Analysis of the $X(5568)$ as scalar tetraquark state in the diquark-antidiquark model with QCD sum rules

Zhi-Gang Wang [1]

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we take the $X(5568)$ as the diquark-antidiquark type tetraquark state with the spin-parity $J^P = 0^+$, construct the scalar-diquark-scalar-antidiquark type current, carry out the operator product expansion up to the vacuum condensates of dimension-10, and study the mass and pole residue in details with the QCD sum rules. We obtain the value $M_X = (5.57 \pm 0.12)$ GeV, which is consistent with the experimental data. The present prediction favors assigning the $X(5568)$ to be the scalar tetraquark state.

PACS number: 12.39.Mk, 12.38.Lg
Key words: Tetraquark states, QCD sum rules

1 Introduction

Recently, the D0 collaboration observed a narrow structure, $X(5568)$, in the decay chains $X(5568) \to B_s^0 \pi^\pm$, $B_s^0 \to J/\psi \phi$, $J/\psi \to \mu^+ \mu^-$, $\phi \to K^+ K^-$ with significance of 5.1σ based on 10.4 fb^{-1} of $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV collected at the Fermilab Tevatron collider [1]. The mass and natural width of the new state are $M_X = 5567.8 \pm 2.9^{+0.9}_{-1.9}$ MeV and $\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5}$ MeV, respectively. The $B_s^0 \pi^\pm$ systems consist of two quarks and two antiquarks of four different flavors. The D0 collaboration fitted the $B_s^0 \pi^\pm$ systems with the Breit-Wigner parameters in relative S-wave, the favored quantum numbers are $J^P = 0^+$. However, the quantum numbers $J^P = 1^+$ cannot be excluded according to decays $X(5568) \to B_s^* \pi^+ \to B_s^0 \pi^+ \gamma$, where the low-energy photon is not detected.

In this article, we assume the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state. There are five types diquarks, scalar diquarks, pseudoscalar diquarks, vector diquarks, axialvector diquarks and tensor diquarks according to the structures in Dirac spinor space. The favored configurations are the scalar diquarks and axialvector diquarks from the QCD sum rules [2, 3]. The heavy scalar and axialvector diquarks have almost degenerate masses [2], while the masses of the light axialvector diquarks lie $(150 - 200)$ MeV above the corresponding light scalar diquarks [3]. We take the scalar light diquark and heavy diquark as the basic constituents [4], construct the scalar-diquark-scalar-antidiquark type current, which is expected to couple potentially to the lowest state, to study the mass and pole residue of the $X(5568)$ with the QCD sum rules [5, 6]. In the charm sector, the $D_s(2307)$ has been studied as the scalar-diquark-scalar-antidiquark type tetraquark state based on the QCD sum rules [7].

The article is arranged as follows: we derive the QCD sum rules for the mass and pole residue of the $X(5568)$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

2 QCD sum rules for the $X(5568)$ as scalar tetraquark state

In the following, we write down the two-point correlation function $\Pi(p)$ in the QCD sum rules,

$$\Pi(p) = i \int d^4 x e^{ip \cdot x} \langle 0 | T \left\{ J(x) J^\dagger(0) \right\} | 0 \rangle \,, \tag{1}$$

$$J(x) = \epsilon^{ijk} \epsilon^{imn} u^j(x) C \gamma_5 s^k(x) \bar{d}^m(x) \gamma_5 C \bar{b}^n(x) \,, \tag{2}$$

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where the i, j, k, m, n are color indexes, the C is the charge conjugation matrix.

We insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J(x)$ into the correlation function $\Pi(p)$ to obtain the hadronic representation [5, 6]. After isolating the ground state contribution of the scalar tetraquark state, we get the following result,

$$\Pi(p) = \frac{\lambda_X^2}{M_X^2 - p^2} + \cdots , \tag{3}$$

where the pole residue λ_X is defined by $\langle 0|J(0)|X(p)\rangle = \lambda_X$.

In the following, we carry out the operator product expansion. We contract the u, d, s and c quark fields in the correlation function $\Pi(p)$ with Wick theorem, and obtain the result:

$$\begin{aligned}
\Pi(p) = {} & i\epsilon^{ijk}\epsilon^{imn}\epsilon^{i'j'k'}\epsilon^{i'm'n'} \int d^4x e^{ip\cdot x} \\
& \mathrm{Tr}\left[\gamma_5 S^{kk'}(x)\gamma_5 C U^{jj'\,T}(x)C\right] \mathrm{Tr}\left[\gamma_5 B^{n'n}(-x)\gamma_5 C D^{m'm\,T}(-x)C\right] ,
\end{aligned} \tag{4}$$

where the $U_{ij}(x)$, $D_{ij}(x)$, $S_{ij}(x)$ and $B_{ij}(x)$ are the full u, d, s and b quark propagators respectively (the $U_{ij}(x)$, $D_{ij}(x)$, $S_{ij}(x)$ can be written as $S_{ij}(x)$ for simplicity, where $q = u$, d, s.),

$$\begin{aligned}
S_{ij}(x) = {} & \frac{i\delta_{ij}\not{x}}{2\pi^2 x^4} - \frac{\delta_{ij}m_q}{4\pi^2 x^2} - \frac{\delta_{ij}\langle\bar{q}q\rangle}{12} + \frac{i\delta_{ij}\not{x}m_q\langle\bar{q}q\rangle}{48} - \frac{\delta_{ij}x^2\langle\bar{q}g_s\sigma Gq\rangle}{192} + \frac{i\delta_{ij}x^2\not{x}m_q\langle\bar{q}g_s\sigma Gq\rangle}{1152} \\
& - \frac{ig_s G_{\alpha\beta}^a t_{ij}^a(\not{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{x})}{32\pi^2 x^2} - \frac{1}{8}\langle\bar{q}_j\sigma^{\mu\nu}q_i\rangle\sigma_{\mu\nu} + \cdots ,
\end{aligned} \tag{5}$$

$$\begin{aligned}
B_{ij}(x) = {} & \frac{i}{(2\pi)^4} \int d^4k e^{-ik\cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_b} - \frac{g_s G_{\alpha\beta}^n t_{ij}^n}{4}\frac{\sigma^{\alpha\beta}(\not{k} + m_b) + (\not{k} + m_b)\sigma^{\alpha\beta}}{(k^2 - m_b^2)^2} \right. \\
& \left. - \frac{g_s^2 (t^a t^b)_{ij} G_{\alpha\beta}^a G_{\mu\nu}^b (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\mu\nu\beta})}{4(k^2 - m_b^2)^5} + \cdots \right\} , \\
f^{\alpha\beta\mu\nu} = {} & (\not{k} + m_b)\gamma^\alpha(\not{k} + m_b)\gamma^\beta(\not{k} + m_b)\gamma^\mu(\not{k} + m_b)\gamma^\nu(\not{k} + m_b) ,
\end{aligned} \tag{6}$$

and $t^n = \frac{\lambda^n}{2}$, the λ^n is the Gell-Mann matrix [6], then compute the integrals both in the coordinate space and momentum space to obtain the correlation function $\Pi(p)$ at the quark level, therefore the QCD spectral density through dispersion relation $\rho(s) = \frac{\mathrm{Im}\Pi(s)}{\pi}$. The explicit expression is neglected for simplicity. In Eq.(5), we retain the terms $\langle\bar{q}_j\sigma_{\mu\nu}q_i\rangle$ come from the Fierz re-arrangement of the $\langle q_i\bar{q}_j\rangle$ to absorb the gluons emitted from other quark lines to extract the mixed condensates $\langle\bar{q}g_s\sigma Gq\rangle$, several new terms appear and play an important role in determining the Borel window.

In this article, we carry out the operator product expansion up to the vacuum condensates of dimension-10, and assume vacuum saturation for the higher dimension vacuum condensates. The condensates $\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{s}s\rangle^2\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle^2$, $\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{s}g_s\sigma Gs\rangle$ and $\langle\bar{s}g_s\sigma Gs\rangle^2$ are the vacuum expectations of the operators of the order $\mathcal{O}(\alpha_s)$. We take the truncations $n \leq 10$ and $k \leq 1$ in a consistent way, the operators of the orders $\mathcal{O}(\alpha_s^k)$ with $k > 1$ are neglected. The condensates $\langle g_s^3 GGG\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle^2$, $\langle\frac{\alpha_s GG}{\pi}\rangle\langle\bar{q}g_s\sigma Gq\rangle$ and $\langle\frac{\alpha_s GG}{\pi}\rangle\langle\bar{s}g_s\sigma Gs\rangle$ have no contributions.

Once the spectral density at the quark level is obtained, we can take the quark-hadron duality below the continuum threshold s_0 and perform Borel transform with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rule:

$$\lambda_X^2 \, \exp\left(-\frac{M_X^2}{T^2}\right) = \int_{m_b^2}^{s_0} ds\, \rho(s) \, \exp\left(-\frac{s}{T^2}\right) , \tag{7}$$

where

$$\rho(s) = \frac{1}{6144\pi^6}\int_\Delta^1 dx\, x(1-x)^4\left(s-\widetilde{m}_b^2\right)^3\left(3s-\widetilde{m}_b^2\right)$$

$$-\frac{m_b\langle\bar{q}q\rangle}{64\pi^4}\int_\Delta^1 dx\,(1-x)^2\left(s-\widetilde{m}_b^2\right)^2$$

$$-\frac{m_s\left[2\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right]}{64\pi^4}\int_\Delta^1 dx\, x(1-x)^2\left(s-\widetilde{m}_b^2\right)\left(2s-\widetilde{m}_b^2\right)$$

$$-\frac{m_b^2}{9216\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\frac{(1-x)^4}{x^2}\left(3s-2\widetilde{m}_b^2\right)$$

$$+\frac{1}{1536\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,(1+2x)(1-x)^2\left(s-\widetilde{m}_b^2\right)\left(2s-\widetilde{m}_b^2\right)$$

$$-\frac{m_b\langle\bar{q}g_s\sigma Gq\rangle}{128\pi^4}\int_\Delta^1 dx\,\frac{(1-x)(1-3x)}{x}\left(s-\widetilde{m}_b^2\right)$$

$$+\frac{m_s\left[3\langle\bar{q}g_s\sigma Gq\rangle-\langle\bar{s}g_s\sigma Gs\rangle\right]}{192\pi^4}\int_\Delta^1 dx\, x(1-x)\left(3s-2\widetilde{m}_b^2\right)$$

$$+\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{12\pi^2}\int_\Delta^1 dx\, x(1-x)\left(3s-2\widetilde{m}_b^2\right)$$

$$+\frac{m_sm_b\langle\bar{q}q\rangle\left[2\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right]}{24\pi^2}\int_\Delta^1 dx$$

$$-\frac{m_b\langle\bar{q}q\rangle}{192\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\frac{(1-x)^2}{x^2}-\frac{m_b\langle\bar{q}q\rangle}{144\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx$$

$$+\frac{m_b^3\langle\bar{q}q\rangle}{576\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\frac{(1-x)^2}{x^3}\delta\left(s-\widetilde{m}_b^2\right)$$

$$+\frac{m_sm_b^2\left[2\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right]}{1152\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\frac{(1-x)^2}{x^2}\left(1+\frac{\widetilde{m}_b^2}{T^2}\right)\delta\left(s-\widetilde{m}_b^2\right)$$

$$-\frac{m_s\left[2\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right]}{384\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,(1-x)\left[2+\widetilde{m}_b^2\left(s-\widetilde{m}_b^2\right)\right]$$

$$-\frac{m_s\langle\bar{q}q\rangle}{576\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\, x\left[2+\widetilde{m}_b^2\left(s-\widetilde{m}_b^2\right)\right]$$

$$-\frac{\langle\bar{q}q\rangle\langle\bar{s}g_s\sigma Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}g_s\sigma Gq\rangle}{48\pi^2}\int_\Delta^1 dx\, x\left[2+\widetilde{m}_b^2\left(s-\widetilde{m}_b^2\right)\right]$$

$$-\frac{m_sm_b\left[12\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle-2\langle\bar{q}q\rangle\langle\bar{s}g_s\sigma Gs\rangle-3\langle\bar{s}s\rangle\langle\bar{q}g_s\sigma Gq\rangle\right]}{288\pi^2}\delta\left(s-m_b^2\right)$$

$$+\frac{m_sm_b\left[2\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right]\langle\bar{q}g_s\sigma Gq\rangle}{96\pi^2}\int_\Delta^1 dx\,\frac{1}{x}\delta\left(s-\widetilde{m}_b^2\right)$$

$$-\frac{m_b\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{9}\delta\left(s-m_b^2\right)+\frac{\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{s}g_s\sigma Gs\rangle}{192\pi^2}\left(1+\frac{m_b^2}{T^2}\right)\delta\left(s-m_b^2\right)$$

$$+\frac{m_sm_b\langle\bar{q}g_s\sigma Gq\rangle\left[\langle\bar{s}g_s\sigma Gs\rangle-3\langle\bar{q}g_s\sigma Gq\rangle\right]}{576\pi^2T^2}\left(1-\frac{m_b^2}{T^2}\right)\delta\left(s-m_b^2\right)$$

$$+\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{144}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\left(1+\frac{\widetilde{m}_b^2}{T^2}\right)\delta\left(s-\widetilde{m}_b^2\right)$$

$$-\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{216T^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\frac{1-x}{x}\widetilde{m}_b^4\delta\left(s-\widetilde{m}_b^2\right)$$

$$+\frac{m_sm_b\langle\bar{q}q\rangle\left[2\langle\bar{q}q\rangle-\langle\bar{s}s\rangle\right]}{144T^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_\Delta^1 dx\,\frac{1}{x^2}\left(1-\frac{\widetilde{m}_b^2}{3T^2}\right)\delta\left(s-\widetilde{m}_b^2\right)$$

$$+\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{216}\langle\frac{\alpha_sGG}{\pi}\rangle\left(1+\frac{m_b^2}{T^2}\right)^3\delta\left(s-m_b^2\right)$$

Parameters	Values
$\langle \bar{q}q \rangle(1\text{GeV})$	$-(0.24 \pm 0.01\,\text{GeV})^3$ [5, 6, 8]
$\langle \bar{s}s \rangle(1\text{GeV})$	$(0.8 \pm 0.1)\langle \bar{q}q \rangle(1\text{GeV})$ [5, 6, 8]
$\langle \bar{q}g_s\sigma Gq \rangle(1\text{GeV})$	$m_0^2 \langle \bar{q}q \rangle(1\text{GeV})$ [5, 6, 8]
$\langle \bar{s}g_s\sigma Gs \rangle(1\text{GeV})$	$m_0^2 \langle \bar{s}s \rangle(1\text{GeV})$ [5, 6, 8]
$m_0^2(1\text{GeV})$	$(0.8 \pm 0.1)\,\text{GeV}^2$ [5, 6, 8]
$\langle \frac{\alpha_s GG}{\pi} \rangle$	$(0.33\,\text{GeV})^4$ [5, 6, 8]
$m_b(m_b)$	$(4.18 \pm 0.03)\,\text{GeV}$ [9]
$m_s(2\text{GeV})$	$(0.095 \pm 0.005)\,\text{GeV}$ [9]

Table 1: The input parameters in the QCD sum rules, the values in the bracket denote the energy scales $\mu = 1\,\text{GeV}$, $2\,\text{GeV}$ and m_b, respectively.

$\Delta = \frac{m_b^2}{s}$, $\widetilde{m}_b^2 = \frac{m_b^2}{x}$, and $\int_{\Delta}^1 dx \to \int_0^1 dx$ when the $\delta\left(s - \widetilde{m}_b^2\right)$ appears.

We differentiate Eq.(7) with respect to $\frac{1}{T^2}$, then eliminate the pole residue λ_X, and obtain the QCD sum rule for the mass of the $X(5568)$,

$$M_X^2 = \frac{\int_{m_b^2}^{s_0} ds \frac{d}{d(-1/T^2)}\rho(s) \exp\left(-\frac{s}{T^2}\right)}{\int_{m_b^2}^{s_0} ds\rho(s) \exp\left(-\frac{s}{T^2}\right)} . \tag{9}$$

3 Numerical results and discussions

The input parameters are shown explicitly in Table 1. The quark condensates, mixed quark condensates and $\overline{MS}$ masses evolve with the renormalization group equation, we take into account the energy-scale dependence according to the following equations,

$$
\begin{aligned}
\langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{4}{9}}, \\
\langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{4}{9}}, \\
\langle \bar{q}g_s\sigma Gq \rangle(\mu) &= \langle \bar{q}g_s\sigma Gq \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{27}}, \\
\langle \bar{s}g_s\sigma Gs \rangle(\mu) &= \langle \bar{s}g_s\sigma Gs \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{27}}, \\
m_b(\mu) &= m_b(m_b) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{25}}, \\
m_s(\mu) &= m_s(2\text{GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})}\right]^{\frac{4}{9}}, \\
\alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1}{b_0^2}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right], \tag{10}
\end{aligned}
$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 213\,\text{MeV}$, $296\,\text{MeV}$ and $339\,\text{MeV}$ for the flavors $n_f = 5$, 4 and 3, respectively [9]. Furthermore, we set the u and d quark masses to be zero.

In Refs.[10, 11], we study the acceptable energy scales of the QCD spectral densities for the hidden charm (bottom) tetraquark states in the QCD sum rules in details for the first time, and

suggest a formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}$ to determine the energy scales, where the X, Y, Z are the four-quark systems, and the $\mathbb{M}_Q$ are the effective heavy quark masses. The energy scale formula has been successfully extended to study the charmed baryon states and hidden-charm pentaquark states [12]. Recently, we re-checked the numerical calculations and found that there exists a small error involving the mixed condensates [11]. The Borel windows are modified slightly and the numerical results are also improved slightly after the small error is corrected, the conclusions survive, the optimal value of the effective mass is $\mathbb{M}_b = 5.17\,\mathrm{GeV}$ instead of $5.13\,\mathrm{GeV}$ for the hidden-bottom tetraquark states. In this article, we use the energy scale formula $\mu = \sqrt{M_X^2 - \mathbb{M}_b^2}$ to determine the optimal energy scale and obtain the value $\mu = 2.1\,\mathrm{GeV}$.

In this article, the QCD spectral density $\rho(s) \propto s^n$ with $n \leq 4$, the integral $\int_0^\infty \rho(s) \exp\left(-\frac{s}{T^2}\right) ds$ converges slowly, it is difficult to obtain the pole contribution larger than 50%. In calculations, we use the QCD spectral density $\rho(s)\theta(s - s_0)$ to approximate the continuum contribution with the value $\sqrt{s_0} = 6.1 \pm 0.1\,\mathrm{GeV}$, where we take it for granted that the energy gap between the ground state and the first radial excited state is about $0.4 - 0.6\,\mathrm{GeV}$, just like the conventional mesons and the hidden-charm tetraquark states, the $Z(4430)$ is assigned to be the first radial excitation of the $Z_c(3900)$ [13, 14]. Now we search for the optimal Borel parameter T^2 according to the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. The optimal Borel parameter $T^2 = (4.5 - 4.9)\,\mathrm{GeV}^2$, the pole contribution is about $(15 - 29)\%$. At the operator product expansion side, the contributions of the vacuum condensates D_i of dimension i have the relations, D_3, D_6, $|D_8|$, $D_9 \gg D_0$, D_4, $|D_5|$, D_7, D_{10}, $D_0 + D_3 \approx 40\%$, $D_4 + D_5 \approx -1\%$, $D_6 \approx 66\%$, $D_7 + D_8 + D_9 \approx -10\%$, $D_{10} \approx 5\%$. The dominant contributions come from the terms $D_0 + D_3 + D_6$, the operator product expansion is convergent, but the pole contribution is smaller than 30%, we cut down the continuum contamination by the threshold parameter s_0. The radial excited states or high resonances have to be included in, if one wants to obtain QCD sum rules with the pole contributions larger than 50% [15].

We take into account all uncertainties of the input parameters, and obtain the values of the mass and pole residue of the $X(5568)$ as the scalar diquark-antidiquark type tetraquark state,

$$
\begin{aligned}
M_X &= (5.57 \pm 0.12)\,\mathrm{GeV}\,, \\
\lambda_X &= (6.7 \pm 1.6) \times 10^{-3}\,\mathrm{GeV}^5\,.
\end{aligned}
\tag{11}
$$

In Fig.1, we plot the predicted mass with variation of the Borel parameter. From the figure, we can see that the platform is rather flat, and we expect to make reasonable prediction. The predicted mass $M_X = (5.57 \pm 0.12)\,\mathrm{GeV}$ is consistent with the experimental data $M_X = 5567.8 \pm 2.9^{+0.9}_{-1.9}\,\mathrm{MeV}$ from the D0 collaboration [1].

In Ref.[16], Zanetti, Nielsen and Khemchandani choose the same interpolating current as the present work, while in Ref.[17], Agaev, Azizi and Sundu choose the axialvector-diquark-axialvector-antidiquark type scalar current. If we apply the identity $\epsilon^{ijk}\epsilon^{imn} = \delta^{jm}\delta^{kn} - \delta^{jn}\delta^{km}$ in the color space to the current $J(x)$, we can obtain the current used in Ref.[18] in studying the $X(5568)$ as a scalar tetraquark state with the QCD sum rules. In Ref.[18], Chen et al also study the $X(5568)$ as the axialvector tetraquark state and reproduce the experimental value of the mass $M_{X(5568)}$. The quantum numbers $J^P = 1^+$ cannot be excluded according to decays $X(5568) \to B_s^* \pi^+ \to B_s^0 \pi^+ \gamma$, where the low-energy photon is not detected. The present article and Refs.[16, 17, 18] appeared in the net http://arxiv.org/ on the same day, and the calculations were done independently. In Refs.[17, 18], the parameters in Table 1 are taken directly to evaluate the QCD spectral densities, while in the present work we evolve the values to a special energy scale $\mu = 2.1\,\mathrm{GeV}$ determined by the energy scale formula. The input parameters chosen in Ref.[16] are different from the values used in the present work and in Refs.[17, 18]. The experimental value of the mass $M_{X(5568)}$ can be well reproduced in the present work and in Refs.[16, 17, 18].

In the following, we perform Fierz re-arrangement to the current J both in the color and
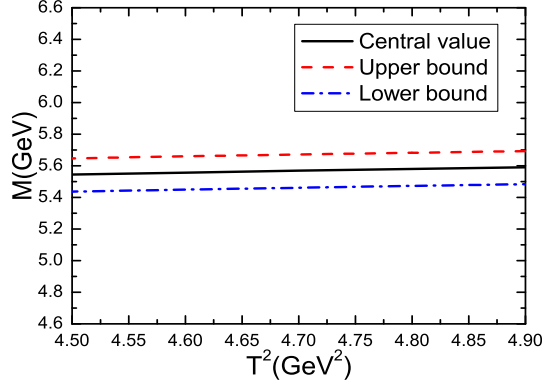
Figure 1: The mass M_X with variation of the Borel parameter T^2.

Dirac-spinor spaces to obtain the result,

$$
\begin{aligned}
J &= \frac{1}{4}\left\{ -\bar{b}s\,\bar{d}u + \bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u - \bar{b}\gamma^\mu s\,\bar{d}\gamma_\mu u - \bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u + \frac{1}{2}\bar{b}\sigma_{\mu\nu}s\,\bar{d}\sigma^{\mu\nu}u \right. \\
&\quad \left. + \bar{b}u\,\bar{d}s - \bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s + \bar{b}\gamma^\mu u\,\bar{d}\gamma_\mu c + \bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s - \frac{1}{2}\bar{b}\sigma_{\mu\nu}u\,\bar{d}\sigma^{\mu\nu}s \right\},
\end{aligned}
\tag{12}
$$

where $\bar{b}\Gamma s\,\bar{d}\Gamma u = \bar{b}^j\Gamma s^j\,\bar{d}^k\Gamma u^k$ and $\bar{b}\Gamma u\,\bar{d}\Gamma s = \bar{b}^j\Gamma u^j\,\bar{d}^k\Gamma s^k$, $\Gamma = 1$, $i\gamma_5$, γ_μ, $\gamma_\mu\gamma_5$, $\sigma_{\mu\nu}$, the j and k are color indexes. The components $\bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u$ and $\bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u$ couple potentially to the meson pair $B_s^0\pi^+$, while the components $\bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s$ and $\bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s$ couple potentially to the meson pair $B^+\bar{K}^0$. The strong decays

$$
X(5568) \quad \to \quad B_s^0\pi^+,
\tag{13}
$$

are Okubo-Zweig-Iizuka super-allowed, while the decays

$$
X(5568) \quad \to \quad B^+\bar{K}^0,
\tag{14}
$$

are kinematically forbidden, which is consistent with the observation of the D0 collaboration [1]. The present work favors assigning the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state.

In Ref.[19], Burns and Swanson argue that it is unusual to assign the $X(5568)$ to be the threshold effect, the $B_s^*\pi - B_s\pi$ cusp with $J^P = 1^-$, the $B_s\pi - B\bar{K}$ cusp with $J^P = 0^+$, the Gamow-Gurney-Condon type resonance induced by the $B_s\pi - B\bar{K}$ interaction, the compact tetraquark state based on phenomenological analysis. In Ref.[20], Guo, Meissner and Zou provide additional arguments using the chiral symmetry and heavy quark symmetry. According to the constituent quark model, the constituent diquark model, the chiral symmetry and heavy quark symmetry, the lowest mass of the $su\bar{b}\bar{d}$ tetraquark state with $J^P = 0^+$ is much larger than the $M_{X(5568)}$ [19, 20, 21], for more references on this subject, one can consult Refs.[19, 20]. In the present work and in Refs.[16, 17, 18], the $X(5568)$ is assigned to be the compact tetraquark state, the experimental value of the mass $M_{X(5568)}$ can be well reproduced based on the QCD sum rules. In Ref.[22], the partial decay width of the strong decay $X(5568) \to B_s^0\pi^+$ is studied with the three-point QCD sum rules. If we saturate the width of the $X(5568)$ with the strong decay $X(5568) \to B_s^0\pi^+$, the experimental value $\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5}$ MeV can be reproduced approximately. More theoretical works and more experimental data are still needed to approve existence or non-existence of the $X(5568)$.

6

4 Conclusion

In this article, we take the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state, construct the scalar-diquark-scalar-antidiquark type current, carry out the operator product expansion up to the vacuum condensates of dimension-10, and study the mass and pole residue of the $X(5568)$ in details with the QCD sum rules. In calculations, we use the formula $\mu = \sqrt{M_X^2 - \mathbb{M}_b^2}$ to determine the energy scale of the QCD spectral density. The present prediction favors assigning the $X(5568)$ to be the scalar tetraquark state. The pole residue can be taken as basic input parameter to study relevant processes of the $X(5568)$ with the three-point QCD sum rules.

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