

Primordial Magnetic Field Via Weibel Instability In The Quark Gluon Plasma Phase

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Abstract

The origin of the observed large scale magnetic fields in the Universe is a mystery. The seed of these magnetic fields has been attributed to physical process in the early universe. In this work we provide a mechanism for the generation of a primordial magnetic field in the early universe via the Weibel instability in the quark gluon plasma. The Weibel instability occurs in the plasma if there is an anisotropy in the particle distribution function of the particles. In early universe, the velocity anisotropy required for Weibel instability to operate is generated in the quark gluon plasma by the collapse of closed $Z(3)$ domain walls that arise in the deconfined phase of the QCD (above $T \sim 200$ MeV). Such large domains can arise in the context of certain low energy scale inflationary models. The closed domains undergo supersonic collapse and the velocity anisotropy is generated in the shocks produced in the wake of the collapsing domain walls. This results in a two stream Weibel instability in the ultra-relativistic quark gluon plasma. The instability in turn generates strong magnetic fields in the plasma. We find that the field strengths generated can be comparable to the equipartition energy density at the QCD scale which is of the order of 10^{18} G.

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I. INTRODUCTION

One of the main unsolved puzzles of modern cosmology is to explain the origin of the observed magnetic fields in the Universe. For the galaxies and galaxy clusters, the observed magnetic fields at the length scales of the order of a few kpc is about $1 - 10 \mu\text{G}$. At Mpc scales, the observations point towards a magnetic field of the order of $10^{-15} - 10^{-18}\text{G}$. The observations for kpc scales can be explained by producing the seed magnetic fields by a Biermann battery mechanism in the proto galaxy, which are then amplified by a galactic dynamo. However this process barely works for the large scale magnetic fields. An appealing alternative to the above scenario is to argue that the seed magnetic field has primordial origin which gets amplified as the proto galactic cloud undergoes collapse. This would naturally provide magnetic fields at all scales. We refer to [1–3] and citations therein for a comprehensive review.

The universe has undergone various stages during its evolution and each of these stages has the potential to provide the seed required to produce the observed magnetic field. In this work we focus on the epoch of the Quark-Hadron (Q-H) transition. This is expected to occur when the universe was roughly micro-seconds old. Till the turn of the century the Q-H transition was supposed to be a first order transition. The bubble wall dynamics associated with the first order transition provided a rich spectrum of possibilities like quark nuggets as dark matter candidates [4], production of baryon inhomogeneities [5] and also the primordial magnetic fields [6–8].

We very briefly recall the various scenarios of magnetic field generation which assume that the QCD phase transition is of the first order. If the universe underwent a first order QCD phase transition, then as the hadron bubbles expand in the quark gluon plasma (QGP) phase the latent heat is released in the surrounding plasma. Under such a situation a Biermann battery can operate in the early universe. The estimates in [6] gave a magnetic field strength of 5G at 100cm scales, which is very low. Cheng and Olinto [7] looked at the baryon density contrast between the shrinking quark regions and expanding hadronic bubbles. Such a contrast is created because of the “snowplow” effect due to the expanding bubble wall. They found that the generated magnetic fields were of the order of $10^6 - 10^8\text{G}$. This translates to galactic magnetic fields of the order of 10^{-11}G today which is 5 orders of magnitude smaller than the observed fields. In ref [8] the hydrodynamic instabilities on the

bubble walls were studied. Such instabilities can be present if the latent heat transport is primarily due to the fluid flow. These instabilities coupled with the baryon density contrast can then create the magnetic fields. They found field strengths of 10^{-20}G on the 10Mpc comoving scale. None of the above scenarios hold in the light of results from lattice gauge theory showing that a first order quark-hadron transition is very unlikely. The transition, for the range of chemical potentials relevant for the early universe, is most likely a crossover. See ref. [9] and references therein for details on the discussion of the order of the transition.

Our proposal for an alternate mechanism for the magnetic field generation utilizes the relativistic generalization of the Weibel instability in a plasma. Weibel [10] had argued that a plasma with anisotropic particle distribution is inherently unstable against infinitesimal magnetic field perturbations. As a result the infinitesimal small field perturbation grow until they reach a saturation. The relativistic generalization of Weibel instability was provided by Yoon and Davidson [11]. Various computer simulations of Weibel instability have also been performed for both the non-relativistic as well as relativistic plasma, see [12, 13] and references cited therein. It was argued in [14] that the Weibel instability could operate in GRB shocks where anisotropy in the particle distribution function is provided by the direction of shock propagation. In the early universe, the anisotropy could have come from the hydrodynamic instabilities of the hadronic bubbles in the QGP background if the transition was of the first order. However, as discussed above, that is not the case. As it turns out, the quark-hadron phase boundary is only one of the possibility for the interface. In addition to the phase boundary, the possibility of extended topological objects in the quark-gluon plasma (QGP) phase of QCD has been extensively discussed in the literature [15–17]. These are domain wall defects that arise from the spontaneous breaking of $Z(3)$ symmetry in the high temperature phase (QGP phase) of QCD. Assuming that the collapsing $Z(3)$ regions are spherical, we find that they inevitably undergo a supersonic collapse. This would lead to a shock front in the wake of the collapsing wall.

Moreover, previous works [18, 19] on these collapsing domains indicate that the transmission coefficients of the quarks through the domain walls depend on their mass. Thus more strange quarks are reflected by the domain wall as compared to the up/down quarks. This gives rise to a build up of charge asymmetry and baryon concentration across the wall. The resulting baryon inhomogeneity and the charge asymmetry will give rise to a velocity anisotropy distribution in the plasma. The presence of the shock coupled with this

anisotropy leads to a large anisotropy in the average kinetic energy of the plasma particles. Such anisotropies in the kinetic energy lead to kinetic instabilities. The Weibel instability is one such instability which is usually generated for such an anisotropic plasma.

For nonrelativistic plasma, details of the Weibel instability have been calculated for a wide range of equilibrium distribution functions. However, for a relativistic anisotropic plasma an analytical solution was obtained in [11] for a specific distribution based on the Water Bag (WB) model. Yoon et al. [11] worked out the solution for a distribution where the flow was along the z -axis and the two velocities are parallel and perpendicular to the flow direction. The plasma in the early universe does not have a specific flow direction. The particles move randomly in all directions. The presence of the shock wave gives a direction to the flow of the particles in the vicinity of the collapsing domain wall. We take that to be our flow direction. So we are able to define the two velocities similar to those used by Yoon et al. [11]. The two velocities are parallel and perpendicular to the velocity of the shock. This distribution leads to the Weibel instability being generated which subsequently leads to the generation of the magnetic field.

Since we are dealing with the high temperature plasma of the early universe, we need to account for the strong interactions between the quarks too. The non-Abelian analogue of the Weibel instability for QCD is the Chromo-Weibel instability. The Chromo-Weibel instability has been studied in quite detail in the context of relativistic heavy ion collision (RHIC) experiments [20–23], as there is an inherent anisotropy in the initial stages of the collisions. As is well known, the QCD interactions dominate over the QED interactions and hence the Chromo-Weibel would isotropize the plasma faster than the electro-Weibel. However, the early universe plasma is very different from the plasma generated during the heavy ion collisions. Early universe plasma will not have the large gluonic contribution as observed in the initial conditions of the heavy ion collision since the densities of the quarks and gluons in the early universe is given by the equilibrium temperature T . Moreover, there will be a significant contribution to the electromagnetic sector from the charged leptons. The leptonic contribution to the plasma may affect its thermodynamical properties [24] and the hydrodynamical properties as well. The quark-lepton scattering cross-section would lead to the instability being transferred to the leptonic sector. As mentioned before, the $Z(3)$ domain walls, generate a charge and baryon asymmetry through preferential transmission of the quarks, hence we look at the electromagnetic Weibel instability only in the quark

sector. A more precise picture would involve the contribution of the charged leptons too. However since such a multicomponent plasma instability is difficult to handle, we focus on the QGP component of the early universe. The argument being that the isotropization of the anisotropy in the momentum is primarily by strong interactions. We argue that that this will lead to a chromo-magnetic energy density which roughly is three times as large as the magnetic electromagnetic sector. It indicates that the magnetic field generated in the electromagnetic sector will not be negligible, it might be close to the equipartition values of the magnetic field in the early universe.

An important point to note is that $Z(3)$ symmetry is broken in the QGP phase, which is the high temperature phase of QCD and restored in the confined phase which is the low temperature phase. This is in contrast to the other symmetry breaking phase transitions like GUT or electro-weak, where the symmetry is broken in the low temperature phase. As a result the $Z(3)$ defects vanish below the confinement transition temperature (~ 200 MeV, unlike the GUT defects which are created in the low temperature phase) and, consequently, the constraints on topological defects coming from CMBR observations by WMAP or Planck [25, 26] do not apply to $Z(3)$ defects. It is then interesting to look for the possible signatures these defects might have left if they were present in the early universe. This work is one such effort in that direction.

In the presence of quarks, questions have been raised on the existence of these objects [27, 28]. However, lattice studies by Deka et al. [29] of QCD with quarks show strong possibility of the existence of non-trivial, metastable, $Z(3)$ vacua for high temperatures of order 700 MeV. The above studies are exciting as such high temperatures occur naturally in the early universe. It may also be possible to probe the existence of these defects in the ongoing relativistic heavy-ion collision experiments at LHC-CERN. These are the only topological defects in a relativistic quantum field theory which can be probed in lab conditions with the present day accelerators. Detailed simulations have been performed to study the formation and evolution of these objects in these experiments [30, 31].

The organization of the paper is as follows. In section II we take a quick look at the $Z(3)$ symmetry, the symmetry of the Polyakov loop which is the order parameter of the confinement transition. We then discuss the formation of $Z(3)$ structures in the early universe. There we discuss in detail the effects of quarks in the context of inflationary cosmology and how in certain low energy inflationary models, these $Z(3)$ domains can survive long enough

to have interesting cosmological implications. In section III we focus on the dynamical evolution of these domains and show that these collapsing domains indeed undergo supersonic collapse. The generation of magnetic fields from resulting shocks via Weibel instability is the subject of section IV. In section V we discuss the role of Chromo-Weibel instability in the saturation of magnetic fields. We conclude the paper with some discussions in section VI.

II. $Z(3)$ DEFECTS IN EARLY UNIVERSE

A. $Z(3)$ domains in QGP

We start our discussion with pure $SU(N)$ gauge theory. In pure gauge $SU(N)$ system, in thermal equilibrium at temperature T , the Polyakov loop [32–34] is defined as

$$L(x) = \frac{1}{N} \text{Tr} \left[\mathbf{P} \exp \left(ig \int_0^\beta A_0(\vec{x}, \tau) d\tau \right) \right]. \quad (1)$$

Here, $\beta = T^{-1}$ and $A_0(\vec{x}, \tau) = A_0^a(\vec{x}, \tau) T^a$, ($a = 1, \dots, N$) are the $SU(N)$ gauge fields satisfying the periodic boundary conditions in the Euclidean time direction τ , viz $A_0(\vec{x}, 0) = A_0(\vec{x}, \beta)$. T^a are the generators of $SU(N)$ in the fundamental representation. \mathbf{P} denotes the path ordering in the Euclidean time τ , and g is the gauge coupling. The trace denotes the summing over color degrees of freedom. Thermal average of the Polyakov loop, $\langle L(\vec{x}) \rangle$, is related to the free energy of a test quark in a pure gluonic medium, $\langle L(\vec{x}) \rangle \propto e^{-\beta F}$. In confined phase, the free energy of a test quark is infinite hence $\langle L(\vec{x}) \rangle = 0$ (i.e. system is below T_c). In deconfined phase a test quark has finite free energy and hence $\langle L(\vec{x}) \rangle \neq 0$. Thus $\langle L(\vec{x}) \rangle$ acts as the order parameter for the confinement-deconfinement phase transition. For brevity, we will use $l(x)$ to denote $\langle L(\vec{x}) \rangle$ from now on. Under $Z(N)$ transformation (which is the center of $SU(N)$), the Polyakov Loop transforms as

$$l(x) \longrightarrow Z \times l(x), \quad \text{where } Z = e^{i\phi}, \quad (2)$$

where, $\phi = 2\pi m/N$; $m = 0, 1 \dots (N-1)$. This leads to the spontaneous breaking of $Z(N)$ symmetry with N degenerate vacua in the deconfined phase or QGP phase. For QCD, $N = 3$ hence it has three degenerate $Z(3)$ vacua resulting from the spontaneous breaking of $Z(3)$ symmetry at $T > T_c$. This leads to the formation of interfaces between regions of

different $Z(N)$ vacua. These vacua are characterized by,

$$l(\vec{x}) = 1, e^{i2\pi/3}, e^{i4\pi/3}. \quad (3)$$

Even though these $Z(3)$ domains contribute to the thermodynamics of $SU(N)$ gauge theory, it has been argued that these $Z(3)$ domains do not have a “physical” meaning [27, 28]. The inclusion of dynamical quarks further complicates the issue, as they do not respect the $Z(N)$ symmetry. It has been argued that the effect of addition of quarks can be interpreted as the explicit breaking of $Z(N)$ symmetry, see, for example, refs. [35–38]. This leads to the lifting of degeneracy of the vacuum, with $l(\vec{x}) = 1$ as the true vacuum and $l(\vec{x}) = e^{i2\pi/3}, e^{i4\pi/3}$ as the metastable ones. We will follow this approach. This interpretation finds support in the lattice QCD studies with quarks [29]. These result strongly favor these metastable $Z(3)$ vacua at high temperature. These $Z(3)$ vacua can have important consequences in the case of early universe where these high temperatures occur quite naturally. The aim of this work is to bring to light one such possibility, namely the generation of a primordial magnetic field, due to the evolution of these domains in the quark-gluon plasma.

In the last decade or so, there has been some effort in understanding various properties of these defects. In [39], the existence of topological string defects at the junction of $Z(3)$ defects was established. The reflection of quarks/antiquarks from $Z(3)$ walls was first studied in ref. [18] and it was shown that baryon inhomogeneities can be produced in the QGP phase by the collapsing $Z(3)$ walls. It was also argued that the collapsing $Z(3)$ domains can concentrate enough baryon number (in certain late time inflationary models) to form quark nuggets thus providing us with an alternate scenario of quark nuggets formation in early universe, which is independent of the order of phase transition. This analysis was extended in ref. [19] by incorporating an interesting possibility arising from the spontaneous CP violation from $Z(3)$ interfaces. This was first discussed by Altes et al [40], who showed that spontaneous CP violation can arise from $Z(N)$ structures due to the non-trivial background gauge field configuration associated with the Polyakov loop. The first quantitative estimates of the CP violation in the quark scattering were made in [41] for the case of pure gauge QCD. These were extended to QCD with quarks in ref [42].

B. Formation of $Z(3)$ domains in early universe

One important difference for the formation of $Z(3)$ walls compared to the formation of other topological defects in the early universe arises from the fact that here symmetry is broken in the high temperature phase, and is restored as the universe cools while expanding. In the standard picture of defect formation, the symmetry is broken in the low temperature phase and defects are formed during the transition to the symmetry broken phase via Kibble mechanism [43]. The question then arises as to how these defects were formed if the universe was in the symmetry broken phase to start with. To discuss the detailed formation of $Z(3)$ structures using standard defect formation scenario, one would require a situation where the universe undergoes the transition from the hadronic (confined/low temperature) phase to the QGP (deconfined/high temperature) phase. Kibble mechanism [43] can then be invoked to study the formation of these defects. This idea was first discussed in detail in [18].

The quarks and gluons were deconfined before inflation as the universe was at a very high temperature ($T \gg T_c$). During inflation the universe cools exponentially due to the rapid expansion. During this cooling, as the temperature drops below the critical temperature T_c (if universe remains in quasi-equilibrium during this period) or as the energy density drops below Λ_{QCD} due to expansion (in a standard out of equilibrium scenario) any previously existing $Z(3)$ interfaces disappear. As the universe starts reheating, after inflation, the temperature eventually becomes higher than the critical temperature for confinement-deconfined transition. $Z(3)$ symmetry will then break spontaneously, and $Z(3)$ walls and associated QGP string will form via the standard Kibble mechanism. However, in presence of quarks, there is an explicit breaking of $Z(3)$ symmetry. Two of the vacua, with $l(x) = z, z^2$, become metastable leading to a pressure difference between the true vacuum and the metastable vacua [40, 44]. This leads to a preferential shrinking of the metastable vacua. Note that the pressure difference between the true vacuum and metastable vacuum may affect the formation of these domains. For example, there may be a bias in the formation of these domains as temperature crosses T_c due to this pressure difference. Though such a bias may get washed out by the thermal fluctuations and the continued rapid reheating at the end of inflation when equilibrium concepts may not strictly apply.

As we will see in section III the collapse of these regions can be very fast (the numerical simulations conducted in context of heavy-ion collisions too indicate $v_w \sim 1$ [30, 31]), they

are unlikely to survive until late times, say until QCD scale. However there are certain situations in which these domains can survive till late times. We will discuss those scenarios next. If these domains survive till late times (which cannot be below the QCD transition epoch), then the magnetic fields will be generated near the QCD transition epoch.

C. Survivability Scenarios for $Z(3)$ Defects till QCD Scale

Friction Dominated Dynamics:- The simplest possibility is that the collapse of $Z(3)$ domains may be slower due to the friction experienced by domain wall. For large friction, the walls may even remain almost frozen in the plasma. It has been discussed in the literature that dynamics of light cosmic strings can be dominated by friction which strongly affects the coarsening of string network [45, 46]. For example, it is plausible that the dynamics of these $Z(3)$ walls is friction dominated because of the non-trivial scattering of quarks across the wall. This can lead to significant friction in wall motion. Only when the surface tension and the pressure difference between the true and metastable vacua dominates over the friction at the late times, the walls will start collapsing. In such a scenario the anisotropy, and hence the magnetic fields, can be generated even near the QCD transition epoch.

Low energy inflationary models:- Even if the dynamics of the domain walls is not strongly friction dominated, it is still possible for these $Z(3)$ domains to survive until the QCD scale, in certain low energy inflationary models [47–49]. In these models the reheating temperature can be quite low ($\sim 1 \text{ TeV}$, or preferably, even lower)). With inclusion of some small friction in the dynamics of domain walls, it is then possible for the walls to survive until QCD transition. For detailed discussion of these issues regarding formation of $Z(3)$ walls in the early Universe see ref.[18].

Effective Restoration of $Z(3)$ at very high temperatures:- An interesting result has been obtained in lattice studies of $SU(N)$ with Higgs by Biswal et al [50] where they show that $Z(N)$ is restored at high temperatures. They find that breaking of $Z(N)$ is directly related to Higgs symmetry breaking. It is effectively restored in the Higgs symmetric phase and $Z(N)$ domains start appearing only in the Higgs broken phase. They suggest that a similar mechanism can operate for $SU(N)$ with fermions where $\bar{\psi}\gamma^0\psi$ can play the role of the scalar. Another situation could be when the thermal fluctuations are larger than the energy barrier. In that case there will be effectively no $Z(3)$ domains until the Universe cools down to the

temperatures where the fluctuations in the energy are smaller than the potential barrier between the two regions.

After formation, the domain wall network undergoes coarsening, leading to only a few domain walls within the horizon volume. Detailed simulation of the formation and evolution of these $Z(3)$ walls in the context of RHICE is discussed in ref [30, 31]. The evolution of these $Z(3)$ domain walls, once they are formed, can be understood quite well from these simulations. Even though the simulations rely on the bubble nucleation, the domain wall network obtained is reasonably independent of that. This is because the basic physics of the Kibble mechanism only requires formation of uncorrelated domains which happens in any transition.

III. SHOCK PRODUCTION BY COLLAPSING DOMAIN WALLS

In this section, we describe the generation of shocks by collapsing domain walls. We start by describing the collapse of bubbles which have already been discussed in great detail in ref. [51]. As mentioned by the authors, the methodology also works for the collapse of spherical domain walls in the “thin wall” limit, see [52–54]. We ignore the effects of gravity and work with flat space-time. The following discussion is along the lines of ref. [53], which is quite general and can be applied to both the first order phase transition bubbles as well as topological defects such as closed domain walls. The starting point is the Lagrangian for the surface of the collapsing interface,

$$\mathcal{L} = -\sigma[\det(-g^{(3)})]^{1/2} - U \quad , \quad (4)$$

where σ is the surface tension of the wall, U is the potential energy due to non-degeneracy of the vacua. $g^{(3)}$ is the three-metric on the surface of the domain wall. If x^μ are usual spacetime coordinates and χ^a are the coordinates for the domain wall hypersurface, then

$$g_{ab}^{(3)} = g_{\mu\nu} x_{,a}^\mu x_{,b}^\nu \quad , \quad (5)$$

with $x_{,a}^\mu \equiv \partial x^\mu / \partial \chi^a$. For a spherical case the convenient coordinate choice is spherical polar coordinates i.e $x^\mu = (t, r, \theta, \phi)$ and $\chi^a = (t, \theta, \phi)$. Expanding r in spherical harmonics viz,

$$r = R(t) + \sum_{l,m} \Delta_{lm} Y_{lm}(\theta, \phi) \quad , \quad (6)$$

and writing the Lagrangian given by eq. 4 upto the first order in Δ_{lm}/R , one gets

$$\mathcal{L} = -\sigma r^2 (1 - \dot{r}^2)^{1/2} \sin(\theta) + \frac{\epsilon}{3\sigma} r^3 \sin(\theta) \quad , \quad (7)$$

where ϵ is difference between the true vacua and the metastable vacua, with $\epsilon > 0$ for the true vacua expanding in the background of metastable vacua. The equation of motion for wall at the zeroth order is then given by

$$\ddot{R} = \frac{\epsilon}{\sigma} (1 - \dot{R}^2)^{3/2} - \frac{2}{R} (1 - \dot{R}^2) \quad , \quad (8)$$

which can be integrated to give

$$(1 - \dot{R}^2)^{1/2} = \frac{R^2}{\epsilon R^3/3\sigma + C} \quad , \quad (9)$$

where C is the constant of integration. We are interested in the asymptotic solutions of eq. 9 for the collapsing domains ($R \rightarrow 0$). In this limit, we find that $|\dot{R}| \rightarrow 1$. As a result when a metastable spherical domain collapses, it will produce shocks.

We now return to the scenario of the collapsing $Z(3)$ domains. As has been discussed previously in II, collapsing $Z(3)$ domain walls lead to the concentration of baryon numbers. As the reflection of quarks/antiquarks depend upon the masses of the quarks. This leads to a larger number of strange quarks being reflected by the domain walls as compared to up/down quarks [18]. This gives rise to a charge imbalance across the domain wall. This is a very local effect and only affects those particles in the plasma which are close to the domain wall. Now the QGP is a multiparticle plasma and will have several distribution functions for the individual particles (quarks/antiquarks, gluons and leptons). Due to the charge on the domain wall some charges are repelled, while the opposite charges are accelerated. Thus the local phase velocity of the different streams of particles do not remain the same. There will be a distortion in the MHD wave travelling in the plasma as the wall collapses. As the velocity of the wall increases, and $|\dot{R}| \rightarrow 1$, shocks will be generated. These shocks will however result in an anisotropic velocity distribution with the particles moving in the direction of shock moving with almost close to the speed of light.

Weibel [10] has shown that an anisotropic velocity distribution in a non-relativistic plasma would give rise to an instability. The instability, which is a two stream instability (called the Weibel instability) gives rise to a magnetic field in the plasma. The Weibel instability was generalized by Yoon and Davidson, ref. [11], for an ultra-relativistic collisionless plasma.

Medvedev and Loeb [14] used the relativistic Weibel instability to generate strong magnetic fields in shocks produced by Gamma Ray Bursts (GRBs). The instability was driven by the anisotropic particle distribution function in the shock. Similarly the shocks generated in the wake of the collapsing domain wall will also have an anisotropic particle distribution. This means that a Weibel instability will be generated in the quark gluon plasma. In the next section, we would like to calculate the magnetic field generated due to the Weibel instability in the wake of the collapsing domain wall.

IV. MAGNETIC FIELDS GENERATED BY THE WEIBEL INSTABILITY

While for the non-relativistic case, the instability is well understood, for the relativistic plasma only approximate analytical solutions are allowed. This is because the parallel and perpendicular motions of the particles in the plasma are coupled by the Lorentz factor γ . Since we are dealing with a very high temperature plasma here, we work with the ultrarelativistic plasma.

As argued previously, due to the charge concentration inside the collapsing $Z(3)$ region, some particles are accelerated while others are decelerated as they pass near the domain wall. The net particle distribution can only be obtained from detailed simulations. However, since we are interested in obtaining a preliminary estimate of the magnetic field generated by the Weibel instability, we just use the particular choice of Particle Distribution Function (PDF) used by Yoon and Davidson, [11]. They had adopted the Water Bag (WB) model to obtain an analytic solution to the dispersion relations obtained from the flow of an anisotropic unmagnetized plasma. They had also studied the detailed properties of the Weibel instability generated in the plasma. Their PDF is given by,

$$F(\vec{p}) = \frac{1}{2\pi p_{\perp}} \delta(p_{\perp} - p_1) \frac{1}{2p_{\parallel}} H(p_2^2 - p_{\parallel}^2), \quad (10)$$

where $H(x)$ is the Heaviside step function. This PDF assumes that the charged particles move on a surface with perpendicular momenta p_{\perp} and are uniformly distributed in parallel momenta between $p_{\parallel} = -p_2$ and $p_{\parallel} = p_2$. Since the particles flow along the shock generated, therefore $|p_{\parallel}| = p_2$ is the shock velocity. The perpendicular momenta is taken to be p_1 . This PDF can be solved exactly by substituting it in the kinetic equation and the dispersion relations obtained. The detailed calculations are available in ref.[11]. Finally, it can be

shown that the instability occurs for the range of wave numbers given by

$$0 \leq k^2 \leq k_0^2 \equiv \left(\frac{\omega_p^2}{\gamma} \right) \left[\frac{v_{\parallel}^2}{2v_{\parallel}^2(1-v_{\perp}^2)} - G(v_{\perp}) \right], \quad (11)$$

where $\omega_p = (4\pi nq^2/m)^{1/2}$ is the non relativistic plasma frequency and $G(v_{\perp}) = (2v_{\perp})^{-1} \log[(1+v_{\perp})/(1-v_{\perp})]$. Of these modes the mode with maximum growth rate, Γ_{max} , dominates the evolution of magnetic field and sets the characteristic length scale, $\lambda \sim k_{max}^{-1}$, for the magnetic field. The growth rate is given by the expression [14]

$$k_{max}^2 = \frac{\omega_p^2}{\gamma(1-v_{\perp}^2)} \left[-\frac{v_{\parallel}^2}{2(1-v_{\perp}^2)} - G(v_{\perp}) + \frac{(1+v_{\perp}^2)v_{\parallel}^2}{\sqrt{2}(1-v_{\perp}^2)^{3/2}} \left(\frac{v_{\parallel}^2}{(1-v_{\perp}^2)} + \frac{1-2v_{\perp}^2-v_{\perp}^4}{v_{\perp}^2} G(v_{\perp}) \right) \right]. \quad (12)$$

In this work, we are working with natural units and hence $c = 1$. Eq. 12 gets much simplified if one takes into account the fact that the early universe plasma is ultra relativistic, so that $\gamma_{\perp} \gg 1$. Also as the collapse velocity of domain walls approaches the speed of light, $\gamma_{\parallel} \gg \gamma_{\perp}$. So under these approximations, the expression for k_{max}^2 is

$$k_{max}^2 \simeq \frac{\omega_p^2}{\sqrt{2}\gamma_{\perp}} \left(1 - \frac{3\gamma_{\perp}}{\sqrt{2}\gamma_{\parallel}} \right). \quad (13)$$

This gives the correlation length of the magnetic field due to the most dominant mode as

$$\lambda_{\text{correlation}} \sim k_{max}^{-1} \simeq 2^{1/4} \left(\frac{\gamma_{\parallel}^{1/2}}{\omega_p} \right). \quad (14)$$

As the magnetic field grows, the deflection of the particles grows leading to an increase in the cyclotron radius of the particles. Only the particles travelling along the field lines can travel distances larger than the cyclotron radius. Also the cyclotron radius cannot be larger than the correlation length scale of the magnetic field. Hence

$$\frac{v_{\perp}m}{qB} \leq 2^{1/4} \left(\frac{\gamma_{\parallel}^{1/2}}{\omega_p} \right). \quad (15)$$

In the above relation $m = m_0\gamma$ is the relativistic mass. Only when $v_{\perp} \sim v_{\parallel}$, the cyclotron radius of the particles will be comparable to the correlation length scale of the magnetic field. Essentially as pointed out in ref.[11], the growth of the instability is due to the anisotropy of the energy in the plasma. The evolution of the instability gradually leads to the equipartition of the energy, which stabilizes the plasma. The magnetic field gets saturated at this point

and the distribution function becomes isotropic. The saturation field strength is then given by

$$B \sim \frac{v_{\parallel} m \omega_p}{\sqrt{2} q \gamma_{\parallel}^{1/2}}. \quad (16)$$

As the $Z(3)$ domains are characterized by the thermal expectation value of the Polyakov loop (section II A), which is related to the free energy of the test quark, the domain wall is characterized by the change in the free energy of a test quark as one moves from one $Z(3)$ region to another. This essentially means that the interaction of the domain wall is only with quarks and not with leptons. Therefore the Weibel instability generated in the shock would be due to the anisotropy in the velocity of the quarks. The charge built up inside the collapsing regions due to the baryon concentration, as shown in ref. [18, 19], would also affect the charged leptons. Since that is a secondary effect we first concentrate on the magnetic field calculations for the quark sector only and come back to the leptonic sector later.

Since we are interested in the saturation value of magnetic fields, i.e when the particles have become isotropic, the baryon number density is given by the equilibrium baryon number density around $T = 200$ MeV, which is roughly of the order 1 fm^{-3} . Since the magnetic force will affect the lightest quark most, we take the mass of the particle appropriate for the u/d quark i.e $m \sim 10$ MeV. Also the plasma is an ultra-relativistic one $v_{\parallel} \sim 1$. Substituting all these in the equation we get an approximate value for the magnetic field B as 10^{19} G.

Actually, the expression for the magnetic field can be put in a useful form by squaring and rearranging it to get,

$$\frac{B^2/8\pi}{m_0 n (\gamma_{\parallel} - 1)} = \frac{(\gamma_{\parallel} - 1)}{2\sqrt{2}\gamma_{\parallel}}, \quad (17)$$

which for $\gamma_{\parallel} \gg 1$, indicates that the magnetic field energy is close to the equipartition energies. This is also borne out by our approximate estimate since the equipartition value of the magnetic field at 100 MeV is about $B_{eq} = 10^{18} \text{ G}$ [7].

V. ROLE OF CHROMO-WEIBEL INSTABILITY

In the previous section we focused our attention to only the electromagnetic interaction of quarks. However we know that quarks predominantly interact via strong interactions. So an anisotropy in the quark distribution function should lead to a Weibel-like instability

in the color sector too. The non-abelian analogue of Weibel instability for QCD is the Chromo-Weibel instability. The Chromo-Weibel instability has been studied in quite detail in the context of relativistic heavy ion collision (RHIC) experiments [20–23], as there is an inherent anisotropy in the initial stages of the collisions. Since in the scenario discussed in previous section, the Weibel instability acts on quarks, the Chromo-Weibel instability would also operate in the early universe plasma.

Since QCD interactions dominate over QED interactions Chromo-Weibel would be the major contributor to the isotropization of the quark momentum distribution function. In almost all likelihood it would be the chromo-magnetic energy that would reach the equipartition values and not the magnetic energy as we discussed in the last section. Then the pertinent question to ask is: What is the magnetic field in the EM sector when the Chromo-Weibel saturates? The answer to this question would tell us the magnetic field produced in the early universe near QCD transition epoch.

To answer this question in detail one would need to study the evolution of Chromo-Weibel instability with the distribution function given by Eqn. (10). A detailed study of growth rate calculations can be found in [57]. The entire approach can be repeated to obtain the growth rate for water-bag distribution function that we have used in this work. However, for obtaining the order of magnitude estimates we can just focus on the form of the expression of k_{max}^2 . From eq (13) we can see that $k_{max}^2 \propto \omega_p^2 \sim g^2 n$, where $g^2 = (4\pi\alpha)$ is the coupling constant and n is the number density of particles. This feature is present for the Chromo-Weibel instability also (see Eqn. (25) in ref. [57]). Since the most dominant mode sets the length scale (k_{max}^{-1}) for the magnetic field, thus setting a limit on the gyromagnetic radius (proportional to $(qB/m)^{-1}$) for the charged particles, we get $B \propto mk_{max}/q$. Also, as the mass times number density is just the energy density, we get $B \propto g\rho^{1/2}/q \sim \rho^{1/2}$. Here we have used $q^2 = 4\pi\alpha = g^2$. We thus obtain the ratio of the magnetic field energy in the color and electromagnetic sectors as

$$\frac{B_{chromo}^2}{B_{em}^2} \sim \left(\frac{\rho_{QGP}}{\rho_{EM}} \right). \quad (18)$$

We write $\rho_{QGP} = \rho_q^c + \rho_{\bar{q}}^c + \rho_g$, where the subscript ‘c’ denotes including color degrees of freedom, and $\rho_{em} = \rho_q + \rho_{\bar{q}}$ as only quarks, not gluons, contribute to electromagnetic field. To get an idea of the values we use the relativistic ideal gas approximation and find, for 2 flavor QGP, $\rho_{QGP} \sim 100T^4$ and $\rho_{EM} \sim 30T^4$. This implies that the magnetic field energy in the color sector is roughly three times larger than in the electromagnetic sector. This

would mean that the magnetic fields in the color sector and the electromagnetic sector of the plasma are of similar strength. It is thus possible to have close to equipartition values of magnetic field in the early universe plasma due to the collapsing $Z(3)$ domains.

This result is surprising, to say the least, at the first look. The problem is that the natural time scales of the strong interaction is of the order of $1 \text{ fm}/c$ which is much smaller than the natural time scales of the electromagnetic interactions. Then why the chromo-magnetic and electromagnetic energies have similar strengths? The answer to this puzzle lies in the realisation that we are not doing a comparison between two different plasmas. If it was a comparison between QGP and the standard electron-positron type electromagnetic plasma we would have got the Chromo-Weibel saturation much earlier than the electromagnetic Weibel saturation. However, we are looking at only QGP and a major component of QGP, namely quarks and anti-quarks, carry electric charge too. Since the filamentation of charges in QGP is due to strong interactions, the electric charge is also filamented, along with the color charge, at the strong interaction time scales which would not happen if it was an electromagnetic plasma.

To understand the above point let us consider only u -quark first. It has positive charge (irrespective of the color) and the filamentation of u -quark based on color would mean that each filament has a specific color. That would also mean that each filament has positive charge flowing. This would mean that magnetic field grows at the QCD time scales. Now add d -quark to the system too. It has negative charge and all the three color. Now, since the electric charge of d -quark is half of that of u -quark, in magnitude, a color filament say red, will have u and d quarks of red color but also a net positive electric charge, which would be half of what it would have been if there was no d -quark. The color filament would be charge neutral only if the density of d -quark is twice that of u -quark which is not possible owing to its larger mass than u . Even in the massless limit the densities of both types of quarks would be equal otherwise d -quark would always be less abundant than u -quark, at any temperature in the early universe QGP. One cannot make up for the lack of negative charge by adding s -quark as it is almost 30 times heavier than the u -quark and hence would not be as abundant. We thus reiterate that the filamentation of color charges leads to the filamentation of the electric charge too. Hence the magnetic field grows with roughly the same rate as chromo-magnetic field. We thus conclude that it is the Chromo-Weibel instability that is responsible for filamentation of electric charge within the QGP, albeit

indirectly. An important thing to note is that in the entire discussion above, the leptonic sector of the early universe plasma has been consistently ignored.

VI. DISCUSSION

We now discuss a few finer points that were glossed over in the previous two sections. The discussion on Weibel instability in previous section was based on the linear analysis. However in the late stages of the evolution, the non-linearity kicks in and hence the magnetic field is unable to attain the equipartition value. The numerical simulations performed in the electro-magnetic (EM) plasma indicate that magnetic field density obtained is lower than the equipartition energies, $B \sim 0.1B_{eq}$. In the case of the QGP, it is known that the Equation of State (EoS) deviates strongly from the ideal case near the transition point. So it is natural to question the validity to map the simulations of an EM plasma exactly onto a QGP plasma. The earlier studies of the shrinking $Z(3)$ domains [18, 19], have shown that near the baryon over-densities in the collapsing region can be quite large ($\sim 10^6$ times the average baryon density). Under such large densities it is quite possible that the quark-gluon plasma is quite well described by the perturbative QCD. Thus it is reasonable to expect that in that regime the EoS follows the ideal gas relation. The magnetic fields in that case could be as large as 10^{17} G. In the initial stages of the collapse, when the baryon concentration inside the shrinking region is not very large, the non-ideal effects are important. Though one cannot apply the results of EM simulations at those stages, it is possible to discuss the effects that a non ideal fluid can have on the generation of the magnetic field. Usually in non-ideal fluids, it is seen that shocks and instabilities are damped due to the presence of energy dissipating effects such as viscosity and thermal conductivity. It might so happen that due to the high thermal conductivity, the magnetic field would reach saturation at an early stage but that will only affect our final estimates by an order of magnitude or so. That would imply that the fields generated would be of lesser magnitude, as saturation point is reached earlier.

Another very important point is the assumption that we implicitly made in section V. The assumption is that the mechanism of the growth of Weibel and Chromo-Weibel instability is similar. This assumption is expected to hold in the initial stages of instability evolution but that cannot be guaranteed when the non-abelian interactions become important. However

it has been conjectured by Arnold and Lenaghan [58] that non-abelian fields become approximately abelian during the growth as the non-abelian self interactions are not sufficient to stop the growth of instability. This is termed as the “*abelianization conjecture*”. In the light of abelianization conjecture it seems reasonable to use the similar approach to study non-abelian plasma as those used to study the traditional plasma physics. One important factor that we have glossed over is the fact that the QGP is a multi-component plasma. Our estimation of the magnetic field is for the u/d component of the plasma. In different components, the magnetic field will grow at a different rate. For example, the masses of u/d and s quark differ considerably, hence the instability may saturate for the u/d quarks but could still continue for the s quark.

Now we briefly discuss the effect of the collapsing $Z(3)$ domain on the leptons. As previously mentioned, the baryon concentration inside the closed $Z(3)$ regions increases with the collapse of the domains. This leads to a net charge accumulation within the domains. Initially the charge build-up is not very large but towards the later stages of the evolution there is a sudden build-up of charge which can be as large as 10^6 times the surrounding baryon densities. The leptons would respond to this sudden build up of this electric charge and this response could lead to an anisotropic distribution in the leptonic component of the early universe plasma. The quark-lepton interactions would also be present and may play an important role in creating an anisotropic momentum distribution for leptons. For example if we look at the decay of Charm quark, which would still be present in reasonable amount around 200 – 300 MeV, then one may possibly get an anisotropy in the leptonic sectors thus fuelling the EM Weibel instability purely in the leptonic sector. We have not discussed these complications but we would like to emphasize that the spherically collapsing $Z(3)$ domains would always generate shocks and will ensure that two stream instabilities are generated in the QGP. Finally, of course, numerical simulations of a Weibel instability in the quark gluon plasma would give us a better estimate of the magnetic field.

The only drawback of our model is the small correlation length of the generated magnetic field. This is the general problem for most primordial magnetic fields generated in very early times (except during inflation). The large scale growth of magnetic fields has been seen in numerical simulations of Weibel instability in the EM plasma by modelling the upstream and the downstream of particles as the current carrying filaments [55]. They find the field scale grows similar to that of inverse cascade of MHD simulations even though the process

are entirely kinetic in nature in a two stream Weibel instability. It would be interesting to see how the numerical results fare in the QGP case.

In MHD, the magnetic field can be amplified by the inverse cascade mechanism if it is a helical magnetic field. Since the magnetic field here is generated by a spherically collapsing domain wall, the bulk flow will not be helical. The only possibility of generating a helical magnetic field will be if the average density of the Chern-Simons number turns out to be non-zero. Even a small helicity in the magnetic field would ensure its amplification to maximally helical fields in the present epoch. This could be an actual possibility in our case, since CP violating quark scattering does occur from asymmetric $Z(3)$ interfaces [41]. We would like to explore this further in a future work. Detailed magneto-hydrodynamic studies have been carried out [56] which show that a rapid growth of the correlation length can occur due to decaying turbulence in the plasma. Such a growth can occur even for a non-helical magnetic field, however the growth is slower than in the helical case.

The magnetic fields present during the QCD phase transitions (deconfinement and/or Chiral) can affect the dynamics of transition. It would again be quite interesting to study the possible implications of such altered dynamics of phase transition in the presence of magnetic fields.

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