Flavor $SU(3)$ properties of beauty tetraquark states with three different light quarks

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Abstract

Beauty tetraquark states $X(\bar{b}q'q''\bar{q})$ composed of $\bar{b}su\bar{d}$, $\bar{b}ds\bar{u}$, and $\bar{b}ud\bar{s}$, are unique that all the four valence quarks are different. Although the claim of existence of the first two states by D0 was not confirmed by data from LHCb, the possibility of such states still generated a lot of interests and should be pursued further. Non-observation of $X(\bar{b}q'q''\bar{q})$ states by LHCb may be just due to a still lower production rate than the limit of LHCb or at some different mass ranges. In this work we use light quark $SU(3)$ flavor symmetry as guideline to classify symmetry properties of beauty tetraquark states. The multiplets which contain states with three different light quarks must be one of $\bar{\mathbf{6}}$ or $\mathbf{15}$ of $SU(3)$ representations. We study possible decays of such a tetraquark state into a B meson and a light pesudoscalar octet meson by constructing a leading order chiral Lagrangian, and also provide search strategies to determine whether a given tetraquark state of this type belongs to $\bar{\mathbf{6}}$ or $\mathbf{15}$. If $X(\bar{b}q'q''\bar{q})$ belongs to $\mathbf{15}$, there are new doubly charged tetraquark states $\bar{b}uu\bar{d}$ and $\bar{b}uu\bar{s}$.

I. INTRODUCTION

D0 collaboration recently claimed to have discovered a new tetraquark state $X(5568)$ with a mass of 5567.8 MeV by studying the invariant energy spectrum of B_s^0 and $\pi^\pm$ based on 10.4 fb^{-1} $p\bar{p}$ collision at $\sqrt{s} = 1.96$ GeV[1]. The significance is 5.1 σ. The states may be tetraquark states composed of $\bar{b}su\bar{d}$ and $\bar{b}ds\bar{u}$. If true, this is the first observation of tetraquark states with four different valence quark flavors. This claim generated a lot of interesting theoretical studies. However, a few weeks later, the LHCb collaboration announced their searching results based on $3\mathrm{fb}^{-1}$ of pp collision data at $\sqrt{s}$= 7 and 8 TeV which did not find the same states[2]. The existence of $X(5568)$ has not be confirmed. In recent years, there have been a lot of researches in exotic states possibly composed of four quarks, such as the X, Y and Z states[3]. There may be more different types of exotic states in Nature. The $X(\bar{b}q'q''\bar{q})$ state, if it exist, may be a tightly bound state of di-quark anti-quark pair such as $[su][\bar{b}\bar{d}]$ or $[sd][\bar{b}\bar{u}]$, or a "molecular state" of the loosely bound $B_{u,d}$ and K or B_s and π mesons. We will refer such states generically as X_b states. Whether they are really bound states or due to some other effect should be carefully analyzed once data become available[4]. Several theoretical papers have appeared discussing the D0 data focusing on the mass estimate, width and J^{CP} nature[5], and also possible partner states in tetraquark model[6].

Although the claim of existence of beauty tetraquark X_b states by D0 was quickly ruled out by data from LHCb, these states are still interesting to study. Non-observation of these states by LHC may just be due to a still lower production rate than the limit of LHCb or at some different mass ranges. In this work we use light quark $SU(3)$ flavor symmetry as the guideline to classify their symmetry properties. From the viewpoint of diquark picture of four quark states[7], they can form 0^{++} or 1^{++} states depending on whether the two anti-quarks are in flavor symmetric or antisymmetric anti-diquark system. Their J^{CP} properties can be probed by studying their decays, such as $X_b \to B_s^0\pi^\pm$ carried out at $D0$. The S-wave decays of these final states will indicate the $\bar{b}su\bar{d}$ or $\bar{b}ds\bar{u}$ are $J^{CP} = 0^{++}$ states. The photon from $B^* \to B\gamma$ is soft and thus not discovered. Then the results for X_b being a 0^{++} or 1^{++} will be the same if only $B_s^0\pi^\pm$ measurement is concerned. Therefore we will concentrate the properties of $J^{CP} = 0^{++}$ states in this paper.

The multiplets which contain states with three different light quarks must be one of the $\bar{\mathbf{6}}$

or the **15** of $SU(3)$ representation. We will study possible decays of such a tetraquark state into a B meson and a light pesudoscalar octet meson by constructing a leading order chiral Lagrangian, and also provide searching strategies to determine whether a given tetraquark state of this type belongs to $\bar{\mathbf{6}}$ or **15**. If X_b belongs to **15**, there are new doubly charged tetraquark states $\bar{b}uu\bar{d}$ and $\bar{b}uu\bar{s}$. Detection of these states will provide unique signatures for beauty tetraquark states belong to the **15** representation. We also provide other methods which can provide crucial information about whether beauty tetraquark states belong to $\bar{\mathbf{6}}$ or **15** multiplet.

II. $SU(3)$ MULTIPLETS CONTAINING FOUR DIFFERENT QUARKS

The beauty tetraquark $X(\bar{b}q'q''\bar{q})$ states are constructed from an anti-b quark $\bar{b}$, two light quarks $q_i q_j$ and one light anit-quark $\bar{q}_k$. Under the flavor $SU(3)$ symmetry, the anti-b quark is a singlet, the light anti-quark is a $\bar{\mathbf{3}}$, and the light quark is a **3**. As far as the $SU(3)$ group structure is concerned, the irreducible states are

$$\mathbf{3}\times\mathbf{3}\times\bar{\mathbf{3}} = \mathbf{3}+\mathbf{3}+\bar{\mathbf{6}}+\mathbf{15}\ . \tag{1}$$

The **3** representations do not contain a component with three different light quarks (anti-quark). Therefore the tetraquark states containing four different quarks cannot be in the **3** representation. In the $\bar{\mathbf{6}}$ representation, the two light quarks are anti-symmetric, while in the **15** representation the two light quarks are symmetric. Both representations have states with three different quark flavors (including an anit-quark). Let us denote the states with four different quarks in $\bar{\mathbf{6}}$ and **15** as X' and X, respectively:

$$\begin{aligned}
\bar{\mathbf{6}}\ &:\ X'_{ds\bar{u}}\sim\bar{b}\bar{u}(ds-sd)\ ,\ \ X'_{su\bar{d}}\sim\bar{b}\bar{d}(su-us)\ ,\ \ X'_{ud\bar{s}}\sim\bar{b}\bar{s}(ud-du)\ ,\\
\mathbf{15}\ &:\ X_{ds\bar{u}}\sim\bar{b}\bar{u}(ds+sd)\ ,\ \ X_{su\bar{d}}\sim\bar{b}\bar{d}(su+us)\ ,\ \ X_{ud\bar{s}}\sim\bar{b}\bar{s}(ud+du)\ .
\end{aligned} \tag{2}$$

There are also other states come along with those with four different quarks. The representations $\bar{\mathbf{6}}$ and **15** can be written in tensor notations as, $X^k_{[i,j]}$ and $X^k_{\{i,j\}}$, respectively. Here $[i,j]$ and $\{i,j\}$ indicate anti-symmetric and symmetric combinations under exchange of flavor indices i and j. Both $X(\bar{\mathbf{6}})$ and $X(\mathbf{15})$ are traceless, that is, $X^i_{[i,j]} = X^i_{\{i,j\}} = 0$. Writing $X^k_{[i,j]}$ in terms of the properly normalized component fields, the component fields in

$\bar{\mathbf{6}}$ can be written as

$$X^1_{[2,3]} = \frac{1}{\sqrt{2}} X^{'}_{ds\bar{u}} \ , \ \ X^2_{[3,1]} = \frac{1}{\sqrt{2}} X^{'}_{su\bar{d}} \ , \ \ X^3_{[1,2]} = \frac{1}{\sqrt{2}} X^{'}_{ud\bar{s}} \ ,$$
$$X^1_{[1,2]} = X^3_{[2,3]} = \frac{1}{2} Y^{'}_{(u\bar{u},s\bar{s})d} \ , \ \ X^1_{[3,1]} = X^2_{[2,3]} = \frac{1}{2} Y^{'}_{(u\bar{u},d\bar{d})s} \ , \ \ X^2_{[1,2]} = X^3_{[3,1]} = \frac{1}{2} Y^{'}_{(d\bar{d},s\bar{s})u} \ . \quad (3)$$

The component fields in $\mathbf{15}$ are

$$X^1_{\{2,3\}} = \frac{1}{\sqrt{2}} X_{ds\bar{u}} \ , \ \ X^2_{\{3,1\}} = \frac{1}{\sqrt{2}} X_{su\bar{d}} \ , \ \ X^3_{\{1,2\}} = \frac{1}{\sqrt{2}} X_{ud\bar{s}} \ ,$$
$$X^1_{\{1,1\}} = (\frac{Y_{\pi u}}{\sqrt{2}}) + \frac{Y_{\eta u}}{\sqrt{6}}) \ , \ \ X^1_{\{1,2\}} = \frac{1}{\sqrt{2}} (\frac{Y_{\pi d}}{\sqrt{2}}) + \frac{Y_{\eta d}}{\sqrt{6}}) \ , \ \ X^1_{\{1,3\}} = \frac{1}{\sqrt{2}} (\frac{Y_{\pi s}}{\sqrt{2}} + \frac{Y_{\eta s}}{\sqrt{6}}) \ ,$$
$$X^2_{\{2,1\}} = \frac{1}{\sqrt{2}} (-\frac{Y_{\pi u}}{\sqrt{2}} + \frac{Y_{\eta u}}{\sqrt{6}}) \ , \ \ X^2_{\{2,2\}} = (-\frac{Y_{\pi d}}{\sqrt{2}} + \frac{Y_{\eta d}}{\sqrt{6}}) \ , \ \ X^2_{\{2,3\}} = \frac{1}{\sqrt{2}} (-\frac{Y_{\pi s}}{\sqrt{2}} + \frac{Y_{\eta s}}{\sqrt{6}}) \ ,$$
$$X^3_{\{3,1\}} = -\frac{1}{\sqrt{3}} Y_{\eta u} \ , \ \ X^3_{\{3,2\}} = -\frac{1}{\sqrt{3}} Y_{\eta d} \ , \ \ X^3_{\{3,3\}} = -\frac{1}{\sqrt{3}} Y_{\eta s} \ ,$$
$$X^1_{\{2,2\}} = Z_{dd\bar{u}} \ , \ \ X^1_{\{3,3\}} = Z_{ss\bar{u}} \ , \ \ X^2_{\{1,1\}} = Z_{uu\bar{d}} \ ,$$
$$X^2_{\{3,3\}} = Z_{ss\bar{d}} \ , \ \ X^3_{\{1,1\}} = Z_{uu\bar{s}} \ , \ \ Z^3_{\{2,2\}} = Z_{dd\bar{s}} \ . \quad (4)$$

It is clear that if $SU(3)$ flavor symmetry is a good (approximate) symmetry to describe the properties of beauty tetraquark X_b states, there are other states come along with the states with four different quarks depending on which representation the tetraquark states with four different quarks belong to. If a tetraquark state composed of four different quarks is found, it is important to determine whether it belongs to $\bar{\mathbf{6}}$ or $\mathbf{15}$. It is interesting to note that if X_b belongs to $\mathbf{15}$, there are new doubly charged tetraquark states $\bar{b}uu\bar{d}$ and $\bar{b}uu\bar{s}$. Detection of these states will provide unique signatures for beauty tetraquark states belong to the $\mathbf{15}$ multiplets.

III. CHIRAL EFFECTIVE THEORY FOR $X(\bar{b}q'q''\bar{q})$

In the following we study possible decays of a tetraquark X_b state into a B meson and a light pseudoscalar octet meson by constructing a leading order chiral Lagrangian, and also provide searching strategies to determine whether a given tetraquark state of this type belongs to $\bar{\mathbf{6}}$ or $\mathbf{15}$. One can construct a chiral perturbation theory[8] to describe a X_b decays into a B meson and a light octet pseudoscalar Π. Here Π is the normalized pseudoscalar octect containing $\pi^0,\ \pi^\pm,\ K^\pm, K^0,\ \bar{K}^0,\ \eta$ fields,

$$\Pi = \begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} . \tag{5}$$

Under chiral $SU(3)_L \times SU(3)_R$, the Goldstone boson field $\Sigma \equiv \exp(2i\pi(x)/f)$ transforms as $\Sigma \to L\Sigma R^\dagger$ with $\pi(x) = \Pi/\sqrt{2}$. $f \approx 93$ MeV is the pion decay constant. It is convenient to define another field $\xi(x)$ by $\Sigma(x) \equiv \xi(x)^2$, which transforms as $\xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$ under global chiral $SU(3)_L \times SU(3)_R$ transformation. The 3×3 matrix field $U(x)$ depends on Goldstone fields $\pi(x)$ as well as on the global $SU(3)$ transformation matrices L and R, thereby being x-dependent field describing hidden local SU(3) transformation in chiral symmetric theories. The fields X_b and $B_i = (B_u, B_d, B_s)$ transforms as $\bar{\mathbf{6}}$ or $\mathbf{15}$, and $\mathbf{3}$ of $SU(3)$, respectively:

$$X^i_{jk} \to U^i{}_{i'} X^{i'}_{j'k'} (U^\dagger)^{j'}{}_j (U^\dagger)^{k'}{}_k, \quad B_i \to U_i{}^j B_j.$$

It is convenient to define vector and axial vector fields (V_μ and A_μ) with following properties under chiral transformations:

$$V_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right) \to UV_\mu U^\dagger + U\partial_\mu U^\dagger\ ,\ \ A_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right) \to UA_\mu U^\dagger\ . \tag{6}$$

Note that the vector field V_μ transforms like a gauge field for local $SU(3)$.

For the pionic transitions $X_b \to B\pi$, the mass difference between X_b and B is very small and the pion is soft. Therefore we can use the heavy particle effective theory for X_b and B, assuming their velocity v is conserved [8]. We should keep in mind that X_b and B fields have v-dependence so that they are actually $X(v)$ and $B(v)$. The $SU(3)$ properties of the relevant fields are given by X^i_{jk} and B_i. One then has

$$\mathcal{L} = \overline{B} iv\cdot DB + \overline{X} iv\cdot DX - \overline{X}M_\xi X - \overline{B} v\cdot AX \tag{7}$$

where we assume that all the $SU(3)$ flavor indices are contracted appropriately in order to respect chiral symmetry.

$$\begin{aligned} (D_\mu B)_i &= \partial_\mu B_i + (V_\mu)^{i'}_i B_{i'}\ , \\ (D_\mu X)^k_{ij} &= \partial_\mu X^k_{ij} + (V_\mu)^k_{k'} X^{k'}_{ij} - (V_\mu)^{i'}_i X^k_{i'j} - (V_\mu)^{j'}_j X_{ij'}\ . \end{aligned} \tag{8}$$

Here X_{ij}^k can be $X_{[i,j]}^k$ or $X_{\{i,j\}}^k$ depending on whether it is a $\bar{\mathbf{6}}$ or a $\mathbf{15}$. When constructing the above leading order chiral Lagrangian, we have assumed that the Lagrangian respects parity, charge-conjugation, Lorentz symmetries, and chiral symmetry in terms of local $SU(3)$ hidden gauge symmetry mentioned before. The explicit forms of the last two operators shall be explained below in detail. Note that the velocity v dependent scalar fields X_b and B we are using now include a wavefunction normalization factors $\sqrt{M_X}$ and $\sqrt{M_B}$ respectively. Therefore their mass dimensions are $[X] = [B] = 3/2$, and not 1, in this formulation.

We would like to comment that if X_b turns out to be a multiplet with $J^P = 1^{++}$ the analysis is similar. One just needs to replace the 0^{++} state for X by a 1^{++} state X_μ with a Lorentz index. In our discussions below we will concentrate on the $SU(3)$ symmetry properties and therefore will not show the Lorentz indices which should always be contracted.

In the exact $SU(3)$ limit, all the X_b states in each representation have the same mass, either $m_{\bar{6}}$ or m_{15}, which have been removed in the velocity-dependent heavy particle effective theory formalism. This degeneracy is broken by light quark masses, which is schematically represented as $\overline{X} M_\xi X$ in Eq. (7). Here $M_\xi \equiv \xi M \xi + \xi^\dagger M \xi^\dagger$ which transforms as an $SU(3)$ octet: $M_\xi \to U(x) M_\xi U^\dagger(x)$ and $M = \mathrm{diag}(m_u, m_d, m_s) \to L M R^\dagger$. Including all possible combinations, the corrections come from the following terms

$$
\begin{aligned}
&\text{For } \bar{\mathbf{6}}: \quad \frac{1}{2}\left(a' \overline{X}_k^{[i,j]} (M_\xi)_j^l X_{[i,l]}^k + b' \overline{X}_k^{[i,j]} (M_\xi)_l^k X_{[i,j]}^l \right) \,, \\
&\text{For } \mathbf{15}: \quad \frac{1}{2}\left(a \overline{X}_k^{\{i,j\}} (M_\xi)_j^l X_{\{i,l\}}^k + b \overline{X}_k^{\{i,j\}} (M_\xi)_l^k X_{\{i,j\}}^l \right) \,,
\end{aligned}
\tag{9}
$$

The parameters $a(a')$ and $b(b')$ have dimensionless $\sim O(1)$ couplings in this formalism.

IV. $X(\bar{b}q'q''\bar{q})$ MASS SPLITTING AND TWO-BODY DECAYS

Expanding the expressions in Eq.(9), we obtain the masses of the states in $\bar{\mathbf{6}}$ and $\mathbf{15}$ representations including the leading $SU(3)$ breaking corrections from a non-zero strange quark mass m_s:

$$
\begin{aligned}
&m(X^{'}_{ds\bar{u}}) = m(X^{'}_{su\bar{d}}) = m_{\bar{6}} + \frac{1}{2}a^{'}m_s\ ,\\
&m(X^{'}_{ud\bar{s}}) = m_{\bar{6}} + b^{'}m_s\ ,\\
&m(Y^{'}_{(uu,dd)\bar{s}}) = m_{\bar{6}} + \frac{1}{2}a^{'}m_s + \frac{1}{2}b^{\prime}m_s\ ,\\
&m(Y^{'}_{(u\bar{u},s\bar{s})d}) = m_{\bar{6}} + \frac{1}{4}a^{'}m_s\ ,\\
&m(Y^{'}_{(d\bar{d},s\bar{s})u}) = m_{\bar{6}} + \frac{1}{4}a^{'}m_s + \frac{1}{2}b^{'}m_s\ ,
\end{aligned}
\tag{10}
$$

for the $\bar{\mathbf{6}}$ representation, and

$$
\begin{aligned}
&m(Z_{uu\bar{d}}) = m(Z_{dd\bar{u}}) = m(Y_{\pi\bar{u}}) = m(Y_{\pi\bar{d}}) = m_{15}\ ,\\
&m(X_{su\bar{d}}) = m(X_{ds\bar{u}}) = m(Y_{\pi\bar{s}}) = m_{15} + \frac{1}{2}am_s\ ,\\
&m(Z_{ss\bar{u}}) = m(Z_{ss\bar{d}}) = m_{15} + am_s\ ,\\
&m(X_{ud\bar{s}}) = m(Z_{uu\bar{s}}) = m(Z_{dd\bar{s}}) = m_{15} + bm_s\ ,\\
&m(Y_{\eta\bar{u}}) = m(Y_{\eta\bar{d}}) = m_{15} + \frac{1}{3}am_s + \frac{2}{3}bm_s\ ,\\
&m(Y_{\eta\bar{s}}) = m_{15} + \frac{5}{6}am_s + \frac{2}{3}bm_s\ ,
\end{aligned}
\tag{11}
$$

for the $\mathbf{15}$ representation.

The above equations imply that, when u and d quark mass contributions are neglected, $X^{'}_{ds\bar{u}}(X_{ds\bar{u}})$ and $X^{'}_{su\bar{d}}(X_{su\bar{d}})$ are degenerate in mass. But the correction to $X^{'}_{ud\bar{s}}$ $(X_{ud\bar{s}})$ will be different. $m_{\bar{6}}$ and m_{15} are the common masses for the states in the $\bar{\mathbf{6}}$ and $\mathbf{15}$ representations respectively. The states $Z_{uu\bar{d}}$, $Z_{dd\bar{u}}$, $Y_{\pi\bar{u}}$ and $Y_{\pi\bar{d}}$ in the $\mathbf{15}$ representation do not receive any corrections and are degenerate in mass. Mass measurement for any one of them will determine m_{15}. While $m_{\bar{6}}$ can be determined by measuring $2m(Y^{'}_{(u\bar{u},s\bar{s})d}) - m(X^{'}_{ds\bar{u}})$.

One can derive sum rules for the masses of tetraquark states in the $\bar{\mathbf{6}}$ or $\mathbf{15}$ analogous to Gell-Mann-Okubo formulae. The sum rules obtained from Eqs.(11) and (12) will provide useful information to reconstruct the masses such as the mass of $X(X')_{ud\bar{s}}$. We will come back to this later.

The properties of $X(\bar{b}q'q''\bar{q})$ can be probed by studying their decays. Among the simplest dacy modes are the two-body decay with a B meson and a light pesudoscalar meson in Π which can be obtained from the leading order chiral Lagrangian constructed in the previous section. The terms responsible for these decays come from the last operator of Eq.(7), which are renormalizable interactions with dimensionless couplings α and $\alpha^{'}$ of $\sim O(1)$:

$$\alpha^{'}\overline{B}^{i}(v\cdot A)^{j}_{k}X^{k}_{[i,j]}\ ,\ \ \alpha\overline{B}^{i}(v\cdot A)^{j}_{k}X^{k}_{\{i,j\}}\ , \tag{12}$$

for X_b in the $\bar{\mathbf{6}}$ and $\mathbf{15}$ representations, respectively. Note that there is a unique operator for X_b' and X_b that is relevant to the pionic transitions, $X_b\to B\Pi$, at leading order in chiral expansion.

The term inducing X_b to a triplet B_i and the pseudoscalar octet Π^i_j from the above can be parameterized as

$$\alpha^{'}X^{k}_{[i,j]}\overline{B}^{i}\Pi^{j}_{k}\ ,\ \ \alpha X^{k}_{\{i,j\}}\overline{B}^{i}\Pi^{j}_{k}\ . \tag{13}$$

Depending on values for the beauty tetraquark X_b mass, there are different possible decay modes allowed. If the mass of a beauty tetraquark has a mass around 5568 MeV as what D0 found, only $X_b\to B_i\pi$ modes are open kinematically. In this case, expanding the above we obtain the decay amplitudes for $A(X(X')\to B_i\Pi^j_k)$ with a π in the final states :

$$\begin{aligned}
&A(X^{'}_{su\bar{d}}\to B^0_s\pi^+)=-A(X^{'}_{ds\bar{u}}\to B^0_s\pi^-)=A(Y^{'}_{(u\bar{u},d\bar{d})s}\to B^0_s\pi^0)=\frac{1}{\sqrt{2}}\alpha^{'}\ ,\\
&A(Y^{'}_{(u\bar{u},s\bar{s})d}\to B^0\pi^0)=A(Y^{'}_{(d\bar{d},s\bar{s})u}\to B^+\pi^0)=-\frac{1}{2\sqrt{2}}\alpha^{'}\ ,\\
&A(Y^{'}_{(u\bar{u},s\bar{s})d}\to B^+\pi^-)=-A(Y^{'}_{(d\bar{d},s\bar{s})u}\to B^0\pi^0)=\frac{1}{2}\alpha^{'}\ ,
\end{aligned} \tag{14}$$

for the states in the $\bar{6}$, and

$$\begin{aligned}
&A(X_{su\bar{d}}\to B^0_s\pi^+)=A(X_{ds\bar{u}}\to B^0_s\pi^-)=A(Y_{\pi s}\to B^0_s\pi^0)=\frac{1}{\sqrt{2}}\alpha\ ,\\
&A(Y_{\pi d}\to B^0\pi^0)=A(Y_{\pi u}\to B^+\pi^0)=\frac{1}{2}(1+\frac{1}{\sqrt{2}})\alpha\ ,\\
&A(Y_{\pi d}\to B^+\pi^-)=-A(Y_{\pi u}\to B^0\pi^+)=\frac{1}{2}\alpha\ ,\\
&A(Y_{\eta u}\to B^+\pi^0)=-A(Y_{\eta d}\to B^0\pi^0)=\frac{1}{2\sqrt{3}}(1-\frac{1}{\sqrt{2}})\alpha\ ,\\
&A(Y_{\eta u}\to B^0\pi^+)=A(Y_{\eta d}\to B^+\pi^-)=\frac{1}{2\sqrt{3}}\alpha\ ,\\
&A(Z_{dd\bar{u}}\to B^0\pi^-)=A(Z_{uu\bar{d}}\to B^+\pi^+)=\alpha\ ,
\end{aligned} \tag{15}$$

for the states in $\mathbf{15}$.

Although the decay channels are limited, they can be used to study many properties of beauty tetraquark X_b. We will show in the next section that with the above limited decay channels, one can extract a lot of information about the properties of X_b.

Several theoretical estimate of X_b masses are higher than 5568 MeV. As pointed out in Ref.[9] it is hard to understand that the bsu baryons Ξ_b and Ξ_b^* have masses of 5794 and 5945 MeV, but the X_b states composed of $\bar{b}su\bar{d}$, $\bar{b}ds\bar{u}$, and $\bar{b}ud\bar{s}$ with an additional valence quark would be hundreds of MeV lighter. Without a detailed dynamic model, this cannot be addressed. If it turns out that some of the states in X_b have a mass larger than 5568 MeV, so that $X \to BK$ or even $X \to B\eta$ are also allowed kinematically. In this case, we have additional decay channels to study:

$$\begin{aligned}
&A(X'_{ds\bar{u}} \to B^0 K^-) = A(X'_{ud\bar{s}} \to B^+ K^0) \\
=&-A(X'_{su\bar{d}} \to B^+ \bar{K}^0) = -A(X'_{ud\bar{s}} \to B^0 K^+) = \frac{1}{\sqrt{2}}\alpha' \ , \\
&A(Y'_{(d\bar{d},s\bar{s})u} \to B^0_s K^+) = A(Y'_{(u\bar{u},d\bar{d})s} \to B^0 \bar{K}^0) \\
=&-A(Y'_{(u\bar{u},d\bar{d})s} \to B^+ K^-) = -A(Y'_{(u\bar{u},s\bar{s})d} \to B^0_s K^0) = \frac{1}{2}\alpha' \ ,
\end{aligned} \tag{16}$$

$$A(Y'_{(d\bar{d},s\bar{s})u} \to B^+\eta) = -A(Y'_{(u\bar{u},s\bar{s})d} \to B^0\eta) = \sqrt{\frac{3}{8}}\alpha' \ , \tag{17}$$

for the states in the $\bar{6}$, and

$$\begin{aligned}
&A(X_{su\bar{d}} \to B^+ \bar{K}^0) = A(X_{ds\bar{u}} \to B^0 K^-) \\
=&A(X_{ud\bar{s}} \to B^+ K^0) = A(X_{ud\bar{s}} \to B^0 K^+) = \frac{1}{\sqrt{2}}\alpha \ , \\
&A(Y_{\pi s} \to B^+ K^-) = -A(Y_{\pi s} \to B^0 \bar{K}^0) = \frac{1}{2}\alpha \ , \\
&A(Y_{\eta d} \to B^0_s K^0) = A(Y_{\eta s} \to B^0_s K^+) = -\frac{1}{\sqrt{3}}\alpha \ , \\
&A(Y_{\eta s} \to B^0 \bar{K}^0) = A(Y_{\eta s} \to B^+ K^-) = \frac{1}{2\sqrt{3}}\alpha \ , \\
&A(Z_{uu\bar{s}} \to B^+ K^+) = A(Z_{dd\bar{s}} \to B^0 K^0) \\
=&A(Z_{ss\bar{d}} \to B^0_s \bar{K}^0) = A(Z_{ss\bar{u}} \to B^0_s K^-) = \alpha \ ,
\end{aligned} \tag{18}$$

$$\begin{aligned}
&A(Y_{\pi u} \to B^+\eta) = -A(Y_{\pi d} \to B^0\eta) = \frac{1}{2\sqrt{3}}\alpha \ , \\
&A(Y_{\pi d} \to B^0\eta) = -A(Y_{\pi u} \to B^+\eta) = \frac{1}{2\sqrt{6}}\alpha \ , \\
&A(Y_{\eta u} \to B^+\eta) = A(Y_{\eta d} \to B^0\eta) = \frac{1}{6}(1 + \frac{5}{\sqrt{2}})\alpha \ , \\
&A(Y_{\eta s} \to B^0_s\eta) = \frac{1}{3}(2 + \frac{1}{\sqrt{2}})\alpha \ ,
\end{aligned} \tag{19}$$

for the states in **15**.

Note that in the above equations for decay amplitudes, there is a common overall factor $E_\pi/\sqrt{2}f_\pi$ from $v\cdot A$ term, which we do not show explicitly in the above expressions. Also one needs to include the wavefunction normalization factors, $\sqrt{M_X M_B}$ for the X_b and B fields as mentioned before.

From the leading order chiral Lagrangian with all the aforementioned factors correctly included, one can use any one of the decay width, if measured, to determine the coupling constant $\alpha'(\alpha)$. For example we can evaluate the decay rate for $X_b \to B_s\pi^\pm$ as follows:

$$\mathcal{M}(X \to B_s\pi^\pm) = \frac{\alpha'}{\sqrt{2}f_\pi}E_\pi\sqrt{M_X M_{B_s}}$$
$$\Gamma(X \to B_s\pi^\pm) = \frac{\alpha'^2}{16\pi}\,|\vec{p}_\pi|\,\frac{M_{B_s}}{M_X}\left(\frac{E_\pi}{f_\pi}\right)^2\ , \tag{20}$$

We have used an approximation $E_\pi \approx (M_X - M_{B_s})$, ignoring the recoil energy of B_s, and $p_\pi = \sqrt{E_\pi^2 - m_\pi^2}$. Measurement of the decay width, the parameter α' can be determined. Had the D0 data to be correct for the decay width to be 21.9 MeV for this decay, one would obtain $|\alpha'| \approx 1.3$ if X_b belongs to $\bar{\mathbf{6}}$. Similarly $|\alpha| \approx 1.3$ if X_b belongs to the **15** representation.

V. DISCUSSIONS

We now discuss how the $SU(3)$ properties of the beauty tetraquark states can be obtained by studying possible two-body decays with a B meson and a light pesudoscalar octet meson in the final states. If $SU(3)$ is at work in organizing the beauty tetraquark X_b states, the X_b states should come in the form of a complete irreduceable representation. This means that if a X_b state composed of $\bar{b}su\bar{d}$ or $\bar{b}ds\bar{u}$ as claimed by D0 is found, there should be other state come along in $\bar{\mathbf{6}}$ or **15**. Then one should determined whether X_b belongs to $\bar{\mathbf{6}}$ or **15**. The obvious way is to find all possible states in a given multiplet. To know whether a particular states are produced one has to look for its decay product the analyze the properties. Therefore the decay channels are crucial in establishing the properties of the beauty tetraquark states. We now using the chiral Lagrangian constructed in section III to outline the strategy of carrying out the analysis.

From Eqs.(14) to (19), we see that if kinematically allowed, each of the beauty tetraquark state in $\bar{\mathbf{6}}$ or **15** can have two-body decays of the type $X_b \to B\Pi$. By analyzing the invariant mass of the final product one can identify whether there are resonant states which can be identified with a state in X_b. The doubly charged tetraquark states $Z_{uu\bar{d}}$ and $Z_{uu\bar{s}}$ are the distinctive feature for that the beauty tetraquark to be a **15** representation.

It D0 data is correct, only $X_b \to B\pi$ decays are kinematically allowed. In this case the available decay channels to analyze the X_b state properties are limited. The allowed decay channels are given in eqs. (15) and (16). We however note that a lot of information about X_b can be extracted, in particular it is sufficient to determine whether the X_b belongs to $\bar{\mathbf{6}}$ or **15**.

Assuming $X_{su\bar{d},ds\bar{u}}$ have been discovered by analyzing $X_{su\bar{d}}(X_{ds\bar{u}}) \to B_s^0\pi^+$ (or $B_s^0\pi^-$), to determine whether X_b belongs to **15** or not is to find the doubly charged unique state $Z_{uu\bar{d}}$ which can decay into $B^+\pi^+$ with a decay width twice as large as $X_{su\bar{d}}(X_{ds\bar{u}}) \to B_s^0\pi^+$ (or $B_s^0\pi^-$). Should this state exist, it can be detected. $Z_{dd\bar{u}} \to B^0\pi^-$ can also add new information. Another type of decays is final states with $B^0\pi^+$ from $Y_{\pi u}$ and $Y_{\eta u}$. This only occurs if X_b belongs to the **15** representation.

Now we turn to final states which are allowed for both $\bar{\mathbf{6}}$ and **15** representations. We note that the ratio $R(B^+\pi^-/B_s^0\pi^0)$ of final states with $B^+\pi^-$ and $B_s^0\pi^0$ can also provide useful information.

If X_b belongs to $\bar{\mathbf{6}}$, $Y'_{(u\bar{u},s\bar{s})d} \to B^+\pi^+$ and $Y'_{(u\bar{u},d\bar{d})s} \to B_s^0\pi^0$ are relevant. The ratio $R(B^+\pi^-/B_s^0\pi^0) = \Gamma(Y'_{(u\bar{u},s\bar{s})d} \to B^+\pi^+)/\Gamma(Y'_{(u\bar{u},d\bar{d})s} \to B_s^0\pi^0)$ is 1/2. While for X_b being in the **15** representation, $Y_{\pi d} \to B^+\pi^-$, $Y_{\eta d} \to B^+\pi^-$ and $Y_{\pi s} \to B_s^0\pi^0$ are relevant. If experiments can find the two separate states $Y_{\pi d}$ and $Y_{\eta d}$, this already tells that $X(5568)$ belongs to the **15** representation. However, if experiments cannot distinguish them, the signal for $B^+\pi^-$ will be counted together. In this case the ratio $R(B^+\pi^-/B_s^0\pi^0)$ is 2/3 which is substantially different than the case with X_b belongs to the $\bar{\mathbf{6}}$.

Once the representation for X_b has been determined, one can obtain all component state masses. For those states which can decay into a pion in the final states, their masses can determined from experimental data. For those which do not have such decay channels, the sum rules derived from Eqs.(11) and (12) can be used to obtain the masses for those states. The masses of those states that cannot be studied by $B\pi$ decays can be expressed in terms of the masses of those states that can be studied as the following for $\bar{\mathbf{6}}$ or $\mathbf{15}$,

$$\begin{aligned}
&m(X'_{ud\bar{s}}) = 2m(Y'_{(d\bar{d},s\bar{s})u}) - m(X'_{ds\bar{u}}) \ , \\
&m(Z_{ss\bar{u}}) = m(Z_{ss\bar{d}}) = 2m(X_{su\bar{d}}) - m(Y_{\pi\bar{u}}) \ , \\
&m(Y_{\eta\bar{s}}) = m(Y_{\pi\bar{s}}) + m(Y_{\eta\bar{u}}) - m(Y_{\pi\bar{u}}) \ , \\
&m(X_{ud\bar{s}}) = m(Z_{uu\bar{s}}) = m(Z_{dd\bar{s}}) = \frac{1}{2}\left(3m(Y_{\eta\bar{s}}) + 4m(Y_{\pi\bar{u}}) - 5m(X_{su\bar{d}})\right) \ .
\end{aligned} \tag{21}$$

We would also like to mention the possibility of using an off-shell light pseudoscalar which subsequent decay into two photons to test whether X belongs to $\bar{\mathbf{6}}$ or $\mathbf{15}$, taking $B^0_s\eta \to B^0_s\gamma\gamma$ final state as an example. This can only happen if X belongs to $\mathbf{15}$ by $Y_{\eta s} \to B^0_s\eta$ which is unlikely to be kinematically allowed. However, the η has a large branching ratio (39%)[10] decay into $\gamma\gamma$, therefore $Y_{\eta s} \to B^0_s\gamma\gamma$ may be possible to be observed. There are other states which can decay into $B^0_s\gamma\gamma$, such as $Y'_{(u\bar{u},d\bar{d})s}, Y_{\pi s} \to B^0_s\pi^0 \to B^0_s\gamma\gamma$ with much larger decay widths. By removing events of the two γ close to pion mass can help to single out events with the two γs from η. This, however, also results in a much suppressed event number with $\Gamma(Y_{\eta s} \to B^0_s\eta \to B_s\gamma\gamma)/\Gamma(Y_{\pi s} \to B^0_s\pi^0 \to B_s\gamma\gamma) \sim 4 \times 10^{-5}$. Although $Y_{\eta s} \to B^0_s\eta \to B^0_s\gamma\gamma$ may be in principle provide useful information about the nature of the X, it is extremely difficult to carry out such tests experimentally.

If it turns out that the $X_b \to BK$ and $X_b \to B\eta$ decay channels are also kinematically open, more information can be obtained. For X_b belongs to $\bar{\mathbf{6}}$, with $X_b \to B\pi$ decay channels only, no direct information on $X'_{ud\bar{s}}$ can be obtained. One needs to use mass relation to infer its mass. Now with $X_b \to BK$ channel opened, one can directly measure this state by looking at the resonant structure of B^+K^0 and B^0K^+ final states. The decay amplitudes for these two decay channels are the same as $X'_{su\bar{d}} \to B^0_s\pi^-$, but with smaller decay widths because of suppression of phase space.

For X_b belongs to $\mathbf{15}$, if only $B\pi$ channels are kinematically allowed there are more states which cannot be studied directly probed by looking at decay products, for example the states, $X_{ud\bar{s}}$, $Y_{\pi s}$, $Y_{\eta s}$, $Z_{ss\bar{u}}$, $Z_{ss\bar{d}}$, $Z_{uu\bar{s}}$ and $Z_{dd\bar{s}}$ do not have $B\pi$ decay channels. We list possible decay channels these states can be studied once $X_b \to BK, \ B\eta$ decay channels are kinematically allowed in the following:

- When $X_b \to BK$ channels are open, $X_{ud\bar{s}} \to B^+K^0, B^0K^+$ with the same decay amplitude as $X_{su\bar{d}} \to B^0_s\pi^-$ can be used to study this state.

- $Y_{\pi s}$ and $Y_{\eta s}$ can be studied by studying B^+K^- and $B^0\bar{K}^0$ final states. If $Y_{\eta s} \to B_s^0\eta$ is also allowed, it can help probe $Y_{\eta s}$ properties.

- The doubly charged state $Z_{uu\bar{s}}$ can be studied by B^+K^+ final state.

- The final states $B_s^0\bar{K}^0$ and $B_s^0K^-$ can provide information about $Z_{ss\bar{d}}$ and $Z_{uu\bar{s}}$ state, respectively.

VI. CONCLUSIONS

In conclusion, we studied properties of beauty tetraquark X_b states with four different quarks and and their associated $SU(3)$ partner states. These states containing four different quarks should be in $\bar{\mathbf{6}}$ or $\mathbf{15}$ of $SU(3)$ representations. We constructed the leading order chiral Lagrangian describing X_b decays into a B meson and a light pesudoscalar octet meson, and identified possible states which can be studied by these decays. Depending on the mass of X_b, the two-body decay channels $BK,\ B\eta$ may or may not be kinematically allowed. We find that evn just $B\pi$ decay channels are allowed, a lot of information about the properties X_b can be extracted. We discussed searching strategies to distinguish whether X_b states belong to $\bar{\mathbf{6}}$ or $\mathbf{15}$. We found that if X_b belongs to $\mathbf{15}$, there must be a new doubly charged four quark state which can decay into $B^+\pi^+$ with a decay width twice as large as that for $X^{(\prime)}_{su\bar{d},ds\bar{u}} \to B_s^0\pi^\pm$. There are several other features which can also distinguish whether X_b belongs to $\bar{\mathbf{6}}$ and $\mathbf{15}$. We derived sum rules for X_b mass useful in constructing all masses in a given representation. If the masses of X_b allow $BK,\ B\eta$ decays to happen, more information about the X_b states can be obtained. As a final comment, we would like to point out that replacing the $\bar{b}$ quark by a $\bar{c}$ quark, one can obtain a similar class of states with the same $SU(3)$ properties[6]. Although the LHCb results did not confirm D0 finding of $X(5568)$ states, the interesting properties of X_b are still worth investigating to understand more about hadron properties, we urge our experimental colleagues to carry out related searches.

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