Precise Determination of Charge Dependent Pion-Nucleon-Nucleon Coupling Constants

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We undertake a covariance error analysis of the pion-nucleon-nucleon coupling constants from the Granada-2013 np and pp database comprising a total of 6713 scattering data. Assuming a unique pion-nucleon coupling constant we obtain $f^2 = 0.0761(3)$. The effects of charge symmetry breaking on the $^3P_0$, $^3P_1$ and $^3P_2$ partial waves are analyzed and we find $f_p^2 = 0.0759(4)$, $f_0^2 = 0.079(1)$ and $f_c^2 = 0.0763(6)$ with minor correlations among the coupling constants. We successfully test normality for the residuals of the fit.

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I. INTRODUCTION

The meson exchange picture is a genuine quantum field theoretical feature which implies, in particular, that the strong force between protons and neutrons at long distances is dominated by the exchange of the lightest hadrons compatible with the conservation laws, namely neutral and charged pions. The strong force acting between nucleons at sufficiently large distances or impact parameters $\sim$ 3fm is solely due to one pion exchange (OPE) and was suggested by Yukawa 80 years ago [1]. The verification of this mechanism not only provides a check of quantum field theory at the hadronic level but also a quantitative insight onto the determination of the forces which hold atomic nuclei [2]. While the mass of the pion may be determined directly from analysis of their tracks or electroweak decays, the determination of the coupling constant to nucleons needs further theoretical elaboration. The pion-nucleon-nucleon coupling constant is rigorously defined as the πNN vertex function when all three particles are on the mass shell and in principle any process involving the elementary vertices $p \to \pi^0 p$, $n \to \pi^0 n$, $p \to \pi^+ n$ and $n \to \pi^- p$ (or their charge conjugated) is suitable for the determination of the corresponding couplings provided all other relevant effects are accounted for at an acceptable confidence level. The combinations entering in NN scattering are (we use the conventions of [3] and when possible, for simplicity, omit the π label),

$$f_p^2 = f_{\pi^0 pp} f_{\pi^0 pp} \tag{1}$$

$$f_0^2 = -f_{\pi^0 nn} f_{\pi^0 pp} \tag{2}$$

$$2f_c^2 = f_{\pi^- pn} f_{\pi^+ np} \tag{3}$$

Usually the charge symmetry breaking is restricted to mass differences by setting $f_p = -f_n = f_c = f$. The relevant relationships between the pseudo-scalar pion coupling constants, $g_{\pi NN}$, and the pseudo-vector one, $f_{\pi NN}$, are given by

$$\frac{g^2_{\pi^a NN'}}{4\pi} = \left(\frac{M_N + M_{N'}}{m_{\pi^+}}\right)^2 f^2_{\pi^a NN'} \tag{4}$$

where $N, N' = n, p$ and $\pi^a = \pi^0, \pi^\pm$ (the factor $m_{\pi^\pm}$ is conventional). Thus, we may define g_0^2, g_c^2, g_p^2 and g_n^2. We take $M_p = 938.272$ MeV the proton mass, $M_n = 939.566$ MeV the neutron mass, and $m_{\pi^\pm} = 139.570$ MeV the mass of the charged pion.

There is a long history of determination of pion-nucleon coupling constants using different approaches. The very first determination was made in 1940 by Bethe by looking at deuteron properties [4, 5] soon after Yukawa proposed his theory and before the pion was experimentally discovered, finding the common value $f^2 = 0.077 - 0.080$. Based on PCAC Goldberger and Treiman deduced $g_{\pi NN}(0) F_\pi = M_N g_A$ [6] strictly valid at the pion off-shell point $q^2 = 0$. The first direct and quantitative evidence for OPE was found in 1960 by Signell [7] by directly fitting the neutral pion mass to differential cross section in p-p Scattering data. The method of partial wave analysis (PWA) was soon afterwards used by Macgregor *et al.* at Livermore [8]. A variety of methods and reactions have been used since the seminal Yukawa paper and more complete account of the subsequent numerous determinations can be traced from comprehensive overviews [9–11]. During many years πN scattering determination through fixed-t dispersion relations was advocated as a precision tool (see e.g. [12] and references therein). The latest most accurate πN scattering determinations one has [13] based on the GMO rule $g_c^2/(4\pi) = 14.11(20)$ ($f_c^2 = 0.0783(11)$), using fixed-t dispersion relations $g_c^2/(4\pi) = 13.76(8)$ [14] and the most recent determination [15, 16] based on πN scattering lengths and $\pi^- d$ scattering and the GMO sum rule yields $g_c^2/(4\pi) = 13.69(12)(15) = 13.69(19)$.

The modern era of high-quality NN interactions initiated by the Nijmegen group [17] enabled to decrease the reduced χ^2/ν from 2 to 1, thanks to the implementation of charge-dependence, vacuum polarization, relativistic corrections and magnetic moments interactions and a suitable selection criterion for compatible data. Their analysis comprised a total of

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4313 NN scattering data. This promoted the determination of the pion-nucleon coupling constant from np and pp scattering to a competitively accurate approach. The main advantage of an NN analysis as compared to the πN analysis, which has so far been restricted to charged pions, is that one can determine both neutral and charged-pion coupling constants simultaneously and search for isospin breaking effects. The originally recommended charge independent value $f^2 = 0.0750(9)$ [18] was revised [19] and confirmed in the 1997 review on the status of the pion-nucleon-nucleon coupling constant [9]; this is the most accurate NN determination to date. There, it was suggested that with more data and better statistics a charge-independence breaking could be checked.

Most of the analyses determining the pion nucleon coupling constants involve heavy statistical analysis for a large body of experimental data, mostly χ^2-fits, which are subjected to a number of *a posteriori* tests [20]. The verification of these tests buttress a sensible analysis of uncertainties of theoretical models [21]. The Nijmegen group [19, 22] checked the statistical quality of pp fit residuals to the time of their analysis using the moments test, which for increasing orders overweights the tails.

In this paper we study the possible difference among the pion-nucleon coupling constants by analyzing np and pp scattering data using the NN Granada-2013 3σ-self consistent database designed and analyzed recently [23–26]. [1] There, out of 8000 published np and pp experimental data measured in the period 1950-2013 we have selected 6713 which satisfactorily passes the tail-sensitive test based on the quantile-quantile plot for the combined np+pp residuals (see also Ref. [27] for an application of these ideas to $\pi\pi$ scattering).

The paper is organized as follows. In Section II we describe the OPE potential to display our notation and discuss the conditions under which we naturally expect to unveil charge dependence in the pion-nucleon coupling constants. In Section III we review the main aspects of our partial wave analysis and the Granada-2013 database as well as some motivation for incorporate charge dependence in the P-waves, besides the customary charge dependence on S-waves implemented in all modern high quality fits. Our numerical results are presented and discussed in Section IV. Finally, in Section V conclusions are presented. In the Appendix we show the extended operator basis accommodating both S- and P-waves charge dependence.

II. CHARGE-DEPENDENT ONE PION EXCHANGE

The charge dependent one pion exchange (CD-OPE) potential incorporates charge symmetry breaking by considering the mass difference of the neutral and charged pions as well as assuming different coupling constants. Using the convention for πNN Lagrangians defined in the review [3], the quantum mechanical potential which reproduces the corresponding Feynman diagrams for on-shell static nucleons in the Born approximation in the pp, nn and np channels is expressed as

$$V_{\mathrm{OPE},pp}(r) = f_p^2 V_{m_{\pi^0},\mathrm{OPE}}(r), \tag{5}$$

$$V_{\mathrm{OPE},nn}(r) = f_n^2 V_{m_{\pi^0},\mathrm{OPE}}(r), \tag{6}$$

$$V_{\mathrm{OPE},np}(r) = -f_n f_p V_{m_{\pi^0},\mathrm{OPE}}(r) - (-)^T 2 f_c^2 V_{m_{\pi^\pm},\mathrm{OPE}}(r), \tag{7}$$

respectively. Here $V_{m,\mathrm{OPE}}$ is given by

$$V_{m,\mathrm{OPE}}(r) = \left(\frac{m}{m_{\pi^\pm}}\right)^2 \frac{1}{3} m \left[Y_m(r)\sigma_1\cdot\sigma_2 + T_m(r)S_{1,2}\right], \tag{8}$$

being Y_m and T_m are the usual Yukawa functions,

$$Y(r) = \frac{e^{-mr}}{mr} \tag{9}$$

$$T(r) = \frac{e^{-mr}}{mr}\left[1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right] \tag{10}$$

and tensor, σ_1 and σ_2 the single nucleon Pauli matrices and $S_{12} = 3\sigma_1\cdot\hat{\mathbf{r}}\sigma_2\cdot\hat{\mathbf{r}} - \sigma_1\cdot\sigma_2$ the tensor operator. Unfortunately, the CD-OPE potential by itself cannot be directly compared to experimental data, and the only way we know how to determine these pion-nucleon couplings is by carrying out a PWA.

From a purely classical viewpoint, in order to *measure* the nuclear force directly it would just be enough to hold and pull two nucleons apart at distances larger than their elementarity size, which is or the order of 2fm [23]. For such an ideal experiment the behavior of the system at shorter distances would be largely irrelevant, as nucleons would behave as point-like particles. This situation would naturally occurs if nucleons were truly infinitely heavy in which case the potential would correspond to the static energy of the system with baryon number $B = 2$ and total charge $Q = 2,1,0$ depending on whether we have pp,pn and nn [2]. Of course, the quantum mechanical nature of the nucleons prevents such a situation experimentally and we are left with scattering experiments. Good operating conditions are achieved when the maximum relative CM momentum, $p_{\max}$, is small enough to avoid complications due to inelastic channels and large enough to contain as many data as possible. This generates a resolution ambiguity of the order of the minimal relative de Broglie wavelength, $\lambda_{\min} = \Delta r \sim 1/p_{\max}$. Since the $NN \to \pi NN$ channel opens up at $p_{\max} \sim \sqrt{m_\pi M_N} \sim 360$MeV, we have $\Delta r \sim 0.6$fm. Unfortunately, in the quantum mechanical NN scattering problem scales are somewhat intertwined, and thus some information of the short distance and unknown components of the potential have to be considered before a scattering amplitude, cross section or polarization asymmetry can be evaluated. We naturally expect that the low energy behavior depends more strongly on the long distance properties of

[1] The Granada database is located at the website `http://www.ugr.es/~amaro/nndatabase/`.

[2] This is the case ,e.g. in lattice calculations, where static sources are placed at a fixed distance [28, 29]. In the quenched approximation it has been found that for a pion mass of $m_\pi = 380$ MeV the value $g^2/(4\pi) = 12.1 \pm 2.7$ which is encouraging [30] but still a crude estimate.

the interaction, some coarse grained information of the unknown contribution is actually needed and can indeed be deduced from experiment with an overall *sufficient* accuracy as to determine differences in the pion-nucleon couplings. This viewpoint allows to determine *a priori* the number of independent parameters $N_{\rm Par}$ needed for a successful fit [23] [3]. These ideas where introduced by Aviles long ago [31] and underly the recent NN analysis carried out by the present authors within recent times where a large database comprising about 8000 published experimental data measured in the period 1950-2013 was considered [23, 24].

There is no symmetry reason why the strong force between protons and between neutrons should be exactly identical, so one should see the difference with a sufficiently large amount of experimental data, $N_{\rm Dat}$. These differences are in fact small and hard to pin down since *a priori* electromagnetic corrections should scale with the fine structure constant $\delta g/g \sim \alpha \sim 1/137$ and strong (QCD) corrections should scale with the $u-d$ quark mass differences (relative to the s-quark mass) which means $\delta g/g \sim (m_u - m_d)/\Lambda_{\rm QCD} \sim (M_p - M_n)/\Lambda_{\rm QCD} \sim 1/100$ for $\Lambda_{\rm QCD} \sim 250$MeV. This simple estimates suggest that in order to witness isospin violations in the couplings we should determine them with a target accuracy better than $1-2\%$, which is not too far from the most recent values. On a purely statistical basis the relative uncertainty due to N independent measurements is $\Delta g/g \sim 1/\sqrt{N}$. If we have some extra parameters $(\lambda_1, \dots, \lambda_{N_{\rm Par}})$,the condition $\Delta g \sim \delta g \sim 0.01 - 0.02$ would require $N = N_{\rm Dat} - N_{\rm Par} \sim 7000 - 10000$ *independent* degrees of freedom. Since $N_{\rm Dat} \gg N_{\rm Par}$ this is comparable to the total amount of existing elastic np and pp scattering data. While these are rough estimates, we stress the independence character of the measurements in order to make these estimates credible; it is not just a question of having more data. From the point of view of χ^2-fits this requires passing satisfactorily normality tests guaranteeing the self-consistency of the fit. In particular, adding many incompatible data invalidates this analysis.

III. THE GRANADA-2013 ANALYSIS

In a series of works we have upgraded the NN database to include a total of 6713 np and pp published experimental data by using a coarse grained representation of the interaction and applying stringent statistical tests on the residuals of the χ^2-fits after a 3σ self consistent selection process has been implemented [25]. The resulting Granada-2013 is at present the largest NN database which can be described by a CD-OPE contribution. The about 60% more data than the 4313 data used in the latest Nijmegen upgrade [9], suggests that we can improve on the errors for the pion-nucleon couplings as discussed in the previous section.

We have discussed in detail the many issues in carrying out the data selection, fitting and the corresponding joint np+pp partial wave analysis. We review here the main aspects as a guideline and refer to those works for further details.

We separate the potential into two well defined regions depending on a chosen cut-off radius r_c fixed in such a way that for $r > r_c$ the CD-OPE is the only strong contribution. In addition, for $r > r_c$ we also have em (Coulomb,vacuum polarization,magnetic moments) [24] and relativistic pieces which we simply add to the strong potential.

$$V(\mathbf{r}) = V_{\rm OPE}(\mathbf{r}) + V_{\rm EM}(\mathbf{r}), \qquad r > r_c \qquad (11)$$

Below the cut-off radius, $r < r_c$ we regard the NN force as unknown, and we use delta-shells located at equidistant points separated by $\Delta r = 0.6$fm and corresponding to the shortest de Broglie wavelength at about pion production threshold. The fitting parameters are the real coefficients $(\lambda_i)^{JS}_{ll'}$ for each partial wave:

$$V^{JS}_{l,l'}(r) = \frac{1}{2\mu} \sum_{i=1}^{N} (\lambda_i)^{JS}_{ll'} \delta(r - r_i), \qquad r \le r_c. \qquad (12)$$

where μ is the NN reduced mass. Alternatively the potential can be expanded in an operator basis extending the AV18 potentials in coordinate space, see the appendix. The transformation between partial wave and operator basis was given in Ref. [24].

It turns out that $r_c = 3$fm provides statistically satisfactory fits to the selected 3σ-self consistent Granada-2013 database. While it would be interesting to separate explicitly the known from the unknown pieces of the interaction below the cut-off radius r_c, this is actually a complication in the fitting procedure, and will not change the values of the most-likely pion-nucleon coupling constants. Another advantage of taking $r_c = 3$fm is that in our analysis there is no need of form factors of any kind, and thus we are relieved from disentangling finite size effects, quark exchange and the intrinsic resolution Δr inherent to any finite energy PWA [4]

The possible A_y problem for np scattering raised by the data of Ref. [33] suggested a sizable isospin breaking of coupling constants. The problem was re-analyzed theoretically by [34] motivated the reanalysis of the data [35] in particular the disentanglement between systematic and statistical errors. Actually, [34] found that these data might be explained in an isolated fashion when isospin was broken thus, we allow this isospin breaking to foresee the possibility of recovering the data.

[3] There is is found that for $r_c = 3$fm , $N_{\rm Par} \sim 60$. The argument is based on the idea that if we take the CD-OPE potential above r_c we can estimate the number of independent potential values $V(r_n)$ below r_c in any partial wave channel, with $r_n = n\Delta r$. Since the maximum angular momentum in the partial wave expansion is $l_{\rm max} \sim p_{\rm max} r_c$ and we have four independent waves for each l we would have $N_{\rm Par} \sim 4 l_{\rm max}(r_c/\Delta r)$. Excluding the points r_n below the centrifugal barrier the number becomes $N_{\rm Par} \sim 2(p_{\rm max} r_c)^2$.

[4] An Explanation of the Apparent Charge Dependence of the Pion Nucleon Coupling was attributed to the strong form factor [32].

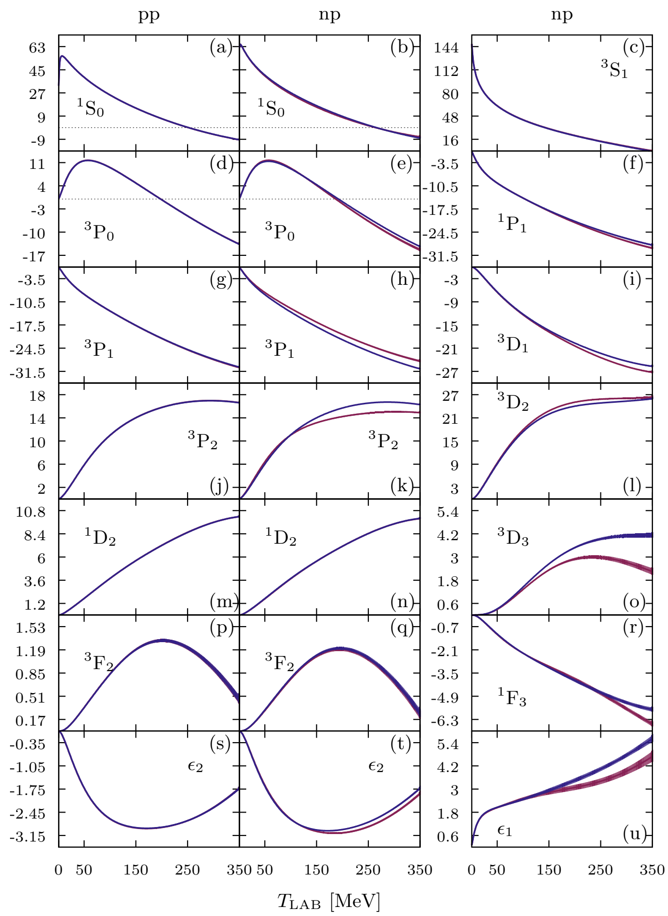

FIG. 1. (Color online) Phaseshifts obtained from a partial waves analysis to pp and np data and statistical uncertainties. Blue band from [24] and red band from a fit with charge symmetry breaking on the $^3P_0$, $^3P_1$ and $^3P_2$ partial waves.

IV. NUMERICAL RESULTS

In our previous analysis we took a fixed and the common value for the coupling constant suggested by the Nijmegen group. When we relax this assumption and assume a unique pion-nucleon coupling constant we obtain $f^2 = 0.07611(33)$ which is 1σ compatible with the latest Nijmegen recommendation [9] of $f^2 = 0.0750(8)$ but almost three times more accurate.

Following the common practice of other analyses [17, 36, 37] , we have previously allowed different pp and np parameters only on the $^1S_0$ partial wave [23, 24, 38, 39] and found that this symmetry breaking is indeed necessary to obtain an accurate description of the pp and np scattering data. The large collection of about 8000 available data also makes it possible to test charge symmetry breaking on the parameterization of higher partial waves, e.g. $^3P_0$, $^3P_1$ and $^3P_2$.

To carry out such a test we used different parameters on those partial waves and performed a full PWA and selection process as described in [24, 39] by fitting the delta-shell potential parameters to the complete database and then applying the 3σ rejection criterion iteratively until a self consistent database is obtained. The consistent database obtained in this case has 3001 pp data and 3727 np data including normalizations and the value for the chi square per number of data is $\chi^2/N_{\mathrm{data}} = 1.03$ When comparing with our previous consistent data base [24] this symmetry breaking can only describe 15 additional data out of more than 1000 rejected data. Fig. 1 compares the low angular momentum phaseshifts of the PWA in [24] (blue bands) with this new analysis (red bands). The pp phaseshifts show no significant difference, while the np ones are statistically different and the differences are even greater for higher angular momentum partial waves.

Usually the charge symmetry breaking is restricted to mass differences by setting $f_p = -f_n = f_c = f$ and the value $f^2 = 0.075$ recommended by the Nijmegen group [40] has been used in most of the potentials since the seminal 1993 partial wave analysis [17]. We test this charge independence with the large body of data available today by using f_p, f_0, and f_c as extra fitting parameters along with the previous 46 delta-shell parameters. We carry out the complete process of fitting the full data base first and iteratively apply the 3σ rejection criterion to obtain a consistent data base. The final data base has a total of 6712 data, just one less than our previous analysis [24], with $\chi^2/N_{\mathrm{data}} = 1.04$.

We show our results in Table I depending on different strategies regarding isospin breaking in S-waves and S- and P-waves as well as in the coupling constants. Assuming a unique pion-nucleon coupling constant we obtain $f^2 = 0.07611(33)$ which is 1σ compatible with the latest Nijmegen recommendation [9] of $f^2 = 0.0750(8)$ but almost three times more accurate. The working group summary of 1999 provides a recent compilation of coupling constants chronological display [10]. The most recent determination [15, 16] based on πN scattering lengths and $\pi^- d$ scattering and the GMO sum rule yields $g_c^2/(4\pi) = 13.69(12)(15) = 13.69(20)$. From our full covariance matrix analysis we get $g_n^2/(4\pi) = 14.91(39)$, $g_p^2/(4\pi) = 13.72(7)$ and $g_c^2/(4\pi) = 13.81(11)$. The last value is compatible with these determinations, but slightly more accurate.

The fitting delta-shell parameters obtained in our different strategies regarding CD breaking in just S-waves and CD breaking in S and P waves can be seen in Tables II and III respectively. We use the resulting parameters along with their covariance matrix to calculate f_p^2, f_0^2 and f_c^2, and propagate the corresponding statistical uncertainties and test charge independence. Fig. 2 shows the correlation ellipses obtained with the fits with (dashed red line) and without (solid blue line) charge symmetry on the P waves.

The standard assumption underlying a conventional χ^2-fit is that the sum of ν-independent gaussian variables belonging to the normal distribution $N(0,1)$ has a χ^2 distribution with ν-degrees of freedom [20]. One can actually check *a posteriori* if the outcoming residuals do indeed fulfill the initial assumption with a given confidence level. The self-consistency of the fit is an important test, since it validates the analysis made in Section II, and provides some confidence on the increase in accuracy that we observed as compared to previous works. The normality test of the three fits presented on this

TABLE I. The pion-nucleon coupling constants f_p^2, f_0^2 and f_c^2 determined from different fits to the Granada-2013 database and their characteristics. We indicate the partial waves where charge dependence is allowed.

f_p^2	f_0^2	f_c^2	CD-waves	χ^2_{pp}	χ^2_{np}	χ^2	$N_{\rm Dat}$	$N_{\rm Par}$	χ^2/ν
0.075	idem	idem	$^1S_0$	3051.64	3958.08	7009.72	6713	46	1.051
0.0761(3)	idem	idem	$^1S_0$	3051.54	3951.30	7002.84	6713	46+1	1.051
0.0759(4)	0.079(1)	0.0763(6)	$^1S_0$	2999.17	3951.40	6950.57	6713	46+3	1.043
0.0758(4)	0.080(2)	0.0765(6)	$^1S_0, P$	3043.64	3863.77	6907.40	6727	46+3+9	1.036

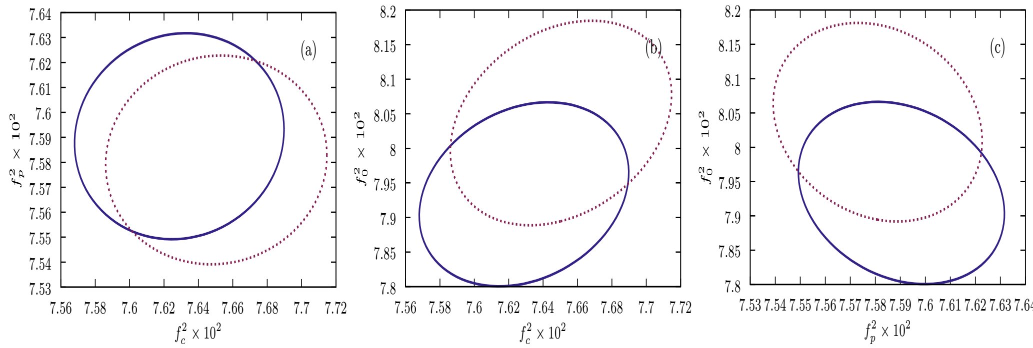

FIG. 2. (Color Online) Correlation ellipses for the coupling constants f_c^2 , f_p^2 and f_0^2 appearing in the OPE potential from a PWA with (solid blue line) and without (dashed red line) charge independence on the P waves and a 3σ consistent database

TABLE II. Fitting delta-shell parameters $(\lambda_n)^{JS}_{l,l'}$ (in fm^{-1}) with their errors for all states in the JS channel for a fit with isospin symmetry breaking on the $^1S_0$ partial wave parameters only and the pion-nucleon coupling constants f_0^2, f_p^2 and f_c^2 as fitting parameters We take $N = 5$ equidistant points with $\Delta r = 0.6$fm. − indicates that the corresponding fitting $(\lambda_n)^{JS}_{l,l'} = 0$. The lowest part of the table shows the resulting OPE coupling constants with errors

Wave	λ_1	λ_2	λ_3	λ_4	λ_5
$^1S_{0\rm np}$	1.20(6)	−0.78(2)	−0.15(1)	−	−0.024(1)
$^1S_{0\rm pp}$	1.31(2)	−0.721(5)	−0.189(2)	−	−0.0208(4)
$^3P_0$	−	0.95(2)	−0.320(7)	−0.062(3)	−0.023(1)
$^1P_1$	−	1.20(2)	−	0.073(2)	−
$^3P_1$	−	1.351(5)	−	0.0577(5)	−
$^3S_1$	1.67(6)	−0.46(1)	−	−0.073(1)	−
ε_1	−	−1.65(2)	−0.32(2)	−0.242(8)	−0.015(3)
$^3D_1$	−	−	0.35(1)	0.106(9)	0.011(3)
$^1D_2$	−	−0.21(1)	−0.203(3)	−	−0.0190(3)
$^3D_2$	−	−1.09(3)	−0.13(2)	−0.251(6)	−0.013(2)
$^3P_2$	−	−0.482(1)	−	−0.0288(7)	−0.0037(4)
ε_2	−	0.31(2)	0.191(4)	0.050(2)	0.0127(6)
$^3F_2$	−	3.50(6)	−0.229(5)	−	−0.0142(5)
$^1F_3$	−	−	0.17(2)	0.076(8)	−
$^3D_3$	−	0.53(2)	−	−	−

$f^2_{\rm p}$	f^2_0	$f^2_{\rm c}$
0.0759(4)	0.079(1)	0.0763(6)

TABLE III. Same as Table II for a fit with isospin symmetry breaking on the $^1S_0$, $^3P_0$, $^3P_1$ and $^3P_2$ partial waves parameters

Wave	λ_1	λ_2	λ_3	λ_4	λ_5
$^1S_{0\rm np}$	0.99(6)	−0.69(2)	−0.20(1)	−	−0.020(2)
$^1S_{0\rm pp}$	1.32(2)	−0.721(5)	−0.189(2)	−	−0.0207(4)
$^3P_{0\rm np}$	−	0.98(4)	−0.31(1)	−0.083(5)	−0.019(1)
$^3P_{0\rm pp}$	−	0.95(2)	−0.323(7)	−0.062(3)	−0.023(1)
$^1P_1$	−	1.25(2)	−	0.070(3)	−
$^3P_{1\rm np}$	−	1.21(2)	−	0.049(1)	−
$^3P_{1\rm pp}$	−	1.366(5)	−	0.0571(6)	−
$^3S_1$	1.51(7)	−0.40(1)	−	−0.070(1)	−
ε_1	−	−1.67(2)	−0.37(2)	−0.236(8)	−0.015(3)
$^3D_1$	−	−	0.45(2)	0.07(1)	0.014(3)
$^1D_2$	−	−0.20(1)	−0.205(3)	−	−0.0188(3)
$^3D_2$	−	−0.96(5)	−0.22(2)	−0.234(8)	−0.016(3)
$^3P_{2\rm np}$	−	−0.435(4)	−	−0.046(2)	−0.0022(7)
$^3P_{2\rm pp}$	−	−0.483(1)	−	−0.0279(8)	−0.0041(4)
ε_2	−	0.30(2)	0.193(4)	0.050(2)	0.0127(6)
$^3F_2$	−	3.40(7)	−0.222(5)	−	−0.0141(6)
$^1F_3$	−	−	0.26(3)	0.06(1)	−
$^3D_3$	−	0.77(3)	−	−	−

$f^2_{\rm p}$	f^2_0	$f^2_{\rm c}$
0.0758(4)	0.080(2)	0.0765(6)

work are summarized on Fig. 3 as rotated quantile-quantile plots. The tail-sensitive test compares the empirical quantiles of the residuals with the expected ones from an equally sized sample from the standard normal distribution. The red bands represent the 95% confidence interval of the normality test. For more details of the Tail-Sensitive test see [26].

Since the strong proton-proton and neutron-neutron potentials correspond to the exchange of a neutral pion, the difference in the couplings manifests in the difference of the potentials above the estimated exclusive domain of the CD-OPE

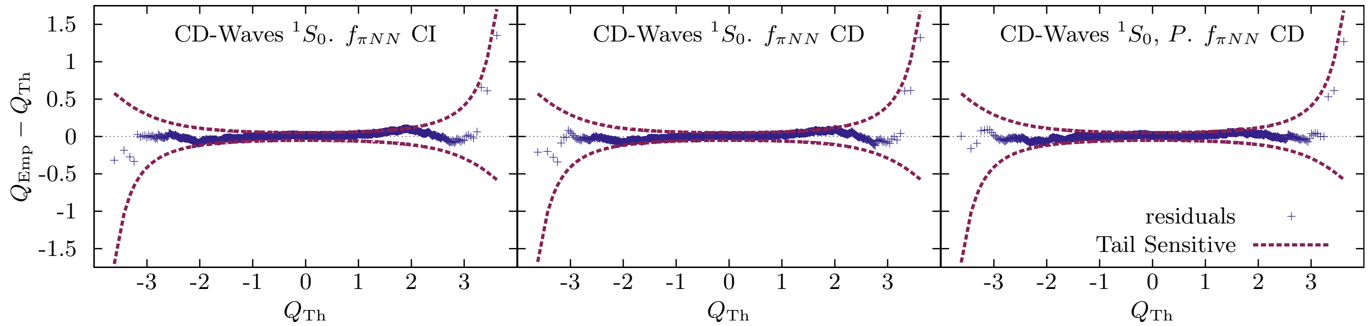

FIG. 3. (Color Online) Rotated quantile-quantile plots for the fits introduced in this work. All points should be inside the confidence band to state that residuals of the fit follow a normal distribution $N(0,1)$, in which case the fit is self-consistent *a posteriori*. Left panel, assuming a charge independent pion-nucleon constant used as a fitting parameter and charge symmetry breaking only on the $^1S_0$ partial wave parameters. Central panel, assuming three different charge dependent pion-nucleon constants used as a fitting parameters and charge symmetry breaking only on the $^1S_0$ partial wave parameters. Right panel, assuming three different charge dependent pion-nucleon constants used as a fitting parameters and charge symmetry breaking on the $^1S_0$ and P partial wave parameters

interaction. We can illustrate the main result pictorially in Fig. 4 by choosing the transversely and longitudinally polarized protons and neutrons. So we see that in any of the cases considered the strength of the nn potential is stronger than the pp potential, for instance $|V_{n\uparrow,n\uparrow}| > |V_{p\uparrow,p\uparrow}|$ for $r > r_c = 3$fm. Note that we cannot determine the neutron-neutron interaction below r_c, and in particular the corresponding neutron-neutron scattering length cannot be determined from the present calculation.

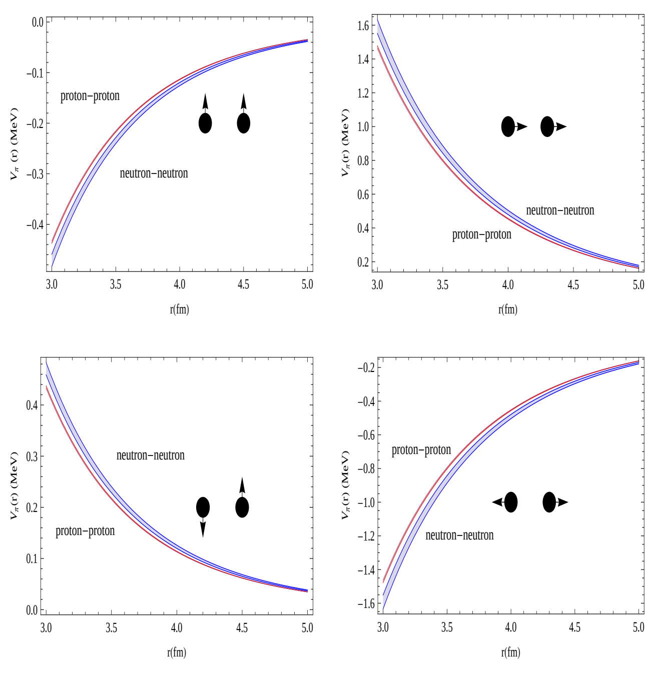

FIG. 4. Proton-proton and neutron-neutron interaction above 3fm due to exchange of a neutral pion for different spin polarization states. The bands correspond to the statistical uncertainties from a fit to 6713np+pp scattering data below $T_{\rm LAB} = 350$MeV with $\chi^2/\nu = 1.039$.

V. CONCLUSIONS

We summarize our points. Using the 3σ self-consistent Granada-2013 database for np and pp scattering we have investigated isospin breaking in the pion-nucleon coupling constants by separating the nuclear potential in two distinct contributions: Above 3 fm we use charge dependent one pion exchange potential for the strong part and electromagnetic and relativistic corrections. Below 3 fm we regard the interaction as unknown and we coarse grain it down to the shortest de Broglie wavelength corresponding to pion production threshold which is about 0.6 fm. With a total number of 55 parameters plus the three pion-nucleon coupling constants we describe a total number of 6713 np and pp data including normalization factors provided by the experimentalists which a total χ^2 of 6916, which means $\chi^2/\nu = 1.039$. We see clear evidence that the coupling of neutral pions to neutrons is larger than to protons. As a consequence neutrons interact more strongly than protons.

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Appendix A: Operator basis

To incorporate charge dependence on P waves two more operators need to be added to the basis we used previously getting a total of 23 operators O^n. The potential is written as a sum of functions multiplied by each operator

$$V(r) = \sum_{n=1,23} V_n(r) O^n \tag{A1}$$

The first fourteen operators are charge independent and correspond to the ones used in the Argonne v_{14} potential

$$\begin{aligned} O^{n=1,14} = {} & 1, \tau_1\cdot\tau_2, \sigma_1\cdot\sigma_2, (\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2), S_{12}, S_{12}(\tau_1\cdot\tau_2), \\ & \mathbf{L}\cdot\mathbf{S}, \mathbf{L}\cdot\mathbf{S}(\tau_1\cdot\tau_2), L^2, L^2(\tau_1\cdot\tau_2), L^2(\sigma_1\cdot\sigma_2), \\ & L^2(\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2), (\mathbf{L}\cdot\mathbf{S})^2, (\mathbf{L}\cdot\mathbf{S})^2(\tau_1\cdot\tau_2) \; . \end{aligned} \tag{A2}$$

These fourteen components are denoted by c, τ, σ, $\sigma\tau$, t, $t\tau$, ls, $ls\tau$, $l2$, $l2\tau$, $l2\sigma$, $l2\sigma\tau$, $ls2$, and $ls2\tau$. The remaining charge dependent operators are

$$\begin{aligned} O^{n=15,21} = {} & T_{12}, (\sigma_1\cdot\sigma_2)T_{12}, S_{12}T_{12}, (\tau_{z1}+\tau_{z2}) \; , \\ & (\sigma_1\cdot\sigma_2)(\tau_{z1}+\tau_{z2}) \; , L^2 T_{12}, L^2(\sigma_1\cdot\sigma_2)T_{12} \, . \\ & \mathbf{L}\cdot\mathbf{S}T_{12}, (\mathbf{L}\cdot\mathbf{S})^2 T_{12} \end{aligned} \tag{A3}$$

and are labeled as T, σT, tT, τz, $\sigma\tau z$, $l2T$, $l2\sigma T$, lsT and $ls2T$. The first five were introduced by Wiringa, Stoks and Schiavilla in [36]; the following two were included in [24] to restrict the charge dependence to the $^1S_0$ by following certain linear dependence relations between V_T, $V_{\sigma T}$, V_{l2T} and $V_{l2\sigma T}$. The last two terms are required for the charge dependence on the $^3P_0$, $^3P_1$ and $^3P_2$ partial waves.

As in our previous analysis we set $V_{tT} = V_{\tau z} = V_{\sigma\tau z} = 0$ to exclude charge dependence on the tensor terms and charge asymmetries. To restrict the charge dependence to the S and P waves parameters the remaining potential functions must follow

$$48V_{l2T} = -5V_T + 3V_{\sigma T} + 12V_{lsT} - 48V_{ls2T} \tag{A4}$$

$$48V_{\sigma l2T} = V_T - 7V_{\sigma T} + 4V_{lsT} - 16V_{ls2T} \tag{A5}$$

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