# Dynamical supersymmetry breaking on magnetized tori and orbifolds

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#### Abstract

We construct several dynamical supersymmetry breaking (DSB) models within a single tendimensional supersymmetric Yang-Mills (SYM) theory, compactified on magnetized tori with or without orbifolding. We study the case that the supersymmetry breaking is triggered by a strong dynamics of  $SU(N_C)$  SYM theory with  $N_F$  flavors contained in the four-dimensional effective theory. We show several configurations of magnetic fluxes and orbifolds, those potentially yield, below the compactification scale, the field contents and couplings required for triggering DSB. We especially find a class of self-complete DSB models on orbifolds, where all the extra fields are eliminated by the orbifold projection and DSB successfully occurs within the given framework. Comments on some perspectives for associating the obtained DSB models with the other sectors, such as the visible sector and another hidden sector for, e.g., stabilizing moduli, are also given.

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# 1 Introduction

Supersymmetric models for particle physics have been quite actively studied for decades, and they will attract much more attention under the second season of Large Hadron Collider. The most famous and successful one is the minimal supersymmetric standard model (MSSM), which is indeed respected in many of supersymmetric models. In a generic model building, these supersymmetric models are accompanied by a sequestered hidden sector which breaks supersymmetry (SUSY) spontaneously, even when that is not mentioned explicitly. The SUSY breaking sector is certainly a key constituent of SUSY scenarios because of the fact that SUSY is broken in our real world at least below the electroweak scale.

A wide variety of models for SUSY breaking sectors, solely or in association with the visible sector, have been proposed so far. In particular, many models of dynamical supersymmetry breaking (DSB) due to the strong dynamics of non-Abelian gauge theories, were proposed after the Seiberg duality revealed infrared behaviors of strongly coupled  $\mathcal{N} = 1$  SUSY theories [1, 2]. These DSB scenarios are quite promising for completing SUSY models, because a large hierarchy between the Planck scale and the SUSY breaking scale (intrinsic strong scale) is easily generated by a logarithmic running of strong gauge couplings.

In this paper, we construct DSB models in four-dimensional (4D) low-energy effective theory derived from ten-dimensional (10D) supersymmetric Yang-Mills (SYM) theories compactified on three 2-tori with magnetic fluxes. Extra-dimensional space with magnetic fluxes has been addressed as a hopeful candidate for the origin of flavor structure of the quarks and leptons, which is a big mystery of the standard model and its SUSY extensions. Magnetic fluxes on tori lead to gauge symmetry breaking and derive a product gauge group from a single large group, realizing generations of chiral fermions as degenerate zero-modes in the bi-fundamental representations [3, 4] in the 4D effective theory.

Indeed, a semi-realistic flavor structure was obtained in an MSSM-like model derived from the magnetized SYM theories [5, 6], where a suitable Yukawa hierarchy consistent with the observed masses and mixings of quarks and leptons is realized. This hierarchy is essentially due to the quasi-localization of wavefunctions in extra-dimensional space [7] caused by the magnetic fluxes. It was also shown that this model can be consistent with the recent experimental constraints on the Higgs boson mass and SUSY particle spectra, where a certain class of SUSYbreaking mediation mechanism is assumed [6].

With  $Z_2$  orbifolding [8], these attractive properties of magnetic fluxes remain still and threegeneration models of the quarks were studied on orbifolds [9]. In Refs. [10], realistic Yukawa hierarchies were indeed realized on magnetized  $Z_2 \times Z'_2$  orbifolds. These magnetized orbifolds<sup>1</sup> lead to a different flavor structure from the magnetized tori without orbifolding [15, 16, 17]. Besides that, the orbifold projection can eliminate extra adjoint fields (those remain massless on tori and are phenomenologically disfavored in many cases), which would be a great advantage in a realistic model building.

Thus, magnetized toroidal compactification with or without orbifolding is an exciting possibility of realizing the suitable visible (MSSM) sector in extra-dimensional field theories. As a

<sup>&</sup>lt;sup>1</sup> Recently,  $Z_3$ ,  $Z_4$  and  $Z_6$  orbifold models were also studied [11, 12, 13, 14].

second step towards completing these models, it is important to study SUSY breaking mechanisms on magnetized tori and orbifolds, which is the main purpose of this paper.

The following sections are organized as follows.

In Sec. 2, we review the 10D SYM theories compactified on magnetized tori. We adopt a  $4D \mathcal{N} = 1$  description of 10D SYM theories, which is quite useful for the later model building. With this description, we give an overview of zero-mode configurations when the theory is compactified on three 2-tori with magnetic fluxes with/without  $Z_2$  orbifolding.

Sec. 3 is the main part of this paper, where the construction of various DSB models is shown with several concrete magnetized backgrounds. In Sec. 3.1, we show certain aspects for DSB on magnetized tori with a simple configuration of magnetic fluxes which yields the gauge symmetry breaking  $U(N) \to U(N_C) \times U(N_X)$ , by assuming certain vacuum expectation values (VEVs) of the adjoint fields and their masses around them. First, a (metastable) DSB model is constructed in Sec. 3.1.1 respecting the Intriligator-Seiberg-Shih (ISS) model [18], that is,  $SU(N_C)$  SYM theory with  $N_F$  fundamental massive quarks, satisfying  $N_C - 1 \leq N_C$  $N_F < \frac{3}{2}N_C$ . We also construct a DSB model without massive quarks in Sec. 3.1.2, deriving a tadpole term from tri-linear couplings in the superpotential, via a suitable strong dynamics. In Sec. 3.2, we extend the flux configuration in such a way that the gauge symmetry breaking  $U(N) \to U(N_C) \times U(N_X) \times U(N_Y)$  occurs. Then, we show a class of self-complete DSB models on magnetized orbifolds, where all the extra unwanted fields are eliminated by the orbifold projection and DSB successfully occurs within the given framework without any nontrivial assumptions. In Sec. 3.3, we comment on some perspectives for embedding the obtained DSB models into a single whole system including the visible (MSSM) sector and another hidden sector for the moduli stabilization.

We conclude with the future prospects in Sec. 4.

In Appendix A, the other flux configurations are shown for deriving the same class of DSB models as the one demonstrated in Sec. 3.2.

## 2 10D SYM theory on magnetized tori

We review 10D SYM theories on magnetized tori and orbifolds briefly, following Ref. [19]. In this paper, the theories are compactified on  $M^4 \times T^2 \times T^2 \times T^2$  with/without  $Z_2$  orbifolding. First, we introduce a 4D  $\mathcal{N} = 1$  description of higher dimensional SUSY theories which is quite useful for our model building. Using the description, we turn on Abelian magnetic fluxes in extra dimensional space and show an overview of zero-mode configurations on the magnetized tori. Finally, we explain about magnetized  $Z_2$  orbifolds which are one of key ingredients in some of our DSB models.

### **2.1 4D** $\mathcal{N} = 1$ decomposition

The 10D SYM theory consists of a 10D vector field  $A_M$  (M = 0, 1, ..., 9) and a 10D Majorana-Weyl spinor field  $\lambda$ . For the extra dimensional directions, we define complex coordinates  $z^i$  (i = 1, 2, 3) and vectors  $A_i$  with complex structures  $\tau_i$  as

$$z^{i} \equiv \frac{1}{2} \left( x^{2+2i} + \tau_{i} x^{3+2i} \right), \qquad A_{i} \equiv -\frac{1}{\operatorname{Im} \tau_{i}} \left( \tau_{i}^{*} A_{2+2i} - A_{3+2i} \right).$$

The periodic boundary conditions for the three 2-tori are given by  $z_i \sim z_i + 1$  and  $z_i \sim z_i + \tau_i$ . On this complex basis, the metric of three 2-tori is represented by

$$ds_{6\mathrm{D}}^2 \equiv 2h_{\bar{i}j}d\bar{z}^{\bar{i}}dz^j, \qquad h_{\bar{i}j} = \delta_{\bar{i}j}2(2\pi R_i)^2,$$

where  $R_i$  determines the period of *i*-th 2-torus.

Now, the 10D vector field  $A_M$  has been decomposed into a 4D vector and three complex scalar fields,  $A_{\mu}$  and  $A_i$ . The spinor field can also be decomposed into four 4D Weyl spinors, which are distinguished by their chiralities on each 2-torus. We denote them as  $\lambda_{+++}$ ,  $\lambda_{+--}$ ,  $\lambda_{-+-}$  and  $\lambda_{--+}$  where the *i*-th subscript  $\pm$  expresses the chirality on the *i*-th 2-torus, and the others (e.g.,  $\lambda_{---}$ ) are excluded by the 10D Weyl condition. We redefine these four spinors as

$$\lambda_0 \equiv \lambda_{+++}, \quad \lambda_1 \equiv \lambda_{+--}, \quad \lambda_2 \equiv \lambda_{-+-}, \quad \lambda_3 \equiv \lambda_{--+},$$

for later convenience.

These 4D component fields form 4D  $\mathcal{N} = 1$  supermultiplets, which are assigned to a vector V and three chiral superfields  $\phi_i$  as

$$V \equiv -\theta \sigma^{\mu} \bar{\theta} A_{\mu} + i \bar{\theta} \bar{\theta} \theta \lambda_{0} - i \theta \theta \bar{\theta} \bar{\lambda}_{0} + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D, \qquad (1)$$

$$\phi_i \equiv \frac{1}{\sqrt{2}} A_i + \sqrt{2} \theta \lambda_i + \theta \theta F_i.$$
<sup>(2)</sup>

The authors of Refs. [20, 21] proposed an action in the 4D  $\mathcal{N} = 1$  superspace, that is equivalent to the usual component-action of 10D SYM theory with the definitions (1) and (2). Ref. [19] showed its extension to the toroidal compactifications where background magnetic fluxes are turned on. In the superspace formulation, a 4D  $\mathcal{N} = 1$  SUSY out of the full  $\mathcal{N} = 4$  SUSY possessed by 10D SYM theories becomes manifest, which is preserved by the configurations of magnetic fluxes. The  $\mathcal{N} = 1$  SUSY-preserving conditions are read from field equations for the auxiliary fields D and  $F_i$ , those are shown later.

### 2.2 Zero-modes on magnetized tori

Next we show the zero-mode structure on magnetized tori. In U(N) gauge theory, magnetic fluxes on the *i*-th 2-torus can be represented by  $N \times N$  matrix  $M^{(i)}$  in

$$\langle A_i \rangle = \frac{\pi}{\operatorname{Im} \tau_i} M^{(i)} \bar{z}_{\bar{i}}.$$

We consider nonvanishing integer values for only diagonal entries of  $M^{(i)}$ , i.e., the Abelian magnetic fluxes. When some of them are degenerate, the gauge symmetry is broken as  $U(N) \rightarrow U(N)$ 

 $U(N_a) \times U(N_b) \times \cdots$ . We require these magnetic fluxes to satisfy conditions  $\langle F_i \rangle = \langle D \rangle = 0$  to preserve 4D  $\mathcal{N} = 1$  SUSY. These can be rewritten simply as [19, 22]

$$\frac{1}{\mathcal{A}^{(1)}}M^{(1)} + \frac{1}{\mathcal{A}^{(2)}}M^{(2)} + \frac{1}{\mathcal{A}^{(3)}}M^{(3)} = 0,$$
(3)

where  $\mathcal{A}^{(i)}$  represents the area of the *i*-th 2-torus. If this is not satisfied, SUSY is broken at a compactification scale which is, in general, extremely higher than the electroweak scale and some of SUSY particles get tachyonic masses due to  $\langle D \rangle \neq 0$ .

In the following, we denote (a, b)-entries of U(N) adjoint superfield  $\phi_j$  by  $\phi_j^{ab}$ . For such bifundamental fields of  $U(N_a) \times U(N_b)$ , zero-mode equations on the magnetized 2-tori are given by

$$\left[\bar{\partial}_{\bar{i}} + \frac{\pi}{2\mathrm{Im}\,\tau_i} M^{(i)}_{ab} z_i\right] \phi^{ab}_j = 0 \qquad \text{for} \quad i = j, \tag{4}$$

$$\left[\partial_i - \frac{\pi}{2\mathrm{Im}\,\tau_i} M^{(i)}_{ab} \bar{z}_{\bar{i}}\right] \phi^{ab}_j = 0 \quad \text{for} \quad i \neq j, \tag{5}$$

where  $M_{ab}^{(i)} \equiv M_a^{(i)} - M_b^{(i)}$  expresses the difference between two diagonal entries in  $M^{(i)}$ . For positive values of  $M_{ab}^{(i)}$ , we find  $M_{ab}^{(i)}$ -degenerate zero-modes as solutions of Eq. (4), while Eq. (5) has no normalizable solution. On the other hand, for  $M_{ab}^{(i)} < 0$ , only Eq. (5) allows  $|M_{ab}^{(i)}|$ degenerate zero-modes. Thus, magnetic fluxes yield generations of chiral fermions.

### 2.3 Magnetized orbifold

We now consider  $Z_2$  orbifolding on magnetized tori. The superfield description introduced above is compatible with orbifold projections, when we assign the same  $Z_2$  parity to all the component fields contained in a single superfield. For example, we consider a  $Z_2$  orbifold which acts on the first and the second 2-tori as  $(z_1, z_2) \rightarrow (-z_1, -z_2)$ . Under this  $Z_2$  transformation, the superfields behave as

$$V(x_{\mu}, z_{1}, z_{2}, z_{3}) = PV(x_{\mu}, -z_{1}, -z_{2}, z_{3})P^{-1},$$
  

$$\phi_{1}(x_{\mu}, z_{1}, z_{2}, z_{3}) = -P\phi_{1}(x_{\mu}, -z_{1}, -z_{2}, z_{3})P^{-1},$$
  

$$\phi_{2}(x_{\mu}, z_{1}, z_{2}, z_{3}) = -P\phi_{2}(x_{\mu}, -z_{1}, -z_{2}, z_{3})P^{-1},$$
  

$$\phi_{3}(x_{\mu}, z_{1}, z_{2}, z_{3}) = P\phi_{3}(x_{\mu}, -z_{1}, -z_{2}, z_{3})P^{-1},$$

where the projection operator P is given by an  $N \times N$  matrix satisfying  $P^2 = 1$ . Then, all the elements are assigned into either even- or odd-parity mode under this  $Z_2$  transformation.

Orbifold projections reduce the number of degenerate zero-modes generated by magnetic fluxes, or eliminate them completely. Ref. [8] identified the number of degeneracy of each  $Z_2$ -eigenmode with the sequence of magnetic fluxes on  $Z_2$  orbifolds, that is shown in Table 1.

From these results, we find that the orbifold background gives variety to a magnetized model building. In the next section, we construct various DSB models on magnetized tori and orbifolds based on them.

M	0	1	2	3	4	5	6	7	8	9	10	2n	2n + 1
Even	1	1	2	2	3	3	4	4	5	5	6	n+1	n+1
Odd	0	0	0	1	1	2	2	3	3	4	4	n-1	n

Table 1: This shows the number of active zero-modes on the magnetized orbifold.

# 3 Dynamical supersymmetry breaking

In this section, we construct DSB models on a variety of magnetized background.

First, we consider the simple configurations of magnetic fluxes leading to a gauge symmetry breaking  $U(N) \rightarrow U(N_C) \times U(N_X)$ , and show some specific configurations with which the resultant zero-modes contain certain DSB models such as the ISS model [18] and others. In the ISS model, for example, SUSY is broken by a strong dynamics of  $SU(N_C)$  gauge theory with  $N_F$  flavors. In our magnetized model building,  $N_C$  and  $N_F$  are determined by the degeneracies of Abelian magnetic fluxes and the degeneracies of the bi-fundamental zero-modes, respectively. These models seem quite simple but there appear some extra massless modes, those should be eliminated or decoupled somehow to obtain successful DSB. As we will see, orbifold projections are not available for such a purpose, and we have to assume some extrinsic effects to eliminate the extra fields in this simple class of models.

In the second part of this section, we consider more structural flux configurations that cause a gauge symmetry breaking  $U(N) \rightarrow U(N_C) \times U(N_X) \times U(N_{X'})$ . A great advantage of such extended configurations is that all the extra fields can be eliminated by a combination of magnetic fluxes and a certain orbifold projection, within a given framework of magnetized orbifold. They are really promising at least as long as we focus on the SUSY breaking sector.

We finally discuss prospects of our DSB models in association with other sectors, such as the visible (MSSM) and other hidden (especially moduli stabilization) sectors.

## **3.1** Models with $U(N) \rightarrow U(N_C) \times U(N_X)$

#### 3.1.1 ISS-type models

In the first type of our model building, we try to realize the ISS model [18], that is, the magnetized background is required to derive  $SU(N_C)$  SYM theory with  $N_F$  massive fundamental flavors from a single 10D U(N) SYM theory. The IR description of such a model is given by

$$W = \lambda \phi_{in} \Phi^{ij} \bar{\phi}^n_i + \mu^2 \Phi^{ii},$$

where  $\Phi$  and  $\phi$  correspond to baryons and mesons  $(i, j = 1, 2, ..., N_F$  and  $n = 1, 2, ..., N_C)$ . We can see that all the F-terms of  $\Phi^{ij}$ ,  $F_{\Phi_{ij}} \sim \mu^2 \delta_{ij} + \lambda \phi_{in} \bar{\phi}_j^n$ , cannot vanish simultaneously for  $N_F > N_C$ . This is the so-called rank-condition mechanism of SUSY breaking. In generic  $SU(N_C)$  theories with  $N_F$  flavors, SUSY breaking metastable vacua are realized within a range  $N_C - 1 \leq N_F < \frac{3}{2}N_C$ . In particular, they can be long-lived when the quark mass scale is much smaller than the dynamical scale.

We consider the configurations of magnetic fluxes which break the gauge symmetry as  $U(N) \rightarrow U(N_C) \times U(N_X)$ . For a while, we take both the  $U(N_C)$  and  $U(N_X)$  gauge groups to

be non-Abelian  $(N_C, N_X \ge 2)$  for the sake of generality. Such magnetic fluxes are given by

$$M^{(1)} = \begin{pmatrix} 0 \times \mathbf{1}_{N_C} & 0\\ 0 & M \times \mathbf{1}_{N_X} \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 \times \mathbf{1}_{N_C} & 0\\ 0 & -1 \times \mathbf{1}_{N_X} \end{pmatrix},$$
$$M^{(3)} = \begin{pmatrix} 0 \times \mathbf{1}_{N_C} & 0\\ 0 & 0 \times \mathbf{1}_{N_X} \end{pmatrix}, \tag{6}$$

where these matrices represent the  $U(N_C + N_X)$  gauge space. This configuration preserves at least a 4D  $\mathcal{N} = 1$  SUSY with  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = M$  fixed for a positive value of M. The chirality projection caused by these magnetic fluxes eliminates the zero-modes of certain elements of the U(N) adjoint chiral superfields, and we find for M > 0,

$$\phi_1 = \begin{pmatrix} \Xi_1 & 0 \\ q & \Omega_1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \Xi_2 & \tilde{q} \\ 0 & \Omega_2 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} \Xi_3 & 0 \\ 0 & \Omega_3 \end{pmatrix}.$$

The magnetized background (6) induces M-pairs of vector-like quarks  $(q, \tilde{q})$  in off-diagonal entries of  $\phi_1$  and  $\phi_2$ . Diagonal entries,  $\Omega_i$  and  $\Xi_i$ , correspond to  $U(N_C)$  and  $U(N_X)$  adjoint fields respectively.

The 10D SYM theory allows couplings between  $\phi_i$ 's only in the form  $\phi_1\phi_2\phi_3$  in the  $\mathcal{N} = 1$ superpotential. When the Wilson lines for  $U(N_X)$  in the third 2-torus are somehow generated, they lead to a nonvanishing VEV of  $\Omega_3$  and then the mass term  $\langle \Omega_3 \rangle q \tilde{q}$  is generated for the quarks. Then, the ISS-type DSB would occur if the quark mass scale is smaller than the dynamical scale of  $SU(N_C)$  SYM theory. However, in order to realize the ISS model exactly, we need to further assume that the fluctuations of adjoint fields  $\Omega_i$  and  $\Xi_i$  around their VEVs  $\langle \langle \Omega_3 \rangle \neq 0, \langle \Omega_{1,2} \rangle = \langle \Xi_i \rangle = 0$  in the present case) should be eliminated or decoupled from the DSB dynamics. Orbifold projections are not useful for such a purpose, because the nonvanishing (continuous) Wilson lines are generically incompatible with the orbifold background. Here, we just assume these extra fields obtain heavy masses due to some extrinsic effects from, e.g., supergravity/string corrections.

Under the above assumption, we have two gauge theories with massive quarks:  $SU(N_C)$ SYM with  $M \times N_X$  fundamental flavors and  $SU(N_X)$  SYM with  $M \times N_C$  fundamental flavors. In the case with

$$N_C - 1 \leq M \times N_X < \frac{3}{2} N_C, \tag{7}$$

the ISS model is realized by the  $SU(N_C)$  gauge theory. In this scenario, we have another constraint on the values of  $N_C$  and  $N_X$ . The running of  $SU(N_X)$  gauge coupling must be milder than that of  $SU(N_C)$ , which leads to the constraint

$$M \times N_X - 3N_C < M \times N_C - 3N_X \Leftrightarrow N_X < N_C.$$
(8)

While one can easily see that both conditions (7) and (8) cannot be satisfied with M = 1, it becomes easier for  $M \ge 2$  to fulfill them and we can find many successful ansatzes, e.g.,

$$N_C = 3, \qquad N_X = 2, \qquad M = 2.$$

When the extra  $U(N_X)$  gauge theory is Abelian, that is,  $N_X = 1$ , we can realize similar models much easier, because the second condition (8) is not required in this case. Thus, we can construct DSB models concerning about only the first one (7).

#### 3.1.2 Models without massive quarks

We have another scenario on the magnetized background (6) where the nonvanishing Wilsonlines are not required for DSB. As we have noticed, a key ingredient of this background is the following coupling,

$$g\Omega_3 q\tilde{q} = g\left(\langle\Omega_3\rangle + \tilde{\Omega}_3\right)q\tilde{q},\tag{9}$$

where g is a coupling constant. In the previous model, we have assumed a nonvanishing VEV  $\langle \Omega_3 \rangle$  and the absence of its fluctuation  $\tilde{\Omega}_3$  at a low energy to realize the ISS-type DSB which has only the mass term for the vector-like quarks in the superpotential. Alternatively here we consider the case that  $\tilde{\Omega}_3(=\Omega_3)$  is active while the Wilson-line  $\langle \Omega_3 \rangle$  is vanishing.

Without the Wilson lines, the higher dimensional SYM theory does not produce any mass terms for q and  $\tilde{q}$  (as well as  $\Omega_3$ ) at least at the leading order. This allows us to infer that, turning on a VEV  $\langle \Omega_3 \rangle \neq 0$  breaks some kinds of global symmetries of higher-dimensional SYM theory on magnetized tori, which prohibits the masses of bi-fundamental (as well as adjoint) fields. Thus, in the following, we can consider our models to be a chiral theory, as long as we do not introduce the continuous Wilson lines. This will become more clear in the next subsection.

For the purpose to derive a DSB model without massive quarks, let us consider a situation where we can ignore the other block-diagonal entries of  $\phi_i$  than  $\Omega_3$  at a low energy (i.e.,  $\Omega_1$ ,  $\Omega_2$  and  $\Xi_i$  are decoupled) for simplicity. Again, this could not be realized by orbifolds because  $\Omega_3$  and  $\Xi_3$  have the same orbifold parity and both of them survive or vanish simultaneously under the orbifold projection. We should consider some extrinsic mechanisms to make the extra fields heavy as in the previous models. When they are somehow decoupled, the superpotential contains only the above Yukawa coupling (9).

In  $SU(N_C)$  SYM theory with  $N_f$  fundamental flavors, for  $N_C > N_F$ , the Affleck-Dine-Seiberg (ADS) potential [23, 24]

$$W_{\text{ADS}} = C_{N_C, N_F} \left(\frac{\Lambda^{3N_C - N_F}}{\det \hat{M}}\right)^{1/(N_C - N_F)},$$

is obtained, where  $\Lambda$  is the dynamical scale,  $N_F \times N_F$  matrix  $\hat{M}$  is defined as  $\hat{M}^i_{\ j} \equiv q^{in} \tilde{q}_{nj}$ , and  $C_{N_C,N_F}$  are constants. Our magnetized model contains  $SU(N_C)$  SYM with  $M \times N_X$  fundamental flavors and  $SU(N_X)$  SYM with  $M \times N_C$  fundamental flavors. We consider the case that the dynamics of the former non-Abelian gauge theory produces the above ADS potential, that is,  $N_C > M \times N_X$ . The total effective superpotential can be written in terms of the operator  $\hat{M}$  as

$$W_{\text{effective}} = g \operatorname{Tr} \Omega_3 \hat{M} + C_{N_C, N_F} \left( \frac{\Lambda^{3N_C - N_F}}{\det \hat{M}} \right)^{1/(N_C - N_F)}$$

This is almost the simplest DSB model found by Affleck, Dine and Seiberg [25]. This potential makes the operator  $\hat{M}$  develop a nonvanishing VEV,  $\langle \hat{M} \rangle \sim \Lambda$ , and the resulting low-energy superpotential for  $\Omega_3$  is

$$W = g\Lambda^2 \Omega_3 + W_0,$$

which is just like the Polonyi model [26].

When the extra gauge theory is non-Abelian,  $N_X \ge 2$ , we have to concern about the condition (8) on  $N_C$  and  $N_X$ , again. However, this is always satisfied when the ADS potential is generated,  $N_C > M \times N_X$ , for any positive value of M. As for Abelian cases  $N_X = 1$ , such an extra constraint is not of course required. Thus, we can obtain a wide variety of this class of models as well as the previous ISS-type models discussed in Sec. 3.1.1.

For  $N_C = N_F$ , the ADS potential is not generated. In this case, however, it is known that the strong dynamics induces chiral condensations yielding a vacuum with det $\langle \hat{M} \rangle \neq 0$ . Therefore the Yukawa coupling (9) produces a tadpole term for  $\Omega_3$  breaking SUSY. Thus, a DSB model can be also obtained for  $N_C = M \times N_X$ . Here, the consistency condition (8) requires  $M \geq 2$ .

## **3.2** Models with $U(N) \rightarrow U(N_C) \times U(N_X) \times U(N_Y)$

So far, we have assumed that the extra adjoint fields are somehow decoupled. We here propose another class of DSB models on magnetized orbifold, where DSB successfully occurs within the given framework without requiring any extrinsic effects. We will find that more structural configurations of magnetic fluxes which cause a gauge symmetry breaking  $U(N) \rightarrow U(N_C) \times$  $U(N_X) \times U(N_Y)$  lead to self-complete DSB models on  $Z_2 \times Z'_2$  orbifolds, where all the extra unwanted fields are eliminated below the compactification scale.

We first explain an overview of this new class of models before giving a concrete configuration of magnetized background. Let us consider the gauge symmetry breaking due to magnetized backgrounds as  $U(N) \rightarrow U(N_C) \times U(N_X) \times U(N_Y)$ . Field contents responsible for the DSB dynamics here are  $(i \neq j \neq k \neq i)$ 

where three diagonal-blocks represent the product gauge group  $U(N_C) \times U(N_X) \times U(N_Y)$  in  $U(N) = U(N_C + N_X + N_Y)$  adjoint matrices and then the off-diagonal blocks in  $\phi_i$ 's are chiral multiplets in the corresponding bi-fundamental representations. Every mass term for S,  $\tilde{Q}$  and Q is forbidden by the (unbroken) gauge symmetry. In order to avoid chiral anomaly in adjunct  $U(N_X)$  and  $U(N_Y)$  gauge theories, we simply set  $N_X = N_Y = 1$  in the following. Even in this simple setup, the number of flavors can be controlled because the magnetic fluxes produce the degeneracy of zero-modes, enhancing the effective flavors.

Chiral superfields S, Q and Q have a Yukawa coupling in the superpotential,

$$W = gSQQ, \tag{11}$$

where g expresses the effective coupling constant, which is given by an overlap integral of wavefunctions determined by magnetic fluxes and is calculable on magnetized tori (see [4, 9] for reviews). In accordance with the discussion in the previous subsection, for  $N_C \geq N_F$ , the  $U(N_C)$  gauge dynamics enforces the operator  $\hat{M} \equiv Q\tilde{Q}$  to develop a nonvanishing VEV, breaking SUSY.

#### 3.2.1 The essential structure

We here aim to realize a minimal setup, that is,  $N_F$  pairs of quarks  $(Q, \tilde{Q})$  and a singlet S in  $U(N_C)$  SYM theory. We require the degeneracy of S to be one in order to avoid the presence of extra massless fields.

The generation structure of Q and  $\tilde{Q}$  should be produced on a single 2-torus. Otherwise, the rank of their Yukawa matrix is reduced and some fields become irrelevant to the DSB dynamics. Let us suppose that it is produced on the first 2-torus and denote magnetic fluxes felt by Q,  $\tilde{Q}$ and S by  $M_1^Q$ ,  $M_1^{\tilde{Q}}$  and  $M_1^S$ , respectively, where the subscript discriminates the first 2-torus. The gauge invariance enforces them to satisfy  $M_1^Q + M_1^{\tilde{Q}} + M_1^S = 0$ . Furthermore, we find that only one of the three is positive and the others have to be negative. The reason is that the Yukawa coupling (11) originates from the 10D gauge coupling  $\phi_1\phi_2\phi_3$ , and positive (negative) magnetic fluxes are required in order to produce zero-modes in  $\phi_1$  ( $\phi_{2,3}$ ) on the first 2-torus, which can be seen from Eqs. (4) and (5).

On a magnetized orbifold, Q, Q and S are assigned into either even- or odd-parity mode under the  $Z_2$  transformation. The numbers of their zero-modes are determined by the magnetic fluxes  $(M_1^Q, M_1^{\tilde{Q}}, M_1^S)$  and their  $Z_2$  parity. We show the relation between magnetic fluxes and the number of zero-modes on magnetized orbifolds in Table 1. The  $Z_2$  invariance of Yukawa coupling (11) allows us to consider three cases for their  $Z_2$  parity assignments, those are (even-even-even), (odd-odd-even) or (even-odd-odd) for  $(Q, \tilde{Q}, S)$ . Note that (odd-evenodd) is equivalent to (even-odd-odd) under the renaming  $(Q, \tilde{Q}) \leftrightarrow (\tilde{Q}, Q)$ , then we exclude the former.

We eventually found only six patterns satisfy these conditions, which are shown in Table 2.

	$Z_2$ parity of $(Q, \tilde{Q}, S)$	$(M_{1}^{Q}, M_{1}^{Q}, M_{1}^{S})$
Pattern 1	(even, even, even)	(-n, n, 0)
Pattern 2	(even, even, even)	(-2n, 2n+1, -1)
Pattern 3	(even, odd, odd)	(-n, n+3, -3)
Pattern 4	(even, odd, odd)	(-2n, 2n+4, -4)
Pattern 5	(odd, odd, even)	(-n, n, 0)
Pattern 6	(odd, odd, even)	(-2n-1, 2n+2, -1)

Table 2: The six allowed combinations of  $Z_2$  parity assignment and magnetic fluxes  $(M_1^Q, M_1^{\tilde{Q}}, M_1^{\tilde{S}})$  are listed, where *n* is an arbitrary positive integer.

In order to produce the singlet S without its multiplicity, |M| = 0, 1 unit of fluxes are allowed for the even-parity mode and |M| = 3, 4 for the odd-parity mode. The condition  $M_1^Q + M_1^{\tilde{Q}} + M_1^S = 0$  (one is positive and the other two are negative) severely restricts the values of  $M_1^Q$  and  $M_1^{\tilde{Q}}$ , because the zero-mode number of Q and that of  $\tilde{Q}$  have to be equal for a successful DSB. Therefore, we can conclude that any other configurations are excluded.

We find some differences among these six patterns. The first one is the value of coupling constants in  $\lambda_{ij} SQ_i \tilde{Q}_j$ . The Yukawa matrix  $\lambda_{ij}$  is proportional to the identity matrix with

Patterns 1 and 5. In the other cases,  $\lambda_{ij}$  have nonvanishing values in their off-diagonal entries which can be calculated explicitly. The second difference is a constraint on the torus areas given by the SUSY preserving condition (3). In general, Patterns 2, 3, 4 and 6 yield characteristic values of the ratios  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)}$  and  $\mathcal{A}^{(1)}/\mathcal{A}^{(3)}$  as one can see in Appendix A. This might be of importance in combination with, especially, the visible sector. We will discuss about it in the last of this section.

These six patterns allow us to construct several realistic DSB models within the given framework of magnetized orbifold without any nontrivial assumptions, which will be shown below.

#### 3.2.2 A self-complete model

We propose concrete DSB models with explicit configurations of magnetic fluxes and orbifolds in the whole extra compact space. As discussed above, let us suppose that the main structure of our DSB models is produced on the first 2-torus. The configurations on the other two 2-tori are determined in order to eliminate all the extra field contents other than Q,  $\tilde{Q}$  and S in  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  without affecting the generation structure of them realized on the first 2-torus. The  $Z_2$  parity assignments and the magnetic fluxes on the first 2-torus are selected from the six patterns shown in Table 2 and the magnetic fluxes on the second and third 2-tori are enforced to satisfy the SUSY preserving condition (3).

In the following, we construct an illustrating model on the basis of Pattern 1. That is, Q,  $\tilde{Q}$  and S are assigned into the even-parity mode on the first 2-torus with  $(M_1^Q, M_1^{\tilde{Q}}, M_1^S) = (-n, n, 0)$ . With the other five patterns, we can also realize similar models which we show in Appendix A.

Let us consider the following magnetized background,

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$
(12)

which breaks the U(N) gauge symmetry down to  $U(N_C) \times U(1)_X \times U(1)_Y$ , while preserving  $\mathcal{N} = 1$  SUSY with  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = \mathcal{A}^{(1)}/\mathcal{A}^{(3)} = M$ . We take the value of M to be positive. In this case (before orbifolding) zero-mode contents are given by

$$\phi_1 = \begin{pmatrix} \Xi_1 & 0 & 0 \\ \tilde{Q}' & \Xi_1' & 0 \\ Q & 0 & \Xi_1'' \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \Xi_2 & \tilde{Q} & 0 \\ 0 & \Xi_2' & 0 \\ 0 & S' & \Xi_2'' \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} \Xi_3 & 0 & Q' \\ 0 & \Xi_3' & S \\ 0 & 0 & \Xi_3'' \end{pmatrix}.$$

On this magnetized tori, we consider two  $Z_2$  orbifold projections, i.e., a  $Z_2 \times Z'_2$  orbifold. The first one,  $Z_2$  orbifolding, acts on the first and the second 2-tori  $(z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)$  with the projection operator

$$P_{+--} = \begin{pmatrix} + & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & - \end{pmatrix}.$$

This operator successfully assigns the even-parity to all of Q,  $\tilde{Q}$  and S as in Pattern 1, while eliminating S' and all the diagonal entries of  $\phi_1$  and  $\phi_2$ . The second one is  $Z'_2$  projection acting on the second and the third 2-tori  $(z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)$  with the projection operator

$$P_{+-+} = \begin{pmatrix} + & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & + \end{pmatrix},$$

which eliminates Q',  $\tilde{Q}'$  and all the diagonal entries of  $\phi_3$ . Consequently, the remaining zeromode contents are exactly the ideal ones (10). All the extra fields have been completely eliminated in the combination of magnetic fluxes and orbifolding.

The total degeneracy of S is certainly one. As for Q and  $\hat{Q}$ , their degeneracy is counted as the resulting number of  $Z_2$  even modes with |M| units of fluxes, which is read from Table 1. We have obtained desirable  $SU(N_C)$  SYM theory with  $N_F = 1, 2, 3, ...$  flavors, which can satisfy the condition  $N_C = N_F \ge 2$  or  $N_C > N_F$  for a successful DSB. We show similar DSB models with Pattern 2 to 6 in Appendix A.

One might consider that the following configuration of magnetic fluxes is better than Eq. (12),

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The last 2-torus is vacant and the condition  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = M$  ensures that the SUSY is preserved. As for the gauge symmetry, U(N) is broken down to  $U(N_C) \times U(2)$ . This U(2) symmetry can be further broken to  $U(1)_X \times U(1)_Y$  by orbifold projections. We again consider  $Z_2 \times Z'_2$  orbifolds for this magnetized background. The first  $Z_2$  acts on the first and the second 2-tori with the operator  $P_{+--}$ , and the second  $Z'_2$  on the second and the third 2-tori with  $P_{+-+}$ . The surviving zero-modes are described as

$$\phi_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Q & 0 & 0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 & \tilde{Q} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & S \\ 0 & S' & 0 \end{pmatrix}, \tag{13}$$

and their full superpotential is found as

$$W = gSQ\tilde{Q},\tag{14}$$

which has the same form as Eq. (11).

Although there is an extra massless field S', it would not affect the DSB dynamics because this is a singlet under  $U(N_C)$  and has no coupling in the superpotential.<sup>2</sup> In this model, the zero-modes of both S and S' have no multiplicity, while the zero-mode degeneracies of Q and  $\tilde{Q}$  are equivalent to the previous model. Thus, the strong dynamics of  $SU(N_C)$  gauge theory

<sup>&</sup>lt;sup>2</sup>We expect that this S' can get a mass at loop-levels after DSB, because Q,  $\tilde{Q}$  and S have  $U(1)_X$  gauge charges.

can generate DSB depending on the values of  $N_C$  and M, without any nontrivial assumptions for extra field contents. This can be another self-sustained DSB model. Although this model contains a decoupled massless field S', the model has a clear advantage to the previous one. There is no magnetic fluxes on the third 2-torus, then its area  $\mathcal{A}^{(3)}$  is not constrained by SUSY conditions. This can be helpful for associating this model with the other sectors as we will discuss in the next subsection.

### 3.3 Comments on the association with other sectors

We have constructed models for a SUSY breaking (hidden) sector, which must be combined with the MSSM (visible) sector and the other phenomenologically/cosmologically required sectors such as the moduli stabilization sector. Especially, when we consider a moduli stabilization mechanism based on non-perturbative effects such as gaugino condensations, like the Kachru-Kallosh-Linde-Trivedi (KKLT) scenario [27], one or more strong gauge theories in the hidden sector are required for the non-perturbative dynamics. There are some key issues for combining these sectors altogether.

Our models are based on SYM theories compactified on magnetized tori with/without orbifolds. The DSB models shown in subsection 3.1 is constructed without orbifolding, and thus, all the associated sectors such as the visible sector must also be constructed without orbifolding. On the other hand, the other DSB models on orbifolds have to be combined with the visible and the other sectors all constructed on the same orbifold. As the promising candidates for the visible sector, realistic flavor structures of MSSM-like models on magnetized tori [5] and orbifolds [9, 15] were derived so far. It is known that they are drastically different from each other. Therefore, we expect that models with or without orbifolding will be distinguishable phenomenologically.

The values of higher-dimensional gauge coupling g, torus area  $\mathcal{A}^{(i)}$  and complex structure  $\tau_i$ , are universal for all the sectors derived from the single higher-dimensional SYM theory. Thus, we have to choose common values for every sectors to be consistent with each other. We naively expect that most of these values are strongly constrained in the visible sector.

First, the gauge coupling g should be determined as follows. The 4D effective gauge coupling constant at the compactification scale, which is roughly given by a product of g and the volume of extra compact space, must be consistent with the experimental data in the visible sector, i.e., the observed values of standard model (SM) gauge couplings. For example, if we consider MSSM for the visible sector, it automatically leads to a unified value of three SM gauge couplings at around  $10^{16}$  GeV which is usually identified as the compactification scale, and the 4D effective gauge coupling can be fixed by the unified value. Next, the complex structures of tori are very important degrees of freedom to control the hierarchical structure of Yukawa couplings in the visible sector. Their values should be set to realize the quark and lepton masses and mixing angles [5, 10]. Finally, the configurations of magnetic fluxes in the visible sector are extremely limited in order to realize the three generation structure of quarks and leptons, and the ratios of three torus areas,  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)}$  and/or  $\mathcal{A}^{(1)}/\mathcal{A}^{(3)}$ , are determined through the SUSY preserving conditions depending on the flux configuration.

We remark that these constraints on parameters from the visible sector inevitably affect the

model building for hidden sectors. Indeed, the DSB models shown in this paper also restrict the ratios  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)}$  and/or  $\mathcal{A}^{(1)}/\mathcal{A}^{(3)}$ , those must be consistent with the constraints from the visible sector. Therefore, the existence of unconstrained parameters in each sector is a great advantage, when we construct the whole system as a combination of the solely constructed visible and hidden sectors. Note that some of our DSB models with a vanishing flux in the third 2-torus restrict only one of the above two ratios.

The models with two magnetized 2-tori (or even with a single magnetized 2-torus) among three are interesting from another point of view. We expect that our magnetized models based on 10D SYM theories would be completed being embedded into magnetized D9-brane systems, while the economically fluxed models have a potential to be compatible with D7-branes (or even D5-branes).<sup>3</sup>

When we construct the whole system by combining our DSB sector with the certain visible and other sectors, we also have to care about the direct couplings among them. All the sectors should be embedded into a single U(N) gauge theory, if we regard our models as D-brane models with a single stack. On the other hand, with multi-stacks of D-branes (e.g., D3/D7 or D5/D9 systems<sup>4</sup>), we can start from a product of multiple U(N) gauge groups. In general, there exist bi-fundamental fields charged under two different sectors, depending on the configurations of magnetic fluxes and orbifolding. In particular, such bi-fundamental fields charged under the SM gauge groups are phenomenologically dangerous in many cases. We should also require that the strong dynamics of DSB and moduli stabilization sectors do not disturb each other through light fields charged under both sectors.

Although these bi-fundamental fields are troublesome in generic cases, vector-like fields charged under both the MSSM and DSB sectors can be interesting, because they behave as messenger fields which mediate SUSY breaking contributions to the visible sector. In the previous analyses [5] of magnetized models, it has been mostly assumed that the SUSY spectra are dominated by the moduli-mediation and/or anomaly-mediation [29, 30], which depends on how to stabilize the moduli fields in association with the DSB sector. For example, in the KKLT-like moduli stabilization scenarios [27] with some concrete DSB sectors [31, 32], contributions from the above two mediations can be comparable, and the so-called mirage mediation scenario [33, 34, 35, 36] is realized. By assuming such a mediation scenario, the SUSY spectrum was studied in concrete magnetized models of the visible sector and some generic features were obtained [5]. Then, it is interesting to employ one of our DSB models as the concrete hidden sector in this kind of scenario. The previous results can be deflected by the gauge-mediated contributions due to the appearance of messengers in the bi-fundamental representation between the MSSM and DSB sectors. We will study them in another places.

<sup>&</sup>lt;sup>3</sup> It is argued that lower-dimensional D-branes may be derived from a magnetized D-brane in higherdimensions with an infinite number of magnetic fluxes [4]. The effective field theory of such lower-dimensional branes can be derived based on such an argument [28].

<sup>&</sup>lt;sup>4</sup>The superfield formulation to describe such mixed D-brane systems was also constructed [28].

### 4 Summary

We have studied DSB models within the framework of 10D SYM theories compactified on magnetized tori and orbifolds.

First, aspects for DSB on magnetized tori/orbifolds have been shown with the simple configurations of magnetic fluxes which causes the gauge symmetry breaking  $U(N) \rightarrow U(N_C) \times U(N_X)$ , by assuming (non)vanishing VEVs of adjoint fields and (non)decoupling of their fluctuations from the DSB dynamics around the VEVs. Then, in order for the strong dynamics of  $SU(N_C)$  SYM theories with  $N_F$  flavors to trigger a dynamical SUSY breaking, certain relations between  $N_C$  and  $N_F$  are required. It is remarkable that the number of flavors  $N_F$  can be controlled by magnetic fluxes in our model, in other words, the background flux configuration determines whether DSB occurs or not.

At the same time, however, we also find that the decoupling of some extra adjoint fields, those could not be eliminated by orbifold projections in the model building procedures, is necessarily assumed in this simple class of models. Otherwise the existence of them could spoil the successful DSB and/or are already ruled out by phenomenological/cosmological observations. In the case that some extrinsic mechanisms realize the assumed situations, these DSB models on magnetized tori are available for a further model building, while the decoupling of extra adjoint fields is in general a challenging issue in the model building based on SYM theories in higher-dimensional spacetime.

Then, next, we have proposed another class of DSB models on orbifolds by extending the previous configurations of magnetic fluxes which preserve  $U(N_C) \times U(N_X)$  symmetry to those yield  $U(N_C) \times U(N_X) \times U(N_Y)$ , especially, to the simplest one  $U(N_C) \times U(1)_X \times U(1)_Y$ . We have searched such flux configurations that the  $SU(N_C)$  SYM theory contains  $N_F$  vector-like pairs  $(Q, \tilde{Q})$  with their nonvanishing Yukawa couplings to a singlet S. As the result, we found six patterns of suitable configurations.

On the basis of one of these six patterns, we demonstrated the construction of a self-complete DSB model on a  $Z_2 \times Z'_2$  orbifold, where all the extra fields below the compactification scale are eliminated by the combination of chiral projections due to magnetic fluxes and the orbifold projections. In Appendix A, we also show the other five patterns allow us to construct similar feasible models. Therefore, we conclude that, in this class of magnetized orbifold models, we can realize DSB without relying on any extrinsic mechanisms to eliminate extra fields.

Furthermore, we have studied another choice of magnetic fluxes, where only two of the three 2-tori are fluxed. Although this permits a presence of one more singlet S' without the Yukawa couplings to quarks, it of course does not disturb the DSB dynamics and can be another self-sustained DSB model. It is remarkable that the existence of unfluxed 2-torus can be an advantage when we combine the DSB (hidden) sector with the MSSM (visible) sector [28].

As discussed in the previous section, our DSB models should be embedded into a larger unified system being compatible with the MSSM sector and the others, e.g., the moduli stabilization sector. This must be an important task from both theoretical and phenomenological points of view. We expect some of the six patterns we found and their extensions being suitable for such embeddings. Finally, it is an interesting possibility that such a whole system is realized by magnetized D-branes. In this case, we should verify some stringy consistency of the full system containing all the sectors for completing our scenario on magnetized tori/orbifolds. These are remained as future works.

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# A Other self-complete models with Pattern 2 to 6

We have shown a concrete DSB model based on Pattern 1 shown in Table 2. We find that similar models can be realized with the other patterns and demonstrate them here (Models shown here contain just  $N_F$ -pairs of  $(Q, \tilde{Q})$  and one singlet S.).

We start from Pattern 2, where all of Q,  $\tilde{Q}$  and S are assigned into the even-parity mode, and their fluxes are parametrized as  $(M_1^Q, M_1^{\tilde{Q}}, M_1^S) = (-2n, 2n+1, -1)$  with a positive integer n. The suitable magnetic fluxes are given by

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2n & 0 \\ 0 & 0 & 2n+1 \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

which break gauge symmetry as  $U(N) \to U(N_C) \times U(1) \times U(1)$  and satisfy the SUSY preserving condition with  $\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = 2n$  and  $\mathcal{A}^{(1)}/\mathcal{A}^{(3)} = 2n + 1$ . We need two different orbifold projections to eliminate extra fields completely, then consider a  $Z_2 \times Z'_2$  orbifold. The  $Z_2$  orbifolding acts on the first and the second tori with the projection operator  $P_{+--}$ , while the  $Z'_2$ orbifolding acts on the second and the third tori with the projection operator  $P_{+-+}$ . These are consistent with the parity assignment of Pattern 2 and eliminate all the extra entries of  $\phi_i$ . The net number of zero-mode of S is one. That of  $Q(\tilde{Q})$  is identified as the number of even-parity mode with |M| = 2n (2n+1) fluxes. We see from Table 1 that both the degeneracies of Q and  $\tilde{Q}$  are equal to n + 1.

With Pattern 3, (Q, Q, S) are assigned into the (even,odd,odd)-parity mode, and their fluxes are given by (-n, n+3, -3) with a positive integer n. A similar model is obtained on the same  $Z_2 \times Z'_2$  orbifold as Pattern 2 but with the different magnetic fluxes,

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n+3 \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The net number of  $Q(\hat{Q})$  is identified as that of even-parity (odd-parity) mode with n(n+3) fluxes. We find the degeneracies of Q and  $\tilde{Q}$  are equal to each other.

With Pattern 4,  $(Q, \tilde{Q}, S)$  are assigned into the (even,odd,odd)-parity mode, and their fluxes are given by (-2n, 2n+4, -4) with a positive integer n. A similar model is obtained with the following magnetic fluxes,

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2n & 0 \\ 0 & 0 & 2n+4 \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

on the  $Z_2 \times Z'_2$  orbifold, where  $Z_2$  acts on the first and the second tori with  $P_{+-+}$ , and  $Z'_2$  acts on the second and the third tori with  $P_{+-+}$ . The net number of  $Q(\tilde{Q})$  is identified as that of even-parity (odd-parity) mode with 2n(2n+4) fluxes. We find both the degeneracies of Q and  $\tilde{Q}$  are n + 1.

With Pattern 5,  $(Q, \tilde{Q}, S)$  are assigned into the (odd,odd,even)-parity mode, and their fluxes are given by (-n, n, 0) with a positive integer n. A similar model is obtained with the following magnetic fluxes,

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

on the  $Z_2 \times Z'_2$  orbifold, where  $Z_2$  acts on the first and the second tori with  $P_{+++}$ , and  $Z'_2$  acts on the second and the third tori with  $P_{+-+}$ . Both the net numbers of Q and  $\tilde{Q}$  are equal to that of odd-parity mode with n fluxes.

Finally with Pattern 6, (Q, Q, S) are assigned into the (odd,odd,even)-parity mode, and their fluxes are parametrized as (-2n - 1, 2n + 2, -1) with a positive integer n. A similar model is obtained on the same  $Z_2 \times Z'_2$  orbifold as Pattern 5 but with the different magnetic fluxes,

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2n+1 & 0 \\ 0 & 0 & 2n+2 \end{pmatrix}, \quad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The net number of  $Q(\tilde{Q})$  are identified as that of odd-parity mode with |M| = 2n + 1 (2n + 2) fluxes. Both the degeneracies of Q and  $\tilde{Q}$  are equal to n.

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