

A local and renormalizable framework for the gauge-invariant operator A_{\min}^2 in Euclidean Yang-Mills theories in linear covariant gauges

M. A. L. Capri^{*}, D. Fiorentini[†], M. S. Guimaraes[‡],
B. W. Mintz[§], L. F. Palhares[¶], S. P. Sorella^{||}

*UERJ – Universidade do Estado do Rio de Janeiro,
Instituto de Física – Departamento de Física Teórica – Rua São Francisco Xavier 524,
20550-013, Maracanã, Rio de Janeiro, Brasil*

Abstract

We address the issue of the renormalizability of the gauge-invariant non-local dimension-two operator A_{\min}^2 , whose minimization is defined along the gauge orbit. Despite its non-local character, we show that the operator A_{\min}^2 can be cast in local form through the introduction of an auxiliary Stueckelberg field. The localization procedure gives rise to an unconventional kind of Stueckelberg-type action which turns out to be renormalizable to all orders of perturbation theory. In particular, as a consequence of its gauge invariance, the anomalous dimension of the operator A_{\min}^2 turns out to be independent from the gauge parameter α entering the gauge-fixing condition, being thus given by the anomalous dimension of the operator A^2 in the Landau gauge.

1 Introduction

Dimension-two condensates have been object of intensive investigations in recent years. These condensates might play an important role in the non-perturbative regime of Euclidean Yang-Mills theories, as pointed out by the considerable amount of results obtained through theoretical and phenomenological studies as well as from lattice simulations [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

For instance, the gluon condensate $\langle A_\mu^a A_\mu^a \rangle$ has been largely investigated in the Landau gauge. As pointed out in [5], this condensate enters the operator product expansion (OPE) of the gluon propagator. Moreover, a combined OPE and lattice analysis has shown that this condensate can account for the $1/Q^2$ corrections which have been reported [18, 19, 20, 21, 24, 27, 28, 29, 30, 31,

^{*}caprimarcio@gmail.com

[†]diegofiorentinia@gmail.com

[‡]mshguimaraes@uerj.br

[§]bruno.mintz@uerj.br

[¶]leticia.palhares@uerj.br

^{||}silvio.sorella@gmail.com

33, 34, 35] in the running of the coupling constant and in the gluon correlation functions.

An effective potential for $\langle A_\mu^a A_\mu^a \rangle$ in Landau gauge has been obtained and evaluated in analytic form at two loops in [7, 10, 11, 15, 16], showing that a nonvanishing value of $\langle A_\mu^a A_\mu^a \rangle$ is favoured as it lowers the vacuum energy. As a consequence, a dynamical gluon mass is generated. We also recall that, in the Landau gauge, the operator $A_\mu^a A_\mu^a$ is *BRST*-invariant on shell, a property which has allowed for an all-orders proof of its multiplicative renormalizability [36]. Its anomalous dimension is not an independent parameter, being expressed as a combination of the gauge β -function and of the anomalous dimension of the gauge field A_μ^a [36], namely

$$\gamma_{A^2}\Big|_{\text{Landau}} = \left(\frac{\beta(a)}{a} + \gamma_A^{\text{Landau}}(a) \right), \quad a = \frac{g^2}{16\pi^2}, \quad (1)$$

where $(\beta(a), \gamma_A^{\text{Landau}}(a))$ denote, respectively, the β -function and the anomalous dimension of the gauge field A_μ in the Landau gauge. This relation was conjectured and explicitly verified up to three-loop order in [37].

Dimension-two condensates also play an important role within the context of the Gribov-Zwanziger approach to confinement [38, 39, 40, 41, 42] as well as for the formation of a dynamical gluon mass within the framework of the Dyson-Schwinger equations in Landau gauge, as reported in [1, 43, 44]. These non-perturbative effects give rise to the so called decoupling solution for the gluon propagator [1, 38, 39, 40, 43, 45], *i.e.* to a propagator which exhibits positivity violation, while attaining a finite non-vanishing value at zero momentum. Until now, this behaviour is in very good agreement with the most recent lattice numerical simulations [46, 47, 48, 49]. The generalization of these results to the linear covariant gauges has been worked out recently and can be found in [50, 51, 52, 53, 54, 55, 56, 57, 58].

Despite the huge amount of results obtained so far, it seems fair to state that many aspects related to dimension-two operators deserve a better understanding. This is certainly the case of the gauge invariance, a central issue in order to give a precise physical meaning to the corresponding condensates. This is precisely the topic which will be studied in the present work. Let us proceed thus by briefly introducing the genuine gauge-invariant dimension-two operator A_{\min}^2 .

1.1 Construction and properties of the operator A_{\min}^2

The gauge-invariant dimension-two operator A_{\min}^2 * is constructed by minimizing the functional $\text{Tr} \int d^4x A_\mu^u A_\mu^u$ along the gauge orbit of A_μ [59, 60, 61, 62], namely

$$\begin{aligned} A_{\min}^2 &\equiv \min_{\{u\}} \text{Tr} \int d^4x A_\mu^u A_\mu^u, \\ A_\mu^u &= u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \end{aligned} \quad (2)$$

In particular, the stationary condition of the functional (2) gives rise to a non-local transverse field configuration $A_\mu^h, \partial_\mu A_\mu^h = 0$, which can be expressed as an infinite series in the gauge field

*See Appendix (B) for more details.

A_μ , *i.e.*

$$\begin{aligned} A_\mu^h &= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \phi_\nu, & \partial_\mu A_\mu^h &= 0, \\ \phi_\nu &= A_\nu - ig \left[\frac{1}{\partial^2} \partial A, A_\nu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\nu \frac{1}{\partial^2} \partial A \right] + O(A^3). \end{aligned} \quad (3)$$

Remarkably, the configuration A_μ^h turns out to be left invariant by infinitesimal gauge transformations order by order in the gauge coupling g [63] (see also Appendix B) as

$$\begin{aligned} \delta A_\mu^h &= 0, \\ \delta A_\mu &= -\partial_\mu \omega + ig [A_\mu, \omega]. \end{aligned} \quad (4)$$

Thus, from expression (2) it follows that

$$\begin{aligned} A_{\min}^2 &= \text{Tr} \int d^4x A_\mu^h A_\mu^h, \\ &= \frac{1}{2} \int d^4x \left[A_\mu^a \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A_\nu^a - gf^{abc} \left(\frac{\partial_\nu}{\partial^2} \partial A^a \right) \left(\frac{1}{\partial^2} \partial A^b \right) A_\nu^c \right] + O(A^4). \end{aligned} \quad (5)$$

The gauge-invariant nature of expression (5) can be made manifest by rewriting it in terms of the field strength $F_{\mu\nu}$. In fact, as proven in [59], it turns out that

$$\begin{aligned} A_{\min}^2 &= -\frac{1}{2} \text{Tr} \int d^4x \left(F_{\mu\nu} \frac{1}{D^2} F_{\mu\nu} + 2i \frac{1}{D^2} F_{\lambda\mu} \left[\frac{1}{D^2} D_\kappa F_{\kappa\lambda}, \frac{1}{D^2} D_\nu F_{\nu\mu} \right] \right. \\ &\quad \left. - 2i \frac{1}{D^2} F_{\lambda\mu} \left[\frac{1}{D^2} D_\kappa F_{\kappa\nu}, \frac{1}{D^2} D_\nu F_{\lambda\mu} \right] \right) + O(F^4), \end{aligned} \quad (6)$$

from which the gauge invariance becomes apparent. The operator $(D^2)^{-1}$ in expression (6) denotes the inverse of the Laplacian $D^2 = D_\mu D_\mu$ with D_μ being the covariant derivative [59]. Let us also underline that, in the Landau gauge $\partial_\mu A_\mu = 0$, the operator $(A_\mu^h A_\mu^h)$ reduces to the operator A^2

$$(A_\mu^{h,a} A_\mu^{h,a}) \Big|_{\text{Landau}} = A_\mu^a A_\mu^a. \quad (7)$$

1.2 Aim of the paper and its structure

As already mentioned, the main aim of the present work is to face the issue of the gauge invariance of non-Abelian gauge theories in the presence of dimension-two operators. More precisely, we shall provide a general and detailed analysis of the gauge-invariant quantity $(A_\mu^h A_\mu^h)$, eq.(5), within the framework of Euclidean Yang-Mills theories quantized in the class of the linear covariant gauges. We shall be able to show that, despite its non-local character, the operator $(A_\mu^h A_\mu^h)$ can be localized by means of the introduction of an auxiliary Stueckelberg field. Nevertheless, the resulting theory can be seen as a kind of unconventional Stueckelberg model which does not suffer from the known drawbacks, *i.e.* the non-renormalizability, of the usual Stueckelberg mass term. Therefore, we end up with a well-defined framework accounting for the existence of a gauge-invariant dimension-two operator.

Relying on an exact BRST invariance, we shall establish the multiplicative renormalizability

of the operator $(A_\mu^h A_\mu^h)$ to all orders of perturbation theory by means of the algebraic renormalization. Moreover, the anomalous dimension of $(A_\mu^h A_\mu^h)$ can be proven to be independent from the gauge parameter α and turns out to be equal to the anomalous dimension of the operator A^2 in the Landau gauge, namely

$$\underline{\gamma_{(A^h)^2}} = \underline{\gamma_{A^2}} \Big|_{\text{Landau}} = \left(\frac{\beta(a)}{a} + \gamma_A^{\text{Landau}}(a) \right), \quad a = \frac{g^2}{16\pi^2}. \quad (8)$$

We underline that expression (8) is valid to all orders of perturbation theory, thereby extending the previous one-loop results obtained in [64].

The paper is organized as follows. In Section 2 we present the localization procedure for the operator $(A_\mu^h A_\mu^h)$ within the framework of a BRST-invariant action. In Section 3 we derive the Ward identities and we establish the all-order renormalizability of $(A_\mu^h A_\mu^h)$ by means of the algebraic renormalization [65]. In Section 4 we discuss the anomalous dimensions of $(A_\mu^h A_\mu^h)$ and of the composite operator A_μ^h by means of the renormalization group equations. Section 5 contains our conclusion. A few appendices collect more details about the construction and the properties of the operator $(A_\mu^h A_\mu^h)$.

2 A local framework for the operator $(A_\mu^h A_\mu^h)$

Our first task will be that of finding a local framework for the non-local operator $(A_\mu^h A_\mu^h)$ of expression (5). To that purpose we start with the standard Faddeev-Popov action of Yang-Mills theory quantized in linear covariant gauges with the inclusion of the mass operator (5) as well as of a constraint enforcing the transversality of the field configuration A_μ^h , eq.(3), *i.e.* we consider the action

$$S = S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} + \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} \right), \quad (9)$$

where S_{FP} stands for the Faddeev-Popov action in linear covariant gauges

$$S_{FP} = \int d^4x \left(\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right) \quad (10)$$

and where we have introduced the operator $(A_\mu^h A_\mu^h)$ through the mass parameter m^2 . Also, the transversality of A_μ^h is enforced by the Lagrange multiplier τ^a .

Since the expression for $(A_\mu^h A_\mu^h)$ given in (5) is an infinite sum of nonlocal terms in the gauge field, the action (9) should be first put in a local form before it can be of any practical use. Following [63, 66, 67], this goal can be achieved by the introduction of an auxiliary localizing Stueckelberg field ξ^a , whose role is to give, for each gauge field A_μ , its corresponding configuration that minimizes the functional A^2 , *i.e.*, A_μ^h . This is most naturally implemented by defining a field h which effectively acts on A_μ as a gauge transformation would act, in order to provide the minimizing configuration A^h , that is,

$$A_\mu^h \equiv A_\mu^{h,a} T^a = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h. \quad (11)$$

with

$$h = e^{ig\xi} = e^{ig\xi^a T^a}, \quad (12)$$

where $\{T^a\}$ are the generators of the gauge group $SU(N)$ and ξ^a is a Stueckelberg field.

Therefore, by substituting the expression (11) for A^h in the action (9), we now have a local theory in terms of the field ξ . The price one has to pay to have such a local theory is a non-polynomial action. Indeed, by expanding (11), one finds an infinite series whose first terms are

$$(A^h)_\mu^a = A_\mu^a - D_\mu^{ab}\xi^b - \frac{g}{2}f^{abc}\xi^b D_\mu^{cd}\xi^d + \mathcal{O}(\xi^3), \quad (13)$$

where

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - gf^{abc}A_\mu^c \quad (14)$$

is the covariant derivative in the adjoint representation.

The nonlocal expression (3) for A_μ^h in terms of the gauge field A_μ can be recovered by imposing the transversality condition $\partial_\mu A_\mu^h = 0$, *i.e.* after taking the divergence of both sides of (13), equating it to zero and solving for the Stueckelberg field ξ^a (see eqs.(190),(191),(192),(193) of Appendix (B)). This check is not only important for the consistency of the present framework but it also makes it clear that, due to the transversality condition enforced by the Lagrange multiplier τ^a , the Stueckelberg field ξ^a acquires now a specific meaning: it is precisely the field which brings a generic gauge configuration A_μ into the gauge-invariant and transverse field configuration A_μ^h which minimizes the functional A_{min}^2 . As it will become clear in the following, this relevant feature, encoded in the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$, will give rise to deep differences between our construction and the standard Stueckelberg mass term. The latter is known to be a non-renormalizable theory which has to be treated as an effective field theory [66].

An important feature of A_μ^h , as defined by eq.(11), is its gauge invariance, that is,

$$A_\mu^h \rightarrow A_\mu^h, \quad (15)$$

as can be seen from the gauge transformations with $SU(N)$ matrix V

$$A_\mu \rightarrow V^\dagger A_\mu V + \frac{i}{g}V^\dagger \partial_\mu V, \quad h \rightarrow V^\dagger h, \quad h^\dagger \rightarrow h^\dagger V. \quad (16)$$

The local version of the action (9), in terms of the Stueckelberg field ξ^a , is thus given by

$$\begin{aligned} S &= S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} + \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} \right) \\ &= S_{FP} + \int d^4x \left[\tau^a \left(A_\mu^a - D_\mu^{ab}\xi^b - \frac{g}{2}f^{abc}\xi^b D_\mu^{cd}\xi^d \right) \right] + \\ &\quad + \frac{m^2}{2} \int d^4x \left(A_\mu^a - D_\mu^{ab}\xi^b - \frac{g}{2}f^{abc}\xi^b D_\mu^{cd}\xi^d \right) \left(A_\mu^a - D_\mu^{ae}\xi^e - \frac{g}{2}f^{aef}\xi^e D_\mu^{fg}\xi^g \right) \\ &\quad + \dots \end{aligned} \quad (17)$$

Due to the use of the auxiliary Stueckelberg field ξ^a , expression (17) exhibits a non-polynomial character. At first sight, this feature might seem to jeopardize its renormalizability. Nevertheless, this will not be the case, as we shall prove in the following.

Before entering into the detailed proof of the renormalizability, it is worth addressing the issue of the BRST symmetry as well as taking a look at the propagators of the elementary fields in order to achieve a better understanding of our action as compared to the usual standard massive Stueckelberg theory.

2.1 BRST invariance

The local action S , eq.(17), enjoys an exact BRST symmetry:

$$sS = 0 , \quad (18)$$

where the nilpotent BRST transformations are given by

$$\begin{aligned} sA_\mu^a &= -D_\mu^{ab}c^b , \\ sc^a &= \frac{g}{2}f^{abc}c^bc^c , \\ s\bar{c}^a &= ib^a , \\ sb^a &= 0 , \\ s\tau^a &= 0 , \\ s^2 &= 0 . \end{aligned} \quad (19)$$

From [68], for the Stueckelberg field we have, with i, j indices associated with a generic representation,

$$sh^{ij} = -igc^a(T^a)^{ik}h^{kj} , \quad s(A^h)_\mu^a = 0 , \quad (20)$$

from which the BRST transformation of the field ξ^a can be evaluated iteratively, yielding

$$s\xi^a = -c^a + \frac{g}{2}f^{abc}c^bc^c - \frac{g^2}{12}f^{amr}f^{mpq}c^pc^q\xi^r + O(\xi^3) . \quad (21)$$

Let us also present a second, equivalent, way of evaluating the BRST transformation of the Stueckelberg field ξ^a . Owing to the dimensionless character of ξ^a , one starts by writing

$$s\xi^a = g^{ab}(\xi)c^b , \quad (22)$$

where $g^{ab}(\xi)$ stands for a generic dimensionless quantity which can be expanded in power series of ξ^a . Imposing now nilpotency of the BRST operator s , *i.e.*

$$s^2\xi^a = s(g^{ab}(\xi)c^b) = 0 , \quad (23)$$

one gets the condition

$$\left(\frac{\partial g^{ab}(\xi)}{\partial \xi^m} g^{mp}(\xi) - \frac{\partial g^{ap}(\xi)}{\partial \xi^m} g^{mb}(\xi) \right) = -g^{ac}(\xi) (gf^{cpb}) . \quad (24)$$

The above equation can be easily solved order by order by expanding the quantity $g^{ab}(\xi)$ in power series of ξ^a , obtaining

$$g^{ab}(\xi) = -\delta^{ab} + \frac{g}{2}f^{abc}\xi^c - \frac{g^2}{12}f^{acd}f^{cbe}\xi^e\xi^d + O(\xi^3) , \quad (25)$$

which gives back precisely expression (21).

Let us end this section by checking out the explicit BRST invariance of A_μ^h . To that purpose, it is better to employ a matrix notation for the fields, *i.e.*

$$\begin{aligned} sA_\mu &= -\partial_\mu c + ig[A_\mu, c] , & sc &= -igcc , \\ sh &= -igch , & sh^\dagger &= igh^\dagger c , \end{aligned} \quad (26)$$

with $A_\mu = A_\mu^a T^a$, $c = c^a T^a$, $\xi = \xi^a T^a$. From expression (11) we get

$$\begin{aligned}
sA_\mu^h &= igh^\dagger c A_\mu h + h^\dagger(-\partial_\mu c + ig[A_\mu, c])h - igh^\dagger A_\mu ch - h^\dagger c \partial_\mu h + h^\dagger \partial_\mu (ch) \\
&= igh^\dagger c A_\mu h - h^\dagger(\partial_\mu c)h + igh^\dagger A_\mu ch - igh^\dagger c A_\mu h - igh^\dagger A_\mu ch - h^\dagger c \partial_\mu h + h^\dagger(\partial_\mu c)h + h^\dagger c \partial_\mu h \\
&= 0.
\end{aligned} \tag{27}$$

2.2 Comparison with the standard Stueckelberg mass term

Let us proceed now by discussing the existing differences between our approach, as expressed by the local action S of eq.(17), and the usual Stueckelberg mass term. We begin by recalling that the standard Stueckelberg formulation amounts to adding the mass term $\frac{m^2}{2} \int d^4x A_\mu^{h,a} A_\mu^{h,a}$ to the Faddeev-Popov action, yielding thus the following action

$$S_{Stueck} = S_{FP} + \frac{m^2}{2} \int d^4x A_\mu^{h,a} A_\mu^{h,a}, \tag{28}$$

where S_{FP} is the Faddeev-Popov action of the linear covariant gauges, eq.(10).

In particular, with respect to expression (17), one notices the absence, in the standard Stueckelberg action (28), of the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$ enforcing the transversality condition $\partial_\mu A_\mu^h = 0$. This means that the Stueckelberg mass term, $\frac{m^2}{2} \int d^4x A_\mu^{h,a} A_\mu^{h,a}$, refers to a generic gauge invariant field configuration A_μ^h . One sees therefore that, while in the ordinary Stueckelberg action the mass term is related to a generic gauge invariant configuration A_μ^h , in our case, besides gauge invariance, the configuration A_μ^h is further constrained by the transversality condition $\partial_\mu A_\mu^h = 0$. Therefore, unlike the standard Stueckelberg formulation, our action refers to a very particular and specific mass term, which is the one obtained by minimizing the operator A_{min}^2 , as precisely expressed by the presence of the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$. This is a non-trivial feature of our model, which makes it deeply different from the usual Stueckelberg action (28).

It is instructive to give a look at the propagators of the Stueckelberg field ξ^a which follow from both formulations. In the case of the standard Stueckelberg action, eq.(28), one obtains

$$\langle \xi^a(p) \xi^b(-p) \rangle_{Stueck} = \delta^{ab} \left(1 + \frac{\alpha m^2}{p^2} \right) \frac{1}{m^2 p^2}. \tag{29}$$

This expression captures in a direct and simple way all drawbacks of the standard Stueckelberg formulation, as reviewed in [66]. One notices, in particular, the presence of the mass parameter m^2 in the denominator of (29), a feature which persists even in the Landau gauge, corresponding to $\alpha = 0$, namely

$$\langle \xi^a(p) \xi^b(-p) \rangle_{Stueck}^{\alpha=0} = \frac{\delta^{ab}}{m^2 p^2}. \tag{30}$$

As one can easily figure out, this property prevents the renormalizability of the standard Stueckelberg formulation [66]. In fact, due to the presence of the parameter m^2 in the denominator of expressions (29), (30), non power-counting renormalizable divergences in the inverse of the mass m^2 will show up, invalidating the perturbative loop expansion. As discussed in [66], the theory stemming from the action (28) has to be treated within the realm of an effective non-renormalizable quantum field theory.

Instead, the inclusion of the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$ leads to a deep modification of the Stueckelberg propagator. In fact, from the quadratic part of the action S , eq.(17), one gets (see also Appendix (C) where the complete list of propagators has been given)

$$\langle \xi^a(p) \xi^b(-p) \rangle_S = \alpha \frac{\delta^{ab}}{p^4} . \quad (31)$$

Expression (31) displays several properties. First of all, unlike the propagator of eq.(29), one notices the absence of the mass parameter m^2 . As far as the UV behaviour is concerned, expression (31) does not pose any problem for the validity of the power counting, a property which will ensure in fact the all-order renormalizability of the model, as it will be proven in detail in the next Section. Another interesting feature displayed by expression (31) is the decoupling nature of the Stueckelberg field in the Landau gauge, $\alpha = 0$. In fact, from Appendix (C), it turns out that

$$\langle \xi^a(p) \xi^b(-p) \rangle_S^{\alpha=0} = \langle A_\mu^a(p) \xi^b(-p) \rangle_S^{\alpha=0} = \langle A_\mu^a(p) \tau^b(-p) \rangle_S^\alpha = 0 . \quad (32)$$

This is a remarkable property of the Landau gauge, which expresses in terms of Feynman rules the decoupling of the Stueckelberg field ξ^a . It reflects the expected fact that, when $\partial_\mu A_\mu = 0$, the higher order terms of the infinite series (3) become harmless, due to the presence of the divergence $\partial_\mu A_\mu$. Equation (32) reveals in a clear way the deep difference existing between the present formulation and the standard Stueckelberg one for which, even in the Landau gauge, the field ξ^a does not decouple, see eq.(30). To some extent, property (32) makes almost immediate the perturbative renormalizability of the action S , eq.(17), in the Landau gauge.

Before ending this section, it is worth spending a few words on the possible implications of the existence of a double pole, at vanishing Euclidean momentum $p^2 = 0$, in the Stueckelberg propagator (31). Even if such a behaviour does not pose problems for the UV power-counting, it might give rise to unwanted infrared divergences in the explicit loop calculations. For that, a BRST invariant infrared regularization will be presented in the next subsection, relying on a nice property of the BRST transformation of the Stueckelberg field ξ^a . Moreover, we underline the presence, in expression (31), of the gauge parameter α . This is a welcome feature. In fact, owing to the BRST invariance of the theory, it turns out that the correlation functions $\langle O(x)O(y) \rangle$ of BRST-invariant composite operators $O(x)$ are independent from the gauge parameter α , see ref.[67] for a recent algebraic proof of this statement. This property, combined with the aforementioned BRST-invariant infrared regularization and with the decoupling nature of the Stueckelberg field ξ^a in the Landau gauge, ensures that the gauge invariant correlators $\langle O(x)O(y) \rangle$ are infrared safe.

Finally, we restate the Euclidean nature of our construction, *i.e* we shall not attempt to provide a possible Minkowski interpretation for the action S , eq.(17). Without entering into details, it will suffice to mention that we expect a violation of perturbative unitarity in Minkowski space, even if our model displays an exact BRST symmetry. This is precisely corroborated by the presence of a double pole in the propagator of the Stueckelberg field. Multipole fields are known in fact to give problems with perturbative unitarity. A nice example of this is offered by the non-local mass operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ which has been studied in detail in [69, 70, 71]. Similarly to the present case, the non-local operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ can be cast in local form by introducing a set of suitable auxiliary fields, so that a local formulation can be constructed at the end, enjoying

an exact BRST symmetry [69, 70, 71]. The resulting action turns out to be renormalizable [69, 70]. Nevertheless, it violates perturbative unitarity due to the presence of multipole fields [71]. We point out that the operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ is the first term of the infinite series of the gauge invariant expansion for the operator A_{min}^2 , as one sees from eq.(6). We expect thus that the same problems encountered in the analysis of the perturbative unitarity for the operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$ will show up also in the case of A_{min}^2 .

Though, as it stands, the Euclidean action S , eq.(17), turns out to be useful in order to study non-perturbative aspects of confining Euclidean Yang-Mills theories. In particular, expression (17) arises within the context of the BRST-invariant formulation of the Gribov-Zwanziger theory recently achieved in [54, 55, 67], which takes into account the non-perturbative effects of the Gribov copies. In addition, the action S can be seen as the BRST-invariant extension in linear covariant gauges of the effective model considered by Tissier and Wschebor in the Landau gauge in order to study the positivity violation of the gluon propagator [72, 73]. Lastly, as already pointed out in the Introduction, the action (17) might enable us to investigate the formation of the dimension two-condensate $\langle A_\mu^h A_\mu^h \rangle$ in a BRST-invariant and α -independent way.

2.3 Infrared BRST-invariant regularization for the Stueckelberg field ξ

As mentioned before, the propagator for the Stueckelberg field in expression (31) could give rise to potential IR divergences when performing explicit loop calculations. Though, as outlined in [67], it turns out to be possible to introduce an IR regularizing mass term for the Stueckelberg field compatible with the BRST invariance. For the benefit of the reader, let us reproduce here the construction of [67]. It relies on a nice property displayed by the BRST transformation of the field ξ^a given in eqs.(20), (21), namely

$$s\left(\frac{\xi^a \xi^a}{2}\right) = -\xi^a c^a, \quad (33)$$

as it follows from eq.(26), *i.e.*

$$s(e^{ig\xi}) = -igc e^{ig\xi}. \quad (34)$$

Expanding the exponential in Taylor series, one gets

$$s\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \dots\right) = -igc\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \dots\right). \quad (35)$$

Multiplying both sides of eq.(35) by ξ , yields

$$\xi s\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \dots\right) = -ig\xi c\left(1 + ig\xi - \frac{g^2}{2}\xi\xi - i\frac{g^3}{3!}\xi\xi\xi + \dots\right). \quad (36)$$

Equating now order by order in g the expression (36) immediately provides eq.(33).

Due to equation (33), we can introduce the following BRST-exact term

$$S_{IRR} = \int d^4x \frac{1}{2} s(\rho \xi^a \xi^a) = \int d^4x \left(\frac{1}{2} M^4 \xi^a \xi^a + \rho \xi^a c^a \right), \quad (37)$$

where (ρ, M) are constant parameters transforming as

$$s\rho = M^4, \quad sM^4 = 0. \quad (38)$$

As it is apparent, the action $(S + S_{IRR})$ is BRST invariant, *i.e.*

$$s(S + S_{IRR}) = 0 . \quad (39)$$

The parameter ρ has ghost number -1 , while M has ghost number zero. From equation (37), it turns out that the propagator of the Stueckelberg field ξ^a behaves now like

$$\langle \xi^a(p) \xi^b(-p) \rangle_{S+S_{IRR}} = \delta^{ab} \frac{\alpha}{p^4 + \alpha M^4} , \quad (40)$$

showing that the mass parameter M introduces an IR regularization in a BRST-invariant way. In Appendix (C) one finds the whole list of all propagators of the elementary fields evaluated in presence of the parameters (ρ, M) , which have to be set to zero at the very end of the computation of the correlation functions.

3 Renormalizability

We are now ready to face the issue of the all order renormalizability of the action S , eq.(17). For later convenience, it turns out to be helpful to employ a slightly different parametrization, redefining the gauge parameter α as well as the gauge, Lagrange multiplier, and Stueckelberg fields as

$$A_\mu^a \rightarrow \frac{1}{g} A_\mu^a , \quad b^a \rightarrow g b^a , \quad \xi^a \rightarrow \frac{1}{g} \xi^a , \quad \alpha \rightarrow \frac{\alpha}{g^2} . \quad (41)$$

Accordingly, for the field strength and the covariant derivative, we get

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c , \quad (42)$$

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - f^{abc} A_\mu^c , \quad (43)$$

while for the action S

$$S = S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} + \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} \right) , \quad (44)$$

$$S_{FP} = \int d^4x \left(\frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right) \quad (45)$$

where

$$A_\mu^h \equiv A_\mu^{h,a} T^a = h^\dagger A_\mu h + i h^\dagger \partial_\mu h , \quad h = e^{i\xi^a T^a} . \quad (46)$$

Also, for the BRST transformation, we have

$$\begin{aligned} sA_\mu^a &= -D_\mu^{ab} c^b , \\ s c^a &= \frac{1}{2} f^{abc} c^b c^c , \\ s \bar{c}^a &= i b^a , \\ s b^a &= 0 , \\ s \tau^a &= 0 , \end{aligned} \quad (47)$$

and

$$s\xi^a = g^{ab}(\xi) c^b , \quad g^{ab}(\xi) = -\delta^{ab} + \frac{1}{2} f^{abc} \xi^c - \frac{1}{12} f^{acd} f^{cbe} \xi^e \xi^d + O(\xi^3) , \quad (48)$$

with

$$sS = 0 . \quad (49)$$

The usefulness of the new parametrization in eqs.(44),(45) relies on the property that, acting on the action S with the differential operator $g^2 \frac{\partial}{\partial g^2}$, gives directly the gauge invariant quantity $\int d^4x F_{\mu\nu}^a F_{\mu\nu}^a$, *i.e.*

$$g^2 \frac{\partial S}{\partial g^2} = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a , \quad (50)$$

a feature which will be helpful in order to write down the parametric form of the most general counterterm allowed by the quantum corrections.

Let us proceed by identifying the Ward identities of the model. To that purpose, following the algebraic renormalization set up [65], we introduce a set of BRST-invariant external sources $(J(x), \mathcal{J}(x)_\mu^a, \Omega_\mu^a(x), L^a(x), K^a(x))$ coupled to the composite operators $(A_\mu^h(x)A_\mu^h(x))$ and $A_\mu^h(x)$ as well as to the non-linear BRST variation of the fields (A_μ^a, c^a, ξ^a) , namely we consider the classical complete BRST-invariant action Σ defined by

$$\begin{aligned} \Sigma = & S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} + \frac{J}{2} A_\mu^{h,a} A_\mu^{h,a} + \mathcal{J}_\mu^a A_\mu^{h,a} - \Omega_\mu^a D_\mu^{ab} c^b + \frac{1}{2} f^{abc} L^a c^b c^c \right. \\ & \left. + K^a g^{ab}(\xi) c^b + \frac{\zeta}{2} J^2 \right) , \end{aligned} \quad (51)$$

with

$$sJ = s\mathcal{J}_\mu^a = s\Omega_\mu^a = sL^a = sK^a = 0 , \quad (52)$$

which ensure the BRST invariance of Σ

$$s\Sigma = 0 . \quad (53)$$

The action S , eq.(44), can be recovered from Σ , modulo a constant vacuum term $V \frac{\zeta}{2} m^4$, by setting the sources $(J, \mathcal{J}_\mu^a, \Omega_\mu^a, L^a, K^a)$ equal to

$$J \Big|_{phys} = m^2 , \quad \mathcal{J}_\mu^a \Big|_{phys} = \Omega_\mu^a \Big|_{phys} = L^a \Big|_{phys} = K^a \Big|_{phys} = 0 , \quad (54)$$

i.e.

$$\Sigma \Big|_{phys} = S + V \frac{\zeta}{2} m^4 , \quad (55)$$

where V stands for the Euclidean space-time volume. The parameter ζ is a dimensionless free parameter which enables us to take into account possible divergences affecting the vacuum term $J^2(x)$ [7, 10, 11], allowed by power-counting due to the fact that source $J(x)$ has dimension two. Let us also mention that the vacuum term $\frac{\zeta}{2} J^2$ is required in order to investigate the formation of the dimension two condensate $\langle A_\mu^h(x) A_\mu^h(x) \rangle$ via evaluation of the corresponding effective potential, see [7, 10, 11]. In particular, the parameter ζ can be made a function of the coupling constant g in such a way that the generating functional of the correlation functions of the theory obeys a homogeneous renormalization group equation [7, 10, 11], a result which will be employed in Section (4) in order to determine the anomalous dimensions of the operators $(A_\mu^h A_\mu^h)$ and A_μ^h .

3.1 Ward identities

The BRST symmetry stated in the previous section can be immediately written as a functional identity. The complete classical action Σ turns out to fulfill the following Ward identities:

- The Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) \equiv \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega_\mu^a} \frac{\delta\Sigma}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta K^a} \frac{\delta\Sigma}{\delta \xi^a} + ib^a \frac{\delta\Sigma}{\delta \bar{c}^a} \right) = 0. \quad (56)$$

In view of the algebraic characterization of the counterterm, we introduce the so-called linearized Slavnov-Taylor operator \mathcal{B}_Σ [65] defined as

$$\mathcal{B}_\Sigma \equiv \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega_\mu^a} \frac{\delta}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta}{\delta\Omega_\mu^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta K^a} \frac{\delta}{\delta \xi^a} + \frac{\delta\Sigma}{\delta \xi^a} \frac{\delta}{\delta K^a} + ib^a \frac{\delta}{\delta \bar{c}^a} \right), \quad (57)$$

which, as the BRST operator s , turns out to be nilpotent

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0. \quad (58)$$

- The gauge-fixing condition and the anti-ghost equation [65]

$$\frac{\delta\Sigma}{\delta b^a} = i\partial_\mu A_\mu^a + \alpha b^a, \quad (59)$$

$$\frac{\delta\Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta\Sigma}{\delta \Omega_\mu^a} = 0, \quad (60)$$

In particular, the identity (60) ensures that the anti-ghost field \bar{c}^a and the source Ω_μ^a enter only through the combination

$$\widehat{\Omega}_\mu^a = \Omega_\mu^a + \partial_\mu \bar{c}^a. \quad (61)$$

- The τ Ward identity

$$\frac{\delta\Sigma}{\delta \tau^a} - \partial_\mu \frac{\delta\Sigma}{\delta \mathcal{J}_\mu^a} = 0, \quad (62)$$

implying that the field τ^a and the source \mathcal{J}_μ^a appear only in the combination

$$\widehat{\mathcal{J}}_\mu^a = \mathcal{J}_\mu^a - \partial_\mu \tau^a. \quad (63)$$

3.2 Characterization of the most general counterterm

In order to characterize the most general invariant counterterm which can be freely added to all orders in perturbation theory we follow the set up of the algebraic renormalization [65] and perturb the classical action Σ by adding an integrated local quantity in the fields and sources, Σ^{ct} , with dimension bounded by four and vanishing ghost number. We demand thus that the perturbed action, $(\Sigma + \varepsilon\Sigma^{ct})$, where ε is an expansion parameter, fulfills, to the first order in ε ,

| Fields & sources | A | b | c | \bar{c} | ξ | τ | Ω | L | K | J | \mathcal{J} |
|------------------|-----|-----|-----|-----------|-------|--------|----------|-----|-----|-----|---------------|
| Dimension | 1 | 2 | 0 | 2 | 0 | 2 | 3 | 4 | 4 | 2 | 3 |
| Ghost number | 0 | 0 | 1 | -1 | 0 | 0 | -1 | -2 | -1 | 0 | 0 |

Table 1: The quantum numbers of fields and sources.

the same Ward identities obeyed by the classical action Σ , *i.e.* equations (56), (59), (60), and (62). This requirement gives rise to the set of equations

$$\begin{aligned}
\mathcal{S}(\Sigma + \varepsilon \Sigma^{ct}) &= O(\varepsilon^2), \\
\frac{\delta}{\delta b^a}(\Sigma + \varepsilon \Sigma^{ct}) &= i\partial_\mu A_\mu^a + \alpha b^a + O(\varepsilon^2), \\
\left(\frac{\delta}{\delta \bar{c}^a} + \partial_\mu \frac{\delta}{\delta \Omega_\mu^a}\right)(\Sigma + \varepsilon \Sigma^{ct}) &= O(\varepsilon^2), \\
\left(\frac{\delta}{\delta \tau^a} - \partial_\mu \frac{\delta}{\delta \mathcal{J}_\mu^a}\right)(\Sigma + \varepsilon \Sigma^{ct}) &= O(\varepsilon^2),
\end{aligned} \tag{64}$$

yielding the following constraints on Σ^{ct} :

$$\mathcal{B}_\Sigma \Sigma^{ct} = 0, \tag{65}$$

$$\frac{\delta}{\delta b^a} \Sigma^{ct} = 0, \tag{66}$$

$$\left(\frac{\delta}{\delta \bar{c}^a} + \partial_\mu \frac{\delta}{\delta \Omega_\mu^a}\right) \Sigma^{ct} = 0, \tag{67}$$

$$\left(\frac{\delta}{\delta \tau^a} - \partial_\mu \frac{\delta}{\delta \mathcal{J}_\mu^a}\right) \Sigma^{ct} = 0. \tag{68}$$

From the constraint (66) it follows that Σ^{ct} is independent from the Lagrange multiplier b^a , while equations (67) and (68) ensure that Σ^{ct} depends only on the combinations $\widehat{\Omega}_\mu^a = \Omega_\mu^a + \partial_\mu \bar{c}^a$ and $\widehat{\mathcal{J}}_\mu^a = \mathcal{J}_\mu^a - \partial_\mu \tau^a$ of equations (61),(63).

From equation (65) one learns that Σ^{ct} belongs to the cohomology [65] of the linearized Slavnov-Taylor operator \mathcal{B}_Σ in the space of the integrated local quantities in the fields and sources of dimension 4 and ghost number zero. Therefore, we can set

$$\Sigma^{ct} = \Delta + \mathcal{B}_\Sigma \Delta^{(-1)}, \tag{69}$$

where $\Delta^{(-1)}$ denotes a four-dimensional integrated quantity in the fields and sources with ghost number -1 . The term $\mathcal{B}_\Sigma \Delta^{(-1)}$ in equation (69) corresponds to the trivial solution, *i.e.* to the exact part of the cohomology of \mathcal{B}_Σ . Instead, the quantity Δ identifies the non-trivial solution, *i.e.* the cohomology of \mathcal{B}_Σ , meaning that $\Delta \neq \mathcal{B}_\Sigma Q$, for some local integrated Q .

From the general results on the cohomology of Yang-Mills theories [65], and with the help of Table 1, where the dimension and the ghost number of all fields and sources are displayed, it follows that Δ and $\Delta^{(-1)}$ can be written as

$$\begin{aligned}
\Delta &= \int d^4x \left[\frac{c_0}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + c_1 (\partial_\mu A_\mu^{h,a}) \partial_\nu A_\nu^{h,a} + c_2 (\partial_\mu A_\nu^{h,a}) \partial_\mu A_\nu^{h,a} + c_3 f^{abc} A_\mu^{h,a} A_\nu^{h,b} \partial_\mu A_\nu^{h,c} \right. \\
&\quad \left. + \lambda^{abcd} A_\mu^{h,a} A_\mu^{h,b} A_\nu^{h,c} A_\nu^{h,d} + \widehat{\mathcal{J}}_\mu^a \mathcal{O}_\mu^a(A, \xi) + J \mathcal{O}(A, \xi) + c_4 \frac{\zeta}{2} J^2 \right],
\end{aligned} \tag{70}$$

and

$$\Delta^{(-1)} = \int d^4x \left[f_1^{ab}(\xi) \widehat{\Omega}_\mu^a A_\mu^b + f_2^{ab}(\xi) L^a c^b + f_3^{ab}(\xi) K^a \xi^b \right], \quad (71)$$

where we have taken into account the gauge invariant nature of the field A_μ^h , *i.e.*

$$\mathcal{B}_\Sigma A_\mu^{h,a} = s A_\mu^{h,a} = 0, \quad A_\mu^{h,a} \neq \mathcal{B}_\Sigma(\hat{\rho}_\mu^a), \quad (72)$$

for any $\hat{\rho}_\mu^a$. The parameters $(c_0, c_1, c_2, c_3, c_4, \lambda^{abcd})$ in expression (70) are free dimensionless coefficients, while $\mathcal{O}_\mu^a(A, \xi)$ and $\mathcal{O}(A, \xi)$ stand for generic local quantities with dimension 1 and 2 and ghost number zero, respectively, depending only on the fields A_μ^a and ξ^a . Also, the quantities $f_1^{ab}(\xi)$, $f_2^{ab}(\xi)$ and $f_3^{ab}(\xi)$ in expression (71) are arbitrary power series in ξ with ghost number zero, allowed by the dimensionless character of the Stueckelberg field ξ^a .

Imposing now the constraint (65), one immediately gets

$$\mathcal{B}_\Sigma \mathcal{O}_\mu^a(A, \xi) = s \mathcal{O}_\mu^a(A, \xi) = 0, \quad (73)$$

$$\mathcal{B}_\Sigma \mathcal{O}(A, \xi) = s \mathcal{O}(A, \xi) = 0, \quad (74)$$

meaning that $\mathcal{O}_\mu^a(A, \xi)$ and $\mathcal{O}(A, \xi)$ have to be BRST invariant. Let us work out in detail the most general solutions of eqs.(73),(74), beginning with eq.(73). Taking into account that the operator $\mathcal{O}_\mu^a(A, \xi)$ has dimension 1, ghost number 0, and carries both color and Lorentz indices, it can be parametrized as

$$\mathcal{O}_\mu^a(A, \xi) = \sigma^{ab}(\xi) A_\mu^b + \omega^{ab}(\xi) \partial_\mu \xi^b, \quad (75)$$

where $(\sigma^{ab}(\xi), \omega^{ab}(\xi))$ are dimensionless quantities in the Stueckelberg field ξ^a . Making use of expression (46), it turns out to be useful to replace A_μ^a by the gauge invariant field $A_\mu^{h,a}$, upon a redefinition of the quantities $(\sigma^{ab}(\xi), \omega^{ab}(\xi))$, *i.e.*

$$\mathcal{O}_\mu^a(A, \xi) = \hat{\sigma}^{ab}(\xi) A_\mu^{h,b} + \hat{\omega}^{ab}(\xi) \partial_\mu \xi^b. \quad (76)$$

Therefore, from condition (73) one gets

$$\frac{\partial \hat{\sigma}^{ab}}{\partial \xi^c} g^{cd} c^d A_\mu^{h,b} + \left(\frac{\partial \hat{\omega}^{ad}}{\partial \xi^b} g^{bc} + \hat{\omega}^{ab} \frac{\partial g^{bc}}{\partial \xi^d} \right) c^c \partial_\mu \xi^d + \hat{\omega}^{ab} g^{bc} \partial_\mu c^c = 0, \quad (77)$$

which immediately gives

$$\begin{aligned} \frac{\partial \hat{\sigma}^{ab}}{\partial \xi^c} &= 0 \Rightarrow \hat{\sigma}^{ab} = b_1 \delta^{ab}, \\ \hat{\omega}^{ab} &= 0, \end{aligned} \quad (78)$$

where b_1 is a constant. We conclude thus that the most general form for \mathcal{O}_μ^a is given by

$$\mathcal{O}_\mu^a(A, \xi) = b_1 A_\mu^{h,a}. \quad (79)$$

The same reasoning applies as well to the case of the operator $\mathcal{O}(A, \xi)$ in eq.(74). Taking into account now that $\mathcal{O}(A, \xi)$ is of dimension two, we write

$$\mathcal{O}(A, \xi) = \sigma^{ab}(\xi) A_\mu^a A_\mu^b + \omega^a(\xi) \partial_\mu A_\mu^a + \lambda^{ab}(\xi) A_\mu^a \partial_\mu \xi^b + \frac{\rho^{ab}(\xi)}{2} (\partial_\mu \xi^a) \partial_\mu \xi^b + \beta^a(\xi) \partial^2 \xi^a, \quad (80)$$

where $(\sigma^{ab}(\xi), \omega^a(\xi), \lambda^{ab}(\xi), \rho^{ab}(\xi), \beta^a(\xi))$ are dimensionless power series in ξ . Again, employing the gauge invariant variable A_μ^h , we obtain, upon a redefinition of $(\sigma^{ab}, \omega^a, \lambda^{ab}, \rho^{ab}, \beta^a)$,

$$\mathcal{O}(A, \xi) = \hat{\sigma}^{ab}(\xi) A_\mu^{h,a} A_\mu^{h,b} + \hat{\omega}^a(\xi) \partial_\mu A_\mu^{h,a} + \hat{\lambda}^{ab}(\xi) A_\mu^{h,a} \partial_\mu \xi^b + \frac{\hat{\rho}^{ab}(\xi)}{2} (\partial_\mu \xi^a) \partial_\mu \xi^b + \hat{\beta}^a(\xi) \partial^2 \xi^a. \quad (81)$$

From equation (74) we have

$$\begin{aligned} 0 &= \frac{\partial \hat{\sigma}^{ab}}{\partial \xi^d} g^{dc} A_\mu^{h,a} A_\mu^{h,b} c^c + \frac{\partial \hat{\omega}^a}{\partial \xi^c} g^{cb} (\partial_\mu A_\mu^{h,a}) c^b + \left(\frac{\partial \hat{\lambda}^{ab}}{\partial \xi^d} g^{dc} + \hat{\lambda}^{ad} \frac{\partial g^{dc}}{\partial \xi^b} \right) A_\mu^{h,a} (\partial_\mu \xi^b) c^c \\ &+ \hat{\lambda}^{ac} g^{cb} A_\mu^{h,a} \partial_\mu c^b + \left(\frac{1}{2} \frac{\partial \hat{\rho}^{ab}}{\partial \xi^d} g^{dc} + \hat{\rho}^{ad} \frac{\partial g^{dc}}{\partial \xi^b} + \hat{\beta}^d \frac{\partial^2 g^{dc}}{\partial \xi^a \partial \xi^b} \right) (\partial_\mu \xi^a) (\partial_\mu \xi^b) c^c \\ &+ \left(\hat{\rho}^{ac} g^{cb} + 2 \hat{\beta}^c \frac{\partial g^{cb}}{\partial \xi^a} \right) (\partial_\mu \xi^a) \partial_\mu c^b + \left(\frac{\partial \hat{\beta}^a}{\partial \xi^c} g^{cb} + \hat{\beta}^c \frac{\partial g^{cb}}{\partial \xi^a} \right) (\partial^2 \xi^a) c^b \\ &+ \hat{\beta}^b g^{ba} \partial^2 c^a, \end{aligned} \quad (82)$$

from which it follows that

$$\begin{aligned} \hat{\lambda}^{ab} &= \hat{\rho}^{ab} = \hat{\beta}^a = 0, \\ \frac{\partial \hat{\sigma}^{ab}}{\partial \xi^d} &= 0 \Rightarrow \hat{\sigma}^{ab} = \frac{b_2}{2} \delta^{ab}, \\ \frac{\partial \hat{\omega}^a}{\partial \xi^b} &= 0 \Rightarrow \hat{\omega}^a = 0 \text{ (by color invariance)}, \end{aligned} \quad (83)$$

where b_2 is a free coefficient. Finally, for the operator $\mathcal{O}(A, \xi)$, we have

$$\mathcal{O}(A, \xi) = \frac{b_2}{2} A_\mu^{h,a} A_\mu^{h,a}. \quad (84)$$

Therefore, for the most general counterterm, eq.(69), we get

$$\begin{aligned} \Delta &= \int d^4x \left[\frac{c_0}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + c_1 (\partial_\mu A_\mu^{h,a}) \partial_\nu A_\nu^{h,a} + c_2 (\partial_\mu A_\nu^{h,a}) \partial_\mu A_\nu^{h,a} + c_3 f^{abc} A_\mu^{h,a} A_\nu^{h,b} \partial_\mu A_\nu^{h,c} \right. \\ &\left. + \lambda^{abcd} A_\mu^{h,a} A_\mu^{h,b} A_\nu^{h,c} A_\nu^{h,d} + b_1 \hat{\mathcal{J}}_\mu^a A_\mu^{h,a} + \frac{b_2}{2} J A_\mu^{h,a} A_\mu^{h,a} + c_4 \frac{\zeta}{2} J^2 \right], \end{aligned} \quad (85)$$

and $\Delta^{(-1)}$ is given by eq. (71).

It remains now to characterize the coefficients $(c_1, c_2, c_3, \lambda^{abcd})$. To that aim, we rely on an important property of the action S in equation (44). When the mass parameter m^2 is set to zero, *i.e.* $m^2 = 0$, expression reduces to

$$S_{m^2=0} = S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} \right), \quad (86)$$

$$S_{FP} = \int d^4x \left(\frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right), \quad (87)$$

which coincides, modulo the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$, with the Faddeev-Popov action S_{FP} of the linear covariant gauges.

Nevertheless, as shown in detail in Appendix (A), the additional term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$ has no consequences on the evaluation of the Green functions of the elementary fields (A_μ, b, c, \bar{c}) , meaning that the correlation functions $\langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}}$ evaluated with the action $S_{m^2=0}$ coincide with those computed with the Faddeev-Popov action S_{FP} , namely

$$\langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}} = \langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{FP}} . \quad (88)$$

From this property, it follows that when the external fields (J, \mathcal{J}, K) are set to zero, *i.e.* $(J, \mathcal{J}, K) \rightarrow 0$, the counterterm (85), (71) has to reduce to that of the Faddeev-Popov action in presence of the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$, namely to expressions (156),(164),(170) of Appendix (A). This requirement gives

$$c_0 = a_0, \quad c_1 = c_2 = c_3 = 0, \quad \lambda^{abcd} = 0, \quad f_1^{ab} = a_1 \delta^{ab}, \quad f_2^{ab} = a_2 \delta^{ab}. \quad (89)$$

so that for the counterterm Σ^{ct} we obtain

$$\begin{aligned} \Sigma^{ct} = & \int d^4x \left[\frac{a_0}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + b_1 \widehat{\mathcal{J}}_\mu^a A_\mu^{h,a} + \frac{b_2}{2} J A_\mu^{h,a} A_\mu^{h,a} + b_3 \frac{\zeta}{2} J^2 \right] \\ & + \mathcal{B}_\Sigma \int d^4x \left[a_1 \widehat{\Omega}_\mu^a A_\mu^a + a_2 L^a c^a + K^a f^a(\xi) \right], \end{aligned} \quad (90)$$

where we have performed the following redefinitions:

$$f^a(\xi) \equiv f_3^{ab}(\xi) \xi^b, \quad b_3 \equiv c_4. \quad (91)$$

3.3 Parametric form of the counterterm and renormalization factors

Having determined the most general form of the invariant counterterm, eq.(90), it remains to check if Σ^{ct} can be reabsorbed in the starting action Σ through a redefinition of parameters, fields and sources. To that end, let us proceed by casting expression (90) in the so called parametric form. From the expressions of the linearized Slavnov-Taylor operator \mathcal{B}_Σ , eq. (57), we can rewrite the counterterm Σ^{ct} as

$$\begin{aligned} \Sigma^{ct} = & \int d^4x \left(\frac{a_0}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + b_1 \mathcal{J}_\mu^a A_\mu^{h,a} + b_1 \tau^a \partial_\mu A_\mu^{h,a} + \frac{b_2}{2} J A_\mu^{h,a} A_\mu^{h,a} + b_3 \frac{\zeta}{2} J^2 \right. \\ & + a_1 A_\mu^a \frac{\delta \Sigma}{\delta A_\mu^a} - i a_1 b^a \partial_\mu A_\mu^a - a_1 \Omega_\mu^a \frac{\delta \Sigma}{\delta \Omega_\mu^a} + a_1 \bar{c}^a \partial_\mu \frac{\delta \Sigma}{\delta \Omega_\mu^a} - a_2 c^a \frac{\delta \Sigma}{\delta c^a} \\ & \left. + a_2 L^a \frac{\delta \Sigma}{\delta L^a} + f^a(\xi) \frac{\delta \Sigma}{\delta \xi^a} - K^a \frac{\partial f^a}{\partial \xi^b} \frac{\delta \Sigma}{\delta K^b} \right). \end{aligned} \quad (92)$$

where use has been made of the explicit expressions of $\widehat{\mathcal{J}}_\mu^a$ and $\widehat{\Omega}_\mu^a$ given, respectively, in eqs.(63),(61). In order to analyze the different terms of expression (92), we set

$$\Sigma^{ct} = \sum_{n=1}^7 \Sigma_n^{ct}, \quad (93)$$

with

$$\begin{aligned}
\Sigma_1^{ct} &= \int d^4x \frac{a_0}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a, \\
\Sigma_2^{ct} &= \int d^4x b_1 \mathcal{J}_\mu^a A_\mu^{h,a}, \\
\Sigma_3^{ct} &= \int d^4x b_1 \tau^a \partial_\mu A_\mu^{h,a}, \\
\Sigma_4^{ct} &= \int d^4x \left(\frac{b_2}{2} J A_\mu^{h,a} A_\mu^{h,a} + b_3 \frac{\zeta}{2} J^2 \right), \\
\Sigma_5^{ct} &= \int d^4x (-i a_1 b^a \partial_\mu A_\mu^a), \\
\Sigma_6^{ct} &= \int d^4x a_1 \bar{c}^a \partial_\mu \frac{\delta \Sigma}{\delta \Omega_\mu^a}, \\
\Sigma_7^{ct} &= \int d^4x \left(a_1 A_\mu^a \frac{\delta \Sigma}{\delta A_\mu^a} - a_1 \Omega_\mu^a \frac{\delta \Sigma}{\delta \Omega_\mu^a} + a_2 L^a \frac{\delta \Sigma}{\delta L^a} + f^a(\xi) \frac{\delta \Sigma}{\delta \xi^a} - K^a \frac{\partial f^a}{\partial \xi^b} \frac{\delta \Sigma}{\delta K^b} \right). \quad (94)
\end{aligned}$$

By noticing that

$$\frac{\partial \Sigma}{\partial g^2} = \frac{\partial}{\partial g^2} \int d^4x \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a = -\frac{1}{g^2} \int d^4x \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a, \quad (95)$$

the term Σ_1^{ct} can be rewritten as

$$\Sigma_1^{ct} = -a_0 g^2 \frac{\partial \Sigma}{\partial g^2}. \quad (96)$$

Taking the variation of the action Σ with respect to \mathcal{J}_μ^a and τ^a ,

$$\frac{\delta \Sigma}{\delta \mathcal{J}_\mu^a} = A_\mu^{h,a}, \quad \frac{\delta \Sigma}{\delta \tau^a} = \partial_\mu A_\mu^{h,a}, \quad (97)$$

the terms Σ_2^{ct} and Σ_3^{ct} are rewritten as

$$\begin{aligned}
\Sigma_2^{count} &= b_1 \int d^4x \mathcal{J}_\mu^a \frac{\delta \Sigma}{\delta \mathcal{J}_\mu^a}, \\
\Sigma_3^{count} &= b_1 \int d^4x \tau^a \frac{\delta \Sigma}{\delta \tau^a}. \quad (98)
\end{aligned}$$

Also, taking the variation of Σ with respect to J , we obtain

$$\frac{\delta \Sigma}{\delta J} = \frac{1}{2} A_\mu^{h,a} A_\mu^{h,a} + \zeta J, \quad (99)$$

from which it follows that Σ_4^{ct} takes the form

$$\Sigma_4^{ct} = \int d^4x \left(b_2 J \frac{\delta \Sigma}{\delta J} + (b_3 - 2b_2) \frac{\zeta}{2} J^2 \right). \quad (100)$$

On the other hand, we also have that

$$\zeta \frac{\partial \Sigma}{\partial \zeta} = \int d^4x \frac{\zeta}{2} J^2. \quad (101)$$

Thus,

$$\Sigma_4^{ct} = b_2 \int d^4x J \frac{\delta \Sigma}{\delta J} + (b_3 - 2b_2) \zeta \frac{\partial \Sigma}{\partial \zeta}. \quad (102)$$

Now, considering the gauge fixing equation (59), we can rewrite Σ_5^{ct} as

$$\Sigma_5^{ct} = \int d^4x \left(-a_1 b^a \frac{\delta \Sigma}{\delta b^a} + a_1 \alpha b^a b^a \right). \quad (103)$$

Furthermore, from

$$2\alpha \frac{\partial \Sigma}{\partial \alpha} = \int d^4x \alpha b^a b^a, \quad (104)$$

one gets

$$\Sigma_5^{ct} = -a_1 \int d^4x b^a \frac{\delta \Sigma}{\delta b^a} + 2a_1 \alpha \frac{\partial \Sigma}{\partial \alpha}. \quad (105)$$

The term Σ_6^{ct} can be immediately rewritten using the anti-ghost equation (60) as:

$$\Sigma_6^{ct} = -a_1 \int d^4x \bar{c}^a \frac{\delta \Sigma}{\delta \bar{c}^a}. \quad (106)$$

Putting together all expressions, for the parametric form of the counterterm we obtain

$$\begin{aligned} \Sigma^{ct} = & -a_0 g^2 \frac{\partial \Sigma}{\partial g^2} + (b_3 - 2b_2) \zeta \frac{\partial \Sigma}{\partial \zeta} + 2a_1 \alpha \frac{\partial \Sigma}{\partial \alpha} + \int d^4x \left(a_1 A_\mu^a \frac{\delta \Sigma}{\delta A_\mu^a} - a_1 b^a \frac{\delta \Sigma}{\delta b^a} - \right. \\ & -a_1 \bar{c}^a \frac{\delta \Sigma}{\delta \bar{c}^a} - a_2 c^a \frac{\delta \Sigma}{\delta c^a} + b_1 \tau^a \frac{\delta \Sigma}{\delta \tau^a} + f^a(\xi) \frac{\delta \Sigma}{\delta \xi^a} - a_1 \Omega_\mu^a \frac{\delta \Sigma}{\delta \Omega_\mu^a} + a_2 L^a \frac{\delta \Sigma}{\delta L^a} + \\ & \left. + b_1 \mathcal{J}_\mu^a \frac{\delta \Sigma}{\delta \mathcal{J}_\mu^a} + b_2 J \frac{\delta \Sigma}{\delta J} - K^b \frac{\partial f^b}{\partial \xi^a} \frac{\delta \Sigma}{\delta K^a} \right), \end{aligned} \quad (107)$$

which can be finally written as

$$\Sigma^{ct} = \mathcal{R} \Sigma, \quad (108)$$

with \mathcal{R} being the differential operator

$$\begin{aligned} \mathcal{R} = & -a_0 g^2 \frac{\partial}{\partial g^2} + (b_3 - 2b_2) \zeta \frac{\partial}{\partial \zeta} + 2a_1 \alpha \frac{\partial}{\partial \alpha} + \int d^4x \left(a_1 A_\mu^a \frac{\delta}{\delta A_\mu^a} - a_1 b^a \frac{\delta}{\delta b^a} \right. \\ & -a_1 \bar{c}^a \frac{\delta}{\delta \bar{c}^a} - a_2 c^a \frac{\delta}{\delta c^a} + b_1 \tau^a \frac{\delta}{\delta \tau^a} + f^a(\xi) \frac{\delta}{\delta \xi^a} - a_1 \Omega_\mu^a \frac{\delta}{\delta \Omega_\mu^a} + a_2 L^a \frac{\delta}{\delta L^a} \\ & \left. + b_1 \mathcal{J}_\mu^a \frac{\delta}{\delta \mathcal{J}_\mu^a} + b_2 J \frac{\delta}{\delta J} - K^b \frac{\partial f^b}{\partial \xi^a} \frac{\delta}{\delta K^a} \right). \end{aligned} \quad (109)$$

The usefulness of expression (108) relies on the fact that it immediately provides the redefinition of the fields, parameter and sources needed to show that the counterterm Σ^{ct} can be in fact reabsorbed into the starting action, namely

$$\Sigma(\Phi) + \varepsilon \Sigma^{ct}(\Phi) = \Sigma(\Phi_0) + O(\varepsilon^2), \quad (110)$$

where ε is an expansion parameter, Φ is a shorthand notation for the fields, parameters and sources, while Φ_0 stands for the corresponding redefinitions. From equation (108) it is apparent that the redefined fields, parameters and sources are given by

$$\Phi_0 = (1 + \varepsilon \mathcal{R}) \Phi. \quad (111)$$

In fact, using (111), it is almost immediate to prove that

$$\Sigma[\Phi_0] = \Sigma[\Phi + \varepsilon \mathcal{R}\Phi] = \Sigma[\Phi] + \varepsilon \mathcal{R}(\Sigma) + O(\varepsilon^2), \quad (112)$$

showing that the counterterm Σ^{ct} can be reabsorbed into the starting action Σ .

By direct inspection of equation (112), for the renormalization factors one finds:

$$\begin{aligned} A_0 &= Z_A^{1/2} A, & b_0 &= Z_b^{1/2} b, & c_0 &= Z_c^{1/2} c, & \bar{c}_0 &= Z_{\bar{c}}^{1/2} \bar{c}, & \xi_0^a &= Z_\xi^{ab}(\xi) \xi^b, & \tau_0 &= Z_\tau^{1/2} \tau, \\ \Omega_0 &= Z_\Omega \Omega, & L_0 &= Z_L L, & K_0^a &= Z_K^{ab}(\xi) K^b, & J_0 &= Z_J J, & \mathcal{J}_0 &= Z_{\mathcal{J}} \mathcal{J}, \\ g_0 &= Z_g g, & \alpha_0 &= Z_\alpha \alpha, & \zeta_0 &= Z_\zeta \zeta, \end{aligned} \quad (113)$$

where

$$Z_g = 1 + \varepsilon a_0, \quad (114)$$

$$Z_A^{1/2} = 1 + \varepsilon a_1, \quad (115)$$

$$Z_c^{1/2} = 1 - \varepsilon a_2, \quad (116)$$

$$Z_{\mathcal{J}} = 1 + \varepsilon b_1, \quad (117)$$

$$Z_J = 1 + \varepsilon b_2, \quad (118)$$

$$Z_\zeta = 1 + \varepsilon (b_3 - 2b_2), \quad (119)$$

$$Z_\xi^{ab} = \delta^{ab} + \varepsilon f_3^{ab}, \quad (120)$$

$$Z_K^{ab} = \delta^{ab} - \varepsilon \frac{\partial f^b}{\partial \xi^a} = \delta^{ab} - \varepsilon \left(\frac{\partial f_3^{bc}}{\partial \xi^a} \xi^c + f_3^{ba} \right), \quad (121)$$

and

$$Z_\alpha = Z_A, \quad Z_b^{1/2} = Z_{\bar{c}}^{1/2} = Z_\Omega = Z_A^{-1/2}, \quad Z_\tau^{1/2} = Z_{\mathcal{J}}, \quad Z_L = Z_c^{-1/2}. \quad (122)$$

Observe that in equations (120) and (121) we have used the definition $f^a(\xi) = f_3^{ab}(\xi) \xi^b$ introduced in eq.(91). We also underline that, according to (120),(121), the renormalization factors of the Stueckelberg field ξ^a and of the corresponding source K^a are nonlinear, *i.e.* they are power series in ξ^a . This is an expected feature, due to the dimensionless character of the Stueckelberg field, a feature common to other renormalizable models displaying massless fields as, for example, $N = 1$ super Yang-Mills theory in superspace, see [74].

4 The anomalous dimensions of $(A_\mu^h A_\mu^h)$ and of A_μ^h

Let us address now the issue of the anomalous dimensions of the operators $(A_\mu^{h,a} A_\mu^{h,a})$ and $A_\mu^{h,a}$. As a consequence of their gauge invariance, their anomalous dimensions turn out to be independent from the gauge parameter α , a result which can be established at the algebraic level through the use of the extended BRST technique [65]. (See also the recent proof given in [67].)

In particular, due to its α -independence, the anomalous dimension of $(A_\mu^{h,a} A_\mu^{h,a})$ is the same as that computed in the Landau gauge, *i.e.* for $\alpha = 0$. Moreover, taking into account that, in the Landau gauge, the operator $(A_\mu^{h,a} A_\mu^{h,a})$ reduces to $(A_\mu^a A_\mu^a)$, we expect that the anomalous

dimension $\gamma_{(A^h)^2}$ of $(A_\mu^{h,a} A_\mu^{h,a})$ should be equal to the anomalous dimension $\gamma_{A^2}|_{\text{Landau}}$ of the operator $(A_\mu^a A_\mu^a)$ in the Landau gauge, namely

$$\gamma_{(A^h)^2} = \gamma_{A^2}|_{\text{Landau}} = - \left(\frac{\beta(a)}{a} + \gamma_A^{\text{Landau}}(a) \right), \quad a = \frac{g^2}{16\pi^2}, \quad (123)$$

where $(\beta(a), \gamma_A^{\text{Landau}}(a))$ denote, respectively, the β -function and the anomalous dimension of the gauge field A_μ in the Landau gauge[†].

A similar property is expected in the case of the operator A_μ^h , namely

$$\gamma_{A^h} = \gamma_{A^h}|_{\alpha=0} = \gamma_A^{\text{Landau}}(a), \quad (124)$$

i.e. the anomalous dimension of A_μ^h should equal that of the gauge field A_μ^a in the Landau gauge. Therefore, both $\gamma_{(A^h)^2}$ and γ_{A^h} would not be independent parameters of the theory.

Let us give a formal proof of equations (123) and (124) by making use of the renormalization group equation (RGE) which, owing to the renormalizability and to the BRST invariance of the theory, reads

$$\begin{aligned} & \mu \frac{\partial \Gamma}{\partial \mu} + \beta_{g^2} \frac{\partial \Gamma}{\partial g^2} - \gamma_A \mathcal{N}_A \Gamma - \gamma_c \mathcal{N}_c \Gamma - \gamma_{(A^h)^2} \int d^4x J \frac{\delta \Gamma}{\delta J} \\ & - \gamma_{A^h} \int d^4x \left(\mathcal{J}_\mu^a \frac{\delta \Gamma}{\delta \mathcal{J}_\mu^a} + \tau^a \frac{\delta \Gamma}{\delta \tau^a} \right) - \int d^4x \left(\gamma_\xi^{ab}(\xi) \xi^b \frac{\delta \Gamma}{\delta \xi^a} + \gamma_K^{ab}(\xi) K^b \frac{\delta \Gamma}{\delta K^a} \right) = 0, \end{aligned} \quad (125)$$

where

$$\begin{aligned} \mathcal{N}_A &= \int d^4x \left(A_\mu^a \frac{\delta}{\delta A_\mu^a} - b^a \frac{\delta}{\delta b^a} - \bar{c}^a \frac{\delta}{\delta \bar{c}^a} - \Omega_\mu^a \frac{\delta}{\delta \Omega_\mu^a} \right) + 2\alpha \frac{\partial}{\partial \alpha}, \\ \mathcal{N}_c &= \int d^4x \left(c^a \frac{\delta}{\delta c^a} - L^a \frac{\delta}{\delta L^a} \right), \\ \gamma_\xi^{ab} &= (Z_\xi^{-1})^{ac} \mu \frac{\partial}{\partial \mu} Z_\xi^{cb}, \\ \gamma_K^{ab} &= (Z_K^{-1})^{ac} \mu \frac{\partial}{\partial \mu} Z_K^{cb}. \end{aligned} \quad (126)$$

Let us act now on the RGE with the test operator

$$\frac{\delta^2}{\delta \mathcal{J}_\mu^a(x) \delta \mathcal{J}_\nu^b(y)}, \quad (127)$$

and set all fields and sources equal to zero. A simple algebraic calculation gives

$$\begin{aligned} 0 &= \mu \frac{\partial}{\partial \mu} \langle A_\mu^{h,a}(x) A_\nu^{h,b}(y) \rangle + \beta_{g^2} \frac{\partial}{\partial g^2} \langle A_\mu^{h,a}(x) A_\nu^{h,b}(y) \rangle \\ &\quad - 2\gamma_A \alpha \frac{\partial}{\partial \alpha} \langle A_\mu^{h,a}(x) A_\nu^{h,b}(y) \rangle - 2\gamma_{A^h} \langle A_\mu^{h,a}(x) A_\nu^{h,b}(y) \rangle. \end{aligned} \quad (128)$$

[†]For an all order algebraic proof of the relationship

$$\gamma_{A^2}|_{\text{Landau}} = - \left(\frac{\beta(a)}{a} + \gamma_A^{\text{Landau}}(a) \right), \quad a = \frac{g^2}{16\pi^2},$$

see [36].

Moreover, due to the α -independence of the gauge-invariant correlation function $\langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle$, it follows that

$$\frac{\partial}{\partial \alpha} \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle = 0. \quad (129)$$

Thus,

$$\mu \frac{\partial}{\partial \mu} \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle + \beta_{g^2} \frac{\partial}{\partial g^2} \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle - 2\gamma_{A^h} \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle = 0. \quad (130)$$

In addition, from (129) we can make direct use of the Landau gauge, namely

$$\langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle = \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle_{\alpha=0} = \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle_{\text{Landau}}. \quad (131)$$

Therefore

$$\langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle = \frac{\int [D\phi] A_\mu^{h,a}(x)A_\nu^{h,b}(y) e^{-S_{(\alpha,m^2=0)}}}{\int [D\phi] e^{-S_{(\alpha,m^2=0)}}}, \quad (132)$$

with $[D\phi] \equiv DADbDcD\bar{c}D\xi D\tau$ and

$$S_{(\alpha,m^2=0)} = \int d^4x \left(\frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + i\bar{b}^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b + \tau^a \partial_\mu A_\mu^{h,a} \right). \quad (133)$$

Integrating out the fields (τ, b, c, \bar{c}) , we get

$$\langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle = \frac{\int DAD\xi \delta(\partial_\mu A_\mu^h) \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) A_\mu^{h,a}(x)A_\nu^{h,b}(y) e^{-S_{\text{SYM}}}}{\int DAD\xi \delta(\partial_\mu A_\mu^h) \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) e^{-S_{\text{SYM}}}}. \quad (134)$$

Employing the result given in Appendix (B), see eqs.(191),(192), the equation $\partial_\mu A_\mu^h = 0$ can be solved iteratively for ξ^a yielding

$$\xi = \frac{1}{\partial^2} \partial_\mu A_\mu + i \frac{g}{\partial^2} \left[\partial A, \frac{\partial A}{\partial^2} \right] + i \frac{g}{\partial^2} \left[A_\mu, \partial_\mu \frac{\partial A}{\partial^2} \right] + \frac{i}{2} \frac{g}{\partial^2} \left[\frac{\partial A}{\partial^2}, \partial A \right] + O(A^3), \quad (135)$$

so that we can integrate over ξ^a , obtaining

$$\langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle = \frac{\int DA \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) A_\mu^{h,a}(x)A_\nu^{h,b}(y) e^{-S_{\text{SYM}}}}{\int DA \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) e^{-S_{\text{SYM}}}}, \quad (136)$$

where A_μ^h is now given by, see eq.(193) of Appendix (B),

$$\begin{aligned} A_\mu^h &= A_\mu - \frac{1}{\partial^2} \partial_\mu \partial A - ig \frac{\partial_\mu}{\partial^2} \left[A_\nu, \partial_\nu \frac{\partial A}{\partial^2} \right] - i \frac{g}{2} \frac{\partial_\mu}{\partial^2} \left[\partial A, \frac{1}{\partial^2} \partial A \right] \\ &+ ig \left[A_\mu, \frac{1}{\partial^2} \partial A \right] + i \frac{g}{2} \left[\frac{1}{\partial^2} \partial A, \frac{\partial_\mu}{\partial^2} \partial A \right] + O(A^3). \end{aligned} \quad (137)$$

However, due to the presence in eq.(136) of the delta function $\delta(\partial_\mu A_\mu)$, all terms containing a divergence ∂A vanish, namely

$$\begin{aligned} \langle A_\mu^{h,a}(x)A_\nu^{h,b}(y) \rangle &= \frac{\int DA \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) A_\mu^{h,a}(x)A_\nu^{h,b}(y) e^{-S_{\text{SYM}}}}{\int DA \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) e^{-S_{\text{SYM}}}} \\ &= \frac{\int DA \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) A_\mu^a(x)A_\nu^b(y) e^{-S_{\text{SYM}}}}{\int DA \delta(\partial_\mu A_\mu) \det(-\partial \cdot D) e^{-S_{\text{SYM}}}} \\ &= \langle A_\mu^a(x)A_\nu^b(y) \rangle_{\text{Landau}}. \end{aligned} \quad (138)$$

Thus, the RGE for the correlation function $\langle A_\mu^{h,a}(x)A_\nu^{h,b}(y)\rangle$ becomes

$$\mu \frac{\partial}{\partial \mu} \langle A_\mu^a(x)A_\nu^b(y)\rangle_{\text{Landau}} + \beta_{g^2} \frac{\partial}{\partial g^2} \langle A_\mu^a(x)A_\nu^b(y)\rangle_{\text{Landau}} - 2\gamma_{A^h} \langle A_\mu^a(x)A_\nu^b(y)\rangle_{\text{Landau}} = 0, \quad (139)$$

which proves equation (124). Of course, the same reasoning can be applied to equation (123).

5 Conclusions

In this work we have provided a study of the gauge-invariant non-local operator A_{min}^2

$$A_{\text{min}}^2 = \text{Tr} \int d^4x A_\mu^h A_\mu^h, \quad (140)$$

with A_μ^h the transverse configuration, $\partial_\mu A_\mu^h = 0$, given in expression (3).

Despite the highly non-local character, we have shown that a fully local set up for both operators $(A_\mu^h A_\mu^h)$ and A_μ^h can be constructed, giving rise to a local and BRST-invariant action S , eq.(17). The main tool in order to achieve such a local formulation has been the introduction of an auxiliary Stueckelberg field ξ^a , eqs.(11),(12).

As pointed out in Section (2), the transversality condition, $\partial_\mu A_\mu^h = 0$, plays an important role, giving rise to deep differences between our formulation and the conventional Stueckelberg one, which is known to be non-renormalizable. Unlike the conventional Stueckelberg formulation, the novel action S , eq.(17), has been proven to be renormalizable to all orders, as shown in details in Sect.(3). Furthermore, owing to the gauge invariance of $(A_\mu^h A_\mu^h)$ and A_μ^h , the corresponding anomalous dimensions, $(\gamma_{(A^h)^2}, \gamma_{A^h})$, turn out to be independent from the gauge parameter α entering the gauge fixing condition, being given by

$$\begin{aligned} \gamma_{(A^h)^2} &= \gamma_{A^2} \Big|_{\text{Landau}} = - \left(\frac{\beta(a)}{a} + \gamma_A^{\text{Landau}}(a) \right), & a &= \frac{g^2}{16\pi^2}, \\ \gamma_{A^h} &= \gamma_{A^h} \Big|_{\alpha=0} = \gamma_A^{\text{Landau}}(a), \end{aligned} \quad (141)$$

where $(\beta(a), \gamma_A^{\text{Landau}}(a))$ denote, respectively, the β -function and the anomalous dimension of the gauge field A_μ in the Landau gauge. We see therefore that $(\gamma_{(A^h)^2}, \gamma_{A^h})$ are not independent parameters of the theory.

The present results can open the road to several future investigations. For instance, the possibility of having at our disposal a local and renormalizable framework might enable us to investigate the formation, through the computation of the effective potential [7, 10, 11, 15, 16], of the gauge-invariant dimension-two condensate $\langle A_\mu^h A_\mu^h \rangle$. This result might yield a better understanding, within a manifestly BRST-invariant set up, of the relevance of the condensate $\langle A_\mu^h A_\mu^h \rangle$ for the formation of the dynamical gluon mass [7, 10, 11, 15, 16] as well for the analysis of the $\frac{1}{Q^2}$ corrections in the gluon correlation functions within the OPE expansion, as reported in [18, 19, 20, 21, 24, 27, 28, 29, 30, 31, 33, 34, 35] in the case of the Landau gauge.

Another topic worth to be mentioned is the study of the BRST-invariant and α -independent correlation function

$$\langle A_\mu^h(x)A_\mu^h(y)\rangle, \quad (142)$$

within the local present set up. Due to its α -independence, expression (142) can be seen as the natural generalization, in the case of the covariant linear gauges, of the two-point function $\langle A_\mu(x)A_\mu(y) \rangle_{\text{Landau}}$ studied in the renormalizable massive Yang-Mills model in the Landau gauge considered in [72, 73]. As such, expression (142) might provide information about the occurrence of positivity violation, already observed in the Landau gauge [72, 73]. In this sense, expression (142) might be regarded as a powerful and practical tool to detect the positivity violation, in linear covariant gauges, of the two-point gluon correlation function within a BRST-invariant formulation.

Acknowledgements.

The Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil), the Faperj, Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro, the SR2-UERJ and the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) are gratefully acknowledged for financial support.

A A review on the renormalization of the Yang-Mills action in linear covariant gauges

When the mass parameter m^2 is set to zero, the action S in eq.(45) reduces to

$$S_{m^2=0} = S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} \right) \quad (143)$$

where S_{FP} is

$$S_{FP} = \int d^4x \left(\frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right), \quad (144)$$

i.e. $S_{m^2=0}$ coincides, modulo the term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$, with the usual Faddeev-Popov action of the linear covariant gauge. Evidently, the action $S_{m^2=0}$ is left invariant by the BRST transformations given in eqs.(47),(48)

$$sS_{m^2=0} = 0. \quad (145)$$

Nevertheless, when $m^2 = 0$, the additional term $\int d^4x \tau^a \partial_\mu A_\mu^{h,a}$ has no consequences on the evaluation of the Green functions of the elementary fields (A_μ, b, c, \bar{c}) . More precisely, it turns out that the correlation functions $\langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}}$ evaluated with the action $S_{m^2=0}$ coincide with those computed with the Faddeev-Popov action S_{FP} , namely

$$\langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}} = \langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{FP}}. \quad (146)$$

The statement (146) can be checked by means of the functional integral. Let us consider expression

$$\langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}} = \frac{\int [D\phi] A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n) e^{-S_{m^2=0}}}{\int [D\phi] e^{-S_{m^2=0}}}, \quad (147)$$

where $[D\phi]$ stands for integration over all fields, *i.e.* $[D\phi] = DADbDcD\bar{c}D\xi D\tau$. Integrating over the field τ , one gets

$$\langle A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}} = \frac{\int DADbDcD\bar{c}D\xi \delta(\partial_\mu A_\mu^h) A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n) e^{-S_{FP}}}{\int DADbDcD\bar{c}D\xi \delta(\partial_\mu A_\mu^h) e^{-S_{FP}}}, \quad (148)$$

Making use of the result given in Appendix (B), see eqs.(191),(192), the equation $\partial_\mu A_\mu^h = 0$ can be solved iteratively for ξ^a yielding

$$\xi = \frac{1}{\partial^2} \partial_\mu A_\mu + i \frac{g}{\partial^2} \left[\partial A, \frac{\partial A}{\partial^2} \right] + i \frac{g}{\partial^2} \left[A_\mu, \partial_\mu \frac{\partial A}{\partial^2} \right] + \frac{i}{2} \frac{g}{\partial^2} \left[\frac{\partial A}{\partial^2}, \partial A \right] + O(A^3), \quad (149)$$

so that expression (148) can be written as

$$\langle A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}} = \frac{\int DADbDcD\bar{c}D\xi \delta(\xi - [\text{power series in } A]) A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n) e^{-S_{FP}}}{\int DADbDcD\bar{c}D\xi \delta(\xi - [\text{power series in } A]) e^{-S_{FP}}}. \quad (150)$$

Observing now that the Faddeev-Popov action S_{FP} , eq.(143), does not contain any dependence from the Stueckelberg field, it follows that the integration over the variable ξ in equation (148) is straightforward, giving

$$\langle A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n) \rangle_{S_{m^2=0}} = \frac{\int DADbDcD\bar{c} A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n) e^{-S_{FP}}}{\int DADbDcD\bar{c} e^{-S_{FP}}}, \quad (151)$$

which proves the statement (146). The same reasoning applies as well to other Green's functions containing the elementary fields (b, c, \bar{c}) . In summary, all Green's function of the elementary fields (A_μ, b, c, \bar{c}) evaluated with the action (143) are exactly the same as those computed with the Faddeev-Popov action (144).

In particular, from this result it follows that the action (143) is renormalizable, the most general counterterm being given, modulo terms in the variable τ , by the usual counterterm of the linear covariant gauges.

Let us give a closer look at the possible local BRST-invariant counterterm $S_{m^2=0}^{ct}$ affecting the action $S_{m^2=0}$ at the quantum level. $S_{m^2=0}^{ct}$ is a local integrated quantity in the fields bounded by dimension four. Moreover, it is useful to notice that, besides the BRST invariance, eq.(145), the action $S_{m^2=0}$ is constrained by the additional Ward identity

$$\int d^4x \frac{\delta S_{m^2=0}}{\delta \tau^a} = 0, \quad (152)$$

which implies that the variable τ can enter only through a space-time derivative, *i.e.* $\partial_\mu \tau^a$. Therefore, owing to the previous considerations, and taking into account that the field τ has dimension two, for the counterterm $S_{m^2=0}^{ct}$ we write

$$S_{m^2=0}^{ct} = S_{FP}^{ct} - \int d^4x (\partial_\mu \tau^a) \mathcal{O}_\mu^a(A, \xi), \quad (153)$$

where S_{FP}^{ct} is the usual local BRST-invariant counterterm of the Faddeev-Popov action in linear covariant gauges and where $\mathcal{O}_\mu^a(A, \xi)$ is a local quantity of dimension 1. From BRST invariance, we immediately get

$$s \mathcal{O}_\mu^a(A, \xi) = 0, \quad (154)$$

whose general solution, see eqs.(73)-(79), is

$$\mathcal{O}_\mu^a(A, \xi) = b_1 A_\mu^{h,a}, \quad (155)$$

with b_1 being an arbitrary coefficient. Thus, for the most general counterterm corresponding to $S|_{m^2=0}$ we have

$$S_{m^2=0}^{ct} = S_{FP}^{ct} - b_1 \int d^4x (\partial_\mu \tau^a) A_\mu^{h,a}. \quad (156)$$

Let us end this subsection by providing the expression of the Faddeev-Popov counterterm S_{FP}^{ct} , as derived from the algebraic renormalization procedure [65].

A.1 Renormalizability of the Faddeev-Popov action in linear covariant gauges

Following [65], in order to determine the most general invariant counterterm S_{FP}^{ct} affecting the Faddeev-Popov action in linear covariant gauges, eq.(144), we start from the complete classical action

$$\Sigma_0 = S_{FP} + \int d^4x \left(-\Omega_\mu^a D_\mu^{ab} c^b + \frac{1}{2} f^{abc} L^a c^b c^c \right), \quad (157)$$

where we have introduced the external sources (Ω_μ^a, L^a) coupled to the non-linear BRST variations of the fields (A_μ^a, c^a) , see eqs.(47),(48).

The action Σ_0 obeys the following set of Ward identities [65]:

$$\int d^4x \left(\frac{\delta \Sigma_0}{\delta \Omega_\mu^a} \frac{\delta \Sigma_0}{\delta A_\mu^a} + \frac{\delta \Sigma_0}{\delta L^a} \frac{\delta \Sigma_0}{\delta c^a} + i b^a \frac{\delta \Sigma_0}{\delta \bar{c}^a} \right) = 0, \quad (158)$$

$$\frac{\delta \Sigma_0}{\delta b^a} = i \partial_\mu A_\mu^a + \alpha b^a, \quad (159)$$

$$\frac{\delta \Sigma_0}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma_0}{\delta \Omega_\mu^a} = 0, \quad (160)$$

from which it turns out [65] that the most general local invariant counterterm Σ_0^{ct} contains three free parameters (a_0, a_1, a_2) , being given by the expression:

$$\Sigma_0^{ct} = a_0 \int d^4x \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{B}_{\Sigma_0} \int d^4x (a_1 (\Omega_\mu^a + \partial_\mu \bar{c}^a) A_\mu^a + a_2 L^a c^a), \quad (161)$$

where \mathcal{B}_{Σ_0} is the nilpotent linearized Slavnov-Taylor operator

$$\mathcal{B}_{\Sigma_0} = \int d^4x \left(\frac{\delta \Sigma_0}{\delta \Omega_\mu^a} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Sigma_0}{\delta A_\mu^a} \frac{\delta}{\delta \Omega_\mu^a} + \frac{\delta \Sigma_0}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta \Sigma_0}{\delta c^a} \frac{\delta}{\delta L^a} + i b^a \frac{\delta}{\delta \bar{c}^a} \right), \quad (162)$$

$$\mathcal{B}_{\Sigma_0} \mathcal{B}_{\Sigma_0} = 0. \quad (163)$$

Expression (161) can be conveniently written in parametric form [65] as

$$\begin{aligned} \Sigma_0^{ct} = & -a_0 g^2 \frac{\partial \Sigma_0}{\partial g^2} + 2\alpha a_1 \frac{\partial \Sigma_0}{\partial \alpha} + \int d^4x \left(a_1 A_\mu^a \frac{\delta \Sigma_0}{\delta A_\mu^a} - a_1 b^a \frac{\delta \Sigma_0}{\delta b^a} - a_1 \bar{c}^a \frac{\delta \Sigma_0}{\delta \bar{c}^a} \right. \\ & \left. - a_1 \Omega_\mu^a \frac{\delta \Sigma_0}{\delta \Omega_\mu^a} - a_2 c^a \frac{\delta \Sigma_0}{\delta c^a} + a_2 L^a \frac{\delta \Sigma_0}{\delta L^a} \right), \end{aligned} \quad (164)$$

which is suitable for establishing the renormalizability of the starting action Σ_0 , *i.e.* to check that Σ_0^{ct} can be reabsorbed in Σ_0 through a redefinition of the fields, parameters and sources, according to

$$\Sigma_0[A, b, c, \bar{c}, \Omega, L, g^2, \alpha] + \varepsilon \Sigma_0^{ct} = \Sigma_0[A_0, b_0, c_0, \bar{c}_0, \Omega_0, L_0, g_0^2, \alpha_0] + O(\varepsilon^2), \quad (165)$$

with ε stands for an expansion parameter and where the label “0” denotes the redefined parameters, fields and sources. By direct inspection of equation (165), it follows that the counterterm Σ_0^{ct} can be reabsorbed through the following redefinitions:

$$g_0^2 = Z_{g^2} g^2, \quad A_0 = Z_A^{1/2} A, \quad c_0 = Z_c^{1/2} c, \quad (166)$$

with

$$\begin{aligned} Z_{g^2} &= 1 - \varepsilon a_0, \\ Z_A^{1/2} &= 1 + \varepsilon a_1, \\ Z_c^{1/2} &= 1 - \varepsilon a_2 \end{aligned} \quad (167)$$

and

$$\begin{aligned} \alpha_0 &= Z_A \alpha, \\ b_0 &= Z_A^{-1/2} b, \\ \bar{c}_0 &= Z_A^{-1/2} \bar{c}, \\ \Omega_0 &= Z_A^{-1/2} \Omega, \\ L_0 &= Z_c^{-1/2} L, \end{aligned} \quad (168)$$

exhibiting the multiplicative all orders renormalizability of the Faddeev-Popov action in linear covariant gauges.

Finally, setting the external sources (Ω_μ^a, L^a) to zero, for the counterterm S_{FP}^{ct} , eq.(156), one gets

$$\begin{aligned} S_{FP}^{ct} &= \Sigma_0^{ct}|_{\Omega=L=0} \\ &= -a_0 g^2 \frac{\partial S_{FP}}{\partial g^2} + 2\alpha a_1 \frac{\partial S_{FP}}{\partial \alpha} + \int d^4x \left(a_1 A_\mu^a \frac{\delta S_{FP}}{\delta A_\mu^a} - a_1 b^a \frac{\delta S_{FP}}{\delta b^a} - a_1 \bar{c}^a \frac{\delta S_{FP}}{\delta \bar{c}^a} - a_2 c^a \frac{\delta S_{FP}}{\delta c^a} \right). \end{aligned} \quad (169)$$

B Properties of the functional $f_A[u]$.

In this Appendix we recall some useful properties of the functional $f_A[u]$

$$f_A[u] \equiv \text{Tr} \int d^4x A_\mu^u A_\mu^u = \text{Tr} \int d^4x \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \right) \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \right). \quad (170)$$

For a given gauge field configuration A_μ , $f_A[u]$ is a functional defined on the gauge orbit of A_μ . Let \mathcal{A} be the space of connections A_μ^a with finite Hilbert norm $\|A\|$, *i.e.*

$$\|A\|^2 = \text{Tr} \int d^4x A_\mu A_\mu = \frac{1}{2} \int d^4x A_\mu^a A_\mu^a < +\infty, \quad (171)$$

and let \mathcal{U} be the space of local gauge transformations u such that the Hilbert norm $\|u^\dagger \partial u\|$ is finite too, namely

$$\|u^\dagger \partial u\|^2 = \text{Tr} \int d^4x \left(u^\dagger \partial_\mu u \right) \left(u^\dagger \partial_\mu u \right) < +\infty . \quad (172)$$

The following proposition holds [59, 60, 61, 62]

- Proposition

The functional $f_A[u]$ achieves its absolute minimum on the gauge orbit of A_μ .

This proposition means that there exists a $h \in \mathcal{U}$ such that

$$\delta f_A[h] = 0 , \quad (173)$$

$$\delta^2 f_A[h] \geq 0 , \quad (174)$$

$$f_A[h] \leq f_A[u] , \quad \forall u \in \mathcal{U} . \quad (175)$$

The operator A_{\min}^2 is thus given by

$$A_{\min}^2 = \min_{\{u\}} \text{Tr} \int d^4x A_\mu^u A_\mu^u = f_A[h] . \quad (176)$$

Let us give a look at the two conditions (173) and (174). To evaluate $\delta f_A[h]$ and $\delta^2 f_A[h]$ we set[‡]

$$v = h e^{ig\omega} = h e^{ig\omega^a T^a} , \quad (177)$$

$$\left[T^a, T^b \right] = i f^{abc} T^c , \quad \text{Tr} \left(T^a T^b \right) = \frac{1}{2} \delta^{ab} , \quad (178)$$

where ω is an infinitesimal hermitian matrix and we compute the linear and quadratic terms of the expansion of the functional $f_A[v]$ in power series of ω . Let us first obtain an expression for A_μ^v

$$\begin{aligned} A_\mu^v &= v^\dagger A_\mu v + \frac{i}{g} v^\dagger \partial_\mu v \\ &= e^{-ig\omega} h^\dagger A_\mu h e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \left(h^\dagger \partial_\mu h \right) e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \partial_\mu e^{ig\omega} \\ &= e^{-ig\omega} A_\mu^h e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \partial_\mu e^{ig\omega} . \end{aligned} \quad (179)$$

Expanding up to the order ω^2 , we get

$$\begin{aligned} A_\mu^v &= \left(1 - ig\omega - g^2 \frac{\omega^2}{2} \right) A_\mu^h \left(1 + ig\omega - g^2 \frac{\omega^2}{2} \right) + \frac{i}{g} \left(1 - ig\omega - g^2 \frac{\omega^2}{2} \right) \partial_\mu \left(1 + ig\omega - g^2 \frac{\omega^2}{2} \right) \\ &= \left(1 - ig\omega - g^2 \frac{\omega^2}{2} \right) \left(A_\mu^h + ig A_\mu^h \omega - g^2 A_\mu^h \frac{\omega^2}{2} \right) + \\ &+ \frac{i}{g} \left(1 - ig\omega - g^2 \frac{\omega^2}{2} \right) \left(ig \partial_\mu \omega - \frac{g^2}{2} (\partial_\mu \omega) \omega - \frac{g^2}{2} \omega (\partial_\mu \omega) \right) \\ &= A_\mu^h + ig A_\mu^h \omega - \frac{g^2}{2} A_\mu^h \omega^2 - ig\omega A_\mu^h + g^2 \omega A_\mu^h \omega - \frac{g^2}{2} \omega^2 A_\mu^h \\ &+ \frac{i}{g} \left(ig \partial_\mu \omega - \frac{g^2}{2} (\partial_\mu \omega) \omega - \frac{g^2}{2} \omega \partial_\mu \omega + g^2 \omega \partial_\mu \omega \right) + O(\omega^3) , \end{aligned} \quad (180)$$

[‡]The case of the gauge group $SU(N)$ is considered here.

from which it follows

$$A_\mu^v = A_\mu^h + ig[A_\mu^h, \omega] + \frac{g^2}{2}[[\omega, A_\mu^h], \omega] - \partial_\mu \omega + i\frac{g}{2}[\omega, \partial_\mu \omega] + O(\omega^3), \quad (181)$$

We now evaluate

$$\begin{aligned} f_A[v] &= \text{Tr} \int d^4x A_\mu^u A_\mu^u \\ &= \text{Tr} \int d^4x \left[\left(A_\mu^h + ig[A_\mu^h, \omega] + \frac{g^2}{2}[[\omega, A_\mu^h], \omega] - \partial_\mu \omega + i\frac{g}{2}[\omega, \partial_\mu \omega] + O(\omega^3) \right) \times \right. \\ &\quad \left. \left(A_\mu^h + ig[A_\mu^h, \omega] + \frac{g^2}{2}[[\omega, A_\mu^h], \omega] - \partial_\mu \omega + i\frac{g}{2}[\omega, \partial_\mu \omega] + O(\omega^3) \right) \right] \\ &= \text{Tr} \int d^4x \left\{ A_\mu^h A_\mu^h + igA_\mu^h[A_\mu^h, \omega] + g^2 A_\mu^h \omega A_\mu^h - \frac{g^2}{2} A_\mu^h A_\mu^h \omega^2 - \frac{g^2}{2} A_\mu^h \omega^2 A_\mu^h - A_\mu^h \partial_\mu \omega \right. \\ &\quad + i\frac{g}{2} A_\mu^h [\omega, \partial_\mu \omega] + ig[A_\mu^h, \omega] A_\mu^h - g^2 [A_\mu^h, \omega] [A_\mu^h, \omega] - ig[A_\mu^h, \omega] \partial_\mu \omega + g^2 \omega A_\mu^h \omega A_\mu^h \\ &\quad \left. - \frac{g^2}{2} A_\mu^h \omega^2 A_\mu^h - \frac{g^2}{2} \omega^2 A_\mu^h A_\mu^h - \partial_\mu \omega A_\mu^h - ig \partial_\mu \omega [A_\mu^h, \omega] + \partial_\mu \omega \partial_\mu \omega + i\frac{g}{2} [\omega, \partial_\mu \omega] A_\mu^h \right\} + O(\omega^3) \\ &= f_A[h] - \text{Tr} \int d^4x \left\{ A_\mu^h, \partial_\mu \omega \right\} + \text{Tr} \int d^4x \left(g^2 A_\mu^h \omega A_\mu^h - \frac{g^2}{2} A_\mu^h A_\mu^h \omega^2 - \frac{g^2}{2} A_\mu^h \omega^2 A_\mu^h \right. \\ &\quad \left. - g^2 [A_\mu^h, \omega] [A_\mu^h, \omega] + g^2 \omega A_\mu^h \omega A_\mu^h - \frac{g^2}{2} A_\mu^h \omega^2 A_\mu^h - \frac{g^2}{2} \omega^2 A_\mu^h A_\mu^h \right) + \text{Tr} \int d^4x (\partial_\mu \omega \partial_\mu \omega \\ &\quad + i\frac{g}{2} [\omega, \partial_\mu \omega] A_\mu^h - ig \partial_\mu \omega [A_\mu^h, \omega] - ig [A_\mu^h, \omega] \partial_\mu \omega + i\frac{g}{2} A_\mu^h [\omega, \partial_\mu \omega]) + O(\omega^3) \\ &= f_A[h] + 2 \int d^4x \text{tr} (\omega \partial_\mu A_\mu^h) + \int d^4x \text{tr} \left\{ 2g^2 \omega A_\mu^h \omega A_\mu^h - 2g^2 A_\mu^h A_\mu^h \omega^2 \right. \\ &\quad \left. - g^2 (A_\mu^h \omega - \omega A_\mu^h) (A_\mu^h \omega - \omega A_\mu^h) \right\} + \int d^4x \text{tr} \left(\partial_\mu \omega \partial_\mu \omega + i\frac{g}{2} \omega \partial_\mu \omega A_\mu^h - i\frac{g}{2} \partial_\mu \omega \omega A_\mu^h \right. \\ &\quad \left. - ig \partial_\mu \omega A_\mu^h \omega + ig \partial_\mu \omega \omega A_\mu^h - ig A_\mu^h \omega \partial_\mu \omega + ig \omega A_\mu^h \partial_\mu \omega + i\frac{g}{2} A_\mu^h \omega \partial_\mu \omega - i\frac{g}{2} A_\mu^h \partial_\mu \omega \omega \right) + O(\omega^3) \\ &= f_A[h] + 2 \text{Tr} \int d^4x (\omega \partial_\mu A_\mu^h) + \text{Tr} \int d^4x (\partial_\mu \omega \partial_\mu \omega + ig \omega \partial_\mu \omega A_\mu^h - ig \partial_\mu \omega \omega A_\mu^h \\ &\quad - 2ig \partial_\mu \omega A_\mu^h \omega + 2ig \partial_\mu \omega \omega A_\mu^h) + O(\omega^3). \end{aligned} \quad (182)$$

Thus

$$\begin{aligned} f_A[v] &= f_A[h] + 2 \text{Tr} \int d^4x (\omega \partial_\mu A_\mu^h) + \text{Tr} \int d^4x (\partial_\mu \omega \partial_\mu \omega + ig \omega \partial_\mu \omega A_\mu^h - ig \partial_\mu \omega \omega A_\mu^h \\ &\quad - ig (\partial_\mu \omega) A_\mu^h \omega + ig (\partial_\mu \omega) \omega A_\mu^h) + O(\omega^3) \\ &= f_A[h] + 2 \text{Tr} \int d^4x (\omega \partial_\mu A_\mu^h) + \text{Tr} \int d^4x \left\{ \partial_\mu \omega (\partial_\mu \omega - ig [A_\mu^h, \omega]) \right\} + O(\omega^3). \end{aligned} \quad (183)$$

Finally

$$f_A[v] = f_A[h] + 2 \text{Tr} \int d^4x (\omega \partial_\mu A_\mu^h) - \text{Tr} \int d^4x \omega \partial_\mu D_\mu(A^h) \omega + O(\omega^3), \quad (184)$$

so that

$$\begin{aligned}\delta f_A[h] &= 0 \Rightarrow \partial_\mu A_\mu^h = 0, \\ \delta^2 f_A[h] &> 0 \Rightarrow -\partial_\mu D_\mu(A^h) > 0.\end{aligned}\quad (185)$$

We see therefore that the set of field configurations fulfilling conditions (185), *i.e.* defining relative minima of the functional $f_A[u]$, belong to the so called Gribov region Ω , which is defined as

$$\Omega = \{A_\mu | \partial_\mu A_\mu = 0 \text{ and } -\partial_\mu D_\mu(A) > 0\}.\quad (186)$$

Let us proceed now by showing that the transversality condition, $\partial_\mu A_\mu^h = 0$, can be solved for $h = h(A)$ as a power series in A_μ . We start from

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h,\quad (187)$$

with

$$h = e^{ig\phi} = e^{ig\phi^a T^a}.\quad (188)$$

Let us expand h in powers of ϕ

$$h = 1 + ig\phi - \frac{g^2}{2}\phi^2 + O(\phi^3).\quad (189)$$

From equation (187) we have

$$A_\mu^h = A_\mu + ig[A_\mu, \phi] + g^2\phi A_\mu\phi - \frac{g^2}{2}A_\mu\phi^2 - \frac{g^2}{2}\phi^2 A_\mu - \partial_\mu\phi + i\frac{g}{2}[\phi, \partial_\mu] + O(\phi^3).\quad (190)$$

Thus, condition $\partial_\mu A_\mu^h = 0$, gives

$$\begin{aligned}\partial^2\phi &= \partial_\mu A + ig[\partial_\mu A_\mu, \phi] + ig[A_\mu, \partial_\mu\phi] + g^2\partial_\mu\phi A_\mu\phi + g^2\phi\partial_\mu A_\mu\phi + g^2\phi A_\mu\partial_\mu\phi \\ &- \frac{g^2}{2}\partial_\mu A_\mu\phi^2 - \frac{g^2}{2}A_\mu\partial_\mu\phi\phi - \frac{g^2}{2}A_\mu\phi\partial_\mu\phi - \frac{g^2}{2}\partial_\mu\phi\phi A_\mu - \frac{g^2}{2}\phi\partial_\mu\phi A_\mu - \frac{g^2}{2}\phi^2\partial_\mu A_\mu \\ &+ i\frac{g}{2}[\phi, \partial^2\phi] + O(\phi^3).\end{aligned}\quad (191)$$

This equation can be solved iteratively for ϕ as a power series in A_μ , namely

$$\phi = \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A, \frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu, \partial_\mu \frac{\partial A}{\partial^2}\right] + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + O(A^3),\quad (192)$$

so that

$$\begin{aligned}A_\mu^h &= A_\mu - \frac{1}{\partial^2}\partial_\mu\partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu \frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] \\ &+ ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3).\end{aligned}\quad (193)$$

Expression (193) can be written in a more useful way, given in eq.(3). In fact

$$\begin{aligned}
A_\mu^h &= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \left(A_\nu - ig \left[\frac{1}{\partial^2} \partial A, A_\nu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\nu \frac{1}{\partial^2} \partial A \right] \right) + O(A^3) \\
&= A_\mu - ig \left[\frac{1}{\partial^2} \partial A, A_\mu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\mu \frac{1}{\partial^2} \partial A \right] - \frac{\partial_\mu}{\partial^2} \partial A + ig \frac{\partial_\mu}{\partial^2} \partial_\nu \left[\frac{1}{\partial^2} \partial A, A_\nu \right] \\
&\quad - i \frac{g}{2} \frac{\partial_\mu}{\partial^2} \partial_\nu \left[\frac{\partial A}{\partial^2}, \frac{\partial_\nu}{\partial^2} \partial A \right] + O(A^3) \\
&= A_\mu - \frac{\partial_\mu}{\partial^2} \partial A + ig \left[A_\mu, \frac{1}{\partial^2} \partial A \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\mu \frac{1}{\partial^2} \partial A \right] + ig \frac{\partial_\mu}{\partial^2} \left[\frac{\partial_\nu}{\partial^2} \partial A, A_\nu \right] \\
&\quad + i \frac{g}{2} \frac{\partial_\mu}{\partial^2} \left[\frac{\partial A}{\partial^2}, \partial A \right] + O(A^3) \tag{194}
\end{aligned}$$

which is precisely expression (193). The transverse field given in eq.(3) enjoys the property of being gauge invariant order by order in the coupling constant g . Let us work out the transformation properties of ϕ_ν under a gauge transformation

$$\delta A_\mu = -\partial_\mu \omega + ig[A_\mu, \omega]. \tag{195}$$

We have, up to the order $O(g^2)$,

$$\begin{aligned}
\delta \phi_\nu &= -\partial_\nu \omega + ig \left[\frac{1}{\partial^2} \partial A, \partial_\nu \omega \right] - i \frac{g}{2} \left[\omega, \partial_\nu \frac{1}{\partial^2} \partial A \right] - i \frac{g}{2} \left[\frac{\partial A}{\partial^2}, \partial_\nu \omega \right] + O(g^2) \\
&= -\partial_\nu \omega + i \frac{g}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\nu \omega \right] + i \frac{g}{2} \left[\partial_\nu \frac{1}{\partial^2} \partial A, \omega \right] + O(g^2). \tag{196}
\end{aligned}$$

Therefore

$$\delta \phi_\nu = -\partial_\nu \left(\omega - i \frac{g}{2} \left[\frac{\partial A}{\partial^2}, \omega \right] \right) + O(g^2), \tag{197}$$

from which the gauge invariance of A_μ^h is established.

Finally, let us work out the expression of A_{\min}^2 as a power series in A_μ .

$$\begin{aligned}
A_{\min}^2 &= \text{Tr} \int d^4x A_\mu^h A_\mu^h \\
&= \text{Tr} \int d^4x \left[\phi_\mu \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \phi_\nu \right] \\
&= \text{Tr} \int d^4x \left[\left(A_\mu - ig \left[\frac{1}{\partial^2} \partial A, A_\mu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\mu \frac{1}{\partial^2} \partial A \right] \right) \times \right. \\
&\quad \left. \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \left(A_\nu - ig \left[\frac{1}{\partial^2} \partial A, A_\nu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\nu \frac{1}{\partial^2} \partial A \right] \right) \right] \\
&= \frac{1}{2} \int d^4x \left[A_\mu^a \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A_\nu^a - 2gf^{abc} \frac{\partial_\nu \partial A^a}{\partial^2} \frac{\partial A^b}{\partial^2} A_\nu^c - gf^{abc} A_\nu^a \frac{\partial A^b}{\partial^2} \frac{\partial_\nu \partial A^c}{\partial^2} \right] + O(A^4). \tag{198}
\end{aligned}$$

leading to the result quoted in eq.(5).

We conclude this Appendix by noting that, due to gauge invariance, A_{\min}^2 can be rewritten in a manifestly invariant way in terms of $F_{\mu\nu}$ and the covariant derivative D_μ [59], see eq.(6).

C Propagators of the elementary fields

In order to evaluate the tree-level two-point functions of the theory, we start from the local action

$$S = S_{FP} + \int d^4x \left(\tau^a \partial_\mu A_\mu^{h,a} + \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} \right) + S_{IRR} \quad (199)$$

where S_{FP} is the Faddeev-Popov term of the linear covariant gauges, eq.(10), and S_{IRR} stands for the BRST-invariant infrared regularizing mass term for the Stueckelberg field, namely

$$S_{IRR} = \int d^4x \frac{1}{2} s (\rho \xi^a \xi^a) = \int d^4x \left(\frac{1}{2} M^4 \xi^a \xi^a + \rho \xi^a c^a \right) \quad (200)$$

From the quadratic part of expression (199), one finds the following set of tree-level propagators in momentum space

$$\langle A_\mu^a(p) A_\nu^b(-p) \rangle = \frac{1}{p^2 + m^2} \delta^{ab} \mathcal{P}_{\mu\nu} + \frac{\alpha}{p^2} \frac{p_\mu p_\nu}{p^2} \quad (201)$$

$$\langle A_\mu^a(p) b^b(-p) \rangle = -\frac{p^2}{p^4 + \alpha M^4} \delta^{ab} p_\mu \quad (202)$$

$$\langle A_\mu^a(p) \xi^b(-p) \rangle = i \frac{\alpha \delta^{ab}}{p^4 + \alpha M^4} p_\mu \quad (203)$$

$$\langle A_\mu^a(p) \tau^b(-p) \rangle = -i \frac{\alpha M^4}{p^2(p^4 + \alpha M^4)} p_\mu \delta^{ab} \quad (204)$$

$$\langle b^a(p) b^b(-p) \rangle = \frac{M^4}{p^4 + \alpha M^4} \delta^{ab} \quad (205)$$

$$\langle b^a(p) \xi^b(-p) \rangle = i \frac{p^2 \delta^{ab}}{p^4 + \alpha M^4} \quad (206)$$

$$\langle b^a(p) \tau^b(-p) \rangle = -i \frac{M^4}{p^2} \delta^{ab} \quad (207)$$

$$\langle \bar{c}^a(p) A_\mu^b(-p) \rangle = -i \frac{\rho \alpha}{p^2(p^4 + \alpha M^4)} \delta^{ab} p_\mu \quad (208)$$

$$\langle \bar{c}^a(p) b^b(-p) \rangle = i \frac{\rho}{p^4 + \alpha M^4} \delta^{ab} \quad (209)$$

$$\langle \bar{c}^a(p) \tau^b(-p) \rangle = \frac{\rho}{p^4 + \alpha M^4} \delta^{ab} \quad (210)$$

$$\langle \bar{c}^a(p) \xi^b(-p) \rangle = \frac{\rho \alpha}{p^2(p^4 + \alpha M^4)} \delta^{ab} \quad (211)$$

$$\langle \xi^a(p) \xi^b(-p) \rangle = \frac{\alpha \delta^{ab}}{p^4 + \alpha M^4} \quad (212)$$

$$\langle \xi^a(p) \tau^b(-p) \rangle = \frac{p^2}{p^4 + \alpha M^4} \delta^{ab} \quad (213)$$

$$\langle \tau^a(p) \tau^b(-p) \rangle = -\left(\frac{m^2(p^4 - \alpha M^4) + M^4 p^2}{p^2(p^4 + \alpha M^4)} \right) \delta^{ab} \quad (214)$$

$$\langle \bar{c}^a(p) c^b(-p) \rangle = \frac{1}{p^2} \delta^{ab} , \quad (215)$$

with $\mathcal{P}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$ being the transverse projector. All other propagators which have not been listed above are vanishing. Let us also remind that the parameters M and ρ which regularize the propagation of the Stueckelberg field in the infrared have to be set to zero at the end of any actual calculation.

References

- [1] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
- [2] J. Greensite and M. B. Halpern, Nucl. Phys. B **271**, 379 (1986).
- [3] M. Stingl, Phys. Rev. D **34**, 3863 (1986) [Erratum-ibid. D **36**, 651 (1987)].
- [4] M. J. Lavelle and M. Schaden, Phys. Lett. B **208**, 297 (1988).
- [5] F. V. Gubarev and V. I. Zakharov, Phys. Lett. B **501**, 28 (2001) [arXiv:hep-ph/0010096].
- [6] F. V. Gubarev, L. Stodolsky and V. I. Zakharov, Phys. Rev. Lett. **86**, 2220 (2001) [arXiv:hep-ph/0010057].
- [7] H. Vershelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B **516**, 307 (2001) [arXiv:hep-th/0105018].
- [8] K. I. Kondo, Phys. Lett. B **514**, 335 (2001) [arXiv:hep-th/0105299].
- [9] K. I. Kondo, T. Murakami, T. Shinohara and T. Imai, Phys. Rev. D **65**, 085034 (2002) [arXiv:hep-th/0111256].
- [10] D. Dudal, H. Vershelde, R. E. Browne and J. A. Gracey, Phys. Lett. B **562**, 87 (2003) [arXiv:hep-th/0302128].
- [11] R. E. Browne and J. A. Gracey, JHEP **0311**, 029 (2003) [arXiv:hep-th/0306200].
- [12] D. Dudal, H. Vershelde, V. E. R. Lemes, M. S. Sarandy, S. P. Sorella and M. Picariello, Annals Phys. **308**, 62 (2003) [arXiv:hep-th/0302168].
- [13] D. Dudal, H. Vershelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro and S. P. Sorella, JHEP **0401**, 044 (2004) [arXiv:hep-th/0311194].
- [14] D. Dudal, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and H. Vershelde, Phys. Rev. D **70**, 114038 (2004) [arXiv:hep-th/0406132].
- [15] R. E. Browne and J. A. Gracey, Phys. Lett. B **597**, 368 (2004) [arXiv:hep-ph/0407238].
- [16] J. A. Gracey, Eur. Phys. J. C **39**, 61 (2005) [arXiv:hep-ph/0411169].
- [17] X. d. Li and C. M. Shakin, Phys. Rev. D **71**, 074007 (2005) [arXiv:hep-ph/0410404].
- [18] P. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **63**, 114003 (2001) [arXiv:hep-ph/0101302].
- [19] P. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, F. De Soto, A. Donini, H. Moutare and J. Rodriguez-Quintero Phys. Rev. D **66**, 034504 (2002) [arXiv:hep-ph/0203119].

- [20] P. Boucaud, F. de Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, H. Moutarde, O. Pene and J. Rodriguez-Quintero, arXiv:hep-lat/0504017.
- [21] E. Ruiz Arriola, P. O. Bowman and W. Broniowski, Phys. Rev. D **70**, 097505 (2004) [arXiv:hep-ph/0408309].
- [22] T. Suzuki, K. Ishiguro, Y. Mori and T. Sekido, Phys. Rev. Lett. **94**, 132001 (2005) [arXiv:hep-lat/0410001].
- [23] F. V. Gubarev and S. M. Morozov, Phys. Rev. D **71**, 114514 (2005) [arXiv:hep-lat/0503023].
- [24] S. Furui and H. Nakajima, Hideo, Few Body Syst. **40**, 101-128 (2006) [arXiv:hep-lat/0503029].
- [25] P. Boucaud, J. P. Leroy, A. Le Yaouanc, A. Y. Lokhov, J. Micheli, O. Pene, J. Rodriguez-Quintero and C. Roiesnel, arXiv:hep-lat/0507005.
- [26] M. N. Chernodub, K. Ishiguro, Y. Mori, Y. Nakamura, M. I. Polikarpov, T. Sekido, T. Suzuki and V. I. Zakharov, arXiv:hep-lat/0508004.
- [27] P. Boucaud, J. P. Leroy, A. Le Yaouanc, A. Y. Lokhov, J. Micheli, O. Pene, J. Rodriguez-Quintero and C. Roiesnel, JHEP **0601**, 037 (2006) doi:10.1088/1126-6708/2006/01/037 [hep-lat/0507005].
- [28] P. Boucaud, F. De Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **79**, 014508 (2009) doi:10.1103/PhysRevD.79.014508 [arXiv:0811.2059 [hep-ph]].
- [29] O. Pene *et al.*, PoS FACESQCD , 010 (2010) [arXiv:1102.1535 [hep-lat]].
- [30] P. Boucaud, M. E. Gomez, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **82**, 054007 (2010) doi:10.1103/PhysRevD.82.054007 [arXiv:1004.4135 [hep-ph]].
- [31] B. Blossier *et al.* [ETM Collaboration], Phys. Rev. D **82**, 034510 (2010) doi:10.1103/PhysRevD.82.034510 [arXiv:1005.5290 [hep-lat]].
- [32] D. Dudal, O. Oliveira and N. Vandersickel, Phys. Rev. D **81**, 074505 (2010) doi:10.1103/PhysRevD.81.074505 [arXiv:1002.2374 [hep-lat]].
- [33] P. Boucaud, D. Dudal, J. P. Leroy, O. Pene and J. Rodriguez-Quintero, JHEP **1112**, 018 (2011) doi:10.1007/JHEP12(2011)018 [arXiv:1109.3803 [hep-ph]].
- [34] B. Blossier *et al.*, Phys. Rev. D **85**, 034503 (2012) doi:10.1103/PhysRevD.85.034503 [arXiv:1110.5829 [hep-lat]].
- [35] B. Blossier, P. Boucaud, M. Brinet, F. De Soto, V. Morenas, O. Pene, K. Petrov and J. Rodriguez-Quintero, Phys. Rev. D **87**, 074033 (2013) doi:10.1103/PhysRevD.87.074033 [arXiv:1301.7593 [hep-ph]].
- [36] D. Dudal, H. Verschelde and S. P. Sorella, Phys. Lett. B **555**, 126 (2003) [arXiv:hep-th/0212182].

- [37] J. A. Gracey, Phys. Lett. B **552** (2003) 101 [arXiv:hep-th/0211144].
- [38] D. Dudal, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D **72**, 014016 (2005) [arXiv:hep-th/0502183].
- [39] D. Dudal, S. P. Sorella, N. Vandersickel and H. Verschelde, Phys. Rev. D **77**, 071501 (2008) doi:10.1103/PhysRevD.77.071501 [arXiv:0711.4496 [hep-th]].
- [40] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel and H. Verschelde, Phys. Rev. D **78** (2008) 065047 doi:10.1103/PhysRevD.78.065047 [arXiv:0806.4348 [hep-th]].
- [41] D. Dudal, S. P. Sorella and N. Vandersickel, Phys. Rev. D **84**, 065039 (2011) doi:10.1103/PhysRevD.84.065039 [arXiv:1105.3371 [hep-th]].
- [42] N. Vandersickel and D. Zwanziger, Phys. Rept. **520** (2012) 175 doi:10.1016/j.physrep.2012.07.003 [arXiv:1202.1491 [hep-th]].
- [43] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78** (2008) 025010 doi:10.1103/PhysRevD.78.025010 [arXiv:0802.1870 [hep-ph]].
- [44] A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. China **11** (2016) no.2, 111203 doi:10.1007/s11467-015-0517-6 [arXiv:1511.08361 [hep-ph]].
- [45] C. S. Fischer, A. Maas and J. M. Pawłowski, Annals Phys. **324** (2009) 2408 doi:10.1016/j.aop.2009.07.009 [arXiv:0810.1987 [hep-ph]].
- [46] A. Cucchieri and T. Mendes, PoS LAT **2007**, 297 (2007) [arXiv:0710.0412 [hep-lat]].
- [47] A. Cucchieri and T. Mendes, Phys. Rev. Lett. **100**, 241601 (2008) doi:10.1103/PhysRevLett.100.241601 [arXiv:0712.3517 [hep-lat]].
- [48] A. Cucchieri, D. Dudal, T. Mendes and N. Vandersickel, Phys. Rev. D **85** (2012) 094513 doi:10.1103/PhysRevD.85.094513 [arXiv:1111.2327 [hep-lat]].
- [49] O. Oliveira and P. J. Silva, Phys. Rev. D **86**, 114513 (2012) doi:10.1103/PhysRevD.86.114513 [arXiv:1207.3029 [hep-lat]].
- [50] R. F. Sobreiro and S. P. Sorella, JHEP **0506**, 054 (2005) [arXiv:hep-th/0506165].
- [51] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **91**, no. 8, 085014 (2015) doi:10.1103/PhysRevD.91.085014 [arXiv:1501.07150 [hep-ph]].
- [52] M. Q. Huber, Phys. Rev. D **91**, no. 8, 085018 (2015) doi:10.1103/PhysRevD.91.085018 [arXiv:1502.04057 [hep-ph]].
- [53] M. A. L. Capri, A. D. Pereira, R. F. Sobreiro and S. P. Sorella, Eur. Phys. J. C **75**, no. 10, 479 (2015) doi:10.1140/epjc/s10052-015-3707-z [arXiv:1505.05467 [hep-th]].
- [54] M. A. L. Capri *et al.*, Phys. Rev. D **92**, no. 4, 045039 (2015) doi:10.1103/PhysRevD.92.045039 [arXiv:1506.06995 [hep-th]].
- [55] M. A. L. Capri *et al.*, Phys. Rev. D **93** (2016) no.6, 065019 doi:10.1103/PhysRevD.93.065019 [arXiv:1512.05833 [hep-th]].

- [56] A. Cucchieri, T. Mendes and E. M. S. Santos, Phys. Rev. Lett. **103**, 141602 (2009) doi:10.1103/PhysRevLett.103.141602 [arXiv:0907.4138 [hep-lat]].
- [57] A. Cucchieri, T. Mendes, G. M. Nakamura and E. M. S. Santos, AIP Conf. Proc. **1354**, 45 (2011) doi:10.1063/1.3587584 [arXiv:1101.5080 [hep-lat]].
- [58] P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D **92**, no. 11, 114514 (2015) doi:10.1103/PhysRevD.92.114514 [arXiv:1505.05897 [hep-lat]].
- [59] D. Zwanziger, Nucl. Phys. B **345**, 461 (1990).
- [60] G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326**, 333 (1989).
- [61] G. Dell'Antonio and D. Zwanziger, Commun. Math. Phys. **138**, 291 (1991).
- [62] P. van Baal, Nucl. Phys. B **369**, 259 (1992).
- [63] M. Lavelle and D. McMullan, Phys. Rept. **279**, 1 (1997) [arXiv:hep-ph/9509344].
- [64] J. A. Gracey, Phys. Lett. B **651**, 253 (2007) doi:10.1016/j.physletb.2007.06.021 [arXiv:0706.1440 [hep-th]].
- [65] O. Piguet and S. P. Sorella, Lect. Notes Phys. M **28** (1995) 1.
- [66] R. Ferrari and A. Quadri, JHEP **0411**, 019 (2004) doi:10.1088/1126-6708/2004/11/019 [hep-th/0408168].
- [67] M. A. L. Capri *et al.*, arXiv:1605.02610 [hep-th].
- [68] N. Dragon, T. Hurth and P. van Nieuwenhuizen, Nucl. Phys. Proc. Suppl. **56B**, 318 (1997) doi:10.1016/S0920-5632(97)00341-1 [hep-th/9703017].
- [69] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Vershelde, Phys. Rev. D **72**, 105016 (2005) doi:10.1103/PhysRevD.72.105016 [hep-th/0510240].
- [70] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Vershelde, Phys. Rev. D **74**, 045008 (2006) doi:10.1103/PhysRevD.74.045008 [hep-th/0605288].
- [71] D. Dudal, N. Vandersickel and H. Vershelde, Phys. Rev. D **76**, 025006 (2007) doi:10.1103/PhysRevD.76.025006 [arXiv:0705.0871 [hep-th]].
- [72] M. Tissier and N. Wschebor, Phys. Rev. D **82**, 101701 (2010) doi:10.1103/PhysRevD.82.101701 [arXiv:1004.1607 [hep-ph]].
- [73] M. Tissier and N. Wschebor, Phys. Rev. D **84**, 045018 (2011) doi:10.1103/PhysRevD.84.045018 [arXiv:1105.2475 [hep-th]].
- [74] O. Piguet and K. Sibold, Boston, Usa: Birkhaeuser (1986) 346 P. (Progress In Physics, 12)