Decoding the $X(5568)$ as a fully open-flavor $su\bar{b}\bar{d}$ tetraquark state

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We investigate the recent evidence for a charged $X(5568)$ meson as an exotic open-flavor tetraquark state $su\bar{b}\bar{d}$ with $J^P = 0^+/1^+$ in the framework of QCD sum rules. We use the color antisymmetric $[\bar{\mathbf{3}}_\mathbf{c}]_{su} \otimes [\mathbf{3}_\mathbf{c}]_{\bar{b}\bar{d}}$ tetraquark currents in both scalar and axial-vector channels to perform evaluations and numerical analyses. Our results imply that the $X(5568)$ can be interpreted as both the scalar $su\bar{b}\bar{d}$ tetraquark state and the axial-vector one, which are in good agreement with the experimental measurement. We also discuss the possible decay patterns of the $X(5568)$ and suggest to search for its neutral partner in the radiative decay into $B_s^0\gamma$ and $B_s^*\gamma$, which can be used to determine its spin-parity quantum numbers. Moreover, we predict its charmed partner state around 2.55 GeV with the quark content $su\bar{c}\bar{d}$ and $J^P = 0^+/1^+$.

Introduction.—As a long-standing puzzle, understanding the nonperturbative QCD behavior quantitatively is one of the most important and intriguing research topics of the hadron physics. Thoroughly figuring out the map of hadron spectrum is a key step to achieve this goal. The possible hadron configurations not only include conventional mesons and baryons, but also contain exotic states like glueball, hybrid, and multiquark states, etc. However, exotic states have not yet been well established, which is the reason why experimentalists and theorists have already paid many and are still paying more attentions to them. With significant experimental progress in the past decade, more and more charmonium-like and bottomonium-like states (also named as *XYZ* particles) [1], and the hidden-charm pentaquarks $P_c(4380)$ and $P_c(4450)$ [4] were observed, which provide good chance for identifying exotic states (see Refs. [2, 3] for review). It is obvious that this story still continues with recent evidence for a $X(5568)$ state [5].

The $X(5568)$ is a narrow structure seen by the DØ Collaboration [5], which appears in the $B_s^0\pi^\pm$ invariant mass spectrum with 5.1σ significance. Its measured mass and width are $M = 5567.8 \pm 2.9(\text{stat})^{+0.9}_{-1.9}(\text{syst})$ MeV and $\Gamma = 21.9 \pm 6.4(\text{stat})^{+5.0}_{-2.5}(\text{syst})$ MeV, respectively. Its decay final state $B_s^0\pi^\pm$ requires the valence quark component of the $X(5568)$ to be $su\bar{b}\bar{d}$ (or $sd\bar{b}\bar{u}$). Hence, the reported $X(5568)$ state, if exist, cannot be categorized into the conventional meson family, and is a good candidate of exotic tetraquark state with valence quarks of four different flavors.

With the mass of $D^+_{sj}(2632)$ [6] as input, the scalar isoscalar $b\bar{s}q\bar{q}$ tetraquark mass was estimated to be around 5832 MeV in Ref. [7], while the mass of the isovector $b\bar{s}u\bar{d}$ tetraquark state can be similarly estimated to be around 5700 MeV. In Ref. [8], the heavy-light meson resonances with $J^P = 0^+$ and $J^P = 1^+$ and similar flavor configurations as the $X(5568)$ were studied in terms of the non-linear chiral SU(3) Lagrangian and

their masses were estimated around 5750-5790 MeV, which is 200 MeV higher than the mass of the $X(5568)$. The $X(5568)$ was studied as a $su\bar{b}\bar{d}$ tetraquark state with $J^P = 0^+$ in Ref. [10]. Inspired by the charm-strange state $D^*_{s0}(2317)$ [9], various exotic pictures were proposed such as the $cq\bar{s}\bar{q}$ tetraquark state [11–14].

In this work, we investigate the $X(5568)$ as a fully open-flavor $su\bar{b}\bar{d}$ tetraquark state with $J^P = 0^+/1^+$ in the framework of QCD sum rules. Our results suggest that the interpretations of the $X(5568)$ as the scalar $su\bar{b}\bar{d}$ tetraquark state and the axial-vector one are both possible. To differentiate them and determine its spin-parity quantum numbers, we further investigate its possible decay patterns, from which we propose to observe the radiative decay of the neutral partner of the $X(5568)$ into $B_s^0\gamma$ and $B_s^*\gamma$. The charmed partner state of the $X(5568)$, with the fully open-flavor quark content $su\bar{c}\bar{d}$, can be searched for in many current experiments. We predict the mass of this charmed-partner to be around 2.55 GeV for both the cases of $J^P = 0^+$ and 1^+.

Interpretation of the $X(5568)$ state.— We shall first construct the diquark-antidiquark type of tetraquark interpolating currents with quark content $su\bar{b}\bar{d}$. There are five independent diquark fields: $q_a^T C q_b$, $q_a^T C\gamma_5 q_b$, $q_a^T C\gamma_\mu q_b$, $q_a^T C\gamma_\mu\gamma_5 q_b$ and $q_a^T C\sigma_{\mu\nu}q_b$, where a, b are color indices. In general, one can use all these diquarks and the corresponding antidiquarks to compose tetraquark operators. However, the P-wave diquarks $q_a^T C q_b$, $q_a^T C\gamma_\mu\gamma_5 q_b$ and $q_a^T C\sigma_{\mu\nu}q_b$ are not favored configurations [15]. In QCD sum rules, they lead to unstable sum rules and thus unreliable mass predictions [16, 17]. Following our previous works in Refs. [18, 19], we use only the S-wave diquark fields $q_a^T C\gamma_5 q_b$ and $q_a^T C\gamma_\mu q_b$ to compose the tetraquark currents coupling to the lowest lying hadron states with $J^P = 0^+/1^+$. Considering the Lorentz and color struc-

tures, we finally obtain the interpolating currents with $J^P = 0^+$

$$
\begin{aligned}
J_1 &= s_a^T C\gamma_5 u_b(\bar{b}_a\gamma_5 C\bar{d}_b^T + \bar{b}_b\gamma_5 C\bar{d}_a^T), \\
J_2 &= s_a^T C\gamma_\mu u_b(\bar{b}_a\gamma^\mu C\bar{d}_b^T + \bar{b}_b\gamma^\mu C\bar{d}_a^T), \\
J_3 &= s_a^T C\gamma_5 u_b(\bar{b}_a\gamma_5 C\bar{d}_b^T - \bar{b}_b\gamma_5 C\bar{d}_a^T), \\
J_4 &= s_a^T C\gamma_\mu u_b(\bar{b}_a\gamma^\mu C\bar{d}_b^T - \bar{b}_b\gamma^\mu C\bar{d}_a^T),
\end{aligned}
\tag{1}
$$

in which J_1 and J_2 belong to the symmetric color structure $[\mathbf{6_c}]_{su} \otimes [\bar{\mathbf{6}}_{\mathbf{c}}]_{\bar{b}\bar{d}}$, and J_3 and J_4 belong to the antisymmetric color structure $[\bar{\mathbf{3}}_{\mathbf{c}}]_{su} \otimes [\mathbf{3_c}]_{\bar{b}\bar{d}}$. The interpolating currents with $J^P = 1^+$ are

$$
\begin{aligned}
J_{1\mu} &= s_a^T C\gamma_5 u_b(\bar{b}_a\gamma_\mu C\bar{d}_b^T + \bar{b}_b\gamma_\mu C\bar{d}_a^T), \\
J_{2\mu} &= s_a^T C\gamma_\mu u_b(\bar{b}_a\gamma_5 C\bar{d}_b^T + \bar{b}_b\gamma_5 C\bar{d}_a^T), \\
J_{3\mu} &= s_a^T C\gamma_5 u_b(\bar{b}_a\gamma_\mu C\bar{d}_b^T - \bar{b}_b\gamma_\mu C\bar{d}_a^T), \\
J_{4\mu} &= s_a^T C\gamma_\mu u_b(\bar{b}_a\gamma_5 C\bar{d}_b^T - \bar{b}_b\gamma_5 C\bar{d}_a^T),
\end{aligned}
\tag{2}
$$

in which $J_{1\mu}$ and $J_{2\mu}$ belong to the symmetric color structure $[\mathbf{6_c}]_{su} \otimes [\bar{\mathbf{6}}_{\mathbf{c}}]_{\bar{b}\bar{d}}$, and $J_{3\mu}$ and $J_{4\mu}$ belong to the antisymmetric color structure $[\bar{\mathbf{3}}_{\mathbf{c}}]_{su} \otimes [\mathbf{3_c}]_{\bar{b}\bar{d}}$. We note that these currents can be related to those $cq\bar{b}\bar{q}$ currents in Ref. [18] by simply replacing the charm quark c by the strange quark s.

In the following, we use these currents to study the fully open-flavor $su\bar{b}\bar{d}$ tetraquark states in the framework of QCD sum rules. Arising from QCD itself, QCD sum rule techniques provide a model-independent method to study nonperturbative problems in strong interaction physics. In the past several decades, QCD sum-rules have been used to study a vast number of hadronic properties for conventional mesons and baryons as reviewed in Refs. [20–24]. Recently, QCD sum-rules have been also extended to studies of exotic hadron states and the results are encouraging [23]. However, the accuracy of the method is limited by the truncation of the OPE series and the complicated structure of the hadronic dispersion integrals.

We start from the two-point correlation functions

$$
\Pi(p^2) = i \int d^4 x e^{ip\cdot x}\langle 0|T[J(x)J^\dagger(0)]|0\rangle,
\tag{3}
$$

$$
\begin{aligned}
\Pi_{\mu\nu}(p^2) &= i \int d^4 x e^{ip\cdot x}\langle 0|T[J_\mu(x)J_\nu^\dagger(0)]|0\rangle \\
&= \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu}\right)\Pi_1(p^2) + \frac{p_\mu p_\nu}{p^2}\Pi_0(p^2),
\end{aligned}
\tag{4}
$$

for scalar and axial-vector tetraquark systems, respectively. The imaginary parts of the invariant functions $\Pi_1(p^2)$ and $\Pi_0(p^2)$ receive contributions from the pure spin-1 and spin-0 intermediate states, respectively. In this paper we shall use $\Pi(p^2)$ and $\Pi_1(p^2)$ to investigate the scalar and axial-vector channels, respectively.

One can build QCD sum rules on the hypothesis that the two-point correlation function can be evaluated at the quark-gluonic level, which is then equated to that obtained at the hadronic level. The correlation function at the hadronic level

can be described in the form of the dispersion relation

$$
\Pi(p^2) = \frac{(p^2)^N}{\pi} \int_<^\infty \frac{\mathrm{Im}\Pi(s)}{s^N(s - p^2 - i\epsilon)}ds + \sum_{n=0}^{N-1} b_n(p^2)^n,
\tag{5}
$$

where b_n are subtraction constants. These unknown constants can be removed later by performing the Borel transform. The imaginary part of the correlation function is defined as the spectral density $\rho(s) \equiv \mathrm{Im}\Pi(s)/\pi$, which can be written as a sum over δ functions by inserting intermediate hadronic states

$$
\begin{aligned}
\rho(s) &= \sum_n \delta(s - m_n^2)\langle 0|J|n\rangle\langle n|J^\dagger|0\rangle + \text{continuum} \\
&= f_X^2\delta(s - m_X^2) + \text{continuum},
\end{aligned}
\tag{6}
$$

in which a narrow resonance approximation is adopted in the second step. The interpolating current $J(x)$ can couple to all intermediate hadrons $|n\rangle$ with the same quantum numbers. As usual we only investigate the lowest lying resonance $|X\rangle$, and m_X and f_X are its mass and coupling constant, respectively. The coupling constants are defined as

$$
\langle 0|J|X\rangle = f_X,
\tag{7}
$$
$$
\langle 0|J_\mu|X\rangle = f_X\epsilon_\mu,
\tag{8}
$$

for the scalar and axial-vector interpolating currents $J(x)$ and $J_\mu(x)$, respectively. ϵ_μ is a polarization vector ($\epsilon \cdot p = 0$).

At the quark-gluonic level, the correlation functions $\Pi(p^2)$ and $\Pi_1(p^2)$ can be evaluated using the method of the operator product expansion (OPE). In this paper, we shall use the interpolating currents $J_3(x)$ with $J^P = 0^+$ and $J_{3\mu}(x)$ with $J^P = 1^+$ to investigate the $X(5568)$ as a $su\bar{b}\bar{d}$ tetraquark state. The spectral density for the current $J_3(x)$ with $J^P = 0^+$ is

$$
\begin{aligned}
\rho_0(s) =\ & \frac{1}{512\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta(1 - \alpha - \beta)^2 \\
& \frac{(m_b^2\beta + m_s^2\alpha - 3\alpha\beta s)(m_b^2\beta + m_s^2\alpha - \alpha\beta s)^3}{\alpha^3\beta^3},
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\rho_3(s) =\ & -\frac{\langle \bar{q}q\rangle}{16\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta \frac{(m_b^2\beta + m_s^2\alpha - \alpha\beta s)}{\alpha\beta} \\
& \left(\frac{m_b}{\alpha} + \frac{m_s}{\beta}\right)(1 - \alpha - \beta)(m_b^2\beta + m_s^2\alpha - 2\alpha\beta s) \\
& + \frac{m_s\langle \bar{s}s\rangle}{64\pi^4} \int_0^{1-m_b^2/s} d\alpha \frac{\alpha^2}{1-\alpha} \\
& \left[m_b^2 - s(1 - \alpha)\right]\left[m_b^2 - 2s(1 - \alpha)\right],
\end{aligned}
$$

$$
\begin{aligned}
\rho_4(s) =\ & \frac{\langle g_s^2 GG\rangle}{1024\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta\Bigg\{\frac{(1 - \alpha - \beta)^2}{3} \\
& (2m_b^2\beta + 2m_s^2\alpha - 3\alpha\beta s)\left(\frac{m_b^2}{\alpha^3} + \frac{m_s^2}{\beta^3}\right) \\
& + \frac{(1 - \alpha - \beta)(m_b^2\beta + m_s^2\alpha - 2\alpha\beta s)}{\alpha\beta} \\
& \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)(m_b^2\beta + m_s^2\alpha - \alpha\beta s)\Bigg\},
\end{aligned}
$$

$$\rho_5(s) = \frac{\langle \bar{q}g_s\sigma\cdot Gq\rangle}{64\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta$$
$$(2m_b^2\beta + 2m_s^2\alpha - 3\alpha\beta s)$$
$$\left[\left(\frac{m_b}{\alpha} + \frac{m_s}{\beta}\right) - \left(\frac{m_b}{\alpha^2} + \frac{m_s}{\beta^2}\right)(1-\alpha-\beta)\right],$$

$$\rho_6(s) = \frac{m_b m_s\langle\bar{q}q\rangle^2}{12\pi^2}\left[\left(1 + \frac{m_b^2-m_s^2}{s}\right)^2 - \frac{4m_b^2}{s}\right]^{1/2} -$$
$$\frac{\langle\bar{s}s\rangle\langle\bar{q}q\rangle}{24\pi^2} \int_0^{1-\frac{m_b^2}{s}} d\alpha\left[4m_b^2\alpha - 6s\alpha(1-\alpha) + m_b m_s\right],$$

$$\rho_8(s) = \frac{\langle\bar{s}s\rangle\langle\bar{q}g_s\sigma\cdot Gq\rangle + \langle\bar{s}g_s\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{48\pi^2}$$
$$+ \frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma\cdot Gq\rangle}{24\pi^2} \int_0^1 d\alpha$$
$$\left[\frac{m_b m_s^3}{\alpha^2}\delta'\left(s - \frac{m_b^2\alpha + m_s^2(1-\alpha)}{\alpha(1-\alpha)}\right)\right.$$
$$\left.- \frac{m_b m_s}{2\alpha(1-\alpha)}\delta\left(s - \frac{m_b^2\alpha + m_s^2(1-\alpha)}{\alpha(1-\alpha)}\right)\right].$$

The spectral density for the current $J_{3\mu}(x)$ with $J^P = 1^+$ is

$$\rho_0(s) = \frac{1}{1024\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta(1-\alpha-\beta)^2 \qquad (10)$$
$$\frac{(m_b^2\beta + m_s^2\alpha - 5\alpha\beta s)(m_b^2\beta + m_s^2\alpha - \alpha\beta s)^3}{\alpha^3\beta^3},$$

$$\rho_3(s) = \frac{\langle\bar{q}q\rangle}{16\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta(m_b^2\beta + m_s^2\alpha - \alpha\beta s)$$
$$(1-\alpha-\beta)\left[\frac{m_s(m_b^2\beta + m_s^2\alpha - \alpha\beta s)}{2\alpha\beta^2}\right.$$
$$\left.- \left(\frac{m_b}{\alpha} + \frac{m_s}{\beta}\right)\frac{(m_b^2\beta + m_s^2\alpha - 2\alpha\beta s)}{\alpha\beta}\right]$$
$$+ \frac{m_s\langle\bar{s}s\rangle}{128\pi^4} \int_0^{1-m_b^2/s} d\alpha$$
$$\frac{\alpha^2\left[m_b^2 - s(1-\alpha)\right]\left[m_b^2 - 3s(1-\alpha)\right]}{1-\alpha},$$

$$\rho_4(s) = \frac{\langle g_s^2 GG\rangle}{3072\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta\left[(1-\alpha-\beta)^2\right.$$
$$(m_b^2\beta + m_s^2\alpha - 2\alpha\beta s)\left(\frac{m_b^2}{\alpha^3} + \frac{m_s^2}{\beta^3}\right)$$
$$- \frac{(1-\alpha-\beta)(m_b^2\beta + m_s^2\alpha - \alpha\beta s)}{2\alpha\beta}$$
$$\left(\frac{3m_b^2\beta + 3m_s^2\alpha - 5\alpha\beta s}{\alpha}\right.$$
$$\left.\left.- \frac{3m_b^2\beta + 3m_s^2\alpha - 9\alpha\beta s}{\beta}\right)\right],$$

$$\rho_5(s) = \frac{\langle\bar{q}g_s\sigma\cdot Gq\rangle}{64\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{\beta_{max}} d\beta$$

$$\left[\frac{m_b(2m_b^2\beta + 2m_s^2\alpha - 3\alpha\beta s)}{\alpha}\right.$$
$$\left.- \frac{m_s(1-\alpha-2\beta)(m_b^2\beta + m_s^2\alpha - 2\alpha\beta s)}{\beta^2}\right],$$

$$\rho_6(s) = \frac{m_b m_s\langle\bar{q}q\rangle^2}{12\pi^2}\left[\left(1 + \frac{m_b^2-m_s^2}{s}\right)^2 - \frac{4m_b^2}{s}\right]^{1/2} -$$
$$\frac{\langle\bar{s}s\rangle\langle\bar{q}q\rangle}{24\pi^2} \int_0^{1-\frac{m_b^2}{s}} d\alpha\left[2m_b^2\alpha - 4s\alpha(1-\alpha) + m_b m_s\right],$$

$$\rho_8(s) = \frac{\langle\bar{s}s\rangle\langle\bar{q}g_s\sigma\cdot Gq\rangle + \langle\bar{s}g_s\sigma\cdot Gs\rangle\langle\bar{q}q\rangle}{96\pi^2}\left(1 + \frac{m_b^2}{s^2}\right)$$
$$+ \frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma\cdot Gq\rangle}{24\pi^2} \int_0^1 d\alpha$$
$$\left[\frac{m_b m_s^3}{\alpha^2}\delta'\left(s - \frac{m_b^2\alpha + m_s^2(1-\alpha)}{\alpha(1-\alpha)}\right)\right.$$
$$\left.- \frac{m_b m_s}{2\alpha}\delta\left(s - \frac{m_b^2\alpha + m_s^2(1-\alpha)}{\alpha(1-\alpha)}\right)\right],$$

where the subscript i of $\rho_i(s)$ denotes the dimension of the condensate and $\rho_0(s)$ is the perturbative term. The integral limitations are

$$\alpha_{min} = \frac{1}{2}\left\{1 + \frac{m_b^2-m_s^2}{s} - \left[\left(1 + \frac{m_b^2-m_s^2}{s}\right)^2 - \frac{4m_b^2}{s}\right]^{1/2}\right\},$$

$$\alpha_{max} = \frac{1}{2}\left\{1 + \frac{m_b^2-m_s^2}{s} + \left[\left(1 + \frac{m_b^2-m_s^2}{s}\right)^2 - \frac{4m_b^2}{s}\right]^{1/2}\right\},$$

$$\beta_{min} = \frac{\alpha m_s^2}{\alpha s - m_b^2},$$

$$\beta_{max} = 1 - \alpha.$$

We adopt the following parameter values to perform QCD sum rule analysis [1, 21, 25–28] in the chiral limit ($m_u = m_d = 0$):

$$m_s(2\,\text{GeV}) = (95 \pm 5)\,\text{MeV},$$
$$m_c(m_c) = \overline{m}_c = (1.275 \pm 0.025)\,\text{GeV},$$
$$m_b(m_b) = \overline{m}_b = (4.18 \pm 0.03)\,\text{GeV},$$
$$\langle\bar{q}q\rangle = -(0.24 \pm 0.01)^3\,\text{GeV}^3,$$
$$\langle\bar{s}s\rangle = (0.8 \pm 0.1)\langle\bar{q}q\rangle, \qquad (11)$$
$$\langle\bar{q}g_s\sigma\cdot Gq\rangle = -M_0^2\langle\bar{q}q\rangle,$$
$$\langle\bar{s}g_s\sigma\cdot Gs\rangle = -M_0^2\langle\bar{s}s\rangle,$$
$$M_0^2 = (0.8 \pm 0.2)\,\text{GeV}^2,$$
$$\langle g_s^2 GG\rangle = (0.48 \pm 0.14)\,\text{GeV}^4,$$

in which $\overline{m}_c$ and $\overline{m}_b$ are the "running masses" of the heavy quarks in the $\overline{\text{MS}}$ scheme. There is an additional minus sign in mixed condensates due to the different definition of the coupling constant g_s compared to that in Ref. [21].

In the following, we use $J_3(x)$ and $J_{3\mu}(x)$ to perform numerical analyses. To establish the QCD sum rules, the Borel transform is usually performed on the correlation functions to pick out the lowest lying state and remove the unknown subtraction constants b_n in Eq. (5). Comparing $\Pi(p^2)$ at both phenomenological and OPE sides, one can obtain the following QCD sum rules via the quark-hadron duality

$$\mathcal{L}_k\left(s_0, M_B^2\right) = f_X^2 m_X^{2k} e^{-m_X^2/M_B^2} = \int_<^{s_0} ds\, e^{-s/M_B^2} \rho(s) s^k\,, \quad (12)$$

where s_0 and M_B are the continuum threshold and Borel parameter. The hadron mass is then extracted as

$$m_X\left(s_0, M_B^2\right) = \sqrt{\frac{\mathcal{L}_1\left(s_0, M_B^2\right)}{\mathcal{L}_0\left(s_0, M_B^2\right)}}\,. \quad (13)$$

One notes that the hadron mass in Eq. (13) is a function of s_0 and M_B, which are two essential parameters in our following analysis. The QCD sum rule prediction of the hadron mass is only significant and reliable in suitable regions of the parameter space (s_0, M_B^2).

We use two criteria to fix the Borel parameter: a) the lower bound on M_B^2 can be determined by the constraint of the OPE convergence, requiring the contribution of the dominant non-perturbative terms (quark condensates here) to be less than one third of perturbative contribution; b) the limitation of the pole contribution (PC) gives the upper bound on M_B^2

$$PC(s_0, M_B^2) = \frac{\mathcal{L}_0\left(s_0, M_B^2\right)}{\mathcal{L}_0\left(\infty, M_B^2\right)}\,. \quad (14)$$

However, the extracted hadron mass m_X should not depend on the unphysical parameter M_B, which results in the stability criterion: we choose the value of the continuum threshold s_0 to minimize the dependence of m_X with respect to the Borel mass M_B. Altogether, we will obtain a Borel window $M_{min}^2 \leq M_B^2 \leq M_{max}^2$ for a definite value of s_0.

For the interpolating current $J_3(x)$ with $J^P = 0^+$, we show the variation of the extracted mass m_X with respect to s_0 in the left panel of Fig. 1. We find that the M_B dependence of the hadron mass becomes very weak for $32\,\text{GeV}^2 \leq s_0 \leq 36$ GeV^2, which is thus a reasonable working region. Accordingly, we fix $s_0 = 34\,\text{GeV}^2$, which is then used to fix the Borel window $6.0\,\text{GeV}^2 \leq M_B^2 \leq 7.4\,\text{GeV}^2$ using the two criteria discussed above.

Within the above parameter regions, we show the variation of the hadron mass m_X with respect to the Borel parameter in the right panel of Fig. 1. We find that the mass sum rules are very stable in the Borel window $6.0\,\text{GeV}^2 \leq M_B^2 \leq 7.4\,\text{GeV}^2$, and the hadron mass is extracted as

$$m_{X, 0^+} = 5.58 \pm 0.14\,\text{GeV}\,, \quad (15)$$

where the error comes from the uncertainties of the bottom quark mass m_b, the condensates $\langle\bar{q}q\rangle, \langle\bar{s}s\rangle, \langle g_s^2 GG\rangle, \langle\bar{q}g_s\sigma\cdot$
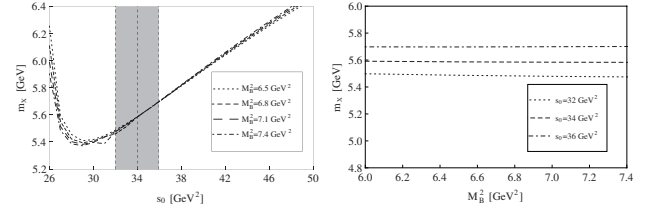

FIG. 1: Variations of the extracted hadron mass with respect to the continuum threshold s_0 (left) and the Borel parameter M_B^2 (right) for $J_3(x)$ with $J^P = 0^+$.
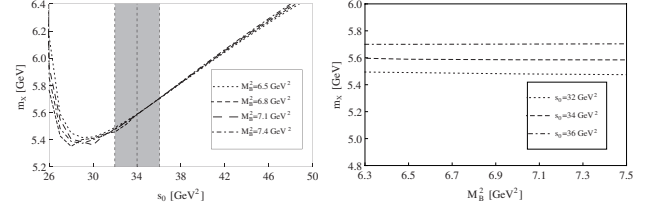

FIG. 2: Variations of the extracted hadron mass with respect to the continuum threshold s_0 (left) and the Borel parameter M_B^2 (right) for $J_{3\mu}(x)$ with $J^P = 1^+$.

$Gq\rangle, \langle\bar{s}g_s\sigma\cdot Gs\rangle$, the parameter M_0^2 in Eq. (11) and the continuum threshold s_0. This mass value is in good agreement with the observed mass of the $X(5568)$ state in Ref. [5], and implies a possible $J^P = 0^+$ $su\bar{b}\bar{d}$ tetraquark interpretation for this exotic state.

For the axial-vector current $J_{3\mu}(x)$ with $J^P = 1^+$, the OPE series has a similar behavior to that of the scalar current $J_3(x)$. After performing the QCD sum rule analyses, we find the working regions for the continuum threshold $32\,\text{GeV}^2 \leq s_0 \leq 36\,\text{GeV}^2$ and Borel parameter $6.3\,\text{GeV}^2 \leq M_B^2 \leq 7.4$ GeV^2. We show the hadron mass m_X as a function of s_0 and M_B in the above working regions in Fig. 2. These mass curves have similar behavior to those in Fig. 1, and the hadron mass is extracted as

$$m_{X, 1^+} = 5.59 \pm 0.15\,\text{GeV}\,. \quad (16)$$

This mass value is also in good agreement with the measured mass of the $X(5568)$ state in Ref. [5], and supports the axial-vector $su\bar{b}\bar{d}$ tetraquark interpretation for this meson.

For the other interpolating currents $J_1(x), J_2(x), J_4(x)$ with $J^P = 0^+$ and $J_{1\mu}(x), J_{2\mu}(x), J_{4\mu}(x)$ with $J^P = 1^+$ in Eqs. (1)-(2), we have also performed QCD sum rule analyses. We shall detailly discuss them in our future studies, but just note that the hadron masses extracted by using these currents are higher than the mass of the $X(5568)$. Especially, the scalar currents $J_1(x), J_2(x)$ and axial-vector currents $J_{1\mu}(x), J_{2\mu}(x)$ lead to hadron masses above 6 GeV, which may be due to that the color structures of these tetraquark currents are symmetric $[\mathbf{6_c}]_{su} \otimes [\bar{\mathbf{6}}_{\mathbf{c}}]_{\bar{b}\bar{d}}$, while those of $J_3(x)$ and $J_{3\mu}(x)$ used above are antisymmetric $[\bar{\mathbf{3}}_{\mathbf{c}}]_{su} \otimes [\mathbf{3_c}]_{\bar{b}\bar{d}}$, and the former color symmetric diquarks and antidiquarks probably have larger masses than the latter antisymmetric ones.

Decay patterns of the X(5568) state.—As discussed in Refs. [28, 29], the diquark-antidiquark tetraquark currents can be transformed to be the mesonic-mesonic currents through the Fierz transformation, from which we can study the decay patterns of the $X(5568)$:

1. The current $J_3(x)$ can be transformed into

$$J_3(x) \rightarrow \bar{b}_a s_a \bar{d}_b u_b \oplus \bar{b}_a \gamma_5 s_a \bar{d}_b \gamma_5 u_b \oplus \bar{b}_a \sigma_{\mu\nu} s_a \bar{d}_b \sigma^{\mu\nu} u_b$$
$$\oplus \ \bar{b}_a \gamma_\mu s_a \bar{d}_b \gamma^\mu u_b \oplus \bar{b}_a \gamma_\mu \gamma_5 s_a \bar{d}_b \gamma^\mu \gamma_5 u_b$$
$$\oplus \ \{s \leftrightarrow u\}. \tag{17}$$

   This suggests that if the $X(5568)$ is a $su\bar{b}\bar{d}$ tetraquark state of $J^P = 0^+$, its kinematically allowed decay channel would be only the S-wave $B_s^0 \pi^+$, which is just its observed channel [5]. Besides this, its neutral partner may also decay into $B_s^* \gamma$ through $\bar{q} \gamma_\mu q \rightarrow \gamma$.

2. The current $J_{3\mu}(x)$ can be transformed into

$$J_{3\mu}(x) \rightarrow \bar{b}_a s_a \bar{d}_b \gamma_\mu \gamma_5 u_b \oplus \bar{b}_a \gamma_5 s_a \bar{d}_b \gamma_\mu u_b$$
$$\oplus \ \bar{b}_a \gamma^\nu \gamma_5 s_a \bar{d}_b \sigma_{\mu\nu} u_b \oplus \bar{b}_a \gamma^\nu s_a \bar{d}_b \sigma_{\mu\nu} \gamma_5 u_b$$
$$\oplus \ \{s \leftrightarrow u\} \oplus \{\bar{b} \leftrightarrow \bar{d}\} \oplus \{s\bar{b} \leftrightarrow u\bar{d}\}. \tag{18}$$

   This suggests that if the $X(5568)$ is a $su\bar{b}\bar{d}$ tetraquark state of $J^P = 1^+$, its kinematically allowed decay channel would be only the S-wave $B_s^* \pi^+$. This channel was also suggested by the DØ Collaboration in the case that the low-energy photon was not detected [5]. Besides this, its neutral partner may decay into $B_s^0 \gamma$.

We notice that the radiative decay of the neutral partner of the $X(5568)$, into $B_s^0 \gamma$ and $B_s^* \gamma$, can be used to determine its spin-parity quantum numbers.

The prediction of the charmed partner state of the X(5568).—The charmed partner state of the $X(5568)$, with the fully open-flavor quark content $su\bar{c}\bar{d}$, can be searched for in many current experiments, such as the Belle, BESIII, LHCb, etc. We can simply replace the bottom quark b by the charm quark c in the interpolating currents $J_3(x)$ with $J^P = 0^+$ and $J_{3\mu}(x)$ with $J^P = 1^+$, and use them to investigate these charmed partner states employing the previously obtained formalism. Similar to $\underline{m_b}$, we use the "running mass" of the charm quark in the $\overline{\text{MS}}$ scheme, as shown in Eq. (11). If the $X(5568)$ is a $su\bar{b}\bar{d}$ tetraquark state of $J^P = 0^+$, its charmed partner state is predicted to have the mass

$$m_{X_c, 0^+} = 2.55 \pm 0.09 \,\text{GeV}, \tag{19}$$

and its possible decay patterns are $D_s \pi$ and DK, etc. While, if the $X(5568)$ is of $J^P = 1^+$, its charmed partner state has the mass

$$m_{X_c, 1^+} = 2.55 \pm 0.10 \,\text{GeV}, \tag{20}$$

and its possible decay patterns are $D_s^* \pi$ and $D^* K$, etc.

Conclusion.—In conclusion, we have investigated the recent reported $X(5568)$ resonance as a fully open-flavor $su\bar{b}\bar{d}$

tetraquark state with $J^P = 0^+/1^+$. In the framework of QCD sum rules, we use the scalar $J_3(x)$ and axial-vector $J_{3\mu}(x)$ tetraquark currents of the color anti-symmetric $[\bar{\mathbf{3}}_{\mathbf{c}}]_{su} \otimes [\mathbf{3}_{\mathbf{c}}]_{\bar{b}\bar{d}}$ to perform QCD sum rule analyses, and extract hadron masses to be 5.58 ± 0.14 GeV and 5.59 ± 0.15 GeV, respectively. These two values are both in good agreement with the experimental mass of the $X(5568)$. Hence, our results suggest that the $X(5568)$ can be interpreted as either a scalar or an axial-vector $su\bar{b}\bar{d}$ tetraquark state. We also investigate its possible decay patterns, and propose to observe the radiative decay of the neutral partner of the $X(5568)$ into $B_s^0 \gamma$ and $B_s^* \gamma$ to determine its spin-parity quantum numbers. Moreover, we predict the mass of the possible charmed partner state of the $X(5568)$, if it exists, to be around 2.55 GeV for both cases of $J^P = 0^+$ and 1^+.

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[1] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).
[2] X. Liu, Chin. Sci. Bull. **59**, 3815 (2014).
[3] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Phys. Rept. 639 (2016) 1-121, arXiv:1601.02092 [hep-ph].
[4] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **115**, 072001 (2015).
[5] V. M. Abazov *et al.* [D0 Collaboration], arXiv:1602.07588 [hep-ex].
[6] A. V. Evdokimov *et al.* [SELEX Collaboration], Phys. Rev. Lett. **93**, 242001 (2004).
[7] Y. R. Liu, S. L. Zhu, Y. B. Dai and C. Liu, Phys. Rev. D **70**, 094009 (2004).
[8] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).
[9] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **90**, 242001 (2003).
[10] S. S. Agaev, K. Azizi and H. Sundu, arXiv:1602.08642 [hep-ph].
[11] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D **71**, 014028 (2005).
[12] K. Terasaki, Phys. Rev. D **68**, 011501 (2003).
[13] Z. G. Wang and S. L. Wan, Nucl. Phys. A **778**, 22 (2006).
[14] J. Vijande, F. Fernandez and A. Valcarce, Phys. Rev. D **73**, 034002 (2006) [Phys. Rev. D **74**, 059903 (2006)].
[15] R. L. Jaffe, Phys. Rept. **409**, 1 (2005).
[16] W. Chen and S. L. Zhu, Phys. Rev. D **83**, 034010 (2011).
[17] R. T. Kleiv, T. G. Steele, A. Zhang and I. Blokland, Phys. Rev.

D **87**, 125018 (2013).

[18] W. Chen, T. G. Steele and S. L. Zhu, Phys. Rev. D **89**, 054037 (2014).

[19] M. L. Du, W. Chen, X. L. Chen and S. L. Zhu, Phys. Rev. D **87**, 014003 (2013).

[20] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979).

[21] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127**, 1 (1985).

[22] P. Colangelo and A. Khodjamirian, *"At the Frontier of Particle Physics/Handbook of QCD"* (World Scientific, Singapore, 2001), Volume 3, 1495.

[23] M. Nielsen, F. S. Navarra and S. H. Lee, Phys. Rept. **497**, 41 (2010)

[24] S. Narison, "QCD as a theory of hadrons from partons to confinement," Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 17 (2002) 1 [hep-ph/0205006]

[25] S. Narison, Phys. Lett. B **707**, 259 (2012).

[26] S. Narison, Phys. Lett. B **693**, 559 (2010) [Phys. Lett. **705**, 544 (2011)].

[27] J. H. Kuhn, M. Steinhauser and C. Sturm, Nucl. Phys. B **778**, 192 (2007).

[28] H. X. Chen, E. L. Cui, W. Chen, T. G. Steele, X. Liu and S. L. Zhu, Phys. Rev. D **91**, 094022 (2015).

[29] H. X. Chen, A. Hosaka and S. L. Zhu, Phys. Rev. D **74**, 054001 (2006).