Mass and decay constant of the newly observed exotic $X(5568)$ state

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The mass and decay constant of the $X(5568)$ state newly observed by D0 Collaboration are computed within the two-point sum rule method using the diquark-antidiquark interpolating current. In calculations, the vacuum condensates up to eight dimensions are taken into account. The obtained result for the mass of the $X(5568)$ state is in a nice agreement with the experimental data.

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I. INTRODUCTION

During the last decade, due to a wide flow of experimental information rushing from Belle, BABAR, BESIII, LHCb, CDF, D0 and some other collaborations, investigation of exotic states, i.e. states that can not be included into the quark-antiquark and three-quark bound schemes of the traditional hadron spectroscopy, became one of the interesting and growing fields in the hadron physics. The period of intensive experimental and theoretical studies of exotic particles started from the discovery of the charmonium-like resonance $X(3872)$ by Belle Collaboration [1], confirmed later in some other experiments [2–4]. The exotic states were produced and observed in B meson decays, in the e^+e^- and $\overline{p}p$ annihilations, in the double charmonium production processes, in the two-photon fusion and pp collisions. Experimental investigations encompass measurements of the masses and decay widths of these states, exploration of their spins, parities and C-parities. The charmonium-like exotic states observed and studied till now form the wide family of XYZ particles.

These discoveries necessitated generation of new theoretical approaches to interpret underlying quark-gluon structure of the exotic states, and invention of methods for calculation of their properties. (see, the reviews [5–12] and references therein). Naturally, efforts were done to consider new charmonium-like resonances as excitations of the ordinary $c\overline{c}$ charmonium and describe their features applying updated quark-antiquark potentials. It should be noted that some of new resonances really allow interpretation as the excited $c\overline{c}$ states. But the main part of the collected experimental data can not be entered into this frame, and therefore for understanding of the phenomenology of XYZ states various quark-gluon models were suggested.

One of the mostly employed models is the four-quark or tetraquark picture of the exotic states. In accordance with this approach new charmonium-like states are formed by two heavy and two light quarks. These quarks may cluster into the colored diquark and antidiquark, which are organized in such a way that to reproduce quantum numbers of the corresponding exotic states [13]. This tetraquark model is known as diquark-antidiquark model of the exotic states. In the meson-molecule model the exotic particle may appear as a bound state of two color-singlet mesons. There are other models within the tetraquark approach Refs.[14, 15], as well as ones that exploit alternative ideas (see, for example Ref. [16]).

Recently, the D0 Collaboration reported the observation of a narrow structure $X(5568)$ in the decay chain $X(5568) \to B_s^0\pi^\pm$, $B_s^0 \to J/\psi\phi$, $J/\psi \to \mu^+\mu^-$, $\phi \to K^+K^-$ [17] based on $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV collected at the Fermilab Tevatron collider. In order to distinguish it from the "traditional" members of the X family of exotic resonances, in what follows we will use for this state the notation $X_b(5568)$. As it was emphasized in Ref. [17] this is the first observation of a hadronic state with four quarks of different flavors. Namely, from the observed decay channel $X_b(5568) \to B_s^0\pi^\pm$ it is not difficult to conclude that the state $X_b(5568)$ consists of b, s, u, d quarks. The assigned quantum numbers for the X_b state are $J^{PC} = 0^{++}$, its mass extracted from the experiment is equal to $m_{X_b} = 5567.8 \pm 2.9(\text{stat})^{+0.9}_{-1.9}(\text{syst})\,\text{MeV}$, and the decay width was estimated as $\Gamma = 21.9 \pm 6.4(\text{stat})^{+5.0}_{-2.5}(\text{syst})\,\text{MeV}$. First suggestions concerning the quark-antiquark organization of the new state were made in Ref. [17], as well. Thus, within the diquark-antidiquark model the X_b may be described as $[bu][\bar{d}\bar{s}]$, $[bd][\bar{s}\bar{u}]$, $[su][\bar{b}\bar{d}]$ or $[sd][\bar{b}\bar{u}]$ bound state. Alternatively, it may be considered as a molecule composed of B and K mesons.

In the present work for the X_b we adopt $[su][\bar{b}\bar{d}]$ diquark model, and calculate for the first time its mass and decay constant using the QCD two-point sum rule.

This article is organized in the following manner. In section II, we calculate the mass and decay constant of the X_b state employing two-point QCD sum rule approach including into analysis the vacuum condensates up to eighth dimension. Our numerical results are presented in Section III and compared with the experimental data of the D0 Collaboration. This section contains also our concluding remarks. The explicit expressions of the spectral density required for calculation of the mass and decay constant are written down in Appendix A.

II. THE SUM RULES FOR THE MASS AND DECAY CONSTANT

To calculate the mass and decay constant of the X_b state in the framework of QCD sum rules, we start from the two-point correlation function

$$\Pi(p)=i\int d^4xe^{ip\cdot x}\langle 0|\mathcal{T}\{J^{X_b}(x)J^{X_b\dagger}(0)\}|0\rangle, \quad (1)$$

where $J^{X_b}(x)$ is the interpolating current with required quantum numbers. We consider $X_b(5568)$ state as a particle with the quantum numbers $J^{PC}=0^{++}$. Then in the diquark model the current $J^{X_b}(x)$ is defined by the following expression

$$J^{X_b}(x)=\varepsilon^{ijk}\varepsilon^{imn}\left[s^j(x)C\gamma_\mu u^k(x)\right]\left[\overline{b}^m(x)\gamma^\mu C\overline{d}^n(x)\right]. \quad (2)$$

In Eq. (2) $i,\ j,\ k, m,\ n$ are color indexes and C is the charge conjugation matrix.

In order to derive QCD sum rule expression we calculate the correlation function in terms of the physical degrees of freedom. Performing integral over x in Eq. (1), we get

$$\Pi^{\rm Phys}(p)=\frac{\langle 0|J^{X_b}|X_b(p)\rangle\langle X_b(p)|J^{X_b\dagger}|0\rangle}{m_{X_b}^2-p^2}+...$$

where m_{X_b} is the mass of the $X_b(5568)$ state, and dots stand for contributions of the higher resonances and continuum states. We define the decay constant f_{X_b} through the matrix element

$$\langle 0|J^{X_b}|X_b(p)\rangle=f_{X_b}m_{X_b}.$$

Then in terms of m_{X_b} and f_{X_b} the correlation function can be written in the form

$$\Pi^{\rm Phys}(p)=\frac{m_{X_b}^2f_{X_b}^2}{m_{X_b}^2-p^2}+\ldots \quad (3)$$

The Borel transformation applied to Eq. (3) yields

$$\mathcal{B}_{p^2}\Pi^{\rm Phys}(p)=m_{X_b}^2f_{Xb}^2e^{-m_{X_b}^2/M^2}+\ldots \quad (4)$$

The theoretical expression for the same function, $\Pi^{\rm QCD}(p)$, has to be determined employing of the quark-gluon degrees of freedom. To this end, we contract the heavy and light quark fields, and for the correlation function $\Pi^{\rm QCD}(p)$ find:

$$\Pi^{\rm QCD}(p)=i\int d^4xe^{ipx}\varepsilon^{ijk}\varepsilon^{imn}\varepsilon^{i'j'k'}\varepsilon^{i'm'n'}$$
$$\times{\rm Tr}\left[\gamma_\mu\widetilde{S}_d^{n'n}(-x)\gamma_\nu S_b^{m'm}(-x)\right]$$
$$\times{\rm Tr}\left[\gamma^\nu\widetilde{S}_s^{jj'}(x)\gamma^\mu S_u^{kk'}(x)\right]. \quad (5)$$

In Eq. (5) we introduce the notation

$$\widetilde{S}_q^{ij}(x)=CS_q^{ijT}(x)C,$$

with $S_q^{ij}(x)$ and $S_b^{ij}(x)$ being the light ($q\equiv u,\ d$ and s) and heavy quark propagators, respectively. We choose the light quark propagator $S_q^{ij}(x)$ in the x-space in the form

$$S_q^{ij}(x)=i\delta_{ij}\frac{\not{x}}{2\pi^2x^4}-\delta_{ij}\frac{m_q}{4\pi^2x^2}-\delta_{ij}\frac{\langle\overline{q}q\rangle}{12}$$
$$+i\delta_{ij}\frac{\not{x}m_q\langle\overline{q}q\rangle}{48}-\delta_{ij}\frac{x^2}{192}\langle\overline{q}g\sigma Gq\rangle+i\delta_{ij}\frac{x^2\not{x}m_q}{1152}\langle\overline{q}g\sigma Gq\rangle$$
$$-i\frac{gG_{ij}^{\alpha\beta}}{32\pi^2x^2}\left[\not{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not{x}\right]-i\delta_{ij}\frac{x^2\not{x}g^2\langle\overline{q}q\rangle^2}{7776}$$
$$-\delta_{ij}\frac{x^4\langle\overline{q}q\rangle\langle g^2GG\rangle}{27648}+\ldots \quad (6)$$

For the heavy quark propagator $S_b^{ij}(x)$ we employ the expression [18]

$$S_b^{ij}(x)=i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\left[\frac{\delta_{ij}\left(\not{k}+m_b\right)}{k^2-m_b^2}\right.$$
$$-\frac{gG_{ij}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}\left(\not{k}+m_b\right)+\left(\not{k}+m_b\right)\sigma_{\alpha\beta}}{(k^2-m_b^2)^2}$$
$$\left.+\frac{g^2}{12}G_{\alpha\beta}^AG^{A\alpha\beta}\delta_{ij}m_b\frac{k^2+m_b\not{k}}{(k^2-m_b^2)^4}+\ldots\right]. \quad (7)$$

In Eqs. (6) and (7) the short-hand notation

$$G_{ij}^{\alpha\beta}\equiv G_A^{\alpha\beta}t_{ij}^A,\quad A=1,\,2\ldots 8,$$

is used, where $i,\,j$ are color indexes, and $t^A=\lambda^A/2$ with λ^A being the standard Gell-Mann matrices. The first term in Eq. (7) is the free (perturbative) massive quark propagator, next ones are nonperturbative gluon corrections. In the nonperturbative terms the gluon field strength tensor $G_{\alpha\beta}^A\equiv G_{\alpha\beta}^A(0)$ is fixed at $x=0$.

In general, the QCD sum rule expressions are derived after fixing the same Lorentz structures in the both phenomenological and theoretical expressions of the correlation function. In the case under consideration this structure is trivial and $\sim$ I. Then there is only one invariant function $\Pi^{\rm QCD}(p^2)$, which can be written down as the dispersion integral

$$\Pi^{\rm QCD}(p^2)=\int_{(m_b+m_s)^2}^{\infty}\frac{\rho^{\rm QCD}(s)}{s-p^2}+..., \quad (8)$$

where $\rho^{\rm QCD}(s)$ is the corresponding spectral density.

The problem posed in this section is calculation of the spectral density $\rho^{\rm QCD}(s)$ necessary for the mass and decay constant analysis. In the present work we include into the sum rule calculations the quark, gluon and mixed vacuum condensates up to and including ones with the dimension 8. For computation of the components of the spectral density we use the technique, essential steps of which and main formulas for their realization were provided in Ref. [19]. This computational scheme includes the following stages: we apply the integral transformation for the terms $\sim 1/(x^2)^n$ coming from the light quark

Parameters	Values
m_b	(4.18 ± 0.03) GeV
m_s	(95 ± 5) MeV
$\langle \bar{q}q \rangle$	$(-0.24 \pm 0.01)^3$ GeV^3
$\langle \bar{s}s \rangle$	$0.8\ \langle \bar{q}q \rangle$
$\langle \frac{\alpha_s G^2}{\pi} \rangle$	(0.012 ± 0.004) GeV^4
m_0^2	(0.8 ± 0.1) GeV^2
$\langle \overline{q}g\sigma Gq \rangle$	$m_0^2 \langle \bar{q}q \rangle$

TABLE I: Input parameters used in calculations

propagators, when necessary replace x_μ by $-i\partial/\partial q_\mu$, and then calculate the obtained x integral. The Dirac delta function appeared in a result of such integration allows us to remove one of the momentum integrals. In order to carry out the remaining integration we use the Feynman parametrization rearranging denominators obtained after this operation, and derive the final expressions applying the well known formulas [19]. The imaginary part of the correlation function can now be extracted by applying in the $D \to 4$ limit the replacement

$$\Gamma\left(\frac{D}{2}-n\right)\left(-\frac{1}{L}\right)^{\frac{D}{2}-n} \to \frac{(-1)^{n-1}}{(n-2)!}(-L)^{n-2}\ln(-L). \tag{9}$$

As a result, we get the imaginary part of the correlation function, and hence the components of the spectral density as the integrals over the Feynman parameter z. The expressions derived for $\rho^{\rm QCD}(s)$ in accordance with these recipes are collected in Appendix A.

Applying the Borel transformation on the variable p^2 to the invariant amplitude $\Pi^{\rm QCD}(p^2)$, equating the obtained expression with the relevant part of $\mathcal{B}_{p^2}\Pi^{\rm Phys}(p)$, and subtracting the continuum contribution, we finally obtain the required sum rule. Thus, the mass of the X_b state can be evaluated from the sum rule

$$m_{X_b}^2 = \frac{\int_{(m_b+m_s)^2}^{s_0} ds s\rho^{\rm QCD}(s)e^{-s/M^2}}{\int_{(m_b+m_s)^2}^{s_0} ds\rho^{\rm QCD}(s)e^{-s/M^2}}, \tag{10}$$

whereas to extract the numerical value of the decay constant f_{X_b} we employ the formula

$$f_{X_b}^2 m_{X_b}^2 e^{-m_{X_b}^2/M^2} = \int_{(m_b+m_s)^2}^{s_0} ds\rho^{\rm QCD}(s)e^{-s/M^2}. \tag{11}$$

The Eqs. (10) and (11) are the sum rules required for evaluating of the X_b state's mass and decay constant, respectively.

III. NUMERICAL RESULTS AND CONCLUSIONS

The QCD sum rules expressions for the mass and decay constant of the X_b contain various parameters that

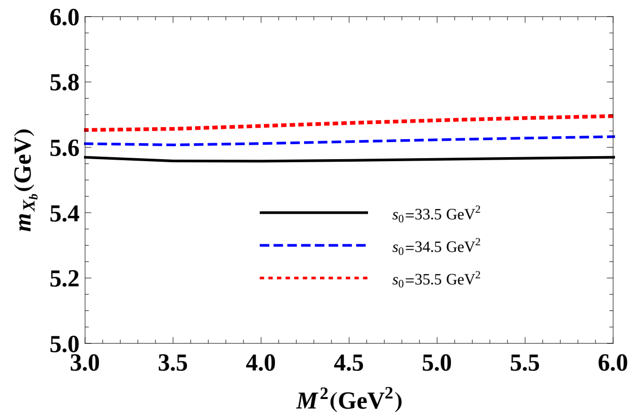

FIG. 1: The mass m_{X_b} as a function of the Borel parameter M^2 for different values of s_0.

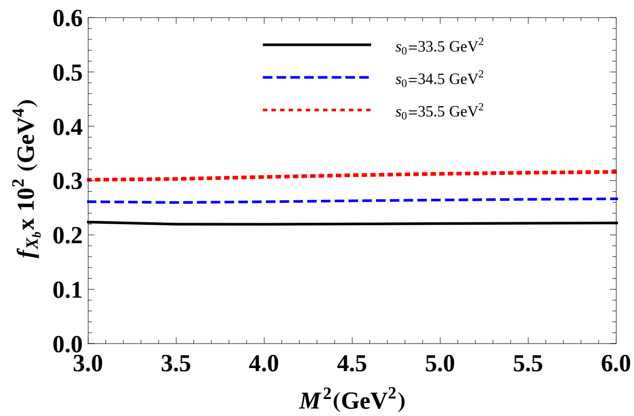

FIG. 2: The decay constant f_{X_b} vs Borel parameter M^2. The values of the parameter s_0 are shown in the figure.

should be fixed in accordance with the standard procedures. Thus, for numerical computation of the X_b state's mass and decay constant we need values of the quark, gluon and mixed condensates. In addition to that, QCD sum rules depend on the b and s quark masses. The values of these parameters can be found in Table I.

Sum rules calculations require fixing of the threshold parameter s_0 and a region within of which it may be varied. For s_0 we employ

$$33.5\ \text{GeV}^2 \le s_0 \le 35.5\ \text{GeV}^2. \tag{12}$$

We also find the range $3\ \text{GeV}^2 < M^2 < 6\ \text{GeV}^2$ as a reliable one for varying the Borel parameter, where the effects of the higher resonances and continuum states, and contributions of the higher dimensional condensates meet all requirements of QCD sum rules calculations. Additionally, in this interval the dependence of the mass and decay constant on M^2 is stable, and we may expect that the sum rules give the correct results. By varying the parameters M^2 and s_0 within the allowed ranges, as well as taking into account ambiguities arising from other input parameters we estimate uncertainties of the whole calculations. The results for the mass m_{X_b} and decay constant f_{X_b} are depicted as the functions of the Borel parameter in Figs. 1 and 2, respectively. The sensitivity of the obtained predictions to the choice of s_0 are also seen in these figures, where three different values for s_0 are employed. Our prediction for the mass m_{X_b} is:

$$m_{X_b} = (5584 \pm 137)\,\mathrm{MeV}. \tag{13}$$

For the decay constant we get:

$$f_{X_b} = (0.24 \pm 0.02) \times 10^{-2}\ \mathrm{GeV}^4. \tag{14}$$

As is seen our prediction for the mass of the $X_b(5568)$ state agrees with experimental data of the D0 Collaboration.

In this paper we have studied the new exotic resonance state with the mass 5568 MeV and quantum numbers $J^{PC} = 0^{++}$, that was observed recently by D0 Collaboration utilizing the collected data of $p\bar{p}$ collision. We have adopted for this state a label $X_b(5568)$ because it is composed of four different quark flavors and differs from the usual charmonium-like members of the X family. We have also accepted the diquark-antidiquark model $[su][\bar{b}\bar{d}]$ for the $X_b(5568)$ state and computed its mass and decay constant employing QCD two-point sum rule method. Our prediction for the mass m_{X_b} is in agreement with the finding of D0 Collaboration. Results of our explorations of the $X_b(5568)$ state obtained by applying other diquark-antidiquark structures and interpolating currents as well as calculation of its decay width, which can be crucial in making decision between various models, will be published elsewhere.

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Appendix: A

In this appendix we have collected the results of our calculations of the spectral density

$$\rho^{\mathrm{QCD}}(s) = \rho^{\mathrm{pert}}(s) + \sum_{k=3}^{k=8} \rho_k(s), \tag{A.1}$$

used for evaluation of the X_b meson mass m_{X_b} and its decay constant f_{X_b} from the QCD sum rule. In Eq. (A.1) by $\rho_k(s)$ we denote the nonperturbative contributions to $\rho^{\mathrm{QCD}}(s)$. In calculations we have neglected the masses of the u and d quarks and taken into account terms $\sim m_s$. The explicit expressions for $\rho^{\mathrm{pert}}(s)$ and $\rho_k(s)$ are presented below as the integrals over the Feynman parameter z:

$$\rho^{\rm pert}(s)=\frac{1}{1536\pi^6}\int\limits_0^a \frac{dzz^4}{(z-1)^3}\left[m_b^2+s(z-1)\right]^3\left[m_b^2+3s(z-1)\right],$$

$$\rho_3(s)=\frac{1}{32\pi^4}\int\limits_0^a \frac{dzz^2}{(z-1)^2}\left[m_b^2+s(z-1)\right]\left\{\langle\overline{d}d\rangle m_b\left[m_b^2+s(z-1)\right]+2m_s(\langle\overline{s}s\rangle-\langle\overline{u}u\rangle)\left[m_b^2+2s(z-1)\right](z-1)\right\},$$

$$\rho_4(s)=\frac{1}{2304\pi^4}\langle\alpha_s\frac{G^2}{\pi}\rangle\int\limits_0^a \frac{dzz^2}{(z-1)^3}\left\{m_b^4\left[z(8z-15)+9\right]+3m_b^2 s(z-1)\left[z(7z-15)+9\right]+6s^2(z-1)^3(2z-3)\right\},$$

$$\rho_5(s)=\frac{m_0^2}{192\pi^4}\int\limits_0^a \frac{dzz}{(1-z)}\left\{3m_b\langle\overline{d}d\rangle\left[m_b^2+s(z-1)\right]+m_s(z-1)(2\langle\overline{s}s\rangle-3\langle\overline{u}u\rangle)\left[2m_b^2+3s(z-1)\right]\right\},$$

$$\rho_6(s)=\frac{1}{324\pi^4}\int\limits_0^a dzzg^2(\langle\overline{u}u\rangle^2+\langle\overline{d}d\rangle^2+\langle\overline{s}s\rangle^2)\left[2m_b^2+3s(z-1)\right],$$

$$\rho_7(s)=\frac{1}{576\pi^2}\langle\alpha_s\frac{G^2}{\pi}\rangle\int\limits_0^a \frac{dz}{(1-z)}\left\{2m_b\langle\overline{d}d\rangle(5z-2)+m_s(z-1)\left[3\langle\overline{s}s\rangle+4\langle\overline{u}u\rangle(4z-1)\right]\right\},$$

$$\rho_8(s)=-\frac{11}{9216\pi^2}\langle\alpha_s\frac{G^2}{\pi}\rangle^2\frac{(m_b^2-s)^2}{s^2}-m_0^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle\int\limits_0^a dz\frac{z-1}{6\pi^2}, \tag{A.2}$$

where $a=(s-m_b^2)/s$.

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