

The $\Upsilon(nS) \rightarrow B_c^*\pi, B_c^*K$ decays with perturbative QCD approach

Junfeng Sun,¹ Yueling Yang,¹ Qin Chang,¹ Gongru Lu,¹ and Jinshu Huang²

¹*Institute of Particle and Nuclear Physics,
Henan Normal University, Xinxiang 453007, China*

²*College of Physics and Electronic Engineering,
Nanyang Normal University, Nanyang 473061, China*

Abstract

Besides the traditional strong and electromagnetic decay modes, $\Upsilon(nS)$ meson can also decay through the weak interactions within the standard model of elementary particle. With anticipation of copious $\Upsilon(nS)$ data samples at the running LHC and coming SuperKEKB experiments, the two-body nonleptonic bottom-changing $\Upsilon(nS) \rightarrow B_c^*\pi, B_c^*K$ decays ($n = 1, 2, 3$) are investigated with perturbative QCD approach firstly. The absolute branching ratios for $\Upsilon(nS) \rightarrow B_c^*\pi$ and B_c^*K decays are estimated to reach up to about 10^{-10} and 10^{-11} , respectively, which might possibly be measured by the future experiments.

I. INTRODUCTION

The upilon $\Upsilon(nS)$ meson is the spin-triplet S -wave state of bottomonium (bound state consisting of bottom quark b and anti-bottom quark \bar{b}) with well-established quantum number of $I^G J^{PC} = 0^{-}1^{-}$ [1]. The characteristic narrow decay widths of $\Upsilon(nS)$ mesons for $n = 1, 2$ and 3 provide insight into the study of strong interactions. [see Table. I, and note that for simplicity, $\Upsilon(nS)$ will denote $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ mesons in the following content if not specified definitely.] The mass of $\Upsilon(nS)$ meson is below the B meson pair threshold. The $\Upsilon(nS)$ meson decays into bottomed hadrons through strong and electromagnetic interactions are forbidden by the law of conservation of flavor number. The bottom-changing $\Upsilon(nS)$ decays can occur only via the weak interactions within the standard model, although with tiny incidence probability. Both constituent quarks of upsilons can decay individually, which provide an alternative system for investigating the weak decay of heavy-flavored hadrons. In this paper, we will study the nonleptonic $\Upsilon(nS) \rightarrow B_c^* P$ ($P = \pi$ and K) weak decays with perturbative QCD (pQCD) approach [2–4].

TABLE I: Summary of mass, decay width, on(off)-peak luminosity and numbers of $\Upsilon(nS)$.

meson	properties [1]		luminosity (fb^{-1}) [5]		numbers (10^6) [5]	
	mass (MeV)	width (keV)	Belle	BaBar	Belle	BaBar
$\Upsilon(1S)$	9460.30 ± 0.26	54.02 ± 1.25	5.7 (1.8)	102 ± 2
$\Upsilon(2S)$	10023.26 ± 0.31	31.98 ± 2.63	24.9 (1.7)	13.6 (1.4)	158 ± 4	98.3 ± 0.9
$\Upsilon(3S)$	10355.2 ± 0.5	20.32 ± 1.85	2.9 (0.2)	28.0 (2.6)	11 ± 0.3	121.3 ± 1.2

Experimentally, (1) over 10^8 $\Upsilon(nS)$ data samples have been accumulated at Belle and BaBar experiments [5]. More and more upilon data samples will be collected at the running hadron collider LHC and the forthcoming e^+e^- collider SuperKEKB^a. There seems to exist a realistic possibility to explore $\Upsilon(nS)$ weak decay at future experiments. (2) Signals of the $\Upsilon(nS) \rightarrow B_c^* \pi, B_c^* K$ decays should be easily distinguished with “charge tag” technique, due to the facts that the back-to-back final states with different electric charges have definite momentum and energy in the rest frame of $\Upsilon(nS)$ meson. (3) The B_c^* meson has not been observed experimentally by now. The B_c^* meson production via the strong interaction are

^a The SuperKEKB has started commissioning test run (<http://www.kek.jp/en/NewsRoom/Release>).

suppressed due to the simultaneous presence of two heavy quarks with different flavors and higher order in QCD coupling constant α_s . The $\Upsilon(nS) \rightarrow B_c^* \pi, B_c^* K$ decays provide a novel pattern to study the B_c^* meson production. The identification of a single explicitly flavored B_c^* meson could be used as an effective selection criterion to detect upsilon weak decays. Moreover, the radiative decay of B_c^* meson provide a useful extra signal and a powerful constraint^b. Of course, any discernible evidences of an anomalous production rate of single bottomed meson from upsilon decays might be a hint of new physics.

Theoretically, many attractive QCD-inspired methods have been developed recently to describe the exclusive nonleptonic decay of heavy-flavored mesons, such as the pQCD approach [2–4], the QCD factorization approach [7–9], soft and collinear effective theory [10–13], and have been applied widely to vindicate measurements on B meson decays. The upsilon weak decay permits one to further constrain parameters obtained from B meson decay, and cross comparisons provide an opportunity to test various phenomenological models. The upsilon weak decay possess a unique structure due to the Cabibbo-Kobayashi-Maskawa (CKM) matrix properties which predicts the channels with one $B_c^{(*)}$ meson are dominant. The $\Upsilon(nS) \rightarrow B_c^* P$ decay belongs to the favorable $b \rightarrow c$ transition, which should, in principle, have relatively large branching ratio among upsilon weak decays. However, there is still no theoretical study devoted to the $\Upsilon(nS) \rightarrow B_c^* P$ decay for the moment. In this paper, we will present a phenomenological investigation on $\Upsilon(nS) \rightarrow B_c^* P$ weak decay with the pQCD approach to supply a ready reference for the future experiments.

This paper is organized as follows. Section II focus on theoretical framework and decay amplitudes for $\Upsilon(nS) \rightarrow B_c^* \pi, B_c^* K$ weak decays. Section III is devoted to numerical results and discussion. The last section is a summary.

^b The investigation on the radiative decay of B_c^* meson can be found in, for example, Ref. [6] with QCD sum rules.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

Theoretically, the $\Upsilon(nS) \rightarrow B_c^*\pi, B_c^*K$ weak decays are described by an effective bottom-changing Hamiltonian based on operator product expansion [14]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb}V_{uq}^* \{C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)\} + \text{h.c.}, \quad (1)$$

where $G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [1] is the Fermi coupling constant; the CKM factors $V_{cb}V_{ud}^*$ and $V_{cb}V_{us}^*$ correspond to $\Upsilon(nS) \rightarrow B_c^*\pi$ and B_c^*K decays, respectively; with the Wolfenstein parameterization, the CKM factors are expanded as a power series in a small Wolfenstein parameter $\lambda \sim 0.2$ [1]:

$$V_{cb}V_{ud}^* = A\lambda^2 - \frac{1}{2}A\lambda^4 - \frac{1}{8}A\lambda^6 + \mathcal{O}(\lambda^7), \quad (2)$$

$$V_{cb}V_{us}^* = A\lambda^3 + \mathcal{O}(\lambda^7). \quad (3)$$

The local tree operators $Q_{1,2}$ are defined as:

$$O_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad (4)$$

$$O_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \quad (5)$$

where α and β are color indices and the sum over repeated indices is understood.

The scale μ factorizes physics contributions into short- and long-distance dynamics. The Wilson coefficients $C_i(\mu)$ summarize the physics contributions at scale higher than μ , and are calculable with the renormalization group improved perturbation theory. The hadronic matrix elements (HME), where the local operators are inserted between initial and final hadron states, embrace the physics contributions below scale of μ . To obtain decay amplitudes, the remaining work is to calculate HME properly by separating from perturbative and nonperturbative contributions.

B. Hadronic matrix elements

Based on Lepage-Brodsky approach for exclusive processes [15], HME is commonly expressed as a convolution integral of hard scattering subamplitudes containing perturbative

contributions with universal wave functions reflecting nonperturbative contributions. In order to effectively regulate endpoint singularities and provide a naturally dynamical cutoff on nonperturbative contributions, transverse momentum of valence quarks is retained and the Sudakov factor is introduced within the pQCD framework [2–4]. Phenomenologically, the pQCD’s decay amplitude could be divided into three parts: the Wilson coefficients C_i incorporating the hard contributions above typical scale of t , process-dependent rescattering subamplitudes T accounting for the heavy quark decay, and wave functions Φ of all participating hadrons, which is expressed as

$$\int dk C_i(t) T(t, k) \Phi(k) e^{-S}, \quad (6)$$

where k is the momentum of valence quarks, and e^{-S} is the Sudakov factor.

C. Kinematic variables

The light cone kinematic variables in the $\Upsilon(nS)$ rest frame are defined as follows.

$$p_\Upsilon = p_1 = \frac{m_1}{\sqrt{2}}(1, 1, 0), \quad (7)$$

$$p_{B_c^*} = p_2 = (p_2^+, p_2^-, 0), \quad (8)$$

$$p_3 = (p_3^-, p_3^+, 0), \quad (9)$$

$$p_i^\pm = (E_i \pm p)/\sqrt{2}, \quad (10)$$

$$k_i = x_i p_i + (0, 0, \vec{k}_{i\perp}), \quad (11)$$

$$\epsilon_1^\parallel = \frac{p_1}{m_1} - \frac{m_1}{p_1 \cdot n_+} n_+, \quad (12)$$

$$\epsilon_2^\parallel = \frac{p_2}{m_2} - \frac{m_2}{p_2 \cdot n_-} n_-, \quad (13)$$

$$\epsilon_{1,2}^\perp = (0, 0, \vec{1}), \quad (14)$$

$$n_+ = (1, 0, 0), \quad (15)$$

$$n_- = (0, 1, 0), \quad (16)$$

$$s = 2 p_2 \cdot p_3, \quad (17)$$

$$t = 2 p_1 \cdot p_2 = 2 m_1 E_2, \quad (18)$$

$$u = 2 p_1 \cdot p_3 = 2 m_1 E_3, \quad (19)$$

$$p = \frac{\sqrt{[m_1^2 - (m_2 + m_3)^2][m_1^2 - (m_2 - m_3)^2]}}{2m_1}, \quad (20)$$

where x_i and $\vec{k}_{i\perp}$ are the longitudinal momentum fraction and transverse momentum of valence quarks, respectively; ϵ_i^\parallel and ϵ_i^\perp are the longitudinal and transverse polarization vectors, respectively, and satisfy relations $\epsilon_i^2 = -1$ and $\epsilon_i \cdot p_i = 0$; the subscript i on variables p_i , E_i , m_i , ϵ_i corresponds to participating hadrons, namely, $i = 1$ for $\Upsilon(nS)$ meson, $i = 2$ for the recoiled B_c^* meson, $i = 3$ for the emitted pseudoscalar meson; n_+ and n_- are positive and negative null vectors, respectively; s , t and u are the Lorentz-invariant variables; p is the common momentum of final states. The notation of momentum is displayed in Fig. 2(a).

D. Wave functions

With the notation in [16, 17], wave functions are defined as

$$\langle 0|b_i(z)\bar{b}_j(0)|\Upsilon(p_1, \epsilon_1^\parallel)\rangle = \frac{f_\Upsilon}{4} \int dk_1 e^{-ik_1 \cdot z} \left\{ \epsilon_1^\parallel [m_1 \Phi_\Upsilon^v(k_1) - \not{p}_1 \Phi_\Upsilon^t(k_1)] \right\}_{ji}, \quad (21)$$

$$\langle 0|b_i(z)\bar{b}_j(0)|\Upsilon(p_1, \epsilon_1^\perp)\rangle = \frac{f_\Upsilon}{4} \int dk_1 e^{-ik_1 \cdot z} \left\{ \epsilon_1^\perp [m_1 \Phi_\Upsilon^V(k_1) - \not{p}_1 \Phi_\Upsilon^T(k_1)] \right\}_{ji}, \quad (22)$$

$$\langle B_c^*(p_2, \epsilon_2^\parallel)|\bar{c}_i(z)b_j(0)|0\rangle = \frac{f_{B_c^*}}{4} \int_0^1 dk_3 e^{ik_3 \cdot z} \left\{ \epsilon_2^\parallel [m_2 \Phi_{B_c^*}^v(k_2) + \not{p}_2 \Phi_{B_c^*}^t(k_2)] \right\}_{ji}, \quad (23)$$

$$\langle B_c^*(p_2, \epsilon_2^\perp)|\bar{c}_i(z)b_j(0)|0\rangle = \frac{f_{B_c^*}}{4} \int_0^1 dk_2 e^{ik_2 \cdot z} \left\{ \epsilon_2^\perp [m_2 \Phi_{B_c^*}^V(k_2) + \not{p}_2 \Phi_{B_c^*}^T(k_2)] \right\}_{ji}, \quad (24)$$

$$\begin{aligned} & \langle P(p_3)|u_i(0)\bar{q}_j(z)|0\rangle \\ &= \frac{i f_P}{4} \int dk_3 e^{ik_3 \cdot z} \left\{ \gamma_5 [\not{p}_3 \Phi_P^a(k_3) + \mu_P \Phi_P^p(k_3) + \mu_P (\not{n}_- \not{n}_+ - 1) \Phi_P^t(k_3)] \right\}_{ji}, \end{aligned} \quad (25)$$

where f_Υ , $f_{B_c^*}$, f_P are decay constants of $\Upsilon(nS)$, B_c^* , P mesons, respectively.

Considering mass relations of $m_{\Upsilon(nS)} \simeq 2m_b$ and $m_{B_c^*} \simeq m_b + m_c$, it might assume that the motion of heavy valence quarks in $\Upsilon(nS)$ and B_c^* mesons is nearly nonrelativistic. The wave functions of $\Upsilon(nS)$ and B_c^* mesons could be approximately described with nonrelativistic quantum chromodynamics (NRQCD) [18–20] and time-independent Schrödinger equation. For an isotropic harmonic oscillator potential, the eigenfunctions of stationary state with quantum numbers nL are written as [21]

$$\phi_{1S}(\vec{k}) \sim e^{-\vec{k}^2/2\beta^2}, \quad (26)$$

$$\phi_{2S}(\vec{k}) \sim e^{-\vec{k}^2/2\beta^2} (2\vec{k}^2 - 3\beta^2), \quad (27)$$

$$\phi_{3S}(\vec{k}) \sim e^{-\vec{k}^2/2\beta^2} (4\vec{k}^4 - 20\vec{k}^2\beta^2 + 15\beta^4), \quad (28)$$

where parameter β determines the average transverse momentum, i.e., $\langle nS | k_\perp^2 | nS \rangle \sim \beta^2$. Employing the substitution ansatz [22],

$$\vec{k}^2 \rightarrow \frac{1}{4} \sum_i \frac{\vec{k}_{i\perp}^2 + m_{q_i}^2}{x_i}, \quad (29)$$

where x_i and m_{q_i} are the longitudinal momentum fraction and mass of valence quark, respectively, then integrating out \vec{k}_\perp and combining with their asymptotic forms, the distribution amplitudes (DAs) for $\Upsilon(nS)$ and B_c^* mesons can be written as [21],

$$\phi_{\Upsilon(1S)}^{v,T}(x) = A x \bar{x} \exp\left\{-\frac{m_b^2}{8\beta_1^2 x \bar{x}}\right\}, \quad (30)$$

$$\phi_{\Upsilon(1S)}^t(x) = B t^2 \exp\left\{-\frac{m_b^2}{8\beta_1^2 x \bar{x}}\right\}, \quad (31)$$

$$\phi_{\Upsilon(1S)}^V(x) = C (1 + t^2) \exp\left\{-\frac{m_b^2}{8\beta_1^2 x \bar{x}}\right\}, \quad (32)$$

$$\phi_{\Upsilon(2S)}^{v,t,V,T}(x) = D \phi_{\Upsilon(1S)}^{v,t,V,T}(x) \left\{1 + \frac{m_b^2}{2\beta_1^2 x \bar{x}}\right\}, \quad (33)$$

$$\phi_{\Upsilon(3S)}^{v,t,V,T}(x) = E \phi_{\Upsilon(1S)}^{v,t,V,T}(x) \left\{\left(1 - \frac{m_b^2}{2\beta_1^2 x \bar{x}}\right)^2 + 6\right\}, \quad (34)$$

$$\phi_{B_c^*}^{v,T}(x) = F x \bar{x} \exp\left\{-\frac{\bar{x} m_c^2 + x m_b^2}{8\beta_2^2 x \bar{x}}\right\}, \quad (35)$$

$$\phi_{B_c^*}^t(x) = G t^2 \exp\left\{-\frac{\bar{x} m_c^2 + x m_b^2}{8\beta_2^2 x \bar{x}}\right\}, \quad (36)$$

$$\phi_{B_c^*}^V(x) = H (1 - t^2) \exp\left\{-\frac{\bar{x} m_c^2 + x m_b^2}{8\beta_2^2 x \bar{x}}\right\}, \quad (37)$$

where $\bar{x} = 1 - x$; $t = x - \bar{x}$. According to NRQCD power counting rules [18], $\beta_i \simeq \xi_i \alpha_s(\xi_i)$ with $\xi_i = m_i/2$ and QCD coupling constant α_s . The exponential function represents k_\perp distribution. Parameters of A, B, C, D, E, F, G, H are normalization coefficients satisfying with the conditions

$$\int_0^1 dx \phi_{\Upsilon(nS)}^i(x) = \int_0^1 dx \phi_{B_c^*}^i(x) = 1 \quad \text{for } i = v, t, V, T. \quad (38)$$

The shape lines of normalized DAs for $\Upsilon(nS)$ and B_c^* mesons are showed in Fig. 1. It is clearly seen that (1) DAs for $\Upsilon(nS)$ and B_c^* mesons fall quickly down to zero at endpoint x, \bar{x}

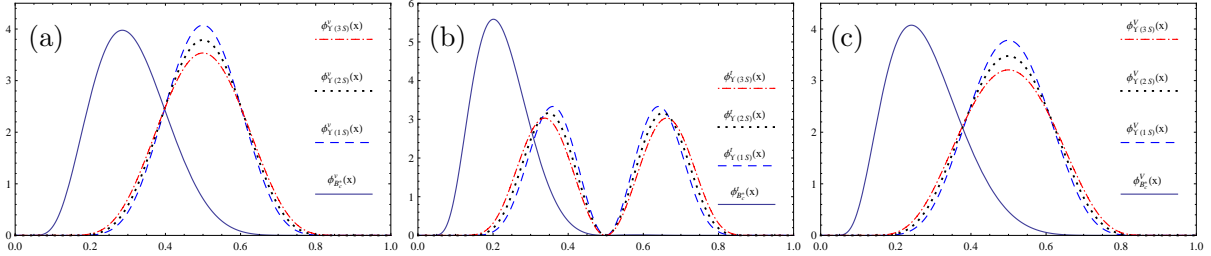


FIG. 1: The normalized distribution amplitudes for $\Upsilon(nS)$ and B_c^* mesons.

$\rightarrow 0$ due to suppression from exponential functions; (2) DAs for $\Upsilon(nS)$ meson are symmetric under the interchange of momentum fractions $x \leftrightarrow \bar{x}$, and DAs for B_c^* meson are basically consistent with the feature that valence quarks share momentum fractions according to their masses.

Our study shows that only the leading twist (twist-2) DAs of the emitted light pseudoscalar meson P is involved in decay amplitudes (see Appendix A). The twist-2 DAs has the expansion [16]:

$$\phi_P^a(x) = 6x\bar{x} \sum_{i=0} a_i C_i^{3/2}(t), \quad (39)$$

and are normalized as

$$\int_0^1 \phi_P^a(x) dx = 1, \quad (40)$$

where $C_i^{3/2}(t)$ are Gegenbauer polynomials,

$$C_0^{3/2}(t) = 1, \quad C_1^{3/2}(t) = 3t, \quad C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), \quad \dots \quad (41)$$

and each term corresponds to a nonperturbative Gegenbauer moment a_i ; note that $a_0 = 1$ due to the normalization condition Eq.(40); the G -parity invariance of the pion DAs requires Gegenbauer moment $a_i = 0$ for $i = 1, 3, 5 \dots$.

E. Decay amplitudes

The Feynman diagrams for $\Upsilon(nS) \rightarrow B_c^* \pi$ weak decay are shown in Fig. 2. There are two types. One is factorizable emission topology where gluon attaches to quarks in the same meson, and the other is nonfactorizable emission topology where gluon connects to quarks between different mesons.

With the pQCD master formula Eq.(6), the amplitude for $\Upsilon(nS) \rightarrow B_c^* P$ decay can be

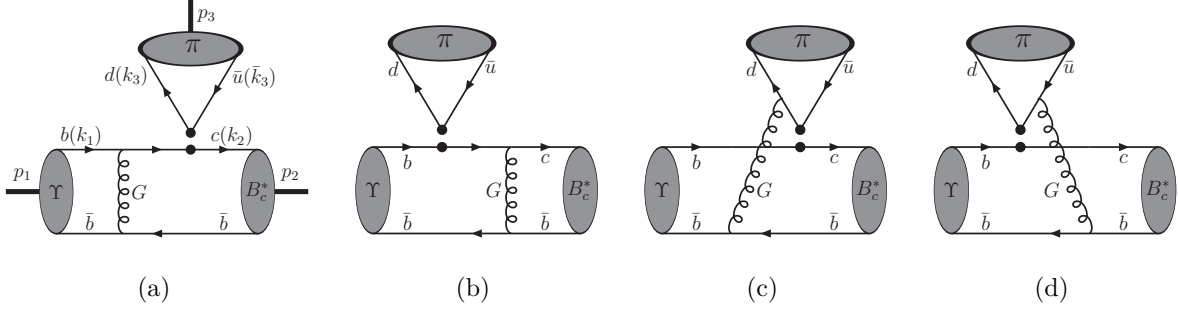


FIG. 2: Feynman diagrams for $\Upsilon(nS) \rightarrow B_c^* \pi$ decay with the pQCD approach, including factorizable emission diagrams (a,b) and nonfactorizable emission diagrams (c,d).

expressed as [23],

$$\mathcal{A}(\Upsilon(nS) \rightarrow B_c^* P) = \mathcal{A}_L(\epsilon_1^\parallel, \epsilon_2^\parallel) + \mathcal{A}_N(\epsilon_1^\perp, \epsilon_2^\perp) + i \mathcal{A}_T \varepsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_2^\nu p_1^\alpha p_2^\beta, \quad (42)$$

which is conventionally written as the helicity amplitudes [23],

$$\mathcal{A}_0 = -C_A \sum_j \mathcal{A}_L^j(\epsilon_1^\parallel, \epsilon_2^\parallel), \quad (43)$$

$$\mathcal{A}_\parallel = \sqrt{2} C_A \sum_j \mathcal{A}_N^j(\epsilon_1^\perp, \epsilon_2^\perp), \quad (44)$$

$$\mathcal{A}_\perp = \sqrt{2} C_A m_1 p \sum_j \mathcal{A}_T^j, \quad (45)$$

$$C_A = i V_{cb} V_{uq}^* \frac{G_F}{\sqrt{2}} \frac{C_F}{N_c} \pi f_\Upsilon f_{B_c^*} f_P, \quad (46)$$

where $C_F = 4/3$ and the color number $N_c = 3$; the subscript i on \mathcal{A}_i^j corresponds to three different helicity amplitudes, i.e., $i = L, N, T$; the superscript j on \mathcal{A}_i^j denotes to indices of Fig. 2. The explicit expressions of building blocks \mathcal{A}_i^j are collected in Appendix A.

III. NUMERICAL RESULTS AND DISCUSSION

In the center-of-mass of $\Upsilon(nS)$ meson, branching ratio $\mathcal{B}r$ for $\Upsilon(nS) \rightarrow B_c^* P$ decay are defined as

$$\mathcal{B}r = \frac{1}{12\pi} \frac{p}{m_\Upsilon^2 \Gamma_\Upsilon} \left\{ |\mathcal{A}_0|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2 \right\}. \quad (47)$$

The input parameters are listed in Table I and II. If not specified explicitly, we will take their central values as the default inputs. Our numerical results are collected in Table. III, where the first uncertainty comes from scale $(1 \pm 0.1)t_i$ and the expression of t_i is given in

Eq.(A20) and Eq.(A21); the second uncertainty is from mass m_b and m_c ; the third uncertainty is from hadronic parameters including decay constants and Gegenbauer moments; the fourth uncertainty is from CKM parameters. The followings are some comments.

TABLE II: The numerical values of input parameters.

The Wolfenstein parameters	
$A = 0.814^{+0.023}_{-0.024}$ [1],	$\lambda = 0.22537 \pm 0.00061$ [1],
Mass, decay constant and Gegenbauer moments	
$m_b = 4.78 \pm 0.06$ GeV [1],	$f_\pi = 130.41 \pm 0.20$ MeV [1],
$m_c = 1.67 \pm 0.07$ GeV [1],	$f_K = 156.2 \pm 0.7$ MeV [1],
$m_{B_c^*} = 6332 \pm 9$ MeV [24],	$f_{B_c^*} = 422 \pm 13$ MeV [26] ^c ,
$a_1^K(1\text{GeV}) = -0.06 \pm 0.03$ [16],	$f_{\Upsilon(1S)} = 676.4 \pm 10.7$ MeV [21],
$a_2^K(1\text{GeV}) = 0.25 \pm 0.15$ [16],	$f_{\Upsilon(2S)} = 473.0 \pm 23.7$ MeV [21],
$a_2^\pi(1\text{GeV}) = 0.25 \pm 0.15$ [16],	$f_{\Upsilon(3S)} = 409.5 \pm 29.4$ MeV [21].

TABLE III: Branching ratio for $\Upsilon(nS) \rightarrow B_c^* P$ decays.

modes	$\Upsilon(1S) \rightarrow B_c^* \pi$	$\Upsilon(2S) \rightarrow B_c^* \pi$	$\Upsilon(3S) \rightarrow B_c^* \pi$
$10^{10} \times \mathcal{B}r$	$4.35^{+0.29+0.19+0.44+0.17}_{-0.24-0.41-0.31-0.30}$	$2.28^{+0.13+0.26+0.40+0.09}_{-0.03-0.35-0.16-0.15}$	$2.14^{+0.12+0.09+0.48+0.07}_{-0.12-0.41-0.15-0.15}$
modes	$\Upsilon(1S) \rightarrow B_c^* K$	$\Upsilon(2S) \rightarrow B_c^* K$	$\Upsilon(3S) \rightarrow B_c^* K$
$10^{11} \times \mathcal{B}r$	$3.45^{+0.23+0.13+0.38+0.13}_{-0.21-0.35-0.27-0.25}$	$1.91^{+0.11+0.07+0.36+0.07}_{-0.09-0.31-0.15-0.14}$	$1.65^{+0.09+0.08+0.40+0.05}_{-0.21-0.33-0.13-0.12}$

(1) Branching ratio for $\Upsilon(nS) \rightarrow B_c^* \pi$ decay is about $\mathcal{O}(10^{-10})$ with pQCD approach, which is well within the measurement potential of LHC and SuperKEKB. For example, experimental studies have showed that production cross sections for $\Upsilon(nS)$ meson in p-p and p-Pb collisions are a few μb at the LHCb [27, 28] and ALICE [29, 30] detectors. Consequently, there will be more than 10^{12} $\Upsilon(nS)$ data samples per ab^{-1} data collected by

^c The decay constant $f_{B_c^*}$ cannot be extracted from the experimental data because of no measurement on B_c^* weak decay at the present time. Theoretically, the value of $f_{B_c^*}$ has been estimated, for example, in Ref. [25] with the QCD sum rules. From Table. 3 of Ref. [25], one can see that the value of $f_{B_c^*}$ are model-dependent. In our calculation, we will take the latest value given by the lattice QCD approach [26] just to offer an order of magnitude estimation on branching ratio for $\Upsilon(nS) \rightarrow B_c^* P$ decays.

the LHCb and ALICE, corresponding to a few hundreds of $\Upsilon(nS) \rightarrow B_c^* \pi$ events. Branching ratio for $\Upsilon(nS) \rightarrow B_c^* K$ decay, $\mathcal{O}(10^{-11})$, is generally less than that for $\Upsilon(nS) \rightarrow B_c^* \pi$ decay by one order of magnitude due to the CKM suppression, $|V_{us}^*/V_{ud}^*|^2 \sim \lambda^2$.

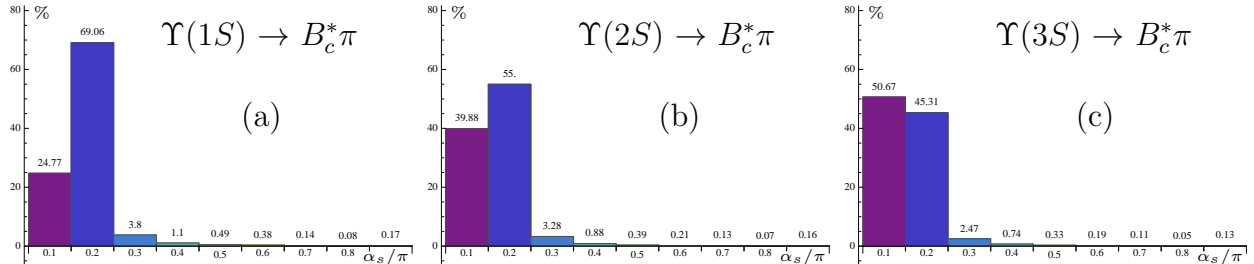


FIG. 3: The contributions to branching ratios for $\Upsilon(1S) \rightarrow B_c^* \pi$ decay (a), $\Upsilon(2S) \rightarrow B_c^* \pi$ decay (b) and $\Upsilon(3S) \rightarrow B_c^* \pi$ decay (c) from different region of α_s/π (horizontal axes), where the numbers over histogram denote the percentage of the corresponding contributions.

(2) As it is well known, due to the large mass of B_c^* , the momentum transition in the $\Upsilon(nS) \rightarrow B_c^* P$ decay may be not large enough. One might naturally wonder whether the pQCD approach is applicable and whether the perturbative calculation is reliable. Therefore, it is necessary to check what percentage of the contributions comes from the perturbative region. The contributions to branching ratio for $\Upsilon(nS) \rightarrow B_c^* \pi$ decay from different α_s/π region are showed in Fig. 3. It can be clearly seen that more than 93% (97%) contributions come from the $\alpha_s/\pi \leq 0.2$ (0.3) region, implying that the $\Upsilon(nS) \rightarrow B_c^* \pi$ decay is computable with the pQCD approach. As the discussion in [2–4], there are many factors for this, for example, the choice of the typical scale, retaining the quark transverse moment and introducing the Sudakov factor to suppress the nonperturbative contributions, which deserve much attention and further investigation.

(3) Because of the relations among masses $m_{\Upsilon(3S)} > m_{\Upsilon(2S)} > m_{\Upsilon(1S)}$ resulting in the fact that phase space increases with the radial quantum number n , in addition, the relations among decay widths $\Gamma_{\Upsilon(3S)} < \Gamma_{\Upsilon(2S)} < \Gamma_{\Upsilon(1S)}$, in principle, there should be relations among branching ratios $\mathcal{B}r(\Upsilon(3S) \rightarrow B_c^* P) > \mathcal{B}r(\Upsilon(2S) \rightarrow B_c^* P) > \mathcal{B}r(\Upsilon(1S) \rightarrow B_c^* P)$ for the same pseudoscalar meson P . But the numerical results in Table. III are beyond such expectation. Why? The reason is that the factor of $p/m_{\Upsilon(nS)}^2$ in Eq.(47) has almost the same value for $n \leq 3$, so branching ratio is proportional to factor $f_{\Upsilon(nS)}^2/\Gamma_{\Upsilon(nS)}$ with the maximal value $f_{\Upsilon(1S)}^2/\Gamma_{\Upsilon(1S)}$ for $n \leq 3$. Besides, contributions from $\alpha_s/\pi \in [0.2, 0.3]$ regions decrease with

n (see Fig. 3), which enhance the decay amplitudes.

(4) Besides the uncertainties listed in Table III, other factors, such as the models of wave functions, contributions of higher order corrections to HME, relativistic effects, and so on, deserve the dedicated study. Our results just provide an order of magnitude estimation.

IV. SUMMARY

The $\Upsilon(nS)$ decay via the weak interaction, as a complementary to strong and electromagnetic decay mechanism, is allowable within the standard model. Based on the potential prospects of $\Upsilon(nS)$ physics at high-luminosity collider experiment, $\Upsilon(nS)$ decay into $B_c^*\pi$ and B_c^*K final states is investigated with the pQCD approach firstly. It is found that (1) the dominant contributions come from perturbative regions $\alpha_s/\pi \leq 0.3$, which might imply that the pQCD calculation is practicable and workable; (2) there is a promising possibility of searching for $\Upsilon(nS) \rightarrow B_c^*\pi$ (B_c^*K) decay with branching ratio about 10^{-10} (10^{-11}) at the future experiments.

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Appendix A: Building blocks for $\Upsilon \rightarrow B_c^*P$ decays

The building blocks \mathcal{A}_i^j , where the superscript j corresponds to indices of Fig. 2 and the subscript i relates with different helicity amplitudes, are expressed as follows.

$$\begin{aligned} \mathcal{A}_L^a = & \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_a, b_1, b_2) E_{ab}(t_a) \phi_\Upsilon^v(x_1) \alpha_s(t_a) \\ & a_1(t_a) \left\{ \phi_{B_c^*}^v(x_2) \left[m_1^2 s - (4 m_1^2 p^2 + m_2^2 u) \bar{x}_2 \right] + \phi_{B_c^*}^t(x_2) m_2 m_b u \right\}, \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \mathcal{A}_N^a = & \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_a, b_1, b_2) E_{ab}(t_a) \phi_\Upsilon^V(x_1) \\ & \alpha_s(t_a) a_1(t_a) m_1 \left\{ \phi_{B_c^*}^V(x_2) m_2 (u - s \bar{x}_2) + \phi_{B_c^*}^T(x_2) m_b s \right\}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \mathcal{A}_T^a = & -2 m_1 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_a, b_1, b_2) E_{ab}(t_a) \\ & \phi_\Upsilon^V(x_1) \alpha_s(t_a) a_1(t_a) \left\{ \phi_{B_c^*}^T(x_2) m_b + \phi_{B_c^*}^V(x_2) m_2 x_2 \right\}, \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \mathcal{A}_L^b &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_b, b_2, b_1) E_{ab}(t_b) \phi_{B_c^*}^v(x_2) \\ &\quad \alpha_s(t_b) a_1(t_b) \left\{ \phi_\Upsilon^v(x_1) \left[m_2^2 u - m_1^2 (s - 4p^2) \bar{x}_1 \right] + \phi_\Upsilon^t(x_1) m_1 m_c s \right\}, \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \mathcal{A}_N^b &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_b, b_2, b_1) E_{ab}(t_b) \phi_{B_c^*}^V(x_2) \\ &\quad \alpha_s(t_b) a_1(t_b) m_2 \left\{ \phi_\Upsilon^V(x_1) m_1 (s - u \bar{x}_1) + \phi_\Upsilon^T(x_1) m_c u \right\}, \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} \mathcal{A}_T^b &= -2 m_2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_b, b_2, b_1) E_{ab}(t_b) \\ &\quad \phi_{B_c^*}^V(x_2) \alpha_s(t_b) a_1(t_b) \left\{ \phi_\Upsilon^V(x_1) m_1 x_1 + \phi_\Upsilon^T(x_1) m_c \right\}, \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \mathcal{A}_L^c &= \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_c, b_2, b_3) \\ &\quad E_{cd}(t_c) \phi_P^a(x_3) \alpha_s(t_c) C_2(t_c) \left\{ \phi_\Upsilon^v(x_1) \phi_{B_c^*}^v(x_2) 4 m_1^2 p^2 (x_1 - \bar{x}_3) \right. \\ &\quad \left. + \phi_\Upsilon^t(x_1) \phi_{B_c^*}^t(x_2) m_1 m_2 (u x_1 - s x_2 - 2 m_3^2 \bar{x}_3) \right\} \delta(b_1 - b_2), \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \mathcal{A}_N^c &= \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_c, b_2, b_3) E_{cd}(t_c) C_2(t_c) \\ &\quad \alpha_s(t_c) \delta(b_1 - b_2) \phi_\Upsilon^T(x_1) \phi_{B_c^*}^T(x_2) \phi_P^a(x_3) \left\{ m_1^2 s (x_1 - \bar{x}_3) + m_2^2 u (\bar{x}_3 - x_2) \right\}, \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} \mathcal{A}_T^c &= \frac{2}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_c, b_2, b_3) E_{cd}(t_c) C_2(t_c) \\ &\quad \alpha_s(t_c) \delta(b_1 - b_2) \phi_\Upsilon^T(x_1) \phi_{B_c^*}^T(x_2) \phi_P^a(x_3) \left\{ m_1^2 (\bar{x}_3 - x_1) + m_2^2 (x_2 - \bar{x}_3) \right\}, \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} \mathcal{A}_L^d &= \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_d, b_2, b_3) \\ &\quad E_{cd}(t_d) \phi_P^a(x_3) \alpha_s(t_d) C_2(t_d) \left\{ \phi_\Upsilon^v(x_1) \phi_{B_c^*}^v(x_2) 4 m_1^2 p^2 (x_3 - x_2) \right. \\ &\quad \left. + \phi_\Upsilon^t(x_1) \phi_{B_c^*}^t(x_2) m_1 m_2 (s x_2 + 2 m_3^2 x_3 - u x_1) \right\} \delta(b_1 - b_2), \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} \mathcal{A}_N^d &= \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_d, b_2, b_3) E_{cd}(t_d) C_2(t_d) \\ &\quad \alpha_s(t_d) \delta(b_1 - b_2) \phi_\Upsilon^T(x_1) \phi_{B_c^*}^T(x_2) \phi_P^a(x_3) \left\{ m_1^2 s (x_3 - x_1) + m_2^2 u (x_2 - x_3) \right\}, \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \mathcal{A}_T^d &= \frac{2}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_d, b_2, b_3) E_{cd}(t_d) C_2(t_d) \\ &\quad \alpha_s(t_d) \delta(b_1 - b_2) \phi_\Upsilon^T(x_1) \phi_{B_c^*}^T(x_2) \phi_P^a(x_3) \left\{ m_1^2 (x_1 - x_3) - m_2^2 (x_2 - x_3) \right\}, \end{aligned} \quad (\text{A12})$$

where $\bar{x}_i = 1 - x_i$; variable x_i is the longitudinal momentum fraction of the valence quark; b_i is the conjugate variable of the transverse momentum $k_{i\perp}$; and $\alpha_s(t)$ is the QCD coupling at the scale of t ; $a_1 = C_1 + C_2/N_c$.

The function H_i are defined as follows [21].

$$H_{ab}(\alpha_e, \beta, b_i, b_j) = K_0(\sqrt{-\alpha}b_i) \left\{ \theta(b_i - b_j) K_0(\sqrt{-\beta}b_i) I_0(\sqrt{-\beta}b_j) + (b_i \leftrightarrow b_j) \right\}, \quad (\text{A13})$$

$$\begin{aligned} H_{cd}(\alpha_e, \beta, b_2, b_3) &= \left\{ \theta(-\beta) K_0(\sqrt{-\beta}b_3) + \frac{\pi}{2} \theta(\beta) [iJ_0(\sqrt{\beta}b_3) - Y_0(\sqrt{\beta}b_3)] \right\} \\ &\times \left\{ \theta(b_2 - b_3) K_0(\sqrt{-\alpha}b_2) I_0(\sqrt{-\alpha}b_3) + (b_2 \leftrightarrow b_3) \right\} \end{aligned} \quad (\text{A14})$$

where J_0 and Y_0 (I_0 and K_0) are the (modified) Bessel function of the first and second kind, respectively; α_e (α_a) is the gluon virtuality of the emission (annihilation) topological diagrams; the subscript of the quark virtuality β_i corresponds to the indices of Fig. 2. The definition of the particle virtuality is listed as follows [21].

$$\alpha = \bar{x}_1^2 m_1^2 + \bar{x}_2^2 m_2^2 - \bar{x}_1 \bar{x}_2 t, \quad (\text{A15})$$

$$\beta_a = m_1^2 - m_b^2 + \bar{x}_2^2 m_2^2 - \bar{x}_2 t, \quad (\text{A16})$$

$$\beta_b = m_2^2 - m_c^2 + \bar{x}_1^2 m_1^2 - \bar{x}_1 t, \quad (\text{A17})$$

$$\begin{aligned} \beta_c &= x_1^2 m_1^2 + x_2^2 m_2^2 + \bar{x}_3^2 m_3^2 \\ &- x_1 x_2 t - x_1 \bar{x}_3 u + x_2 \bar{x}_3 s, \end{aligned} \quad (\text{A18})$$

$$\begin{aligned} \beta_d &= x_1^2 m_1^2 + x_2^2 m_2^2 + x_3^2 m_3^2 \\ &- x_1 x_2 t - x_1 x_3 u + x_2 x_3 s. \end{aligned} \quad (\text{A19})$$

The typical scale t_i and the Sudakov factor E_i are defined as follows, where the subscript i corresponds to the indices of Fig. 2.

$$t_{a(b)} = \max(\sqrt{-\alpha}, \sqrt{-\beta_{a(b)}}, 1/b_1, 1/b_2), \quad (\text{A20})$$

$$t_{c(d)} = \max(\sqrt{-\alpha}, \sqrt{|\beta_{c(d)}|}, 1/b_2, 1/b_3), \quad (\text{A21})$$

$$E_{ab}(t) = \exp\{-S_\Upsilon(t) - S_{B_c^*}(t)\}, \quad (\text{A22})$$

$$E_{cd}(t) = \exp\{-S_\Upsilon(t) - S_{B_c^*}(t) - S_P(t)\}, \quad (\text{A23})$$

$$S_\Upsilon(t) = s(x_1, p_1^+, 1/b_1) + 2 \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q, \quad (\text{A24})$$

$$S_{B_c^*}(t) = s(x_2, p_2^+, 1/b_2) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q, \quad (\text{A25})$$

$$S_{\pi,K}(t) = s(x_3, p_3^+, 1/b_3) + s(\bar{x}_3, p_3^+, 1/b_3) + 2 \int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q, \quad (\text{A26})$$

where $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension; the explicit expression of $s(x, Q, 1/b)$ can be found in the appendix of Ref. [2].

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