The width of the Roper resonance in baryon chiral perturbation theory

Jambul Gegelia,[1,2] Ulf-G. Meißner,[3,1] and De-Liang Yao[1]

[1]*Institute for Advanced Simulation,*
Institut für Kernphysik and Jülich Center for Hadron Physics,
Forschungszentrum Jülich, D-52425 Jülich, Germany
[2]*Tbilisi State University, 0186 Tbilisi, Georgia*
[3]*Helmholtz Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics,*
Universität Bonn, D-53115 Bonn, Germany
(Dated: June 15, 2016)

Abstract

We calculate the width of the Roper resonance at next-to-leading order in a systematic expansion of baryon chiral perturbation theory with pions, nucleons, and the delta and Roper resonances as dynamical degrees of freedom. Three unknown low-energy constants contribute up to the given order. One of them can be fixed by reproducing the empirical value for the width of the Roper decay into a pion and a nucleon. Assuming that the remaining two couplings of the Roper interaction take values equal to those of the nucleon, the result for the width of the Roper decaying into a nucleon and two pions is consistent with the experimental value.

1

I. INTRODUCTION

At low energies, chiral perturbation theory [1, 2] provides a successful description of the Goldstone boson sector of QCD. It turns out that a systematic expansion of loop diagrams in terms of small parameters in effective field theories (EFTs) with heavy degrees of freedom is a rather complicated issue. The problem of power counting in baryon chiral perturbation theory [3] may be solved by using the heavy-baryon approach [4–6] or by choosing a suitable renormalization scheme [7–10]. The Δ resonance and (axial) vector mesons can also be included in EFT (see e.g. Refs. [11–20]). On the other hand, the inclusion of heavier baryons such as the Roper resonance is more complicated.

Despite the fact that the Roper resonance was found a long time ago in a partial wave analysis of pion-nucleon scattering data [21], a satisfactory theory of this state is still missing. The Roper is particularly interesting as it is the first nucleon resonance that exhibits a decay mode into a nucleon and two pions, besides the decay into a nucleon and a pion. Also, the Roper appears unexpectedly low in the spectrum, below the first negative parity nucleon resonance, the $S_{11}(1535)$. It is therefore timely to address this state in a chiral EFT. First steps in this direction have been made in Refs. [22–26].

In this work we calculate the width of the Roper resonance in a systematic expansion in the framework of baryon chiral perturbation theory with pions, nucleons, the delta and Roper resonances as explicit degrees of freedom.

The paper is organised as follows: in Section II we specify the effective Lagrangian, in Section III the pole mass and the width of the Roper resonance are defined and the perturbative calculation of the width is outlined in Section IV. In Section V we discuss the renormalization and the power counting applied to the decay amplitude of the Roper resonance, while Section VI contains the numerical results. We briefly summarize in Section VII.

II. EFFECTIVE LAGRANGIAN

We start by specifying the elements of the chiral effective Lagrangian which are relevant for the calculation of the width of the Roper at next-to-leading order in the power counting specified below. We consider pions, nucleons, the delta and Roper resonances as dynamical degrees of freedom. The corresponding most general effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi NR} + \mathcal{L}_{\pi\Delta R}, \tag{1}$$

where the subscripts indicate the dynamical fields contributing to a given term. From the purely mesonic sector we need the following structures [2, 27]

$$\begin{aligned}
\mathcal{L}_{\pi\pi}^{(2)} &= \frac{F^2}{4}\langle\partial_\mu U \partial^\mu U^\dagger\rangle + \frac{F^2 M^2}{4}\langle U^\dagger + U\rangle, \\
\mathcal{L}_{\pi\pi}^{(4)} &= \frac{1}{8}l_4\langle u^\mu u_\mu\rangle\langle\chi_+\rangle + \frac{1}{16}(l_3 + l_4)\langle\chi_+\rangle^2,
\end{aligned} \tag{2}$$

where $\langle\ \rangle$ denotes the trace in flavor space, F is the pion decay constant in the chiral limit and M is the leading term in the quark mass expansion of the pion mass [2]. The pion fields are contained in the unimodular unitary 2×2 matrix U, with $u = \sqrt{U}$ and

$$u_\mu = i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right],$$

$$\chi^+ = u^\dagger \chi u^\dagger + u \chi^\dagger u \,, \quad \chi = \begin{bmatrix} M^2 & 0 \\ 0 & M^2 \end{bmatrix} . \tag{3}$$

The terms of the Lagrangian with pions and baryons contributing to our calculation read:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i \slashed{D} - m + \frac{1}{2} g \, \slashed{u} \gamma^5 \right\} \Psi_N \,,$$

$$\mathcal{L}_{\pi R}^{(1)} = \bar{\Psi}_R \left\{ i \slashed{D} - m_R + \frac{1}{2} g_R \slashed{u} \gamma^5 \right\} \Psi_R \,,$$

$$\mathcal{L}_{\pi R}^{(2)} = \bar{\Psi}_R \left\{ c_1^R \langle \chi^+ \rangle \right\} \Psi_R \,,$$

$$\mathcal{L}_{\pi NR}^{(1)} = \bar{\Psi}_R \left\{ \frac{g_{\pi NR}}{2} \gamma^\mu \gamma_5 u_\mu \right\} \Psi_N + \text{h.c.} \,,$$

$$\mathcal{L}_{\pi \Delta}^{(1)} = -\bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \left\{ \left(i \slashed{D}^{jk} - m_\Delta \delta^{jk} \right) g^{\mu\nu} - i \left(\gamma^\mu D^{\nu,jk} + \gamma^\nu D^{\mu,jk} \right) + i \gamma^\mu \slashed{D}^{jk} \gamma^\nu + m_\Delta \delta^{jk} \gamma^\mu \gamma^\nu \right.$$
$$\left. + \frac{g_1}{2} \slashed{u}^{jk} \gamma_5 g^{\mu\nu} + \frac{g_2}{2} (\gamma^\mu u^{\nu,jk} + u^{\nu,jk} \gamma^\mu) \gamma_5 + \frac{g_3}{2} \gamma^\mu \slashed{u}^{jk} \gamma_5 \gamma^\nu \right\} \xi_{kl}^{\frac{3}{2}} \Psi_\nu^l \,,$$

$$\mathcal{L}_{\pi N\Delta}^{(1)} = h \, \bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(z_1) \, \omega_\alpha^j \Psi_N + \text{h.c.} \,,$$

$$\mathcal{L}_{\pi\Delta R}^{(1)} = h_R \, \bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(\tilde{z}) \, \omega_\alpha^j \Psi_R + \text{h.c.} \,, \tag{4}$$

where Ψ_N and Ψ_R are isospin doublet fields with bare masses m_{N0} and m_{R0}, corresponding to the nucleon and the Roper resonance, respectively. The vector-spinor isovector-isospinor Rarita-Schwinger field Ψ_ν represents the Δ resonance [28] with bare mass $m_{\Delta 0}$, $\xi^{\frac{3}{2}}$ is the isospin-3/2 projector, $\omega_\alpha^i = \frac{1}{2} \langle \tau^i u_\alpha \rangle$ and $\Theta^{\mu\alpha}(z) = g^{\mu\alpha} + z \gamma^\mu \gamma^\nu$, where z is a so-called off-shell parameter. We fix the off-shell structure of the interactions involving the delta by adopting $g_1 = -g_2 = -g_3$ and $z_1 = \tilde{z} = 0$. Note that these off-shell parameters can be absorbed in LECs and are thus redundant [29, 30]. Leaving out the external sources, the covariant derivatives are defined as follows:

$$D_\mu \Psi_{N/R} = (\partial_\mu + \Gamma_\mu) \Psi_{N/R} \,,$$
$$(D_\mu \Psi)_{\nu,i} = \partial_\mu \Psi_{\nu,i} - 2 \, i \, \epsilon_{ijk} \Gamma_{\mu,k} \Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i} \,,$$
$$\Gamma_\mu = \frac{1}{2} \left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger \right] = \tau_k \Gamma_{\mu,k} \,. \tag{5}$$

Note that a mixing kinetic term of the form $i\lambda_1 \bar{\Psi}_R \gamma_\mu D^\mu \Psi_N - \lambda_2 \bar{\Psi}_R \Psi_N + \text{h.c.}$ can be dropped, since, using field transformations and diagonalising the nucleon-Roper mass matrix, it can be reduced to the form of operators of the Lagrangian presented above [22].

III. THE POLE MASS AND THE WIDTH OF THE ROPER RESONANCE

The dressed propagator of the Roper resonance can be written as

$$iS_R(p) = \frac{i}{\slashed{p} - m_{R0} - \Sigma_R(\slashed{p})} \,, \tag{6}$$

where $-i\,\Sigma_R(\slashed{p})$ is the self-energy, i.e. the sum of all one-particle-irreducible diagrams contributing to the two-point function of the Roper resonance. The pole of the dressed propagator S_R is obtained by solving the equation

$$S_R^{-1}(z) \equiv z - m_{R0} - \Sigma_R(z) = 0 \,. \tag{7}$$

We define the physical mass and the width of the Roper resonance by parameterizing the pole as

$$z = m_R - i\frac{\Gamma_R}{2}. \tag{8}$$

The pertinent topologies of the one- and two-loop diagrams contributing to the self-energy of the Roper resonance are shown in Fig. 1. We use BPHZ renormalization by subtracting loop diagrams in their chiral limit and replace the parameters of the Lagrangian by their renormalized values, i.e. counterterm diagrams are not shown explicitly.
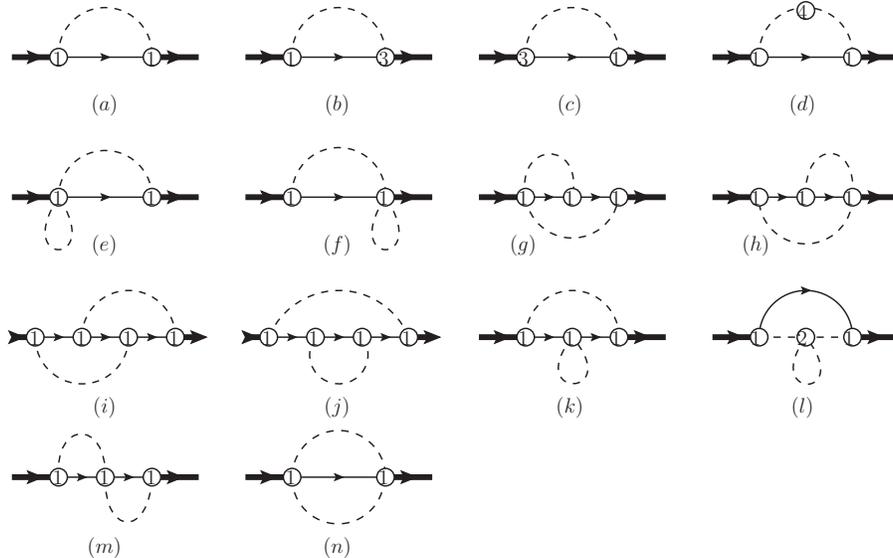

FIG. 1: One and two-loop self-energy diagrams of the Roper resonance up-to-and-including fifth order according to the standard power counting. The dashed and thick solid lines represent the pions and the Roper resonances, respectively. The thin solid lines in the loops stand for either nucleons, Roper or delta resonances. The numbers in the circles give the chiral order of the vertices.

We solve Eq. (7) perturbatively order by order in the loop expansion. We parameterize the pole as

$$z = m_2 + \hbar\delta z_1 + \hbar^2\delta z_2 + \mathcal{O}(\hbar^3), \tag{9}$$

where $m_2 = m_R^0 + 4c_1^R M^2$, with m_R^0 the physical Roper mass in the chiral limit, and substitute in Eq. (7) in which we write the self-energy as an expansion in the number of loops

$$\Sigma_R = \hbar\Sigma_1 + \hbar^2\Sigma_2 + \mathcal{O}(\hbar^3). \tag{10}$$

By expanding in powers of $\hbar$, we get

$$\hbar\delta z_1 + \hbar^2\delta z_2 - \hbar\Sigma_1(m_2) - \hbar^2\delta z_1\Sigma_1'(m_2) - \hbar^2\Sigma_2(m_2) + \mathcal{O}(\hbar^3) = 0. \tag{11}$$

Solving Eq. (11) we obtain

$$\begin{aligned} \delta z_1 &= \Sigma_1(m_2), \\ \delta z_2 &= \Sigma_1(m_2)\,\Sigma_1'(m_2) + \Sigma_2(m_2). \end{aligned} \tag{12}$$

Eq. (12) leads to the following expression for the width

$$
\begin{aligned}
\Gamma_R \;=\;& \hbar\, 2i\,\mathrm{Im}\left[\Sigma_1(m_2)\right] \\
&+\; \hbar^2\, 2i\, \left\{\mathrm{Im}\left[\Sigma_1(m_2)\right]\mathrm{Re}\left[\Sigma_1'(m_2)\right] + \mathrm{Re}\left[\Sigma_1(m_2)\right]\mathrm{Im}\left[\Sigma_1'(m_2)\right]\right\} \\
&+\; \hbar^2\, 2i\,\mathrm{Im}\left[\Sigma_2(m_2)\right] + \mathcal{O}(\hbar^3).
\end{aligned}
\tag{13}
$$

Using the power counting specified in section V, it turns out that the contribution of the second term in Eq. (13) is of an order higher than the accuracy of our calculation, which is δ^5 (where δ is a small expansion parameter). In particular, $\mathrm{Im}\left[\Sigma_1(m_2)\right]$ is of order δ^3, $\mathrm{Re}\left[\Sigma_1'(m_2)\right]$ is of order δ^4, $\mathrm{Re}\left[\Sigma_1(m_2)\right]$ is of order δ^6 and $\mathrm{Im}\left[\Sigma_1'(m_2)\right]$ is of order δ^2. Also, modulo higher order corrections, we can replace m_2 by the physical mass m_R. To calculate the contributions of the one- and two-loop self-energy diagrams to the width of the Roper resonance, specified in the first and third terms of Eq. (13), respectively, we use the Cutkosky cutting rules. As shown in Ref. [31] in quantum field theories with unstable particles the scattering amplitude is unitary in the space of stable particles alone. Thus, to calculate the imaginary part of the self-energy of the Roper resonance at one loop order we need to take into account only the contribution of the diagrams with internal nucleon lines. At two-loop order only contributions obtained by cutting the lines, corresponding to stable particles, are needed. Details of the calculation of the Roper resonance width using the decay amplitudes are given in the next section.

IV. THE WIDTH OF THE ROPER RESONANCE OBTAINED FROM THE DECAY AMPLITUDES

By applying the cutting rules to the diagrams of Fig. 1 we obtain the graphs contributing in the decay amplitudes of the Roper resonance into πN and $\pi\pi N$ systems, specified in Figs. 2 and 3, respectively. The decay amplitude corresponding to $R(p) \to N(p')\pi^a(q)$ can be written as

$$
\mathcal{A}^a = \bar{u}_N(p')\left\{A\,\slashed{q}\gamma_5\tau^a\right\}u_R(p),
\tag{14}
$$

where a is an isospin index of the pion, and the $\bar{u}, u$ are conventional spinors. The corresponding decay width reads

$$
\Gamma_{R\to\pi N} = \frac{\lambda^{1/2}(m_R^2, m_N^2, M^2)}{16\pi\, m_R^3}\,|\mathcal{M}|^2,
\tag{15}
$$

with $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and the unpolarized squared amplitude has the form

$$
|\mathcal{M}|^2 = 3(m_N + m_R)^2\left[(m_N - m_R)^2 - M_\pi^2\right]A^*A.
\tag{16}
$$

Next, we define the kinematical variables for the decay $R(p) \to N(p')\pi^a(q_1)\pi^b(q_2)$ via

$$
s_1 = (q_1 + q_2)^2, \qquad s_2 = (p' + q_1)^2, \qquad s_3 = (p' + q_2)^2,
\tag{17}
$$

subject to the contraint
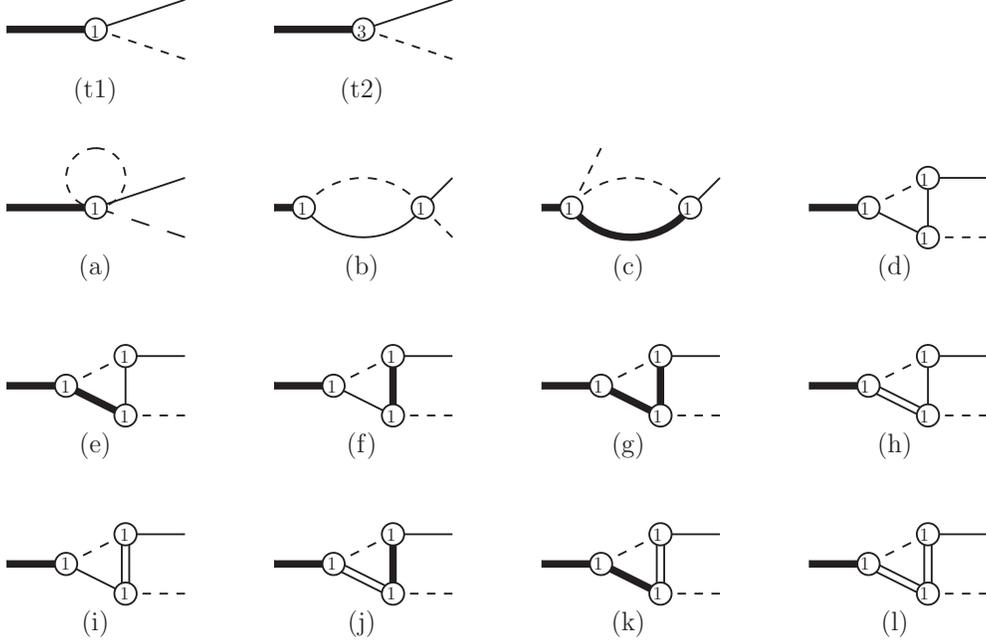
$$
s_1 + s_2 + s_3 = m_R^2 + m_N^2 + 2M_\pi^2.
\tag{18}
$$

FIG. 2: Feynman diagrams contribution to the decay $R \to N\pi$ up to leading one-loop order. Dashed, solid, double and thick solid lines correspond to pions, nucleons, deltas and Roper resonances, respectively. The numbers in the circles give the chiral orders of the vertices.

The isospin and the Lorentz decomposition of the decay amplitude reads

$$\mathcal{A}^{ab} = \chi_N^\dagger \left\{ \delta^{ab} F_+ + i\epsilon^{abc} \tau^c F_- \right\} \chi_R \,, \tag{19}$$

$$F_\pm = \bar{u}_N(p') \left\{ F_\pm^{(1)} - \frac{1}{2(m_N + m_R)} \left[\slashed{q}_1, \slashed{q}_2 \right] F_\pm^{(2)} \right\} u_R(p) \,, \tag{20}$$

with the χ being isospinors, a and b are isospin indices of the pions. The unpolarized squared invariant amplitude is given by

$$|\mathcal{M}|^2 = \sum_{i,j=1}^2 \mathcal{Y}_{ij} \left[\frac{3}{2} F_+^{(i)^*} F_+^{(j)} + 3 F_-^{(i)^*} F_-^{(j)} \right] \,,$$

$$\mathcal{Y}_{11} = 2 \left[(m_N + m_R)^2 - s_1 \right] \,,$$

$$\mathcal{Y}_{12} = \mathcal{Y}_{21} = -s_1 \nu \,,$$

$$\mathcal{Y}_{22} = \frac{1}{2} \left[(4M_\pi^2 - s_1)(s_1 - (m_R - m_N)^2) - s_1 \nu^2 \right] \,, \tag{21}$$

with ν given by

$$\nu = \frac{s_2 - s_3}{m_N + m_R} \,. \tag{22}$$

The decay width corresponding to the $\pi\pi N$ final state is obtained by substituting $|\mathcal{M}|^2$ from Eq. (21) in the following formula

$$\Gamma_{R \to \pi\pi N} = \frac{1}{32 m_R^3 (2\pi)^3} \int_{4M_\pi^2}^{(m_R - m_N)^2} ds_1 \int_{s_{2-}}^{s_{2+}} ds_2 \, |\mathcal{M}|^2 \,, \tag{23}$$

6
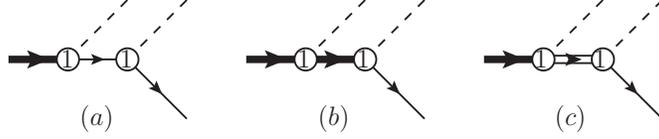
FIG. 3: Tree diagrams contributing to the $R \to \pi\pi N$ decay. Crossed diagrams are not shown. Dashed, solid, double and thick solid lines correspond to pions, nucleons, deltas and Roper resonances, respectively. The numbers in the circles give the chiral orders of the vertices.

where the integration limits over s_2 are given by

$$s_{2\pm} = \frac{m_R^2 + m_N^2 + 2M_\pi^2 - s_1}{2} \pm \frac{1}{2s_1} \lambda^{1/2}(s_1, m_R^2, m_N^2) \lambda^{1/2}(s_1, M_\pi^2, M_\pi^2). \quad (24)$$

Let us emphasize here that to obtain the width of the Roper resonance we need to calculate the imaginary part of the self-energy in the complex region. However, within the accuracy of our calculation, we only need the imaginary parts of the one- and two-loop diagrams for the real mass of the Roper resonance. We can relate these to the decay amplitudes also calculated by putting the Roper external line on the real mass-shell. While this is an useful approximation well suited for our current accuracy, to define the physical properties of unstable particles one needs to use the complex on-shell conditions, see, e.g. Ref. [32].

Thus, the contributions of the one- and two-loop self-energy diagrams in the width of the Roper resonance, specified in the first and third terms of Eq. (13) sum up to

$$\Gamma_R = \Gamma_{R \to \pi N} + \Gamma_{R \to \pi\pi N}, \quad (25)$$

where $\Gamma_{R \to \pi N}$ and $\Gamma_{R \to \pi\pi N}$ are given by Eq. (15) and Eq. (23), respectively. The power counting and the diagrams contributing to each of these decay modes up to a given order of accuracy are discussed in the next section.

V. RENORMALIZATION AND POWER COUNTING

By counting the mass differences $m_R - m_N$, $m_\Delta - m_N$ and $m_R - m_\Delta$ as of the same order as the pion mass and the pion momenta, the standard power counting would apply to all tree and loop diagrams considered in this work. According to the rules of this counting a four-dimensional loop integration is of order q^4, an interaction vertex obtained from an $\mathcal{O}(q^n)$ Lagrangian counts as of order q^n, a pion propagator as order q^{-2}, and a nucleon propagator as order q^{-1}. We would also assign the order q^{-1} to the Δ and the Roper resonance propagators for non-resonant kinematics. The propagators of the delta and the Roper resonance get enhanced for resonant kinematics when they appear as intermediate states outside the loop integration. In this case we would assign the order q^{-3} to these propagators.

As the mass difference $m_R - m_N \sim 400$ MeV, the above mentioned power counting cannot be trusted. By considering $m_R - m_N$ as a small parameter of the order δ^1, it is more appropriate to count $M_\pi \sim \delta^2$. To work out further details of the new counting, it is a more

convenient to work with the kinematical variable ν as defined in Eq. (22) for the $R \to \pi\pi N$ decay. Within the range of integration specified by Eq. (23), ν varies from $m_N - m_R$ to $m_R - m_N$ (for $M_\pi = 0$) and therefore we count $\nu \sim \delta$. As s_1 varies from $4M_\pi^2$ to $(m_R - m_N)^2$, we assign the order δ^2 to it. We also count $m_R - m_\Delta \sim \delta^2$.

The $R \to \pi N$ width of Eq. (16) is of order $\delta^3 \times$ order of A^*A. The tree and one loop diagrams, contributing to the $R \to \pi N$ decay are shown in Fig. 2. The tree order diagram (t2) is proportional to $\not{q}\gamma_5 M_\pi^2$ and therefore it contributes at order δ^4 to A, while diagram (t1) gives an order δ^0 contribution. All one loop diagrams are of the order q^3 in the standard counting. Thus they give order q^2 contributions in A. Expanding these contributions in powers of M_π, we absorb the first, M_π-independent, term in the renormalization of the coupling of the tree diagram (t1), which becomes complex. However, the imaginary part is of the order δ^2 and can be calculated explicitly. The next term in the expansion in powers of M_π is linear in M_π and hence, if non-vanishing, it does not violate the standard power counting (terms, non-analytic in M_π^2 do not violate the standard power counting). Therefore its coefficient must contain at least one power of $(m_R - m_N)$. That is, the term linear in M_π is at least of order δ^3. Further terms are of even higher order. As a result, restricting ourselves to the order δ^2 in A, and thus to order δ^5 in the width of the Roper resonance, the only contribution of one loop diagrams of Fig. 2 which we might need is the M_π-independent imaginary part. However, this imaginary part starts contributing in A^*A only at order δ^4. Thus all contributions of the one-loop diagrams are beyond the accuracy of our calculation. We have checked that for the numerical values of the couplings, as specified below, the individual contributions of the diagrams in Fig. 2 in the decay amplitude are indeed small compared to the one of the tree order diagram.

According to Eq. (23) the $R \to \pi\pi N$ width is of order $\delta^3 \times$ order of $|\mathcal{M}|^2$. The leading order tree diagrams contributing to the $R \to \pi\pi N$ decay are shown in Fig. 3. The delta propagators in these diagrams are to be understood as dressed ones. Expanding these propagators around their pole, we observe that the non-pole parts start contributing at higher orders and therefore can be dropped. The contributions of the loop diagrams are suppressed by additional powers of δ so that they do not contribute at order δ^5. Among these loop diagrams are those contributing to the decay of the Roper resonance to $\pi\pi N$ system with two final pions in the iso-singlet channel. Due to the presence of an iso-scalar scalar resonance $f_0(500)$ in this channel [33] an infinite number of pion-pion finite state interaction diagrams have to be summed up (see, e.g., Refs. [34, 35]). Alternatively one can include the $f_0(500)$ as an explicit degree of freedom in the effective Lagrangian [36]. In both approaches it turns out that the corresponding contributions to $R \to \pi\pi N$ amplitude are of higher order than δ^5, and hence estimated to be within the theoretical uncertainty due to higher order contributions given in Eq. (30) below.

VI. NUMERICAL RESULTS

To calculate the full decay width of the Roper resonance we use the following standard values of the parameters [33]

$$M_\pi = 139 \text{ MeV}, \quad m_N = 939 \text{ MeV}, \quad m_\Delta = 1210 \text{ MeV}, \quad \Gamma_\Delta = 100 \text{ MeV},$$
$$m_R = 1365 \text{ MeV}, F_\pi = 92.2 \text{ MeV}, \tag{26}$$

in Eqs. (15) and (23) and obtain

$$
\begin{aligned}
\Gamma_{R \to \pi N} &= 550 \, g_{\pi N R}^2 \text{ MeV}, \\
\Gamma_{R \to \pi \pi N} &= (1.49 \, g_A^2 \, g_{\pi N R}^2 - 2.76 \, g_A \, g_{\pi N R}^2 \, g_R + 1.48 \, g_{\pi N R}^2 \, g_R^2 \\
&\quad + \ 2.96 \, g_A \, g_{\pi N R} \, h h_R - 3.79 \, g_{\pi N R} \, g_R \, h h_R + 9.93 \, h^2 h_R^2) \text{ MeV}. \qquad (27)
\end{aligned}
$$

Eq. (27) depends on five couplings in total for the two of which we substitute $g_A = 1.27$ [33] and $h = 1.42 \pm 0.02$. The latter value is the real part of this coupling taken from Ref. [37]. As noted before, the imaginary part only contributes to orders beyond the accuracy of our calculations. As for the other unknown parameters, we choose to pin down $g_{\pi N R}$ so that we reproduce the width $\Gamma_{R \to \pi N} = (123.5 \pm 19.0)$ MeV from PDG [33], which yields $g_{\pi N R} = \pm(0.47 \pm 0.04)$. In what follows we take both signs into account which contributes to the error budget. Further we assume $g_R = g_A$ and $h_R = h$. With the values specified above, one can predict the decay width for the decay mode $R \to \pi \pi N$:

$$
\begin{aligned}
\Gamma_{R \to \pi \pi N} &= [0.53(9) - 0.98(17) + 0.53(9) \pm 3.57(31) \mp 4.57(40) + 40.4(1.6)] \text{ MeV} \\
&= [40.5(1.6) \pm 1.0(0.5)] \text{ MeV} , \qquad (28)
\end{aligned}
$$

where the second term is due to the choice of the sign of $g_{\pi N R}$. If we incorporate the second term as error to the first term, then the decay width reads

$$
\Gamma_{R \to \pi \pi N} = 40.5(2.2) \text{ MeV} , \qquad (29)
$$

where the error is obtained in quadrature from Eq. (28). As is clearly seen from Eq. (28), the largest contribution in $\Gamma_{R \to \pi \pi N}$ width comes from the decay with the delta resonance as an intermediate state. Further, we estimate the theoretical error due to the omitting the higher order contributions using the approach of Ref. [38], which leads to

$$
\Gamma_{R \to \pi \pi N} = (40.5 \pm 2.2 \pm 16.8) \text{ MeV}. \qquad (30)
$$

Our estimation is consistent with $\Gamma_{\pi \pi N} = (66.5 \pm 9.5)$ MeV quoted by PDG [33].

VII. SUMMARY

In current work we have calculated the width of the Roper resonance up to next-to-leading order in a systematic expansion of baryon chiral perturbation theory with pions, nucleons, delta and Roper resonances as dynamical degrees of freedom. We define the physical mass and the width of the Roper resonance by relating them to the real and imaginary parts of the complex pole of the dressed propagator. The next-to-leading order calculation of the width requires obtaining the imaginary parts of one- and two-loop self-energy diagrams. We employed the Cutkosky cutting rules and obtained the width up to given order accuracy by squaring the decay amplitudes. Three unknown coupling constants contribute in the corresponding expressions. One of them we fix by reproducing the PDG value for the width of the Roper decay in a pion and a nucleon. Assuming that the remaining two couplings of the Roper interaction take values equal to those of the nucleon, we obtain the result for the width of Roper resonance decaying in the two pions and a nucleon that is consistent with the PDG value, based on the the uncertainties within the given and from higher orders, cf. Eq. (30).

To improve the accuracy of our calculation, three-loop contributions to the self-energy of the Roper resonance need to be calculated. Moreover, contributions of an infinite number of diagrams, corresponding to the scalar-isoscalar pion-pion scattering need to be re-summed either by solving pion-pion scattering equations or including the $f_0(500)$ as an explicit dynamical degree of freedom. Note also that new unknown low-energy coupling constants appear at higher orders, which need to be pinned down.

[1] S. Weinberg, Physica **A96**, 327 (1979).

[2] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).

[3] J. Gasser, M. E. Sainio, and A. Švarc, Nucl. Phys. **B307**, 779 (1988).

[4] E. E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991).

[5] V. Bernard, N. Kaiser, J. Kambor and U.-G. Meißner, Nucl. Phys. B **388**, 315 (1992).

[6] V. Bernard, N. Kaiser and U.-G. Meißner, Int. J. Mod. Phys. E **4**, 193 (1995), [hep-ph/9501384].

[7] H. B. Tang, arXiv:hep-ph/9607436.

[8] T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999).

[9] J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).

[10] T. Fuchs, J. Gegelia, G. Japaridze, and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

[11] T. R. Hemmert, B. R. Holstein, and J. Kambor, J. Phys. G **24**, 1831 (1998).

[12] V. Pascalutsa and D. R. Phillips, Phys. Rev. C **67**, 055202 (2003).

[13] V. Bernard, T. R. Hemmert, and U.-G. Meißner, Phys. Lett. B **565**, 137 (2003).

[14] V. Pascalutsa, M. Vanderhaeghen, and S. N. Yang, Phys. Rept. **437**, 125 (2007).

[15] C. Hacker, N. Wies, J. Gegelia, and S. Scherer, Phys. Rev. C **72**, 055203 (2005).

[16] T. Fuchs, M. R. Schindler, J. Gegelia and S. Scherer, Phys. Lett. B **575**, 11 (2003), [hep-ph/0308006].

[17] P. C. Bruns and U.-G. Meißner, Eur. Phys. J. C **40**, 97 (2005), [hep-ph/0411223].

[18] P. C. Bruns and U.-G. Meißner, Eur. Phys. J. C **58**, 407 (2008), [arXiv:0808.3174 [hep-ph]].

[19] C. Terschlüsen, S. Leupold and M. F. M. Lutz, PoS Bormio **2013**, 046 (2013).

[20] S. Leupold and C. Terschlüsen, PoS BORMIO **2012**, 024 (2012), [arXiv:1206.2253 [hep-ph]].

[21] L. D. Roper, Phys. Rev. Lett. **12**, 340 (1964).

[22] B. Borasoy, P. C. Bruns, U.-G. Meißner and R. Lewis, Phys. Lett. B **641**, 294 (2006).

[23] D. Djukanovic, J. Gegelia, and S. Scherer, Phys. Lett. B **690**, 123 (2010).

[24] B. Long and U. van Kolck, Nucl. Phys. A **870-871**, 72 (2011), [arXiv:1105.2764 [nucl-th]].

[25] T. Bauer, J. Gegelia and S. Scherer, Phys. Lett. B **715**, 234 (2012), [arXiv:1208.2598 [hep-ph]].

[26] E. Epelbaum, J. Gegelia, U.-G. Meißner and D. L. Yao, Eur. Phys. J. C **75**, no. 10, 499 (2015), [arXiv:1510.02388 [hep-ph]].

[27] S. Bellucci, J. Gasser and M. E. Sainio, Nucl. Phys. B **423**, 80 (1994) [Nucl. Phys. B **431**, 413 (1994)], [hep-ph/9401206].

[28] W. Rarita and J. S. Schwinger, Phys. Rev. **60**, 61 (1941).

[29] H. B. Tang and P. J. Ellis, Phys. Lett. B **387**, 9 (1996) [hep-ph/9606432].

[30] H. Krebs, E. Epelbaum and U.-G. Meißner, Phys. Lett. B **683**, 222 (2010). [arXiv:0905.2744 [hep-th]].

[31] M. J. G. Veltman, Physica **29**, 186 (1963).

[32] J. Gegelia and S. Scherer, Eur. Phys. J. A **44**, 425 (2010), [arXiv:0910.4280 [hep-ph]].

[33] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[34] D. Black, A. H. Fariborz, S. Moussa, S. Nasri and J. Schechter, Phys. Rev. D **64**, 014031 (2001), [hep-ph/0012278].

[35] E. Hernandez, E. Oset and M. J. Vicente Vacas, Phys. Rev. C **66**, 065201 (2002), [nucl-th/0209009].

[36] U.-G. Meißner and J. A. Oller, Nucl. Phys. A **673**, 311 (2000), [nucl-th/9912026].

[37] D. L. Yao, D. Siemens, V. Bernard, E. Epelbaum, A. M. Gasparyan, J. Gegelia, H. Krebs and U.-G. Meißner, JHEP **1605**, 038 (2016), [arXiv:1603.03638 [hep-ph]].

[38] E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A **51**, no. 5, 53 (2015), [arXiv:1412.0142 [nucl-th]].