

Gauge-invariant description of Higgs phenomenon and quark confinement

Kei-Ichi Kondo*

Department of Physics, Faculty of Science, Chiba University, Chiba 263-8522, Japan

We propose a novel description for the Higgs mechanism by which a gauge boson acquires the mass. We do not assume spontaneous breakdown of gauge symmetry signaled by a non-vanishing vacuum expectation value of the scalar field. In fact, we give a manifestly gauge-invariant description of the Higgs mechanism in the operator level, which does not rely on spontaneous symmetry breaking. This enables us to discuss the confinement-Higgs complementarity from a new perspective. The “Abelian” dominance in quark confinement of the Yang-Mills theory is understood as a consequence of the gauge-invariant Higgs phenomenon for the relevant Yang-Mills-Higgs model.

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I. INTRODUCTION

The *Brout-Englert-Higgs mechanism* or Higgs phenomenon for short is one of the most well-known mechanisms by which gauge bosons [1] acquire their masses [2–4]. In the conventional wisdom, the Higgs mechanism is understood in such a way that the *spontaneous symmetry breaking* (SSB) generates mass for a gauge boson: The original gauge group G is spontaneously broken down to a subgroup H by choosing a specific vacuum as the physical state from all the possible degenerate ground states (the lowest energy states). Such SSB of the original gauge symmetry is caused by a non-vanishing vacuum expectation value (VEV) $\langle\phi\rangle \neq 0$ of a scalar field ϕ governed by a given potential $V(\phi)$. For a continuous group G , there appear the massless *Nambu-Goldstone bosons* associated with the SSB $G \rightarrow H$ according to the Nambu-Goldstone theorem [5, 6]. When the scalar field couples to a gauge field, however, the massless Nambu-Goldstone bosons are absorbed to provide the gauge boson with the mass. Thus, the massless Nambu-Goldstone bosons disappear from the spectrum. In a semi-classical treatment, the VEV $\langle\phi\rangle$ is identified with one of the minima ϕ_0 of the scalar potential $V(\phi)$, namely, $\langle\phi\rangle = \phi_0 \neq 0$ with $V'(\phi_0) = 0$.

Although this paper focuses on the Higgs phenomenon in the continuum space time, it is very instructive to learn the lattice results, because some non-perturbative and rigorous results are available on the lattice. Especially, the lattice gauge theory *à la* Wilson [7] gives a well-defined gauge theory without gauge fixing. The *Elitzur theorem* [8] tells us that the local continuous gauge symmetry cannot break spontaneously, if no gauge fixing is introduced. In the absence of gauge fixing, all gauge non-invariant Green functions vanish identically. Especially, the VEV $\langle\phi\rangle$ of the scalar field ϕ is rigorously zero,

$$\langle\phi\rangle = 0, \quad (1)$$

no matter what the form of the scalar potential $V(\phi)$.

Therefore, we are forced to fix the gauge to cause the non-zero VEV. Even after the gauge fixing, however, we still have the problem. Whether SSB occurs or not depends on the gauge choice. For instance, in non-compact $U(1)$ gauge-Higgs model under the covariant gauge fixing with a gauge fixing parameter α , the SSB occurs $\langle\phi\rangle \neq 0$ only in the Landau gauge $\alpha = 0$, and no SSB occur $\langle\phi\rangle = 0$ in all other covariant gauges with $\alpha \neq 0$, as rigorously shown in [9, 10]. In an axial gauge, $\langle\phi\rangle = 0$ for compact models [11]. In contrast, it can happen that $\langle\phi\rangle \neq 0$ in a unitary gauge regardless of the shape of the scalar potential. It is obvious that the VEV of the scalar field is not a gauge-independent criterion of SSB.

Even after breaking completely the local gauge symmetry G by imposing a suitable gauge fixing condition, there can remain a global gauge symmetry H' of G . Such a global symmetry H' is called the *remnant global gauge symmetry* [12, 13]. Only a remnant global gauge symmetry H' of the local gauge symmetry G can break spontaneously to cause the Higgs phenomenon [14]. However, such a subgroup H' is not unique and the location of the breaking in the phase diagram depends on H' in the gauge-Higgs model. The relevant numerical evidences are given on a lattice [13] for different H' allowed for various confinement scenarios. Moreover, the transition occurs in the regions where the Fradkin-Shenker-Osterwalder-Seiler theorem [15, 16] assures us that there is no transition in the phase diagram. Thus, the spontaneous gauge symmetry breaking is a rather misleading terminology.

These observations indicate that the Higgs phenomenon should be characterized in a gauge-invariant way without breaking the original gauge symmetry. In this paper, we show that a gauge boson can acquire the mass in a gauge-invariant way without assuming spontaneous breakdown of gauge symmetry which is signaled by the non-vanishing VEV of the scalar field. We demonstrate that the Higgs phenomenon occurs even without such SSB. The spontaneous symmetry breaking is sufficient but not necessary for the Higgs mechanism to work. Remember that quark confinement is realized in the unbroken gauge symmetry phase with *mass gap*. Thus, the gauge-invariant description of the Higgs mechanism can shed new light on the *complementarity* between confine-

*Electronic address: kondok@faculty.chiba-u.jp

ment phase and Higgs phase [17].

II. YANG-MILLS-HIGGS MODEL AND THE CONVENTIONAL HIGGS MECHANISM

In this paper we use the notation for the inner product of the Lie-algebra valued quantities $\mathcal{A} = \mathcal{A}^A T_A$ and $\mathcal{B} = \mathcal{B}^A T_A$; $\mathcal{A} \cdot \mathcal{B} := 2\text{tr}(\mathcal{A}\mathcal{B}) = \mathcal{A}^A \mathcal{B}^B 2\text{tr}(T_A T_B) = \mathcal{A}^A \mathcal{B}^A$ under the normalization $\text{tr}(T_A T_B) = \frac{1}{2}\delta_{AB}$ for the generators T_A of the Lie algebra $su(N)$ ($A = 1, 2, \dots, \dim G = N^2 - 1$) for a gauge group $G = SU(N)$. The $SU(N)$ Yang-Mills field $\mathcal{A}_\mu(x) = \mathcal{A}_\mu^A(x) T_A$ has the field strength $\mathcal{F}_{\mu\nu}(x) = \mathcal{F}_{\mu\nu}^A(x) T_A$ defined by $\mathcal{F}_{\mu\nu} := \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - ig[\mathcal{A}_\mu, \mathcal{A}_\nu]$.

We consider a Yang-Mills-Higgs theory specified by a gauge-invariant action. The Yang-Mills field $\mathcal{A}_\mu(x) = \mathcal{A}_\mu^A(x) T_A$ and the *adjoint scalar field* $\phi(x) = \phi^A(x) T_A$ obey the gauge transformation:

$$\begin{aligned} \mathcal{A}_\mu(x) &\rightarrow U(x)\mathcal{A}_\mu(x)U^{-1}(x) + ig^{-1}U(x)\partial_\mu U^{-1}(x), \\ \phi(x) &\rightarrow U(x)\phi(x)U^{-1}(x), \quad U(x) \in G = SU(N). \end{aligned} \quad (2)$$

For concreteness, consider the $G = SU(N)$ Yang-Mills-Higgs theory with the Lagrangian density:

$$\begin{aligned} \mathcal{L}_{\text{YMH}} &= -\frac{1}{4}\mathcal{F}^{\mu\nu}(x) \cdot \mathcal{F}_{\mu\nu}(x) \\ &\quad + \frac{1}{2}(\mathcal{D}^\mu[\mathcal{A}]\phi(x)) \cdot (\mathcal{D}_\mu[\mathcal{A}]\phi(x)) \\ &\quad - V(\phi(x) \cdot \phi(x)). \end{aligned} \quad (3)$$

where we have defined the covariant derivative $\mathcal{D}_\mu[\mathcal{A}] := \partial_\mu - ig[\mathcal{A}_\mu, \cdot]$ in the adjoint representation. We assume that the adjoint scalar field $\phi(x) = \phi^A(x) T_A$ has the fixed radial length, which is represented by a constraint:¹

$$\phi(x) \cdot \phi(x) \equiv \phi^A(x)\phi^A(x) = v^2. \quad (4)$$

Notice that $\phi(x) \cdot \phi(x)$ is a gauge-invariant combination. Therefore, the potential V as an arbitrary function of $\phi(x) \cdot \phi(x)$ is invariant under the gauge transformation. The covariant derivative $\mathcal{D}_\mu[\mathcal{A}] := \partial_\mu - ig[\mathcal{A}_\mu, \cdot]$ transforms according to the adjoint representation under the gauge transformation: $\mathcal{D}_\mu[\mathcal{A}] \rightarrow U(x)\mathcal{D}_\mu[\mathcal{A}]U^{-1}(x)$. This is also the case for the field strength $\mathcal{F}_{\mu\nu}(x)$. Moreover, the constraint (4) is invariant under the gauge transformation and does not break the gauge invariance

of the theory. Therefore, \mathcal{L}_{YMH} of (3) with the constraint (4) is invariant under the local gauge transformation (2).

For $N = 2$, this theory is nothing but the well-known Georgi-Glashow model which exemplifies the SSB of the local gauge symmetry from $SU(2)$ down to $U(1)$ except for the magnitude of the scalar field being fixed (4). In this paper, we focus our discussions on the $SU(2)$ case.

First, we recall the conventional description for the Higgs mechanism. If the scalar field $\phi(x)$ acquires a non-vanishing VEV $\langle \phi(x) \rangle = \langle \phi \rangle$, then the covariant derivative reduces to

$$\begin{aligned} \mathcal{D}_\mu[\mathcal{A}]\phi(x) &:= \partial_\mu \phi(x) - ig[\mathcal{A}_\mu(x), \phi(x)] \\ &\rightarrow -ig[\mathcal{A}_\mu(x), \langle \phi \rangle] + \dots, \end{aligned} \quad (5)$$

and the Lagrangian density reads

$$\begin{aligned} \mathcal{L}_{\text{YMH}} &\rightarrow -\frac{1}{2}\text{tr}_G\{\mathcal{F}^{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x)\} \\ &\quad - g^2\text{tr}_G\{[\mathcal{A}^\mu(x), \langle \phi \rangle][\mathcal{A}_\mu(x), \langle \phi \rangle]\} + \dots \\ &= -\frac{1}{2}\text{tr}_G\{\mathcal{F}^{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x)\} \\ &\quad - g^2\text{tr}_G\{[T_A, \langle \phi \rangle][T_B, \langle \phi \rangle]\}\mathcal{A}^{\mu A}(x)\mathcal{A}_\mu^B(x) + \dots \end{aligned} \quad (6)$$

To break spontaneously the local continuous gauge symmetry G by realizing the non-vanishing VEV $\langle \phi \rangle$ of the scalar field ϕ , we choose the *unitary gauge* in which the scalar field $\phi(x)$ is pointed to a specific direction $\phi(x) \rightarrow \phi_\infty$ uniformly over the spacetime.

This procedure does not completely break the original gauge symmetry G . Indeed, there may exist a subgroup H of G such that ϕ_∞ does not change under the local H gauge transformation. This is the *partial SSB* $G \rightarrow H$: the mass is provided for the coset G/H (broken parts), while the mass is not supplied for the H -commutative part of \mathcal{A}_μ :

$$\begin{aligned} \mathcal{L}_{\text{YMH}} &\rightarrow -\frac{1}{2}\text{tr}_G\{\mathcal{F}^{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x)\} \\ &\quad - (gv)^2\text{tr}_{G/H}\{\mathcal{A}^\mu(x)\mathcal{A}_\mu(x)\}. \end{aligned} \quad (7)$$

After the partial SSB, therefore, the resulting theory is a gauge theory with the residual gauge group H .

For $G = SU(2)$, by taking the usual *unitary gauge* in which the scalar field $\phi(x) = \phi^A(x) T_A$ ($A = 1, 2, 3$) is chosen so that

$$\langle \phi_\infty \rangle = vT_3, \quad \text{or} \quad \langle \phi_\infty^A \rangle = v\delta^{A3}, \quad (8)$$

the second term of (6) generates the mass term,

$$\begin{aligned} &-g^2v^2\text{tr}_G\{[T_A, T_3][T_B, T_3]\}\mathcal{A}^{\mu A}(x)\mathcal{A}_\mu^B(x) \\ &= \frac{1}{2}g^2v^2(\mathcal{A}^{\mu 1}(x)\mathcal{A}_\mu^1(x) + \mathcal{A}^{\mu 2}(x)\mathcal{A}_\mu^2(x)). \end{aligned} \quad (9)$$

For $SU(2)$, indeed, the off-diagonal gluons \mathcal{A}_μ^a ($a = 1, 2$) acquire the same mass $M_W := gv$, while the diagonal gluon \mathcal{A}_μ^3 remains massless. Even after taking the

¹ After imposing the constraint (4), the subsequent argument should hold irrespective of the form of the potential V . The vacuum manifold in the target space of the scalar field is determined by the minima of the potential V , which also satisfies the constraint (4). However, there are some options as to when and how the constraint is incorporated, see e.g., (39). The potential is omitted in what follows when any confusion does not occur. Moreover, this model is perturbatively non-renormalizable and the non-perturbative treatment is required.

unitary gauge (8), $U(1)$ gauge symmetry described by \mathcal{A}_μ^3 still remains as the residual local gauge symmetry $H = U(1)$, which leaves ϕ_∞ invariant (the local rotation around the axis of the scalar field ϕ_∞).

Thus, the SSB is sufficient for the Higgs mechanism to take place. But, it is not clear whether the SSB is necessary or not for the Higgs mechanism to work.

In the complete SSB $G \rightarrow H = \{1\}$, all components of the Yang-Mills field become massive with no massless components:

$$\mathcal{L}_{\text{YMH}} \rightarrow -\frac{1}{2}\text{tr}_G\{\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\} - (gv)^2\text{tr}_G\{\mathcal{A}^\mu\mathcal{A}_\mu\}, \quad (10)$$

and the resulting theory has no residual gauge symmetry. This case should be separately discussed, see Appendix.

III. GAUGE-INVARIANT HIGGS MECHANISM: $SU(2)$ CASE

Next, we give a novel description, namely, a gauge-invariant (gauge-independent) description of mass generation for gauge bosons without relying on the SSB. We construct a composite vector field $\mathcal{W}_\mu(x)$ from the Yang-Mills field $\mathcal{A}_\mu(x)$ and the (adjoint) scalar field $\phi(x)$ by

$$\mathcal{W}_\mu(x) := -ig^{-1}[\hat{\phi}(x), \mathcal{D}_\mu[\mathcal{A}]\hat{\phi}(x)], \quad (11)$$

with the unit scalar field $\hat{\phi}$ defined by

$$\hat{\phi}(x) := \phi(x)/v, \quad (12)$$

and the covariant derivative in the adjoint representation $\mathcal{D}_\mu[\mathcal{A}]\phi := \partial_\mu\phi - ig[\mathcal{A}_\mu, \phi]$. We find that the kinetic term of the Yang-Mills-Higgs model is identical to the ‘‘mass term’’ of the vector field $\mathcal{W}_\mu(x)$:

$$\begin{aligned} \frac{1}{2}\mathcal{D}^\mu[\mathcal{A}]\phi(x) \cdot \mathcal{D}_\mu[\mathcal{A}]\phi(x) &= \frac{1}{2}M_W^2\mathcal{W}^\mu(x) \cdot \mathcal{W}_\mu(x), \\ M_W &:= gv, \end{aligned} \quad (13)$$

as far as the constraint (4) is satisfied. Indeed, this fact is shown explicitly for $G = SU(2)$:

$$\begin{aligned} g^2v^2\mathcal{W}^\mu \cdot \mathcal{W}_\mu &= v^{-2}2\text{tr}([\phi, \mathcal{D}^\mu[\mathcal{A}]\phi][\phi, \mathcal{D}_\mu[\mathcal{A}]\phi]) \\ &= v^{-2}\{(\phi \cdot \phi)(\mathcal{D}^\mu[\mathcal{A}]\phi \cdot \mathcal{D}_\mu[\mathcal{A}]\phi) \\ &\quad - (\phi \cdot \mathcal{D}^\mu[\mathcal{A}]\phi)(\phi \cdot \mathcal{D}_\mu[\mathcal{A}]\phi)\} \\ &= (\mathcal{D}^\mu[\mathcal{A}]\phi) \cdot (\mathcal{D}_\mu[\mathcal{A}]\phi), \end{aligned} \quad (14)$$

where we have used the constraint (4) and $\phi \cdot \mathcal{D}_\mu[\mathcal{A}]\phi = \phi \cdot \partial_\mu\phi + \phi \cdot (g\mathcal{A}_\mu \times \phi) = g\mathcal{A}_\mu \cdot (\phi \times \phi) = 0$, with $\phi \cdot \partial_\mu\phi = 0$ following from differentiating the constraint (4).

Remarkably, the above ‘‘mass term’’ (13) of \mathcal{W}_μ is gauge invariant, since \mathcal{W}_μ obeys the adjoint gauge transformation:

$$\mathcal{W}_\mu(x) \rightarrow U(x)\mathcal{W}_\mu(x)U^{-1}(x). \quad (15)$$

Therefore, the vector field \mathcal{W}_μ becomes massive without breaking the original gauge symmetry. The above description shows that the SSB of gauge symmetry is not necessary for generating the mass of gauge bosons \mathcal{W}_μ , since we do not need to choose a specific vacuum from all possible degenerate ground states distinguished by the direction of ϕ . The relation (11) gives a gauge-independent definition of the massive gluon mode in the operator level. The relation (11) is also *independent from the parameterization of the scalar field*. See Appendix in which the statement is exemplified for a simpler model.

How is this description related to the conventional one? The constraint $\phi \cdot \phi = v^2$ represents the vacuum manifold in the target space of the scalar field ϕ . The scalar field ϕ subject to the constraint $\phi \cdot \phi = v^2$ is regarded as the *Nambu-Goldstone modes* living in the *flat direction* at the bottom of the potential $V(\phi)$, giving the *degenerate lowest energy states*. Therefore, the massive field \mathcal{W}_μ is formed by combining the *massless (would-be) Nambu-Goldstone modes* with the original massless Yang-Mills field \mathcal{A}_μ . This corresponds to the conventional explanation that the gauge boson acquires the mass by absorbing the Nambu-Goldstone boson appeared in association with the SSB.

Despite its appearance (11) of \mathcal{W}_μ obeying the adjoint gauge transformation, the independent internal degrees of freedom of the new field $\mathcal{W}_\mu = (\mathcal{W}_\mu^A)$ ($A = 1, 2, 3$) is equal to $\dim(G/H) = 2$, since \mathcal{W}_μ has no components parallel to the scalar field, that is to say, \mathcal{W}_μ is orthogonal to the scalar field ϕ :

$$\mathcal{W}_\mu(x) \cdot \phi(x) = 0. \quad (16)$$

Notice that this is a gauge-invariant statement. Thus, $\mathcal{W}_\mu(x)$ represent the massive modes corresponding to the coset space G/H as expected. In this way, we can understand the *residual gauge symmetry* left in the partial SSB: $G = SU(2) \rightarrow H = U(1)$. In fact, by taking the unitary gauge $\phi(x) \rightarrow \phi_\infty = v\hat{\phi}_\infty$, \mathcal{W}_μ reduces to

$$\begin{aligned} \mathcal{W}_\mu(x) &\rightarrow -ig^{-1}[\hat{\phi}_\infty, \mathcal{D}_\mu[\mathcal{A}]\hat{\phi}_\infty] \\ &= [\hat{\phi}_\infty, [\hat{\phi}_\infty, \mathcal{A}_\mu(x)]] \\ &= \mathcal{A}_\mu(x) - (\mathcal{A}_\mu(x) \cdot \hat{\phi}_\infty)\hat{\phi}_\infty. \end{aligned} \quad (17)$$

Then \mathcal{W}_μ agrees with the off-diagonal components for the specific choice $\hat{\phi}_\infty^A = \delta^{A3}$:

$$\mathcal{W}_\mu^A(x) \rightarrow \begin{cases} \mathcal{A}_\mu^a(x) & (A = a = 1, 2) \\ 0 & (A = 3) \end{cases}. \quad (18)$$

This suggests that the original gauge field \mathcal{A}_μ is separated into two pieces:

$$\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{W}_\mu(x). \quad (19)$$

By definition, the field $\mathcal{V}_\mu(x)$ transforms under the gauge transformation just like the original gauge field $\mathcal{A}_\mu(x)$:

$$\mathcal{V}_\mu(x) \rightarrow U(x)\mathcal{V}_\mu(x)U^{-1}(x) + ig^{-1}U(x)\partial_\mu U^{-1}(x). \quad (20)$$

Then the question is how to characterize the first piece $\mathcal{V}_\mu(x)$ which is expected to become dominant in the low-energy $E \ll M_W$ region, where $\mathcal{W}_\mu(x)$ with the mass M_W can be negligible. According to (11), it is shown that $\mathcal{W}_\mu(x) = 0$ is equivalent to

$$\mathcal{D}_\mu[\mathcal{V}]\hat{\phi}(x) = 0. \quad (21)$$

Using the first equation (16) and the second equation (21), we find that a composite vector field \mathcal{V}_μ is constructed from the Yang-Mills field \mathcal{A}_μ and the scalar field ϕ as [18]:

$$\mathcal{V}_\mu(x) = c_\mu(x)\hat{\phi}(x) + ig^{-1}[\hat{\phi}(x), \partial_\mu\hat{\phi}(x)], \quad (22)$$

$$c_\mu(x) := \mathcal{A}_\mu(x) \cdot \hat{\phi}(x). \quad (23)$$

In fact, this form for $\mathcal{V}_\mu(x)$ agrees with $\mathcal{V}_\mu(x) = \mathcal{A}_\mu(x) - \mathcal{W}_\mu(x)$ when eq.(11) is substituted into $\mathcal{W}_\mu(x)$. In the unitary gauge $\phi(x) \rightarrow \phi_\infty = v\hat{\phi}_\infty$, \mathcal{V}_μ reduces to

$$\mathcal{V}_\mu(x) \rightarrow (\mathcal{A}_\mu(x) \cdot \hat{\phi}_\infty)\hat{\phi}_\infty. \quad (24)$$

Then, \mathcal{V}_μ agrees with the diagonal component for $\hat{\phi}_\infty^A = \delta^{A3}$:

$$\mathcal{V}_\mu^A(x) \rightarrow \begin{cases} 0 & (A = a = 1, 2) \\ \mathcal{A}_\mu^3(x) & (A = 3) \end{cases}. \quad (25)$$

Thus, the above arguments go well in the topologically trivial sector.

In the topologically non-trivial sector, the above argument must be improved, since $\partial_\mu\hat{\phi}$ is not identically zero in the presence of singularities related to the topological configuration. Indeed, in order to realize the unitary gauge configuration starting from the hedgehog configuration of the scalar field, we need to perform the singular gauge transformation in the presence of the *'t Hooft-Polyakov magnetic monopole* [19]. This case will be refined later.

Notice that the decomposition equality (19) represents a rather non-trivial statement where $\mathcal{V}_\mu(x)$ is identified with (22) and $\mathcal{W}_\mu(x)$ is identified with (11). We first introduce the fields $\mathcal{V}_\mu(x)$ and $\mathcal{W}_\mu(x)$ as composite field operators of $\mathcal{A}_\mu(x)$ and $\hat{\phi}(x)$. Then we regard a set of field variables $\{c_\mu(x), \mathcal{W}_\mu(x), \hat{\phi}(x)\}$ as obtained from $\{\mathcal{A}_\mu(x), \hat{\phi}(x)\}$ based on *change of variables*:

$$\{c_\mu(x), \mathcal{W}_\mu(x), \hat{\phi}(x)\} \leftarrow \{\mathcal{A}_\mu(x), \hat{\phi}(x)\}, \quad (26)$$

where (23) and (11) give respectively the transformation law of $c_\mu(x)$ and $\mathcal{W}_\mu(x)$ from $\{\mathcal{A}_\mu(x), \hat{\phi}(x)\}$. Indeed, we can calculate the Jacobian associated with this change of variables. See [20, 37, 41] for details. Finally, we identify $c_\mu(x)$, $\mathcal{W}_\mu(x)$ and $\hat{\phi}(x)$ with the fundamental field variables (which are independent up to the constraint (4)) for describing the massive Yang-Mills theory anew. (Here fundamental means that the quantization should be performed with respect to those variables $\{c_\mu(x), \mathcal{W}_\mu(x), \hat{\phi}(x)\}$ which appear e.g., in the path-integral measure.)

According to the decomposition (19), the field strength $\mathcal{F}_{\mu\nu}(x)$ of the gauge field $\mathcal{A}_\mu(x)$ is decomposed as

$$\begin{aligned} \mathcal{F}_{\mu\nu}[\mathcal{A}] &:= \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu - ig[\mathcal{A}_\mu, \mathcal{A}_\nu] \\ &= \mathcal{F}_{\mu\nu}[\mathcal{V}] + \mathcal{D}_\mu[\mathcal{V}]\mathcal{W}_\nu - \mathcal{D}_\nu[\mathcal{V}]\mathcal{W}_\mu \\ &\quad - ig[\mathcal{W}_\mu, \mathcal{W}_\nu]. \end{aligned} \quad (27)$$

By substituting the decomposition (27) into the $SU(2)$ Yang-Mills-Higgs Lagrangian, we obtain

$$\begin{aligned} \mathcal{L}_{\text{YMH}} &= -\frac{1}{4}\mathcal{F}_{\mu\nu}[\mathcal{V}] \cdot \mathcal{F}^{\mu\nu}[\mathcal{V}] \\ &\quad - \frac{1}{4}(\mathcal{D}_\mu[\mathcal{V}]\mathcal{W}_\nu - \mathcal{D}_\nu[\mathcal{V}]\mathcal{W}_\mu)^2 + \frac{1}{2}M_W^2\mathcal{W}^\mu \cdot \mathcal{W}_\mu \\ &\quad + \frac{1}{2}\mathcal{F}_{\mu\nu}[\mathcal{V}] \cdot ig[\mathcal{W}^\mu, \mathcal{W}^\nu] - \frac{1}{4}(ig[\mathcal{W}_\mu, \mathcal{W}_\nu])^2, \end{aligned} \quad (28)$$

where each term is $SU(2)$ invariant. Then it is easy to observe that the vector field \mathcal{W}_μ has the ordinary kinetic term and the mass term. Therefore, there is a massive vector pole in the propagator of \mathcal{W}_μ (after a certain gauge fixing). Thus, \mathcal{W}_μ is not an auxiliary field, but is a propagating field with the mass M_W (up to possible quantum corrections).

IV. CONFINED MASSIVE PHASE: $SU(2)$ CASE

Remarkably, the field strength $\mathcal{F}_{\mu\nu}[\mathcal{V}](x) := \partial_\mu\mathcal{V}_\nu(x) - \partial_\nu\mathcal{V}_\mu(x) - ig[\mathcal{V}_\mu(x), \mathcal{V}_\nu(x)]$ of $\mathcal{V}_\mu(x)$ is shown to be proportional to $\hat{\phi}(x)$ [20]:

$$\begin{aligned} \mathcal{F}_{\mu\nu}[\mathcal{V}](x) &= \hat{\phi}(x)\{\partial_\mu c_\nu(x) - \partial_\nu c_\mu(x) + H_{\mu\nu}(x)\}, \\ H_{\mu\nu}(x) &:= ig^{-1}\hat{\phi}(x) \cdot [\partial_\mu\hat{\phi}(x), \partial_\nu\hat{\phi}(x)]. \end{aligned} \quad (29)$$

We can introduce the Abelian-like $SU(2)$ gauge-invariant field strength $f_{\mu\nu}(x)$ by

$$\begin{aligned} f_{\mu\nu}(x) &:= \hat{\phi}(x) \cdot \mathcal{F}_{\mu\nu}[\mathcal{V}](x) \\ &= \partial_\mu c_\nu(x) - \partial_\nu c_\mu(x) + H_{\mu\nu}(x). \end{aligned} \quad (30)$$

In the low-energy $E \ll M_W$ or the long-distance $r \gg M_W^{-1}$ region, we can neglect the field \mathcal{W}_μ as the first approximation. Then the dominant low-energy modes are described by the restricted Lagrangian density:

$$\mathcal{L}_{\text{YM}}^{\text{rest}} = -\frac{1}{4}\mathcal{F}^{\mu\nu}[\mathcal{V}] \cdot \mathcal{F}_{\mu\nu}[\mathcal{V}] = -\frac{1}{4}f^{\mu\nu}f_{\mu\nu}. \quad (31)$$

The resulting gauge theory with the Lagrangian (31) is called the *restricted Yang-Mills theory*. Consequently, the $SU(2)$ Yang-Mills theory looks like the Abelian gauge theory (31). But, even at this stage the original non-Abelian gauge symmetry $SU(2)$ is not broken.

In the low-energy $E \ll M_W$ or the long-distance $r \gg M_W^{-1}$ region, the massive components $\mathcal{W}_\mu(x)$ become negligible and the restricted theory become dominant.

This is equal to a phenomenon called the “Abelian” dominance [21, 22] in quark confinement. We have shown that the “Abelian” dominance in quark confinement of the Yang-Mills theory is understood as a consequence of the Higgs mechanism defined in a gauge-invariant way for the relevant (or equivalent) Yang-Mills-Higgs model. The Abelian dominance was confirmed for the string tension [23] and for the propagator [24, 25] for the $SU(2)$ Yang-Mills theory on the lattice in the Maximal Abelian gauge [26], and later reconfirmed based on the gauge-invariant formulation on the lattice for the string tension [27] and the full propagator [28].

Notice that $H_{\mu\nu}(x)$ is *locally* closed ($dH = 0$) and hence it can be *locally* exact ($H = dh$) due to the Poincaré lemma. Then $H_{\mu\nu}(x)$ has the Abelian potential $h_\mu(x)$:

$$H_{\mu\nu}(x) = \partial_\mu h_\nu(x) - \partial_\nu h_\mu(x). \quad (32)$$

Therefore, the $SU(2)$ gauge-invariant Abelian-like field strength $f_{\mu\nu}$ is rewritten as

$$f_{\mu\nu}(x) = \partial_\mu G_\nu(x) - \partial_\nu G_\mu(x), \quad G_\mu(x) := c_\mu(x) + h_\mu(x). \quad (33)$$

We call c_μ the *electric potential* and h_μ the *magnetic potential*. Indeed, h_μ agrees with the Dirac magnetic potential, see section 6.10 of [20].

We can define the *magnetic-monopole current* $k^\mu(x)$ in a gauge-invariant way:

$$k^\mu(x) = \partial_\nu {}^* f^{\mu\nu}(x), \quad (34)$$

where $*$ denotes the Hodge dual, e.g., for $D = 4$, the dual tensor $*f^{\mu\nu}$ of $f^{\mu\nu}$ is defined by $*f^{\mu\nu}(x) := \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}f_{\rho\sigma}(x)$. The magnetic current $k^\mu(x)$ is not identically zero, since the Bianchi identity valid for the electric potential c_μ is violated by the magnetic potential h_μ . The contribution of the gauge-invariant magnetic monopole to the Wilson loop average can be detected using the non-Abelian Stokes theorem for the Wilson loop operator, see [29, 30] and section 6 of [20].

The restricted Yang-Mills theory obtained from the original $SU(2)$ Yang-Mills theory has the magnetic part besides the electric part which exists in the usual non-compact $U(1)$ gauge theory. Therefore, the restricted Yang-Mills theory is regarded as the continuum counterpart to the compact $U(1)$ gauge theory on the lattice which involves the magnetic monopoles leading to confinement in the strong coupling region [31, 32]. It is known [33, 34] that the compact $U(1)$ gauge theory on the lattice has two phases: confinement phase due to magnetic monopoles in the strong coupling region [31, 32] which is separated by a critical coupling from the Coulomb phase in the weak coupling region [35, 36].

The Yang-Mills-Higgs model includes the parameters specifying the potential besides the gauge coupling. They are arbitrary and hence the mass gap of the theory is not uniquely determined. In sharp contrast to the Yang-Mills-Higgs model, the mass gap in the Yang-Mills theory

should be generated in a dynamical way without breaking gauge invariance, and it is determined without free parameters to be adjusted.

In the Yang-Mills theory, indeed, the mass M_W can be generated in a dynamical way, e.g., by a gauge-invariant vacuum condensation $\langle \mathcal{W}^\mu \cdot \mathcal{W}_\mu \rangle$ so that $M_W^2 \simeq \langle \mathcal{W}^\mu \cdot \mathcal{W}_\mu \rangle$ due to the quartic self-interactions $-\frac{1}{4}(ig[\mathcal{W}_\mu(x), \mathcal{W}_\nu(x)])^2$ among $\mathcal{W}_\mu(x)$ field, in sharp contrast to the ordinary Yang-Mills-Higgs model. The analytical calculation for such a condensate was done in [37]. Moreover, the mass M_W has been measured by numerical simulations on the lattice in [28] (see also section 9.4 of [20]) as

$$M_W \simeq 2.69\sqrt{\sigma_{\text{phys}}} \simeq 1.19\text{GeV}, \quad (35)$$

where σ_{phys} is the string tension of the linear potential in the quark-antiquark potential.

The mass M_W is used to show the existence of confinement-deconfinement phase transition at a finite critical temperature T_c , separating confinement phase with vanishing Polyakov loop average at low temperature and deconfinement phase with non-vanishing Polyakov loop average at high temperature [38]. The critical temperature T_c is obtained from the calculated ratio T_c/M_W for a given M_W , which provides a reasonable estimate.

Notice that we cannot introduce the ordinary mass term for the field \mathcal{V}_μ , since it breaks the original gauge invariance. But, another mechanism of generating mass for the Abelian gauge field $G_\mu := c_\mu + h_\mu$ could be available, e.g., magnetic mass for photon due to the Debye screening caused by magnetic monopoles, which yields confinement and mass gap in three-dimensional Yang-Mills-Higgs theory as shown in [39]. Moreover, the Abelian gauge field must be confined, which is a problem of gluon confinement. In view of these, the full propagator of the Abelian gauge field must have a quite complicated form, as has been discussed in e.g., [40].

In the Yang-Mills-Higgs model, the gauge field \mathcal{A}_μ and the scalar field ϕ are independent field variables. However, the Yang-Mills theory should be described by the gauge field \mathcal{A}_μ alone and hence the scalar field ϕ must be supplied by the gauge field \mathcal{A}_μ due to the strong interactions. In other words, the scalar field ϕ should be given as a (complicated) functional of the gauge field. This is achieved by imposing the constraint which we call the *reduction condition* [41, 42], see also section 4 of [20]. We choose e.g.,

$$\chi(x) := [\hat{\phi}(x), \mathcal{D}^\mu[\mathcal{A}]\mathcal{D}_\mu[\mathcal{A}]\hat{\phi}(x)] = \mathbf{0}, \quad (36)$$

which is also written as $\mathcal{D}^\mu[\mathcal{V}]\mathcal{W}_\mu(x) = 0$. This condition is gauge covariant,

$$\chi(x) \rightarrow U(x)\chi(x)U^{-1}(x). \quad (37)$$

This is easily shown from the gauge transformation (2) of the scalar field and the Yang-Mills field.

The reduction condition plays the role of eliminating the extra degrees of freedom introduced by the radially fixed scalar field into the Yang-Mills theory [20].

The reduction condition represents as many conditions as the independent degrees of freedom of the radially fixed scalar field $\phi(x)$, since

$$\chi(x) \cdot \hat{\phi}(x) = 0. \quad (38)$$

Therefore, imposing the reduction condition (36) exactly eliminates extra degrees of freedom introduced by the radially fixed scalar field (4), see [20].

Fortunately, the reduction condition is automatically satisfied in the level of field equations. We introduce a Lagrange multiplier field $\lambda(x)$ to incorporate the constraint (4) into the Lagrangian:

$$\mathcal{L}'_{\text{YMH}} = \mathcal{L}_{\text{YMH}} + \lambda(x) (\phi(x) \cdot \phi(x) - v^2). \quad (39)$$

Then the field equations are obtained as

$$\frac{\delta S'_{\text{YMH}}}{\delta \lambda(x)} = \phi(x) \cdot \phi(x) - v^2 = 0, \quad (40)$$

$$\frac{\delta S'_{\text{YMH}}}{\delta \mathcal{A}^\mu(x)} = \mathcal{D}^\nu[\mathcal{A}] \mathcal{F}_{\nu\mu}(x) - ig[\phi(x), \mathcal{D}_\mu[\mathcal{A}]\phi(x)] = 0, \quad (41)$$

$$\begin{aligned} \frac{\delta S'_{\text{YMH}}}{\delta \phi(x)} = & -\mathcal{D}^\mu[\mathcal{A}] \mathcal{D}_\mu[\mathcal{A}]\phi(x) - 2\phi(x)V'(\phi(x) \cdot \phi(x)) \\ & + 2\lambda(x)\phi(x) = 0. \end{aligned} \quad (42)$$

The reduction condition (36) follows by applying the covariant derivative $\mathcal{D}^\mu[\mathcal{A}]$ to (41) as $\mathcal{D}^\mu[\mathcal{A}]\mathcal{D}^\nu[\mathcal{A}]\mathcal{F}_{\nu\mu} = ig\mathcal{D}^\mu[\mathcal{A}][\phi, \mathcal{D}_\mu[\mathcal{A}]\phi] = ig[\phi, \mathcal{D}^\mu[\mathcal{A}]\mathcal{D}_\mu[\mathcal{A}]\phi]$ (this is the covariant version of the current conservation law), since $\mathcal{D}^\mu[\mathcal{A}]\mathcal{D}^\nu[\mathcal{A}]\mathcal{F}_{\nu\mu} = 0$. Taking the commutator of the field equation (42) for the scalar field ϕ with ϕ , we find that the reduction condition (36) is automatically satisfied, irrespective of the choice of the potential function $V(\phi \cdot \phi)$: $[\phi, \mathcal{D}^\mu[\mathcal{A}]\mathcal{D}_\mu[\mathcal{A}]\phi] = [\phi, -2\phi V'(\phi \cdot \phi) + 2\lambda\phi] = 0$.

Notice that the equivalence between the Yang-Mills-Higgs theory and the pure Yang-Mills theory is expected to hold only when the scalar field is radially fixed. If we include the radial degree of freedom for the scalar field, the equivalence is lost. Indeed, the radial degree of freedom for the scalar field corresponds to the Higgs particle with a non-zero mass.

V. CONCLUSION AND DISCUSSION

In this paper we have given a gauge-independent description for the Higgs mechanism by which a gauge boson acquires the mass in a manifestly gauge-invariant way. We have written the resulting massive gauge modes \mathcal{W}_μ explicitly in the operator level. Therefore, we can describe the Higgs mechanism without assuming spontaneous breakdown of gauge symmetry relying on a non-vanishing vacuum expectation value of the scalar field. In this way, we can understand the mass generation of

gauge bosons in the gauge-invariant way without breaking the original gauge symmetry. The spontaneous symmetry breaking is sufficient but not necessary for the Higgs mechanism to work.

The novel description of the Higgs mechanism enables us to discuss the confinement-Higgs complementarity from a new perspective. Our results suggest that the $SU(2)$ Yang-Mills theory in the gapped or massive phase is equivalent to the Yang-Mills-Higgs theory with a radially fixed adjoint scalar field in the Higgs phase which is conventionally considered to be associated to the spontaneous symmetry breaking $G = SU(2) \rightarrow H = U(1)$. The gapped or massive phase is regarded as the confinement phase, which was confirmed on a lattice by numerical simulations for the reformulated Yang-Mills theory [20].

Moreover, we have discussed the implications of the gauge-invariant Higgs mechanism for quark confinement. We have shown that the ‘‘Abelian’’ dominance in quark confinement of the $SU(2)$ Yang-Mills theory is understood as a consequence of the gauge-invariant Higgs phenomenon for the relevant $SU(2)$ Yang-Mills-Higgs model.

The case of larger gauge groups $SU(N)$ ($N \geq 3$) will be treated in a subsequent paper. In particular, some interesting cases $SU(3) \rightarrow U(1) \times U(1)$, $SU(3) \rightarrow U(2)$, and $SU(2) \times U(1) \rightarrow U(1)$ will be discussed in detail.

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Appendix A: Higgs mechanism for the complete SSB

We consider the Abelian-Higgs theory or $U(1)$ gauge-scalar theory with the Lagrangian density:

$$\begin{aligned} \mathcal{L}_{\text{AH}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi), \\ V(\phi^*\phi) = & \frac{\lambda}{2} \left(\phi^*\phi - \frac{\mu^2}{\lambda} \right)^2, \quad \phi \in \mathbb{C}, \quad \lambda > 0, \end{aligned} \quad (\text{A1})$$

where $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the field strength of the $U(1)$ gauge field $A_\mu(x)$ and $D_\mu = \partial_\mu - ieA_\mu(x)$ is a $U(1)$ covariant derivative for the complex scalar field $\phi(x) \in \mathbb{C}$ with q being the electric charge of $\phi(x)$. Here $*$ denotes the complex conjugate. For $\mu^2 > 0$, the minimum of the potential is attained when the magnitude of the scalar field is equal to the value:

$$|\phi(x)| = \frac{v}{\sqrt{2}}, \quad v = \sqrt{\frac{\mu^2}{\lambda/2}}. \quad (\text{A2})$$

If we use a representation of polar decomposition for the radially fixed scalar field:

$$\phi(x) = \frac{v}{\sqrt{2}} e^{i\pi(x)/v} \in \mathbb{C}, \quad \pi(x) \in \mathbb{R}, \quad (\text{A3})$$

the covariant derivative reads

$$D_\mu\phi = (\partial_\mu - ieA_\mu)\phi(x) = -\frac{v}{\sqrt{2}}ie\left(A_\mu - \frac{1}{ev}\partial_\mu\pi\right)e^{i\pi/v}, \quad (\text{A4})$$

and the kinetic term of the scalar field reads

$$(D_\mu\phi)^*(D^\mu\phi) = \frac{1}{2}e^2v^2\left(A_\mu - \frac{1}{ev}\partial_\mu\pi\right)^2. \quad (\text{A5})$$

By introducing a new (massive) vector field W_μ by

$$W_\mu(x) := A_\mu(x) - m^{-1}\partial_\mu\pi(x), \quad m := ev, \quad (\text{A6})$$

\mathcal{L}_{AH} is completely rewritten in terms of W_μ :

$$\mathcal{L}_{\text{AH}} = -\frac{1}{4}(\partial_\mu W_\nu - \partial_\nu W_\mu)^2 + \frac{1}{2}m^2W_\mu W^\mu. \quad (\text{A7})$$

The field π is usually interpreted as the massless Nambu-Goldstone boson associated with the complete SSB $G = U(1) \rightarrow H = \{1\}$, which is absorbed into the massive field W_μ . For $G = U(1)$, we find that the massive vector field W_μ has a manifestly gauge-invariant representation written in terms of A_μ and $\hat{\phi}$ ($\hat{\phi} := \phi(x)/|\phi(x)|$):

$$W_\mu(x) = ie^{-1}\hat{\phi}^*(x)D_\mu\hat{\phi}(x) = -ie^{-1}\hat{\phi}(x)D_\mu\hat{\phi}^*(x). \quad (\text{A8})$$

This reduces to (A6) for the parameterization (A3). The representation (A8) is *independent from the parameterization* of the scalar field. Therefore, a different representation is obtained from another parameterization:

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \varphi(x) + i\chi(x)]. \quad (\text{A9})$$

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