = \frac{\left(1 - c\phi_*^2\right)^2}{\left(1 - \frac{q-1}{q}c\phi_*^2\right)^2} \varepsilon_{\mathcal{S}},\tag{10}
$$

$$
\eta(I) = \frac{1 - c\phi_*^2}{1 - \frac{q-1}{q}c\phi_*^2} \eta_{\rm S} - \frac{\frac{2}{q-1}c\phi_*^2}{1 - \frac{q-1}{q}c\phi_*^2} \varepsilon_{\rm S}.\tag{11}
$$

In the next section, we will calculate I_N numerically, and estimate the spectral index $n_s(I_N)$ and the tensor-to-scalar ratio $r(I_N)$.

IV. SPECTRAL INDEX AND TENSOR-TO-SCALAR RATIO

Let us see how the spectral index n_s and the tensor-to-scalar ratio r will change in the presence of other fields with large amplitude. For given λ , we use the Planck normalization of the primordial density perturbations [\[2\]](#page-5-1),

$$
A_s = \frac{1}{12\pi^2} \frac{V^3(I_N)}{V'^2(I_N)} \approx 2.2 \times 10^{-9},\tag{12}
$$

to determine M.

We show the estimates of n_s and r for various λ in Fig. 1, together with λ dependences for n_s and r in Figs. 2 and 3, respectively. We can see that our n_s and r do not differ from those in the pure Starobinsky model for $\lambda \gtrsim 10^{-4}$. As the amplitude of the ϕ field gets closer to the Planck scale $\phi_*(I_N) \gtrsim 0.7$ (0.8) for $q = 3$ (8), which corresponds to $\lambda \lesssim 10^{-4}$, r quickly decreases, while n_s stays in the current Planck-favored region. Lowering λ further, we have too small n_s , which departs from the region allowed by the present Planck observation for $\lambda \lesssim 10^{-5}$, i.e., $\phi_*(I_N) \gtrsim 0.99$. Notice that there is little dependence on q.

FIG. 1: Spectral index n_s and tensor-to-scalar ratio r for various λ for (a) $q = 3$ and (b) $q = 8$. We also show 1σ and 2σ regions of the Plank result.

FIG. 2: Spectral index n_s for various λ for (a) $q = 3$ and (b) $q = 8$.

In order to understand how n_s and r change in the presence of another field with large amplitude, we show the amplitude of the inflaton I at $N=0$, 20, 40, and 60 for $q=8$ in Fig. 4. The evolution of the inflaton seems changed drastically for $\lambda \lesssim 10^{-4}$. In particular, the excursion of the inflaton gets shorter, because $I_{N=60}$ becomes smaller while I_{end} remains almost unchanged. Moreover, most of the e-folds occur in shorter excursions of the inflaton as λ gets smaller. Note that the same behavior can be seen for any q . We can explain how this happens by considering the potential. The inflaton potential at large amplitude is suppressed by the effect of the other field with large vacuum expectation values, and the potential becomes flatter. Thus, the tensor-to-scalar ratio r decreases as λ gets smaller. Eventually, the local maximum appears as λ becomes smaller than $\sim 10^{-5}$, and most of the e-folds occur at the vicinity of the local maximum, as seen in Fig. 4. The potential of the Starobinsky model V_S and $V(I)$ in Eq. (9) for $\lambda = 10^{-6}$ and $q = 8$ are displayed in Fig. 5. There is a local maximum in $V(I)$ for $\lambda = 10^{-6}$ and $q = 8$, but not in V_S . Once the local maximum develops as λ gets smaller, the curvature of the potential becomes larger, and hence the spectral index n_s deviates considerably from that of the pure Starobinsky model and comes out of the range allowed by the Planck result.

FIG. 3: Tensor-to-scalar ratio r for various λ for (a) $q = 3$ and (b) $q = 8$.

FIG. 4: Amplitude of the inflaton I at $N=0$, 20, 40, and 60 for $q=8$.

FIG. 5: Potential of the inflaton (a) for the Starobinsky model and (b) in the presence of another field for $\lambda = 10^{-6}$ and $q = 8$.

Current CMB observations such as the Planck experiment cannot tell if the Starobinsky inflation is affected by the other field with a large amplitude for $\lambda \gtrsim 10^{-5}$. Future experiments may, however, reveal such effects of other fields for $\lambda = 10^{-5}$ –10⁻⁴. The crucial steps toward this end are removals of the foreground and the lensing effect, and the forecasts on the capabilities of future CMB experiments to constrain the cosmological parameters are reported in Refs. [\[8](#page-5-7)[–10\]](#page-5-8). In particular, Ref. [\[10\]](#page-5-8) divides future experiments into two categories, pre- and post-2020, and obtains the accuracies of the cosmological parameters to be determined as $\sigma(n_s) \sim 2.2 \times 10^{-3}$ and $\sigma(r) \sim 3 \times 10^{-3}$ for the pre-2020 experiments, and $\sigma(n_s) \sim 1.8 \times 10^{-3}$ and $\sigma(r) \sim 1.3 \times 10^{-4}$ for the post-2020 experiments. It seems difficult to distinguish between the Starobinsky inflation with and without the effects of other fields by the pre-2020 experiments, but the post-2020 experiments may have the ability to confirm the existence of other fields with large amplitudes for $\lambda = 10^{-5}$ –10⁻⁴, especially due to the very accurate r determination.

V. STARONBINSKY MODEL IN SUPERGRAVITY AND THE NEGATIVE HUBBLE-INDUCED MASS TERM FOR OTHER FIELDS

The Starobinsky model can be realized in supergravity. For example, it can be derived in superconformal theory, where the Kähler potential and superpotential are written as [\[7](#page-5-6)]

$$
K = S\overline{S} - \frac{\left(\chi - \overline{\chi}\right)^2}{2} - \zeta \left(S\overline{S}\right)^2, \qquad W = \frac{M\sqrt{3}}{2}S\left(1 - e^{-\frac{2\chi}{\sqrt{3}}}\right),\tag{13}
$$

respectively, where S is the Goldstino superfield, χ is the scalar field whose real part is the inflaton I, and ζ is a constant. Another example utilizes no-scale supergravity, where the K¨ahler potential and superpotential are, respectively, given by [\[6\]](#page-5-5)

$$
K = -3\log\left(T + \bar{T} - \frac{|\chi|^2}{3}\right), \qquad W = \frac{\hat{\mu}}{2}\chi^2 - \frac{\hat{\lambda}}{3}\chi^3,
$$
\n(14)

where T and χ are complex scalar fields and $\hat{\mu}$ and $\hat{\lambda}$ are constants. The real part of χ can be regarded as the inflaton with the moduli T being stabilized [\[6\]](#page-5-5).

On the other hand, many light scalar fields may obtain large vacuum expectation values during inflation due to a negative Hubble-induced mass in supergravity [\[5](#page-5-4)], and Eq. (7) is naturally obtained. It can be achieved by a higher-order term in Kähler potential such as $\Delta K = aS\overline{S}\Phi\overline{\Phi}$ for superconformal theory or $\Delta K = a\chi\overline{\chi}\Phi\overline{\Phi}$ for the no-scale model, where $a(>1)$ is a coupling constant, and Φ is a complex scalar field whose real part is ϕ . Together with $\Delta W = \frac{\lambda}{p} \Phi^p$, we obtain Eq. (7), with c being $O(1)$.

Notice that, in no-scale supergravity, a negative Hubble-induced mass term with $c \sim 0.1$ can be realized by one-loop correction in the absence of the above higher-order term in Kähler potential [\[11](#page-5-9)]. In this case, there is less effect of other scalar fields with large amplitude on the inflaton potential and dynamics.

VI. CONCLUSION

We have considered how the inflaton dynamics in the Starobinsky inflation will change because of another scalar field with large amplitude which is naturally obtained due to a negative Hubble-induced mass. This effect suppresses the inflaton potential at large amplitudes, and we have found that the dynamics of the inflaton will be changed if the amplitude of such other field becomes close to the Planck scale, $\phi_*(I_N) \gtrsim 0.7$ (0.8) for $q = 3$ (8), or, in other words, $\lambda \lesssim 10^{-4}$. For $\lambda \lesssim 10^{-5}$, which is $\phi_*(I_N) \gtrsim 0.99$, the inflaton potential is distorted too much, and it is no longer allowed by the current Planck observation.

We focus on the case for $\lambda \gtrsim 10^{-5}$, where it is allowed at present. In that region, although the scalar spectral index
does not above so much the topsor to scalar ratio g quickly degreeses for $\lambda \ge 10^{-4}$. Since the n_s does not change so much, the tensor-to-scalar ratio r quickly decreases for $\lambda \gtrsim 10^{-4}$. Since the post-2020 CMB experiments may reach $\sigma(r) \sim 10^{-4}$ [\[8](#page-5-7)[–10\]](#page-5-8), we may have a chance to confirm the presence of such other fields with large amplitude.

Finally, we comment on the case in which the other scalar field is a flat direction in supersymmetry, which may result in the Affleck-Dine baryogenesis [\[5](#page-5-4), [12\]](#page-5-10). Our result seems to show that the baryon asymmetry can be created almost maximally without affecting the dynamics of the Starobinsky inflation.

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