Analysis of the strong decay $X(5568) \to B_s^0\pi^+$ with QCD sum rules

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Abstract

In this article, we take the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state, study the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension-6 and including both the connected and disconnected Feynman diagrams, then calculate the partial decay width of the strong decay $X(5568) \to B_s^0\pi^+$ and obtain the value $\Gamma_X = (20.5 \pm 8.1)$ MeV, which is consistent with the experimental data $\Gamma_X = \left(21.9 \pm 6.4^{+5.0}_{-2.5}\right)$ MeV from the D0 collaboration.

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1 Introduction

Recently, the D0 collaboration observed a narrow structure, $X(5568)$, in the decay $X(5568) \to B_s^0\pi^\pm$ with significance of 5.1σ [1]. The measured mass and width are $m_X = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right)$ MeV and $\Gamma_X = \left(21.9 \pm 6.4^{+5.0}_{-2.5}\right)$ MeV, respectively. The D0 collaboration fitted the $B_s^0\pi^\pm$ systems with the Breit-Wigner parameters in relative S-wave, the favored quantum numbers are $J^P = 0^+$. However, the quantum numbers $J^P = 1^+$ cannot be excluded according to decays $X(5568) \to B_s^*\pi^+ \to B_s^0\pi^+\gamma$, where the low-energy photon is not detected. There have been several possible assignments, such as the scalar-diquark-scalar-antidiquark type tetraquark state [2, 3, 4, 5, 6, 7], axialvector-diquark-axialvector-antidiquark type tetraquark state [3, 8, 9], $B^{(*)}\bar{K}$ hadronic molecule state [10], threshold effect [11].

The calculations based on the QCD sum rules indicate that both the scalar-diquark-scalar-antidiquark type and axialvector-diquark-axialvector-antidiquark type interpolating currents can give satisfactory mass m_X to reproduce the experimental data [2, 3, 4, 8]. In Ref.[9], Agaev, Azizi and Sundu choose the axialvector-diquark-axialvector-antidiquark type interpolating current, calculate the hadronic coupling constant $g_{XB_s\pi}$ with the light-cone QCD sum rules in conjunction with the soft-π approximation and other approximations, and obtain the partial decay width for the process $X(5568) \to B_s^0\pi^+$. In Ref.[7], Dias et al choose the scalar-diquark-scalar-antidiquark type interpolating current, calculate the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules in the soft-π limit by taking into account only the connected Feynman diagrams in the leading order approximation, and obtain the partial decay width for the decay $X(5568) \to B_s^0\pi^+$. In previous work [2], we choose the scalar-diquark-scalar-antidiquark type interpolating current to study the mass of the $X(5568)$ with the QCD sum rules. In this article, we extend our previous work to study the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension-6 and including both the connected and disconnected Feynman diagrams, then calculate the partial decay width of the strong decay $X(5568) \to B_s^0\pi^+$.

The article is arranged as follows: we derive the QCD sum rule for the hadronic coupling constant $g_{XB_s\pi}$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

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2 QCD sum rule for the hadronic coupling constant $g_{XB_s\pi}$

We can study the strong decay $X(5568) \to B_s^0\pi^+$ with the three-point correlation function $\Pi(p,q)$,

$$\Pi(p,q) = i^2\int d^4x d^4y e^{ip\cdot x}e^{iq\cdot y}\langle 0|T\left\{J_{B_s}(x)J_\pi(y)J_X(0)\right\}|0\rangle\,, \tag{1}$$

where the currents

$$\begin{aligned}
J_{B_s}(x) &= \bar{s}(x)i\gamma_5 b(x)\,,\\
J_\pi(y) &= \bar{u}(y)i\gamma_5 d(y)\,,\\
J_X(0) &= \epsilon^{ijk}\epsilon^{imn}u^j(0)C\gamma_5 s^k(0)\bar{d}^m(0)\gamma_5 C\bar{b}^n(0)\,,
\end{aligned} \tag{2}$$

interpolate the mesons B_s, π and $X(5568)$, respectively, the i, j, k, m, n are color indexes, the C is the charge conjugation matrix. In Ref.[7], the axialvector current is used to interpolate the π meson.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{B_s}(x)$, $J_\pi(y)$ and $J_X(0)$ into the three-point correlation function $\Pi(p,q)$ and isolate the ground state contributions to obtain the following result,

$$\begin{aligned}
\Pi(p,q) &= \frac{f_\pi m_\pi^2 f_{B_s}m_{B_s}^2\lambda_X g_{XB_s\pi}}{(m_u+m_d)(m_b+m_s)}\frac{1}{(m_X^2-p'^2)\left(m_{B_s}^2-p^2\right)(m_\pi^2-q^2)}\\
&+\frac{1}{(m_X^2-p'^2)\left(m_{B_s}^2-p^2\right)}\int_{s_\pi^0}^\infty dt\frac{\rho_{X\pi}(p^2,t,p'^2)}{t-q^2}\\
&+\frac{1}{(m_X^2-p'^2)(m_\pi^2-q^2)}\int_{s_{B_s}^0}^\infty dt\frac{\rho_{XB_s}(t,q^2,p'^2)}{t-p^2}+\cdots\,,
\end{aligned} \tag{3}$$

where $p'=p+q$, the f_{B_s}, f_π and λ_X are the decay constants of the mesons B_s, π and $X(5568)$, respectively, the $g_{XB_s\pi}$ is the hadronic coupling constant.

In the following, we write down the definitions,

$$\begin{aligned}
\langle 0|J_X(0)|X(p')\rangle &= \lambda_X\,,\\
\langle 0|J_{B_s}(0)|B_s(p)\rangle &= \frac{f_{B_s}m_{B_s}^2}{m_b+m_s}\,,\\
\langle 0|J_\pi(0)|\pi(q)\rangle &= \frac{f_\pi m_\pi^2}{m_u+m_d}\,,
\end{aligned} \tag{4}$$

$$\langle B_s(p)\pi(q)|X(p')\rangle = i g_{XB_s\pi}\,. \tag{5}$$

The two unknown functions $\rho_{X\pi}(p^2,t,p'^2)$ and $\rho_{XB_s}(t,q^2,p'^2)$ have complex dependence on the transitions between the ground state $X(5568)$ and the excited states of the π and B_s mesons, respectively. We introduce the parameters $C_{X\pi}$ and C_{XB_s} to parameterize the net effects,

$$\begin{aligned}
C_{X\pi} &= \int_{s_\pi^0}^\infty dt\frac{\rho_{X\pi}(p^2,t,p'^2)}{t-q^2}\,,\\
C_{XB_s} &= \int_{s_{B_s}^0}^\infty dt\frac{\rho_{XB_s}(t,q^2,p'^2)}{t-p^2}\,,
\end{aligned} \tag{6}$$

and rewrite the correlation function $\Pi(p,q)$ into the following form,

$$\begin{aligned}
\Pi(p,q) &= \frac{f_\pi m_\pi^2 f_{B_s}m_{B_s}^2\lambda_X g_{XB_s\pi}}{(m_u+m_d)(m_b+m_s)}\frac{1}{(m_X^2-p'^2)\left(m_{B_s}^2-p^2\right)(m_\pi^2-q^2)}\\
&+\frac{C_{X\pi}}{(m_X^2-p'^2)\left(m_{B_s}^2-p^2\right)}+\frac{C_{XB_s}}{(m_X^2-p'^2)(m_\pi^2-q^2)}+\cdots\,.
\end{aligned} \tag{7}$$

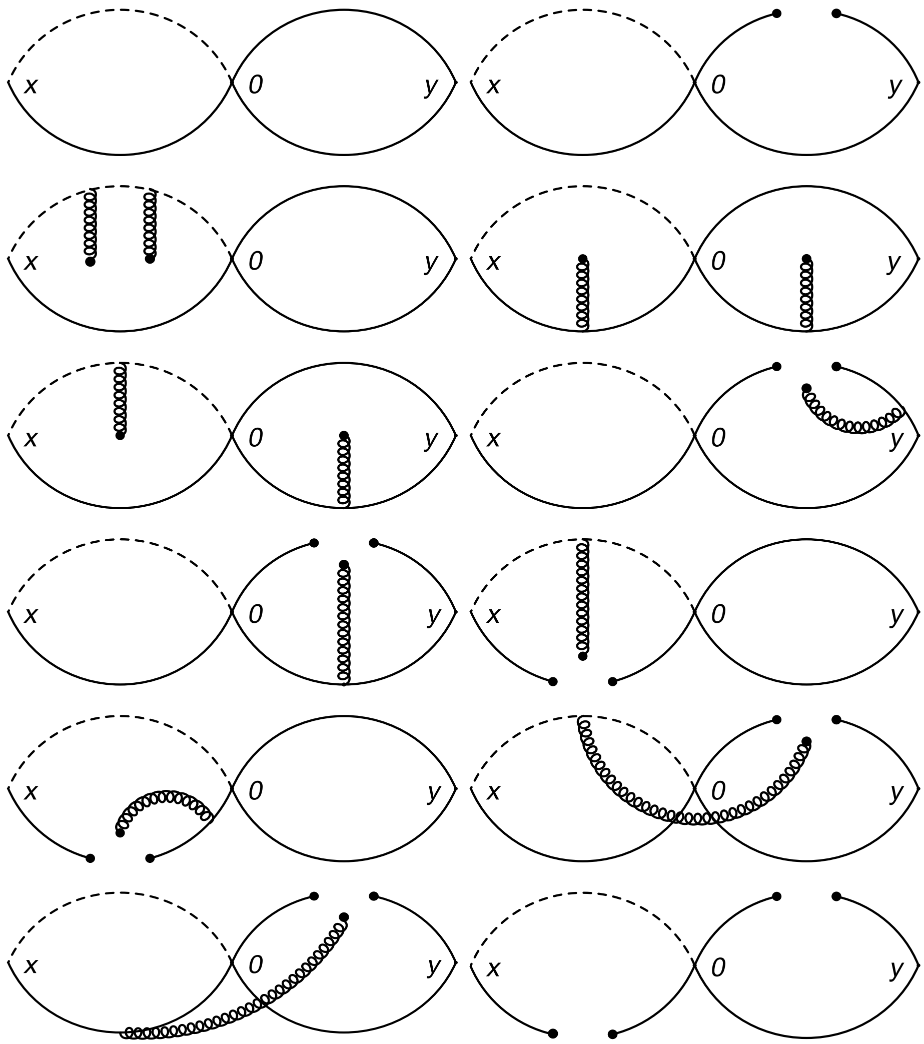

Figure 1: The Feynman diagrams calculated in this article, where the solid lines and dashed lines denote the light quarks and heavy quarks, respectively, the waved lines denote the gluons. Other diagrams obtained by interchanging of the light quark lines are implied.

We set $p'^2 = p^2$ and take the double Borel transform with respect to the variable $P^2 = -p^2$ and $Q^2 = -q^2$ respectively to obtain the QCD sum rule at the left side (LS),

$$\begin{aligned}
\mathrm{LS} &= \frac{f_\pi m_\pi^2 f_{B_s} m_{B_s}^2 \lambda_X g_{XB_s\pi}}{(m_u+m_d)(m_b+m_s)} \frac{1}{m_X^2 - m_{B_s}^2} \left\{ \exp\left(-\frac{m_{B_s}^2}{M_1^2}\right) - \exp\left(-\frac{m_X^2}{M_1^2}\right) \right\} \exp\left(-\frac{m_\pi^2}{M_2^2}\right) \\
&\quad + C_{XB_s} \exp\left(-\frac{m_X^2}{M_1^2}\right) \exp\left(-\frac{m_\pi^2}{M_2^2}\right) . \qquad (8)
\end{aligned}$$

In calculations, we neglect the dependencies of the $C_{X\pi}$ and C_{XB_s} on the variables p^2, p'^2, q^2 therefore the dependencies of the $C_{X\pi}$ and C_{XB_s} on the variables M_1^2 and M_2^2, take the $C_{X\pi}$ and C_{XB_s} as free parameters, and choose the suitable values to eliminate the contaminations so as to obtain the stable sum rules with the variations of the Borel parameters [12, 13].

Now we carry out the operator product expansion at the large Euclidean space-time region $-p^2 \to \infty$ and $-q^2 \to \infty$, take into account the vacuum condensates up to dimension 6 and neglect the contribution of the three-gluon condensate, as the three-gluon condensate is the vacuum expectation of the operator of the order $\mathcal{O}(\alpha_s^{3/2})$. In other words, we calculate the Feynman diagrams shown in Fig.1. For example, the first diagram is calculated in the following ways,

$$\begin{aligned}
\Pi(p,q) &= -\frac{6}{(2\pi)^8}\int d^4k d^4l \frac{\mathrm{Tr}\left\{\gamma_5(\not{k}+m_s)\gamma_5(\not{k}+\not{p}+m_b)\gamma_5(\not{l}+\not{q})\gamma_5\not{l}\right\}}{k^2\left[(k+p)^2-m_b^2\right]^2(l+q)^2l^2} \\
&= -\frac{6}{(2\pi)^8}\frac{(-2\pi i)^2}{2\pi i}\int_{m_b^2}^{\infty}ds\frac{1}{s-p^2}\int d^4k\,\delta\left[k^2\right]\delta\left[(k+p)^2-m_b^2\right]\frac{(-2\pi i)^2}{2\pi i}\int_0^{\infty}du\frac{1}{u-q^2} \\
&\quad \int d^4l\,\delta\left[l^2\right]\delta\left[(l+q)^2\right]\mathrm{Tr}\left\{\gamma_5(\not{k}+m_s)\gamma_5(\not{k}+\not{p}+m_b)\gamma_5(\not{l}+\not{q})\gamma_5\not{l}\right\} \\
&= \frac{3}{128\pi^4}\int_{m_b^2}^{\infty}ds\frac{1}{s-p^2}\frac{(s-m_b^2)^2}{s}\int_0^{\infty}du\frac{u}{u-q^2} \\
&\quad +\frac{3m_sm_b}{64\pi^4}\int_{m_b^2}^{\infty}ds\frac{1}{s-p^2}\frac{s-m_b^2}{s}\int_0^{\infty}du\frac{u}{u-q^2}\,. \qquad (9)
\end{aligned}$$

The operator product expansion converges for large $-p^2$ and $-q^2$, it is odd to take the limit $q^2\to 0$.

Then we set $p'^2=p^2$, take the quark-hadron duality below the continuum thresholds, and perform the double Borel transform with respect to the variables $P^2=-p^2$ and $Q^2=-q^2$ respectively to obtain the perturbative term,

$$\begin{aligned}
B_{M_1^2,M_2^2}\Pi(p,q) &= \frac{3}{128\pi^2}\int_{m_b^2}^{s_0}ds\int_0^{u_0}du\frac{(s-m_b^2)^2}{s}u\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad +\frac{3m_sm_b}{64\pi^2}\int_{m_b^2}^{s_0}ds\int_0^{u_0}du\frac{s-m_b^2}{s}u\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\,, \qquad (10)
\end{aligned}$$

where the s_0 and u_0 are the continuum threshold parameters for the $X(5568)$ and π, respectively.

Other Feynman diagrams are calculated in analogous ways, finally we obtain the QCD sum rules at the right side (RS),

$$\begin{aligned}
\mathrm{RS} &= \frac{3}{128\pi^2}\int_{m_b^2}^{s_0}ds\int_0^{u_0}du\frac{(s-m_b^2)^2}{s}u\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad +\frac{3m_sm_b}{64\pi^2}\int_{m_b^2}^{s_0}ds\int_0^{u_0}du\frac{s-m_b^2}{s}u\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad +\frac{1}{192\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{m_b^2}^{s_0}ds\int_0^{u_0}du\left(2-\frac{m_b^2}{s}\right)\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad -\frac{m_b\langle\bar{s}s\rangle}{16\pi^2}\int_0^{u_0}du\,u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad -\frac{m_s\langle\bar{s}s\rangle}{32\pi^2}\left(1+\frac{m_b^2}{M_1^2}\right)\int_0^{u_0}du\,u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad +\frac{1}{192\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_0^{u_0}du\,u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right) \\
&\quad +\frac{1}{128\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{m_b^2}^{s_0}ds\frac{(s-m_b^2)^2}{s}\exp\left(-\frac{s}{M_1^2}\right) \\
&\quad -\frac{m_b\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^2}\int_0^{u_0}du\left(1+\frac{u}{M_1^2}-\frac{um_b^2}{2M_1^4}-\frac{um_sm_b^3}{6M_1^6}\right)\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right)\,. \qquad (11)
\end{aligned}$$

The terms $\langle\bar{q}q\rangle\langle\bar{s}s\rangle$ disappear after performing the double Borel transform, the last Feynman diagram in Fig.1 have no contribution.

In Refs.[13, 14], the width of the $Z_c(4200)$ is studied with the three-point QCD sum rules by including both the connected and disconnected Feynman diagrams, which is contrary to Ref.[15], where only the connected Feynman diagrams are taken into account to study the width of the $Z_c(3900)$. In this article, the contributions come from the connected diagrams can be written as RS_c,

$$\begin{aligned}\mathrm{RS}_c &= \frac{1}{192\pi^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{m_b^2}^{s_0} ds\int_0^{u_0} du\left(2-\frac{m_b^2}{s}\right)\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\\ &\quad -\frac{m_b\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^2}\int_0^{u_0} du\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right), \end{aligned} \tag{12}$$

which is too small to account for the experimental data [1].

Finally, we obtain the QCD sum rule,

$$\mathrm{LS} = \mathrm{RS}. \tag{13}$$

There appear some energy scale dependence at the hadron side (or LS) of the QCD sum rule according to the factors m_u+m_d and m_b+m_s, we can eliminate the energy scale dependence by using the currents $\widehat{J}_{B_s}(x)$ and $\widehat{J}_\pi(y)$,

$$\begin{aligned}\widehat{J}_{B_s}(x) &= (m_b+m_s)\,\bar{s}(x)i\gamma_5 b(x)\,,\\ \widehat{J}_\pi(y) &= (m_u+m_d)\,\bar{u}(y)i\gamma_5 d(y)\,,\end{aligned} \tag{14}$$

then

$$\begin{aligned}\langle 0|\widehat{J}_{B_s}(0)|B_s(p)\rangle &= f_{B_s}m_{B_s}^2\,,\\ \langle 0|\widehat{J}_\pi(0)|\pi(q)\rangle &= f_\pi m_\pi^2\,,\end{aligned} \tag{15}$$

and

$$\begin{aligned}C_{X\pi} &\to C_{X\pi}\,(m_b+m_s)\,(m_u+m_d)\,,\\ C_{XB_s} &\to C_{XB_s}\,(m_b+m_s)\,(m_u+m_d)\,,\end{aligned} \tag{16}$$

the resulting QCD sum rule at the right side also acquires a factor $(m_b+m_s)\,(m_u+m_d)$, a equivalent QCD sum rule is obtained, the predicted hadronic coupling constant $g_{XB_s\pi}$ is not changed.

We can also study the strong decay $X(5568)\to B_s^0\pi^+$ with the three-point correlation function $\Pi_{\mu\nu}(p,q)$,

$$\Pi_{\mu\nu}(p,q) = i^2\int d^4x d^4y e^{ip\cdot x}e^{iq\cdot y}\langle 0|T\left\{\eta_\mu^{\bar{s}b}(x)\eta_\nu^{\bar{u}d}(y)J_X(0)\right\}|0\rangle\,, \tag{17}$$

where the currents

$$\begin{aligned}\eta_\mu^{\bar{s}b}(x) &= \bar{s}(x)\gamma_\mu\gamma_5 b(x)\,,\\ \eta_\nu^{\bar{u}d}(y) &= \bar{u}(y)\gamma_\nu\gamma_5 d(y)\,,\end{aligned} \tag{18}$$

interpolate the mesons B_s and π, respectively. At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $\eta_\mu^{\bar{s}b}(x)$ and $\eta_\nu^{\bar{u}d}(y)$ into the three-point correlation function $\Pi_{\mu\nu}(p,q)$ and isolate the ground state contributions to obtain the following result,

$$
\begin{aligned}
\Pi_{\mu\nu}(p,q) &= \frac{f_{B_s} f_\pi \lambda_X g_{XB_s\pi}}{(m_X^2 - p'^2)\left(m_{B_s}^2 - p^2\right)(m_\pi^2 - q^2)} \left(-p_\mu q_\nu\right) \\
&+ \frac{f_{B_{s1}} m_{B_{s1}} f_\pi \lambda_X g_{XB_{s1}\pi}}{(m_X^2 - p'^2)\left(m_{B_{s1}}^2 - p^2\right)(m_\pi^2 - q^2)} \left(-q_\mu q_\nu + \frac{p\cdot q}{p^2} p_\mu q_\nu\right) \\
&+ \frac{f_{B_{s1}} m_{B_{s1}} f_{a_1} m_{a_1} \lambda_X g_{XB_{s1}a_1}}{(m_X^2 - p'^2)\left(m_{B_{s1}}^2 - p^2\right)\left(m_{a_1}^2 - q^2\right)} \left(g_{\mu\nu} - \frac{1}{p^2} p_\mu p_\nu - \frac{1}{q^2} q_\mu q_\nu + \frac{p\cdot q}{p^2 q^2} p_\mu q_\nu\right) + \cdots ,
\end{aligned}
\tag{19}
$$

where $p' = p + q$, the $f_{B_{s1}}$, f_{B_s}, f_{a_1} and f_π are the decay constants of the mesons $B_{s1}(5830)$, B_s, $a_1(1260)$ and π, respectively, the $g_{XB_{s1}\pi}$ and $g_{XB_{s1}a_1}$ are the hadronic coupling constants.

In the following, we write down the definitions,

$$
\begin{aligned}
\langle 0|\eta_\mu^{\bar{s}b}(0)|B_s(p)\rangle &= i f_{B_s} p_\mu \ , \\
\langle 0|\eta_\nu^{\bar{u}d}(0)|\pi(q)\rangle &= i f_\pi q_\nu \ , \\
\langle 0|\eta_\mu^{\bar{s}b}(0)|B_{s1}(p)\rangle &= f_{B_{s1}} m_{B_{s1}} \varepsilon_\mu \ , \\
\langle 0|\eta_\nu^{\bar{u}d}(0)|a_1(q)\rangle &= f_{a_1} m_{a_1} \epsilon_\nu \ , \qquad (20) \\
\langle B_{s1}(p)\pi(q)|X(p')\rangle &= \varepsilon^* \cdot q \, g_{XB_{s1}\pi} \ , \\
\langle B_{s1}(p)a_1(q)|X(p')\rangle &= i\varepsilon^* \cdot \epsilon^* \, g_{XB_{s1}a_1} \ , \qquad (21)
\end{aligned}
$$

where the ε_μ and ϵ_ν are polarization vectors of the axialvector mesons $B_{s1}(5830)$ and $a_1(1260)$, respectively. From the values $m_X = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right)$ MeV [1], $m_{B_{s1}} = (5828.40 \pm 0.04 \pm 0.41)$ MeV, $m_{B_s} = (5366.7 \pm 0.4)$ MeV [16], we can obtain $m_{B_{s1}} - m_{B_s} \approx 462\,\text{MeV}$ and $m_{B_{s1}} - m_X \approx 261\,\text{MeV}$. If we take the interpolating currents $\eta_\mu^{\bar{s}b}(x)$ and $\eta_\nu^{\bar{u}d}(y)$, there are contaminations from the axialvector mesons $B_{s1}(5830)$ and $a_1(1260)$. We should multiply both sides of Eq.(19) by $p^\mu q^\nu$ to eliminate the contaminations of the axialvector mesons $B_{s1}(5830)$ and $a_1(1260)$,

$$
p^\mu q^\nu \Pi_{\mu\nu}(p,q) = \frac{f_{B_s} f_\pi \lambda_X g_{XB_s\pi}}{(m_X^2 - p'^2)\left(m_{B_s}^2 - p^2\right)(m_\pi^2 - q^2)} \left(-p^2 q^2\right) + \cdots , \tag{22}
$$

which corresponds to taking the pseudoscalar currents $\widehat{J}_{B_s}(x)$ and $\widehat{J}_\pi(y)$ according to the following identities,

$$
\begin{aligned}
\partial^\mu \eta_\mu^{\bar{s}b}(x) &= (m_b + m_s)\, \bar{s}(x) i\gamma_5 b(x) = \widehat{J}_{B_s}(x) \, , \\
\partial^\nu \eta_\nu^{\bar{u}d}(y) &= (m_u + m_d)\, \bar{u}(y) i\gamma_5 d(y) = \widehat{J}_\pi(y) \, .
\end{aligned}
\tag{23}
$$

The axialvector currents $\eta_\mu^{\bar{s}b}(x)$ and $\eta_\nu^{\bar{u}d}(y)$ can also be chosen to study the strong decay $X(5568) \to B_s^0\pi^+$.

We also expect to study the strong decay $X(5568) \to B_s^0\pi^+$ with the light-cone QCD sum rules using the two-point correlation function $\overline{\Pi}(p,q)$,

$$
\overline{\Pi}(p,q) = i\int d^4x e^{ip\cdot x} \langle \pi(q)| T\left\{J_{B_s}(x) J_X(0)\right\}|0\rangle \, , \tag{24}
$$

where the $\langle \pi(q)|$ is an external π state.

At the QCD side, we obtain the following result after performing the wick's contraction,

$$
\overline{\Pi}(p,q) = i\int d^4x e^{ip\cdot x} \langle \pi(q)| \epsilon^{ijk}\epsilon^{imn} u_j^T(0) C\gamma_5 S_s^{kl}(-x) i\gamma_5 S_b^{ln}(x)\gamma_5 C\bar{d}_m^T(0)|0\rangle \, , \tag{25}
$$

where the $S_s^{kl}(-x)$ and $S_b^{ln}(x)$ are the full s and b quark propagators, respectively. The u and $\bar{d}$ quarks stay at the same point $x = 0$, the light-cone distribution amplitudes of the π meson are almost useless, the integrals over the π meson's light-cone distribution amplitudes reduce to overall normalization factors. In the light-cone QCD sum rules, such a situation is possible only in the soft pion limit $q \to 0$, and the light-cone expansion reduces to the short-distance expansion [17]. In Ref.[9], Agaev, Azizi and Sundu take the soft pion limit $q \to 0$, and choose the $C\gamma_\mu \otimes \gamma^\mu C$ type current to interpolate the $X(5568)$, and use the light-cone QCD sum rules to study the strong decay $X(5568) \to B_s^0\pi^+$. The light-cone QCD sum rules are reasonable only in the soft pion approximation.

3 Numerical results and discussions

The hadronic parameters are taken as $m_X = 5.5678\,\rm{GeV}$ [1], $\lambda_X = 6.7\times 10^{-3}\,\rm{GeV}^5$, $\sqrt{s_0} = (6.1\pm 0.1)\,\rm{GeV}$ [2], $m_\pi = 0.13957\,\rm{GeV}$, $m_{B_s} = 5.3667\,\rm{GeV}$ [16], $f_\pi = 0.130\,\rm{GeV}$, $\sqrt{u_0} = (0.85\pm0.05)\,\rm{GeV}$ [18], $f_{B_s} = 0.231\,\rm{GeV}$ [19], $f_\pi m_\pi^2/(m_u+m_d) = -2\langle\bar{q}q\rangle/f_\pi$ from the Gell-Mann-Oakes-Renner relation, and $M_2^2 = (0.8-1.2)\,\rm{GeV}^2$ from the QCD sum rules [18]. At the QCD side, the vacuum condensates are taken to be standard values, $\langle\bar{q}q\rangle = -(0.24\pm 0.01\,\rm{GeV})^3$, $\langle\bar{s}s\rangle = (0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8\pm0.1)\,\rm{GeV}^2$ and $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\rm{GeV})^4$ at the energy scale $\mu = 1\,\rm{GeV}$ [20, 21]. The quark condensates and mixed quark condensates evolve with the renormalization group equation, $\langle\bar{q}q\rangle(\mu) = \langle\bar{q}q\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{4}{9}}$ and $\langle\bar{q}g_s\sigma Gq\rangle(\mu) = \langle\bar{q}g_s\sigma Gq\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{27}}$, where $q = u, d, s$.

In the article, we take the $\overline{MS}$ masses $m_b(m_b) = (4.18\pm0.03)\,\rm{GeV}$ and $m_s(\mu = 2\,\rm{GeV}) = (0.095\pm0.005)\,\rm{GeV}$ from the Particle Data Group [16], and take into account the energy-scale dependence of the $\overline{MS}$ masses from the renormalization group equation,

$$
\begin{aligned}
m_b(\mu) &= m_b(m_b)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{23}}\, , \\
m_s(\mu) &= m_s(2\rm{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\rm{GeV})}\right]^{\frac{4}{9}}\, , \\
\alpha_s(\mu) &= \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0^2}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right]\, , \qquad (26)
\end{aligned}
$$

where $t = \log\frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 213\,\rm{MeV}$, $296\,\rm{MeV}$ and $339\,\rm{MeV}$ for the flavors $n_f = 5$, 4 and 3, respectively [16]. Furthermore, we set the u and d quark masses to be zero. In the heavy quark limit, the b-quark can be taken as a static potential well, and unchanged in the decay $X(5568) \to B_s^0\pi^+$. In this article, we take the typical energy scale $\mu = m_b$.

The unknown parameter is chosen as $C_{XB_s} = -0.00059\,\rm{GeV}^8$. There appears a platform in the region $M_1^2 = (4.5-5.5)\,\rm{GeV}^2$. Now we take into account the uncertainties of the input parameters and obtain the value of the hadronic coupling constant $g_{XB_s\pi}$, which is shown explicitly in Fig.2,

$$
g_{XB_s\pi} = (10.6\pm2.1)\ \rm{GeV}\, . \qquad (27)
$$

Now we obtain the partial decay width,

$$
\begin{aligned}
\Gamma\left(X(5568)\to B_s^0\pi^+\right) &= \frac{g_{XB_s\pi}^2}{16\pi M_X^3}\sqrt{\left[m_X^2-(m_{B_s}+m_\pi)^2\right]\left[m_X^2-(m_{B_s}-m_\pi)^2\right]} \\
&= (20.5\pm8.1)\,\rm{MeV}\, . \qquad (28)
\end{aligned}
$$

The decays $X(5568) \to B^+\bar{K}^0$ are kinematically forbidden, so the width Γ_X can be saturated by the partial decay width $\Gamma\left(X(5568)\to B_s^0\pi^+\right)$, which is consistent with the experimental value

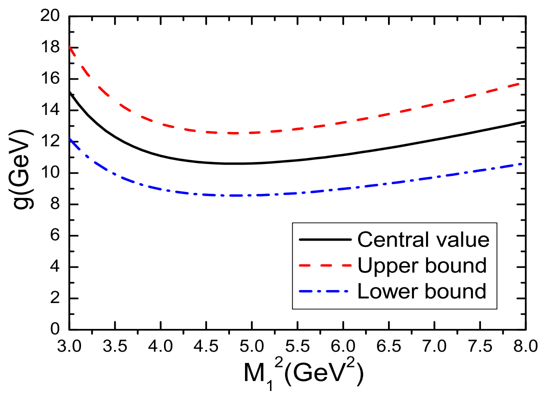

Figure 2: The hadronic coupling constant $g_{XB_s\pi}$ with variation of the Borel parameter M_1^2.

$\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5}\,\text{MeV}$ from the D0 collaboration [1]. The present work favors assigning the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state.

In the following, we perform Fierz re-arrangement to the current J_X both in the color and Dirac-spinor spaces to obtain the result,

$$\begin{aligned} J_X &= \frac{1}{4}\Big\{ -\bar{b}s\,\bar{d}u + \bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u - \bar{b}\gamma^\mu s\,\bar{d}\gamma_\mu u - \bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u + \frac{1}{2}\bar{b}\sigma_{\mu\nu}s\,\bar{d}\sigma^{\mu\nu}u \\ &\quad +\bar{b}u\,\bar{d}s - \bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s + \bar{b}\gamma^\mu u\,\bar{d}\gamma_\mu c + \bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s - \frac{1}{2}\bar{b}\sigma_{\mu\nu}u\,\bar{d}\sigma^{\mu\nu}s \Big\}\,, \end{aligned} \tag{29}$$

the components $\bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u$ and $\bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u$ couple potentially to the meson pair $B_s\pi^+$, while the components $\bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s$ and $\bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s$ couple potentially to the meson pair $B^+\bar{K}^0$. The strong decays

$$X(5568) \rightarrow B_s\pi^+\,, \tag{30}$$

are Okubo-Zweig-Iizuka super-allowed, while the decays

$$X(5568) \rightarrow B^+\bar{K}^0\,, \tag{31}$$

are kinematically forbidden, which is consistent with the observation of the D0 collaboration [1]. In previous works, we observed that the $C\gamma_5 \otimes \gamma_5 C$ type hidden-charm tetraquark states have slight smaller masses than that of the $C\gamma_\mu \otimes \gamma^\mu C$ type hidden-charm tetraquark states, the predicted lowest masses are $m_{C\gamma_5\otimes\gamma_5 C} = \left(3.82^{+0.08}_{-0.08}\right)$ GeV and $m_{C\gamma_\mu\otimes\gamma^\mu C} = \left(3.85^{+0.15}_{-0.09}\right)$ GeV [22]. We expect that a $C\gamma_\mu \otimes \gamma^\mu C$ type current can also reproduce the experimental value $m_X = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right)$ MeV approximately [3, 8].

Now we construct the current η_X and perform Fierz re-arrangement both in the color and Dirac-spinor spaces to obtain the following result,

$$\begin{aligned} \eta_X &= \epsilon^{ijk}\epsilon^{imn}u^j C\gamma_\mu s^k \bar{d}^m\gamma^\mu C\bar{b}^n\,, \\ &= \bar{b}s\,\bar{d}u + \bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u + \frac{1}{2}\bar{b}\gamma_\mu s\,\bar{d}\gamma^\mu u - \frac{1}{2}\bar{b}\gamma_\mu\gamma_5 s\,\bar{d}\gamma^\mu\gamma_5 u \\ &\quad +\bar{b}u\,\bar{d}s + \bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s + \frac{1}{2}\bar{b}\gamma_\mu u\,\bar{d}\gamma^\mu s - \frac{1}{2}\bar{b}\gamma_\mu\gamma_5 u\,\bar{d}\gamma^\mu\gamma_5 s\,, \end{aligned} \tag{32}$$

the components $\bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u$ and $\bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u$ couple potentially to the meson pair $B_s\pi^+$, while the components $\bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s$ and $\bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s$ couple potentially to the meson pair $B^+\bar{K}^0$, which is analogous to the current J_X. It is also sensible to assign the $X(5568)$ to be an axialvector-diquark-axialvector-antidiquark type tetraquark state or the $X(5568)$ has some axialvector-diquark-axialvector-antidiquark type tetraquark components.

The $C\otimes C$ type current $\widetilde{J}_X$ and $C\gamma_\mu\gamma_5\otimes\gamma_5\gamma^\mu C$ type current $\widetilde{\eta}_X$ are expected to couple potentially to the scalar tetraquark with much larger masses,

$$\begin{aligned}
\widetilde{J}_X &= \epsilon^{ijk}\epsilon^{imn}u^j C s^k \bar{d}^m C\bar{b}^n\,, \\
\widetilde{\eta}_X &= \epsilon^{ijk}\epsilon^{imn}u^j C\gamma_\mu\gamma_5 s^k \bar{d}^m \gamma_5\gamma^\mu C\bar{b}^n\,,
\end{aligned} \tag{33}$$

as the favored configurations are the scalar diquarks ($C\gamma_5$-type) and axialvector diquarks ($C\gamma_\mu$-type) from the QCD sum rules [23, 24].

4 Conclusion

In this article, we take the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state, study the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules, then calculate the partial decay width of the strong decay $X(5568)\to B_s^0\pi^+$ and obtain the value $\Gamma_X=(20.5\pm8.1)$ MeV, which is consistent with the experimental data $\Gamma_X=\left(21.9\pm6.4^{+5.0}_{-2.5}\right)$ MeV from the D0 collaboration. In calculation, we carry out the operator product expansion up to the vacuum condensates of dimension-6, and take into account both the connected and disconnected Feynman diagrams. The present prediction favors assigning the $X(5568)$ to be the diquark-antidiquark type tetraquark state with $J^P=0^+$. However, the quantum numbers $J^P=1^+$ cannot be excluded according to decays $X(5568)\to B_s^*\pi^+\to B_s^0\pi^+\gamma$, where the low-energy photon is not detected.

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