

Radiative decays of negative parity heavy baryons in QCD

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Abstract

The transition form factors responsible for the radiative $\Sigma_Q \rightarrow \Lambda_Q \gamma$ and $\Xi'_Q \rightarrow \Xi \gamma$ decays of the negative parity baryons are examined within light cone QCD sum rules. The decay widths of the radiative transitions are calculated using the obtained results of the form factors.

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1 Introduction

Baryons with single heavy quarks receive a lot of attention in the high energy physics community. This is due particularly to the development of the heavy quark effective theory and its application to this baryonic system.

Despite considerable development on the theoretical side, noticeable progress has only recently been made on the experimental side. Lately, all hadrons containing single heavy quark with positive parity, except Ω_b^* , as well as several heavy baryons possessing negative parity have been discovered in experiments (for a review, see [1]).

Baryons containing only a single heavy quark are usually described in terms of SU(3) multiplets, which can be represented as a subgroup of the larger SU(4) group including all baryons with zero, one two or three charmed quarks. In this case baryon multiplet structures appear in the form $4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus 4$. The symmetric subgroup 20 contains the decuplet with positive parity, the mixed-symmetric subgroup 20 contains the octets of the lowest level with positive parity, and the antisymmetric subgroup 4 contains Ξ_c^0 , Λ_c , Ξ_c^+ and Λ . Note that the singlet Λ has the quantum number $J^P = \frac{1}{2}^-$. Similar construction takes place for the baryons containing single b quark.

After experimental observation of the negative parity heavy baryons, the next step is the study of their electromagnetic, weak and strong decays. In this sense the study of electromagnetic decays plays exceptional role, which provide us information about the internal structure of negative parity baryons, as well as about the nonperturbative aspects of QCD.

The radiative decays between positive parity baryons in framework of the relativistic quark model [2], in heavy baryon chiral perturbation theory [3], in the formalism that incorporates both heavy quark symmetry and chiral symmetry [4], in static quark model [5], in bag model [6], in light cone QCD sum rules incorporated with the heavy quark effective theory [7], etc. The analysis of similar decays for the negative parity baryons has recently started. Therefore the study of radiative decays of negative parity baryons represent proves to be very useful in order to get information about their properties.

In the present work we study the radiative decays $\Sigma_Q \rightarrow \Lambda_Q \gamma$ and $\Xi'_Q \rightarrow \Xi \gamma$ between negative parity heavy baryons in framework of the light cone QCD sum rules method. Note that the same transitions for the positive parity baryons have been studied in the same method in [8].

The paper is organized as follows. In section 2, the light cone QCD sum rules for the electromagnetic form factors responsible for the $\Sigma_Q \rightarrow \Lambda_Q \gamma$ and $\Xi'_Q \rightarrow \Xi \gamma$ transitions are derived. The results numerical analysis of the sum rules for the form factors is presented in Section 3. Using the values of these form factors at evaluated $Q^2 = 0$ which corresponds to the real photon emission, the corresponding decay widths are calculated.

2 Sum rules for the transition form factors between negative parity heavy baryons

In this section we shall construct the light cone QCD sum rules for the $\Sigma_Q \rightarrow \Lambda_Q \gamma$ and $\Xi'_Q \rightarrow \Xi \gamma$ transition form factors between negative parity heavy baryons. For this purpose

we consider the following correlation function,

$$\Pi_\mu(p, q) = - \int d^4x \int d^4y e^{i(px+qy)} \langle 0 | T \{ \eta_{Q_1}(0) J_\mu^{el}(y) \bar{\eta}_{Q_2}(x) \} | 0 \rangle , \quad (1)$$

where η_{Q_1} and η_{Q_2} are the interpolating currents of the initial and final heavy baryons which interact simultaneously with both positive and negative parity baryons, $j_\mu^{el} = e_q \bar{q} \gamma_\mu q + e_Q \bar{Q} \gamma_\mu Q$ is the electromagnetic current with, and e_q and e_Q are the electric charges for the light and heavy quarks, respectively.

The radiated photon can be absorbed into the electromagnetic background field which is defined as $F_{\mu\nu} = i(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) e^{iqx}$. The correlation function then can be written as,

$$\Pi_\mu(p, q) \varepsilon^\mu = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_{Q_1}(x) \bar{\eta}_{Q_2}(0) \} | 0 \rangle_F , \quad (2)$$

where the subscript F means that vacuum expectation values of the corresponding operators are evaluated in the presence of the background field. Here we note that, Eq. (1) can be obtained by expanding the correlation function in powers of $F_{\mu\nu}$, and keeping only terms linear in $F_{\mu\nu}$ which corresponds to the single photon emission (more about the background field method and its applications can be found in [9] and [10]).

It follows from Eq. (2) that, in order to calculate the correlation function, the expressions of the interpolating currents of heavy baryons are needed. The general form of the interpolating currents of the spin-1/2 heavy baryons entering into the symmetric sextet and antisymmetric antitriplet representations are [11],

$$\begin{aligned} \eta_Q^{(6)} &= -\frac{1}{\sqrt{2}} \epsilon^{abc} \left\{ (q_1^{aT} C Q^b) \gamma_5 q_2^c - (Q^{aT} C q_2^b) \gamma_5 q_1^c + \beta (q_1^{aT} C \gamma_5 Q^b) q_2^c - \beta (Q^{aT} C \gamma_5 q_2^b) q_1^c \right\} , \\ \eta_Q^{(\bar{3})} &= \frac{1}{\sqrt{6}} \epsilon^{abc} \left\{ 2(q_1^{aT} C q_2^b) \gamma_5 Q^c + (q_1^{aT} C Q^b) \gamma_5 q_2^c + (Q^{aT} C q_2^b) \gamma_5 q_1^c \right. \\ &\quad \left. + 2\beta (q_1^{aT} C \gamma_5 q_2^b) Q^c + (q_1^{aT} C \gamma_5 Q^b) q_2^c + (Q^{aT} C \gamma_5 q_2^b) q_1^c \right\} , \end{aligned} \quad (3)$$

where β is the arbitrary auxiliary parameter. The light quark contents of the heavy baryons in sextet and antitriplet representations are presented in Table 1.

	$\Sigma_{b(c)}^{+(++)}$	$\Sigma_{b(c)}^{0(+)}$	$\Sigma_{b(c)}^{- (0)}$	$\Xi'_{b(c)}{}^{- (0)}$	$\Xi_{b(c)}^{0(+)}$	$\Xi_{b(c)}^{- (0)}$	$\Xi_{b(c)}^{0(+)}$	$\Lambda_{b(c)}^{0(+)}$
q_1	u	u	d	d	u	d	u	u
q_2	u	d	d	s	s	s	s	d

Table 1: Light quark contents of the heavy spin-1/2 baryons.

According to the general strategy of QCD sum rules method, the correlation function (2) should be evaluated in two different kinematical domains. On the one side Eq. (2) should be dominated by the decays $\Sigma_Q \rightarrow \Lambda_Q \gamma$ and $\Xi'_Q \rightarrow \Xi \gamma$ if the virtualities p^2 and $(p+q)^2$ are close to the heavy baryon masses, i.e., $p^2 \simeq m_{final}^2$, $(p+q)^2 \simeq m_{initial}^2$. On the other side, it can be calculated in the kinematical domain $p^2 \ll 0$ and $(p+q)^2 \ll 0$ as an expansion in terms of the photon distribution amplitudes with increasing twist.

We proceed now calculating the correlation function from the hadronic part. For this aim the complete set of hadrons carrying the same quantum numbers as the interpolating current are inserted. At this point it is useful noting that the interpolating current can interact both with positive and negative parity baryons. Taking this fact into account, and calculating the correlation function from the hadronic side, we get

$$\Pi_\mu(p, q)\epsilon^\mu = \varepsilon^\mu \sum_{\substack{i=+,- \\ j=+,-}} \frac{\langle 0|\eta_{Q_2}|B_{2^i}(p, s)\rangle}{p^2 - m_{2^i}^2} \langle B_{2^i}(p, s)|j_\mu^{el}|B_{1^j}\rangle \frac{\langle B_{1^j}(p+q, s)|\bar{\eta}_{Q_1}|0\rangle}{(p+q)^2 - m_{1^j}^2}, \quad (4)$$

where $+(-)$ means positive(negative) parity baryon.

The matrix element entering into Eq. (4) is defined as,

$$\begin{aligned} \langle 0|\eta_{Q_2}|B_{2^i}(p, s)\rangle &= \lambda_i \Gamma^i u^i(p, q), \\ \langle B_{2^i}(p, s)|j_\mu^{el}|B_{1^j}(p+q, s)\rangle &= \bar{u}^i \left[\left(\gamma_\mu - \frac{\not{q}q_\mu}{q^2} \right) f_1^\alpha - \frac{i\sigma_{\mu\nu}q^\nu}{m_{1^j} + m_{2^i}} f_2^\alpha \right] \Gamma^\alpha u^j(p+q, s), \end{aligned} \quad (5)$$

where

$$\Gamma^\alpha = \begin{cases} I, & \text{for the } + \rightarrow +, \text{ and } - \rightarrow - \text{ transitions,} \\ \gamma_5, & \text{for the } + \rightarrow -, \text{ and } - \rightarrow + \text{ transitions.} \end{cases}$$

Using the equation of motion, the matrix element (5) can be written as,

$$\begin{aligned} \langle B_{2^i}(p, s)|j_\mu^{el}|B_{1^j}(p+q, s)\rangle &= \bar{u}^i \left\{ \left(\gamma_\mu - \frac{\not{q}q_\mu}{q^2} \right) f_1^\alpha \right. \\ &\quad \left. + \frac{\gamma_\mu [2\beta m_{1^j} + 2m_{2^i}] - 2(p+q)_\mu - 2p_\mu}{2(m_{1^j} + m_{2^i})} f_2^\alpha \right\} \Gamma^\alpha u^j(p+q, s), \end{aligned} \quad (6)$$

where

$$\beta = \begin{cases} +1, & \text{for the } + \rightarrow +, - \rightarrow - \text{ transitions,} \\ -1, & \text{for the } + \rightarrow -, - \rightarrow + \text{ transitions.} \end{cases}$$

Imposing the conservation of the electromagnetic current, it can easily be shown that the radiative decays under consideration is described by the form factor f_2^- . Using the condition $q\varepsilon = 0$, we see from Eq. (6) that only the structure p_μ is needed for the estimation of the form factor f_2^- .

Using Eqs. (5) and (6), and performing summation over spins of the initial and final states of heavy baryons we obtain the following expression of the correlation function from the hadronic side,

$$\begin{aligned} \Pi_\mu \varepsilon^\mu &= -A(p\varepsilon)(\not{p}_2 + m_{2^+})(\not{p}_1 + m_{1^+}) \\ &\quad + B(p\varepsilon)(\not{p}_2 - m_{2^-})(\not{p}_1 - m_{1^-}) \\ &\quad + C(p\varepsilon)(\not{p}_2 - m_{2^-})(\not{p}_1 + m_{1^+}) \\ &\quad + D(p\varepsilon)(\not{p}_2 + m_{2^+})(\not{p}_1 - m_{1^-}) + \dots, \end{aligned} \quad (7)$$

where dots represent contributions of higher states and continuum. A , B , C , and D in Eq. (7) are given as,

$$\begin{aligned}
A &= \frac{2\lambda_{1+}\lambda_{2+}}{m_{1+} + m_{2+}} \frac{f_2^+}{(p^2 - m_{2+}^2)[(p+q)^2 - m_{1+}^2]}, \\
B &= \frac{2\lambda_{1-}\lambda_{2-}}{m_{1-} + m_{2-}} \frac{f_2^-}{(p^2 - m_{2-}^2)[(p+q)^2 - m_{1-}^2]}, \\
C &= \frac{2\lambda_{1+}\lambda_{2-}}{m_{1+} + m_{2-}} \frac{f^T}{(p^2 - m_{2-}^2)[(p+q)^2 - m_{1+}^2]}, \\
D &= \frac{2\lambda_{1-}\lambda_{2+}}{m_{1-} + m_{2+}} \frac{f^T}{(p^2 - m_{2+}^2)[(p+q)^2 - m_{1-}^2]}.
\end{aligned}$$

It follows from Eq. (7) that the correlation function contains four different contributions coming from $+\rightarrow+$, $-\rightarrow-$, $+\rightarrow-$, and $-\rightarrow+$ transitions. As has already been mentioned the radiative decay between the negative parity baryons under consideration is described only by the form factor f_2^- . Therefore, the unwanted contributions coming from three transitions should be eliminated. This can be achieved by solving the set of four equations that result from four different Lorentz structures $(p\varepsilon)\not{p}\not{q}$, $(p\varepsilon)\not{p}$, $(p\varepsilon)\not{q}$, and $(p\varepsilon)I$.

In order to construct sum rules for the form factor f_2^- calculation of the correlation function from the QCD side is needed. The correlation function given in Eq. (2) can be calculated from the QCD side using the operator product expansion (OPE) over the twist of the nonlocal operators. The expansion of the nonlocal operators up to twist-4 is calculated in [12], which gets contribution from the two-particle $\bar{q}q$, three-particle $\bar{q}Gq$, and four-particle $\bar{q}G^2q$, $\bar{q}q\bar{q}q$ nonlocal operators. In the present work we take into account the contributions coming only from two- and three-particle nonlocal operators. Indeed, taking higher Fock-space component into account demands simultaneous calculations of the corrections with conformal spin $j = 5$ to both two- and three-particle distribution amplitudes. The contributions of higher conformal spin terms should be small. For this reason, neglecting contributions of the four-particle nonlocal operators is justified on the basis of an expansion in conformal spin (for more details see [12]). The long distance contributions are taken into account by introducing the matrix elements of the two- or three-particle nonlocal operators between the vacuum and one particle states. These matrix elements are determined with the help of the photon distribution amplitudes (DAs). The parametrization of the aforementioned matrix elements in terms of the photon DAs are given as,

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \bar{q}q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\
-\frac{i}{2(qx)} e_q \langle \bar{q}q \rangle &\left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \epsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u)
\end{aligned}$$

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -i e_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -i e_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q}q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&\quad - \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&\quad - \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&\quad \left. + \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&\quad + \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&\quad - \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&\quad - \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&\quad \left. + \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&\quad + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
&\quad \left. + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \right\} , \tag{8}
\end{aligned}$$

where $\varphi_\gamma(u)$ is the leading twist-2, $\psi^v(u)$, $\psi^a(u)$, \mathcal{A} and \mathcal{V} are the twist-3, and $h_\gamma(u)$, \mathbb{A} , \mathcal{T}_i ($i = 1, 2, 3, 4$) are the twist-4 photon DAs, and χ is the magnetic susceptibility. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g) .$$

Calculating the correlation function from the QCD side, and separating the coefficients of the Lorentz structures $(p\varepsilon) \not{p} \not{q}$, $(p\varepsilon) \not{p}$, $(p\varepsilon) \not{q}$, and $(p\varepsilon)I$ from both hadronic and QCD side, we get the following four equations for determination of the form factor f_2^- ,

$$\begin{aligned}
-A + B + C + D &= \Pi_1^{th} , \\
-m_{2+}(A - D) - m_{2-}(B + C) &= \Pi_2^{th} , \\
-(m_{1+} + m_{2+})A - (m_{1-} + m_{2-})B &= \Pi_3^{th} ,
\end{aligned}$$

$$\begin{aligned}
& +(m_{1+} - m_{2-})C - (m_{1-} - m_{2+})D = \Pi_3^{th} , \\
& -m_{2+}(m_{1+} + m_{2+})A + m_{2-}(m_{1-} + m_{2-})B \\
& -m_{2-}(m_{1+} - m_{2-})C - m_{2+}(m_{1-} - m_{2+})D = \Pi_4^{th} .
\end{aligned} \tag{9}$$

Solving Eq. (9) for the form factor f_2^- , and performing the Borel transformation over the variables $-p^2$ and $-(p+q)^2$ using

$$\mathcal{B}\left\{\frac{1}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)}\right\} \rightarrow e^{-m_1^2/M_1^2 - m_2^2/M_2^2} ,$$

in order to suppress higher states and continuum contributions, we finally get the following sum rules for the form factor f_2^- at the point $-q^2 = Q^2 = 0$,

$$\begin{aligned}
f_2^-(Q^2 = 0) = & \frac{m_{1-} + m_{2-}}{2\lambda_{1-}\lambda_{2-}} e^{-m_1^2/M_1^2 - m_2^2/M_2^2} \left\{ \frac{1}{(m_{1-} + m_{1+})(m_{2-} + m_{2+})} \left[(m_{1+} - m_{2-})m_{2+}\Pi_1^B \right. \right. \\
& \left. \left. - m_{2+}\Pi_2^B - (m_{1+} - m_{2-})\Pi_3^B - \Pi_4^B \right] \right\} + \int ds_1 ds_2 \rho^h(s_1, s_2) e^{-s_1/M_1^2 - s_2/M_2^2} . \tag{10}
\end{aligned}$$

The last term in Eq. (10) represents the contributions of the higher states, as well as continuum. This contribution is usually estimated by using the quark–hadron duality ansatz, which states that above some threshold in the (s_1, s_2) plane the hadronic spectral density $\rho^h(s_1, s_2)$ is equal to the spectral density calculated from the QCD side.

The explicit expressions of Π_i^B for the $\Sigma_Q \rightarrow \Lambda_Q \gamma$ and $\Xi'_Q \rightarrow \Xi_Q \gamma$ transitions are presented in Appendix.

Few words about the continuum subtraction procedure are in order. This procedure is explained in detail in [13], where use has been made of the quark–hadron duality. In the case $M_1^2 = M_2^2 = 2M^2$, and $u_0 = 1/2$ the subtraction can be carried out with the help of the formula,

$$M^{2n} e^{-m^2/M^2} \rightarrow \frac{1}{\Gamma(n)} \int_{m^2}^{s_0} ds e^{-s/M^2} (s - m^2)^{n-1} , \quad (n \geq 1) ,$$

from which for the leading twist terms we have,

$$M^2 e^{-m^2/M^2} \rightarrow M^2 \left(e^{-m^2/M^2} - e^{-s_0/M^2} \right) ,$$

where m is the heavy quark mass.

In the present work the subtraction procedure is not performed for the higher order twist terms which are proportional to the zeroth or negative powers of M^2 , since non of these contributions are small (see for example [13]). It should be emphasized here that, in principle, single and double dispersion integrals coming from subtraction procedure can appear. But such terms disappear after the double Borel transformations.

The residues of the negative parity are calculated in [14]. In the transitions under consideration, the masses of the initial and final state heavy baryons are very close to each other, and hence we set $M_1^2 = M_2^2 = 2M^2$.

After carrying out the numerical analysis for the form factor f_2^- the decay widths of the transitions under consideration can easily be calculated, whose expression is given as,

$$\Gamma(B_{1-} \rightarrow B_{2-}\gamma) = \frac{4\alpha|\vec{q}|^3}{m_{1-} + m_{2-}} |f_2^-(0)|^2, \quad (11)$$

where α is the fine structure constant, and

$$|\vec{q}| = \frac{m_{1-}^2 - m_{2-}^2}{2m_{1-}},$$

is the magnitude of the photon momentum.

3 Numerical analysis

Present section is devoted to the numerical analysis of the sum rules for the form factor $f_2^-(0)$. The main nonperturbative input parameters in the sum rules are the photon DAs, whose expressions are given in [9]. In addition to the photon DAs the sum rules contain other input parameters such as, the quark condensate $\langle\bar{q}q\rangle$, vacuum expectation value of the dimension-5 operator $m_0^2\langle\bar{q}q\rangle$, magnetic susceptibility χ of the quark fields. In our analysis we shall use the following values these parameters: $\langle\bar{u}u\rangle|_{1\text{ GeV}} = \langle\bar{d}d\rangle|_{1\text{ GeV}} = -(0.243)^3\text{ GeV}^3$, $\langle\bar{s}s\rangle|_{1\text{ GeV}} = 0.8\langle\bar{u}u\rangle|_{1\text{ GeV}}$, $m_0^2 = (0.8 \pm 0.2)\text{ GeV}^2$ (the value of m_0^2 is determined from the analysis of the two point sum rules for baryons, as well as from the B, B^* system) [15–17], $f_{3\gamma} = -0.0039\text{ GeV}^2$ [9]. The magnetic susceptibility is calculated in numerous works [18–20], and in our numerical calculations we use $\chi(1\text{ GeV}) = -0.2.85\text{ GeV}^{-2}$.

Having determined the input parameters we can now perform the numerical analysis of the sum rule for the form factor $f_2^-(0)$. The sum rule includes three auxiliary parameters, namely, the continuum threshold s_0 , Borel mass parameter M^2 , and the arbitrary auxiliary parameter β in the expression of the interpolating currents. Obviously, the measurable quantity $f_2^-(0)$ should be independent of these parameters. Therefore we need to find such regions for all these parameters where the form factor $f_2^-(0)$ is practically independent of them. This program can be implemented in the following way. As far as the continuum threshold is concerned, analysis of various sum rules shows that the difference $\sqrt{s_0} - m$, where m is the ground state mass, varies in the region $0.3\text{ GeV} \leq \sqrt{s_0} - m \leq 0.8\text{ GeV}$, and in our calculations we use the average value $\sqrt{s_0} - m = 0.5\text{ GeV}$. In determination of the domain for the Borel mass parameter M^2 , we demand that the following two conditions must be satisfied: a) The upper bound is obtained by requiring that the higher states and continuum contributions constitute at most 40% of the contributions coming from the perturbative parts. The lower bound is determined by imposing the requirement that the higher twist contributions are less than the leading twist contributions. The analysis of the sum rules for the negative parity heavy baryons which is studied in [14], leads to the following working regions for M^2 ,

$$\begin{aligned} 2.5\text{ GeV}^2 \leq M^2 \leq 4.0\text{ GeV}^2, & \text{ for } \Sigma_c, \Xi'_c, \Lambda_c, \Xi_c, \\ 4.5\text{ GeV}^2 \leq M^2 \leq 7.0\text{ GeV}^2, & \text{ for } \Sigma_b, \Xi'_b, \Lambda_b, \Xi_b. \end{aligned}$$

Finally, the domain for the auxiliary parameter β is determined by requesting that the form factor $f_2^-(0)$ shows good stability with respect to its variation in this region. As an example, in Figs. (1) and (2) we present the dependence of the form factor $f_2^-(0)$ for the $\Sigma_b \rightarrow \Lambda_b$ transition, on $\cos \theta$, where $\beta = \tan \theta$, at $s_0 = 42 \text{ GeV}^2$ and $s_0 = 44 \text{ GeV}^2$, and at several fixed values of M^2 . We observe from these figures that, in the region $-1.0 \leq \cos \theta \leq -0.7$, the form factor $f_2^-(0)$ seems to be practically independent to the variation in $\cos \theta$, and also it is insensitive to the different choices of the values of s_0 and M^2 . Performing similar analysis for all remaining transition channels, we get the following values for the form factor $f_2^-(0)$ which are summarized in the following table:

$$f_2^-(0) = \begin{cases} (1.2 \pm 0.3) & \text{for } \Sigma_c^+ \rightarrow \Lambda_c^+ \gamma, \\ (0.8 \pm 0.2) & \text{for } \Xi_c'^+ \rightarrow \Xi_c^+ \gamma, \\ (-0.030 \pm 0.008) & \text{for } \Xi_c'^0 \rightarrow \Xi_c^0 \gamma, \\ (1.4 \pm 0.2) & \text{for } \Sigma_b^0 \rightarrow \Lambda_b^0 \gamma, \\ (1.2 \pm 0.2) & \text{for } \Xi_b'^0 \rightarrow \Xi_b^0 \gamma, \\ (-0.18 \pm 0.03) & \text{for } \Xi_b'^- \rightarrow \Xi_b^- \gamma. \end{cases}$$

The uncertainties coming from the errors of input parameters are taken into account quadratically.

Using these values of the form factors $f_2^-(0)$, and Eq. (11), we estimate the decay widths of the decays under consideration whose values are given as,

$$\begin{aligned} \Gamma_{\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma} &= 1.2 \times (1.0 \pm 0.5) \text{ keV}, \\ \Gamma_{\Xi_c'^+ \rightarrow \Xi_c^+ \gamma} &= 0.7 \times (1.0 \pm 0.5) \text{ keV}, \\ \Gamma_{\Xi_c'^0 \rightarrow \Xi_c^0 \gamma} &= 0.002 \times (1.00 \pm 0.5) \text{ keV}, \\ \Gamma_{\Sigma_b^0 \rightarrow \Lambda_b^0 \gamma} &= 1.4 \times (1.0 \pm 0.3) \text{ keV}, \\ \Gamma_{\Xi_b'^0 \rightarrow \Xi_b^0 \gamma} &= 0.6 \times (1.0 \pm 0.3) \text{ keV}, \\ \Gamma_{\Xi_b'^- \rightarrow \Xi_b^- \gamma} &= 0.018 \times (1.0 \pm 0.3) \text{ keV}, \end{aligned}$$

The predictions presented for the transition form factors, as well as decay widths constitute the main objective of the present work. It follows from these results that the decay widths of $\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma$, $\Sigma_b^0 \rightarrow \Lambda_b^0 \gamma$, $\Xi_c'^+ \rightarrow \Xi_c^+ \gamma$, and $\Xi_b'^0 \rightarrow \Xi_b^0 \gamma$ transitions are quite large and can be measured in the near future; while the widths of $\Xi_c'^0 \rightarrow \Xi_c^0 \gamma$, and $\Xi_b'^- \rightarrow \Xi_b^- \gamma$ transitions are very small.

In conclusion, we employ light cone QCD sum rules in calculating the form factor $f_2^-(0)$ for the magnetic-dipole transition $M1$ between the negative parity heavy spin-1/2 baryons. Using the values of the form factors $f_2^-(0)$ for the transitions under consideration, we also estimate their decay widths. Our results predict that $\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma$, $\Sigma_b^0 \rightarrow \Lambda_b^0 \gamma$, $\Xi_c'^0 \rightarrow \Xi_c^0 \gamma$, and $\Xi_c'^+ \rightarrow \Xi_c^+ \gamma$ decays have large widths and can be measured in future experiments.

Appendix

1) Coefficient of the $(p\varepsilon)\not{p}\not{q}$ structure

$$\begin{aligned}
& \Pi_1^B = \\
& - \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}M^{10}}(1-\beta)f_{3\gamma}\langle g_s^2 G^2 \rangle m_0^2 m_b^4 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+\beta)\tilde{j}_1(\psi^v) + \beta\psi^a(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{3456\sqrt{3}M^8}(1-\beta)f_{3\gamma}\langle g_s^2 G^2 \rangle m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+\beta)\tilde{j}_1(\psi^v) + \beta\psi^a(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}\pi^2 M^6}(1-\beta)\langle g_s^2 G^2 \rangle m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left\{ 3\beta m_0^2 \right. \\
& + 8f_{3\gamma}\pi^2 \left[2(3+\beta)\tilde{j}_1(\psi^v) + \beta\psi^a(u_0) \right] \left. \right\} \\
& + \frac{e^{-m_b^2/M^2}}{96\sqrt{3}M^4}(1-\beta)f_{3\gamma}m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+\beta)\tilde{j}_1(\psi^v) + \beta\psi^a(u_0) \right] \\
& - \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}\pi^2 M^2}(1-\beta)\langle g_s^2 G^2 \rangle (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ (-1+\beta)\mathbb{A}(u_0) - 3(1+\beta)i_2(\mathcal{S}, 1) \right. \\
& - 3i_2(\tilde{\mathcal{S}}, 1) - 2i_2(\mathcal{T}_1, 1) - 3i_2(\mathcal{T}_2, 1) + 2i_2(\mathcal{T}_3, 1) + 3i_2(\mathcal{T}_4, 1) + 6i_2(\mathcal{S}, v) + 2i_2(\tilde{\mathcal{S}}, v) \\
& + 4i_2(\mathcal{T}_2, v) - 4i_2(\mathcal{T}_3, v) + \beta \left[-3i_2(\tilde{\mathcal{S}}, 1) + 2i_2(\mathcal{T}_1, 1) - 3i_2(\mathcal{T}_2, 1) - 2i_2(\mathcal{T}_3, 1) \right. \\
& + 3i_2(\mathcal{T}_4, 1) + 2i_2(\mathcal{S}, v) + 6i_2(\tilde{\mathcal{S}}, v) + 4i_2(\mathcal{T}_3, v) - 4i_2(\mathcal{T}_4, v) - 8\tilde{j}_2(h_\gamma) \left. \right] - 16\tilde{j}_2(h_\gamma) \left. \right\} \\
& - \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}\pi^2 M^2}(1-\beta)(e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left\{ \beta \langle g_s^2 G^2 \rangle - 2f_{3\gamma}m_0^2\pi^2 \left[2(11+5\beta)\tilde{j}_1(\psi^v) \right. \right. \\
& + (2+5\beta)\psi^a(u_0) \left. \left. \right] \right\} \\
& + \frac{1}{64\sqrt{3}\pi^2}(1-\beta)(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left[(1-\beta)\mathcal{I}_0 - (5+\beta)m_b^2\mathcal{I}_1 + 2(2+\beta)m_b^4\mathcal{I}_2 \right] \left[i_2(\mathcal{S}, 1) - i_2(\mathcal{T}_4, 1) \right] \\
& - \frac{1}{64\sqrt{3}\pi^2}(1-\beta)(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left[(1-\beta)\mathcal{I}_0 + (1+5\beta)m_b^2\mathcal{I}_1 - 2(1+2\beta)m_b^4\mathcal{I}_2 \right] \left[i_2(\tilde{\mathcal{S}}, 1) + i_2(\mathcal{T}_2, 1) \right] \\
& - \frac{1}{32\sqrt{3}\pi^2}(1-\beta)(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ (1-\beta)m_b^2(\mathcal{I}_1 - m_b^2\mathcal{I}_2)i_2(\mathcal{T}_1, 1) - (1-\beta)(\mathcal{I}_0 - m_b^2\mathcal{I}_1)i_2(\mathcal{T}_3, 1) \right. \\
& - m_b^2(\mathcal{I}_1 - m_b^2\mathcal{I}_2) \left[(3+\beta)i_2(\mathcal{S}, v) + (1+3\beta)i_2(\tilde{\mathcal{S}}, v) \right] + 2(1-\beta)(\mathcal{I}_0 - m_b^2\mathcal{I}_1)i_2(\mathcal{T}_3, v) \left. \right\} \\
& + \frac{1}{32\sqrt{3}\pi^2}(1-\beta)(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ \left[(1-\beta)\mathcal{I}_0 + 2\beta m_b^2\mathcal{I}_1 - (1+\beta)m_b^4\mathcal{I}_2 \right] i_2(\mathcal{T}_2, v) \right. \\
& + \left. \left[(1-\beta)\mathcal{I}_0 - 2m_b^2\mathcal{I}_1 + (1+\beta)m_b^4\mathcal{I}_2 \right] i_2(\mathcal{T}_4, v) \right\} \\
& + \frac{3}{128\sqrt{3}\pi^4}\sqrt{3}(1-\beta^2)(e_s - e_u)m_b^3\mathcal{I}_{\ell n} \\
& + \frac{1}{384\sqrt{3}\pi^4}(1-\beta)\mathcal{I}_{-1} \left[3(1+\beta)(e_s - e_u)m_b + 4(1-\beta)\pi^2(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle)\chi\varphi_\gamma(u_0) \right] \\
& - \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2 m_b}(1-\beta) \left\{ 36\beta m_0^2 m_b (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) + 2(3+\beta)f_{3\gamma} \left[(e_s - e_u)\langle g_s^2 G^2 \rangle \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 96m_b\pi^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\tilde{j}_1(\psi^v) + e_uf_{3\gamma}\left[(1+\beta)\langle g_s^2G^2\rangle + 96\beta m_b\pi^2\langle\bar{s}s\rangle\right] \\
& - e_sf_{3\gamma}\left[(1+\beta)\langle g_s^2G^2\rangle + 96\beta m_b\pi^2\langle\bar{u}u\rangle\right]\psi^a(u_0)\Big\} \\
& - \frac{1}{512\sqrt{3}\pi^4}(1-\beta)m_b\mathcal{I}_1\left\{(1+\beta)(e_s - e_u)(\langle g_s^2G^2\rangle + 12m_b^4) - 32e_b m_b\pi^2(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\right. \\
& + 16m_b^2\pi^2\left[2(3+\beta)(e_s - e_uf_{3\gamma}\tilde{j}_1(\psi^v) - (1-\beta)m_b(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\chi\varphi_\gamma(u_0)\right. \\
& \left. - (1+\beta)(e_s - e_uf_{3\gamma}\psi^a(u_0))\right]\Big\} \\
& + \frac{1}{1536\sqrt{3}\pi^4m_b}(1-\beta)\mathcal{I}_0\left\{(1+\beta)(e_s - e_u)(\langle g_s^2G^2\rangle + 18m_b^4) + 48\beta e_uf_{3\gamma}\pi^2\langle\bar{s}s\rangle\right. \\
& - 48e_b m_b\pi^2(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle) - 48\beta e_s m_b\pi^2\langle\bar{u}u\rangle - 12\pi^2m_b\left[(1-\beta)(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\mathbb{A}(u_0)\right. \\
& - 4(3+\beta)(e_s - e_uf_{3\gamma}m_b\tilde{j}_1(\psi^v) + 4(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)(2(2+\beta)\tilde{j}_2(h_\gamma) \\
& \left. + (1-\beta)m_b^2\chi\varphi_\gamma(u_0)) + 2(1+\beta)(e_s - e_uf_{3\gamma}m_b\psi^a(u_0))\right]\Big\} \\
& - \frac{1}{2304\sqrt{3}\pi^4}(1-\beta)m_b\mathcal{I}_2\left\{3m_b\left[-(1+\beta)(e_s - e_uf_{3\gamma}m_b(\langle g_s^2G^2\rangle + 3m_b^4)\right.\right. \\
& + 2\pi^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left((2-\beta)m_0^2 + 12\beta m_b^2) - 2\pi^2e_b\left((2-\beta)m_0^2 - 12m_b^2\right)(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\right] \\
& + \pi^2\left[-18(1-\beta)m_b^3(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\mathbb{A}(u_0) - 6(3+\beta)(e_s - e_uf_{3\gamma}(\langle g_s^2G^2\rangle\right. \\
& + 12m_b^4)\tilde{j}_1(\psi^v) + 2m_b(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\left(-72(2+\beta)m_b^2\tilde{j}_2(h_\gamma)\right. \\
& \left. \left. + (1-\beta)(\langle g_s^2G^2\rangle + 12m_b^4)\chi\varphi_\gamma(u_0)\right) + 3(1+\beta)(e_s - e_uf_{3\gamma}(\langle g_s^2G^2\rangle + 12m_b^4)\psi^a(u_0))\right]\Big\}.
\end{aligned}$$

(1)

2) Coefficient of the $(p\varepsilon)\not{p}$ structure

$$\begin{aligned}
& \Pi_2^B = \\
& - \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2M^4}(2-\beta-\beta^2)\langle g_s^2G^2\rangle m_0^2 m_b(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}\pi^2M^2}(2-\beta-\beta^2)\langle g_s^2G^2\rangle m_0^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \\
& + \frac{e^{-m_b^2/M^2}}{576\sqrt{3}\pi^2m_b}(1-\beta)(2+\beta)\langle g_s^2G^2\rangle(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \\
& - \frac{1}{16\sqrt{3}\pi^2}(1-\beta)(2+\beta)m_b[(e_b + e_u)\langle\bar{s}s\rangle - (e_b + e_s)\langle\bar{u}u\rangle]\mathcal{I}_0 \\
& - \frac{1}{64\sqrt{3}\pi^2}(1-\beta)m_b\left\{\left[3(1+\beta)e_uf_{3\gamma}m_0^2 - 8(2+\beta)(e_b + e_uf_{3\gamma}m_b^2)\right]\langle\bar{s}s\rangle\right. \\
& \left. - \left[3(1+\beta)e_s m_0^2 - 8(2+\beta)(e_b + e_s)m_b^2\right]\langle\bar{u}u\rangle\right\}\mathcal{I}_1
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{192\sqrt{3}\pi^2}(1-\beta)m_b \left\{ \left[(2+\beta)e_u \langle g_s^2 G^2 \rangle - 3 \left((3+2\beta)e_b + (7+5\beta)e_u \right) m_0^2 m_b^2 \right. \right. \\
& + 12(2+\beta)(e_b + e_u)m_b^4 \left. \left. \langle \bar{s}s \rangle - \left[(2+\beta)e_s \langle g_s^2 G^2 \rangle - 3 \left((3+2\beta)e_b + (7+5\beta)e_s \right) m_0^2 m_b^2 \right. \right. \right. \\
& \left. \left. \left. + 12(2+\beta)(e_b + e_s)m_b^4 \right] \langle \bar{u}u \rangle \right\} \mathcal{I}_2 .
\end{aligned} \tag{2}$$

3) Coefficient of the $(p\varepsilon)\not{q}$ structure

$$\begin{aligned}
\Pi_3^B = & \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}M^{10}}(1-\beta)f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^5 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[4(2+\beta)\tilde{j}_1(\psi^v) - (1+2\beta)\psi^a(u_0) \right] \\
& - \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}M^8}(1-\beta)f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^3 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[4(2+\beta)\tilde{j}_1(\psi^v) - (1+2\beta)\psi^a(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}\pi^2 M^6}(1-\beta) \langle g_s^2 G^2 \rangle m_b (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left\{ 3(1+2\beta)m_0^2 m_b^2 \right. \\
& \left. + 4f_{3\gamma}(3m_0^2 - 2m_b^2)\pi^2 \left[4(2+\beta)\tilde{j}_1(\psi^v) - (1+2\beta)\psi^a(u_0) \right] \right\} \\
& - \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}\pi^2 M^4}(1-\beta)m_b (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left\{ (5+7\beta) \langle g_s^2 G^2 \rangle m_0^2 \right. \\
& \left. - 8f_{3\gamma}(\langle g_s^2 G^2 \rangle - 6m_0^2 m_b^2)\pi^2 \left[4(2+\beta)\tilde{j}_1(\psi^v) - \psi^a(u_0) - 2\beta\psi^a(u_0) \right] \right\} \\
& - \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}\pi^2 M^2}(1-\beta) \langle g_s^2 G^2 \rangle m_b (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ 3(1+\beta)\mathbb{A}(u_0) + 4(1-\beta)i_2(\mathcal{S}, 1) \right. \\
& + 4(1-\beta)i_2(\tilde{\mathcal{S}}, 1) + 6i_2(\mathcal{T}_1, 1) + 4i_2(\mathcal{T}_2, 1) - 6i_2(\mathcal{T}_3, 1) - 4i_2(\mathcal{T}_4, 1) - 4i_2(\tilde{\mathcal{S}}, v) - 12i_2(\mathcal{T}_2, v) \\
& + 12i_2(\mathcal{T}_3, v) + 2\beta \left[3i_2(\mathcal{T}_1, 1) - 2i_2(\mathcal{T}_2, 1) - 3i_2(\mathcal{T}_3, 1) + 2i_2(\mathcal{T}_4, 1) + 2i_2(\tilde{\mathcal{S}}, v) - 2i_2(\mathcal{T}_2, v) \right. \\
& \left. + 6i_2(\mathcal{T}_3, v) - 4i_2(\mathcal{T}_4, v) \right] \left. \right\} \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2 m_b M^2} \left\{ 2(2-\beta-\beta^2) \langle g_s^2 G^2 \rangle m_b^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) i_2(\mathcal{S}, v) \right. \\
& + (1-\beta) \left[144(1+\beta)f_{3\gamma} m_0^2 m_b^2 \pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \tilde{j}_1(\psi^v) \right. \\
& - 2(3+\beta) \langle g_s^2 G^2 \rangle m_b^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_2(h_\gamma) + (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left((2+\beta) \langle g_s^2 G^2 \rangle m_0^2 \right. \\
& \left. \left. - 2(1+2\beta) \langle g_s^2 G^2 \rangle m_b^2 - 36(1+\beta)f_{3\gamma} m_0^2 m_b^2 \pi^2 \psi^a(u_0) \right) \right] \left. \right\} \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}\pi^2 m_b}(1-\beta)(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left[(1-\beta) \langle g_s^2 G^2 \rangle + 18(7-\beta)m_b^2 e^{m_b^2/M^2} (\mathcal{I}_0 - m_b^2 \mathcal{I}_1) \right. \\
& \left. - 18(5+\beta)m_b^2 e^{m_b^2/M^2} \mathcal{I}_{ln} \right] \left[i_2(\mathcal{S}, 1) - i_2(\mathcal{T}_4, 1) \right] \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}\pi^2 m_b}(1-\beta)(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left[(1-\beta) \langle g_s^2 G^2 \rangle + 18(1-7\beta)m_b^2 e^{m_b^2/M^2} (\mathcal{I}_0 - m_b^2 \mathcal{I}_1) \right.
\end{aligned}$$

$$\begin{aligned}
& + 18(1 + 5\beta)m_b^2 e^{m_b^2/M^2} \mathcal{I}_{\ell n} \left[i_2(\tilde{\mathcal{S}}, 1) + i_2(\mathcal{T}_2, 1) \right] \\
& + \frac{e^{-m_b^2/M^2}}{768\sqrt{3}\pi^2 m_b} (1 - \beta^2) (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ \left[\langle g_s^2 G^2 \rangle + 72m_b^2 e^{m_b^2/M^2} (\mathcal{I}_0 - m_b^2 \mathcal{I}_1) \right. \right. \\
& - 36m_b^2 e^{m_b^2/M^2} \mathcal{I}_{\ell n} \left. \right] i_2(\mathcal{T}_1, 1) - \left[\langle g_s^2 G^2 \rangle + 36m_b^2 e^{m_b^2/M^2} \mathcal{I}_{\ell n} \right] i_2(\mathcal{T}_3, 1) \left. \right\} \\
& - \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}\pi^2 m_b} (1 - \beta) (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left[\langle g_s^2 G^2 \rangle + 36m_b^2 e^{m_b^2/M^2} (\mathcal{I}_0 - m_b^2 \mathcal{I}_1) \right] \\
& \times \left\{ 2(2 + \beta) i_2(\mathcal{S}, v) + i_2(\tilde{\mathcal{S}}, v) + 3i_2(\mathcal{T}_2, v) - 3i_2(\mathcal{T}_3, v) - \beta \left[i_2(\tilde{\mathcal{S}}, v) - i_2(\mathcal{T}_2, v) \right. \right. \\
& + 3i_2(\mathcal{T}_3, v) - 2i_2(\mathcal{T}_4, v) \left. \right] \left. \right\} \\
& - \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}\pi^2} (e_s - e_u) f_{3\gamma} \left\{ (1 - \beta)^2 \langle g_s^2 G^2 \rangle - 72e^{m_b^2/M^2} \left[2(1 + \beta + \beta^2) \mathcal{I}_{-1} - (1 - \beta)^2 m_b^2 \mathcal{I}_0 \right. \right. \\
& - (1 + 4\beta + \beta^2) m_b^4 \mathcal{I}_1 \left. \right] - 216(1 + \beta)^2 m_b^2 e^{m_b^2/M^2} \mathcal{I}_{\ell n} \left. \right\} i_3(\mathcal{V}, v) \\
& + \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}\pi^2} (e_s - e_u) f_{3\gamma} \left\{ (1 - \beta)^2 \langle g_s^2 G^2 \rangle + 72e^{m_b^2/M^2} \left[(1 + 4\beta + \beta^2) \mathcal{I}_{-1} + (1 - \beta)^2 m_b^2 \mathcal{I}_0 \right. \right. \\
& - 2(1 + \beta + \beta^2) m_b^4 \mathcal{I}_1 \left. \right] + 216(1 + \beta)^2 m_b^2 e^{m_b^2/M^2} \mathcal{I}_{\ell n} \left. \right\} i_3(\mathcal{A}, v) \\
& - \frac{1}{768\sqrt{3}\pi^4 m_b^2} (1 + \beta + \beta^2) (e_s - e_u) \left\{ 12\mathcal{I}_{-3} - m_b^2 \left[48\mathcal{I}_{-2} - 72m_b^2 \mathcal{I}_{-1} + (\langle g_s^2 G^2 \rangle + 48m_b^4) \mathcal{I}_0 \right] \right\} \\
& - \frac{1}{32\sqrt{3}\pi^2} (1 - \beta) m_b \left[(7 + 2\beta) e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) - \beta (3e_u \langle \bar{s}s \rangle - 3e_s \langle \bar{u}u \rangle) \right] \mathcal{I}_0 \\
& + \frac{1}{128\pi^2} \sqrt{3} (1 - \beta^2) m_b ((e_s \langle \bar{s}s \rangle) - e_u \langle \bar{u}u \rangle) \mathbb{A}(u_0) \mathcal{I}_0 \\
& + \frac{1}{768\sqrt{3}\pi^4} m_b \left\{ - (1 + \beta + \beta^2) (e_s - e_u) m_b (\langle g_s^2 G^2 \rangle + 12m_b^4) \right. \\
& - 3(1 - \beta) e_b [(5 + \beta) m_0^2 - 24(3 + \beta) m_b^2] \pi^2 (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \\
& + 48(1 - \beta)^2 m_b^2 \pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) - 18(1 - \beta^2) m_b^2 \pi^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathbb{A}(u_0) \left. \right\} \mathcal{I}_1 \\
& - \frac{1}{384\sqrt{3}\pi^2} (1 - \beta) m_b \left\{ \left[(2 + \beta) e_u \langle g_s^2 G^2 \rangle - 3m_0^2 m_b^2 (3 + 2\beta) e_b - 3m_0^2 m_b^2 (7 + 5\beta) e_u \right. \right. \\
& + 12(2 + \beta) (e_b + e_u) m_b^4 \left. \right] \langle \bar{s}s \rangle - \left[(2 + \beta) e_s \langle g_s^2 G^2 \rangle - 3m_0^2 m_b^2 (3 + 2\beta) e_b - 3m_0^2 m_b^2 (7 + 5\beta) e_s \right. \\
& + 12(2 + \beta) (e_b + e_s) m_b^4 \left. \right] \langle \bar{u}u \rangle \left. \right\} \mathcal{I}_2 \\
& + \frac{1}{384\sqrt{3}\pi^2} (1 - \beta) (5 + \beta) e_b m_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \mathcal{I}_{\ell n} \\
& + \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}\pi^4 m_b^2} \left\{ 4(1 - \beta) m_b \pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[(4 + 5\beta) \langle g_s^2 G^2 \rangle - 18(1 + 2\beta) m_0^2 m_b^2 \right] \right. \\
& + 3(1 - \beta^2) \langle g_s^2 G^2 \rangle m_b \pi^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathbb{A}(u_0) \\
& \left. + 36e^{m_b^2/M^2} (1 + \beta + \beta^2) (e_s - e_u) \left[2\mathcal{I}_{-3} - 9m_b^2 \mathcal{I}_{-2} + 18m_b^4 \mathcal{I}_{-1} - 11m_b^6 \mathcal{I}_0 + 6m_b^6 \mathcal{I}_{\ell n} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{e^{-m_b^2/M^2}}{6\sqrt{3}} f_{3\gamma} m_b (2 - \beta - \beta^2) (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \tilde{j}_1(\psi^v) \\
& + \frac{e^{-m_b^2/M^2}}{576\sqrt{3}\pi^2 m_b} (1 - \beta)(3 + \beta) (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left[\langle g_s^2 G^2 \rangle + 36e^{m_b^2/M^2} m_b^2 (\mathcal{I}_0 - m_b^2 \mathcal{I}_1) \right] \tilde{j}_2(h_\gamma) \\
& - \frac{1}{768\sqrt{3}\pi^2 m_b} (1 - \beta^2) (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi \left\{ 2 \langle g_s^2 G^2 \rangle \mathcal{I}_0 + 3m_b^2 \left[12\mathcal{I}_{-1} - (\langle g_s^2 G^2 \rangle + 12m_b^4) \mathcal{I}_1 \right] \right. \\
& + \left. 72m_b^4 \mathcal{I}_{\ell n} \right\} \varphi_\gamma(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{24\sqrt{3}} f_{3\gamma} m_b (1 + \beta - 2\beta^2) (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^a(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}\pi^4} (1 + \beta + \beta^2) (e_s - e_u) f_{3\gamma} \left\{ \langle g_s^2 G^2 \rangle - 36e^{m_b^2/M^2} \left[\mathcal{I}_{-1} - 2m_b^2 \mathcal{I}_0 + m_b^4 \mathcal{I}_1 \right] \right\} \\
& \times \left[4\tilde{j}_1(\psi^v) - \psi^a(u_0) \right].
\end{aligned} \tag{3}$$

4) Coefficient of the $(p\varepsilon)I$ structure

$$\begin{aligned}
& \Pi_4^B = \\
& + \frac{e^{-m_b^2/M^2}}{3456\sqrt{3}M^8} (1 - \beta^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^4 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{1728\sqrt{3}M^6} (1 - \beta^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}\pi^2 M^4} (1 - \beta) \langle g_s^2 G^2 \rangle m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[3(2 + \beta) m_0^2 - 8(1 + \beta) f_{3\gamma} \pi^2 \psi^v(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{48\sqrt{3}M^2} (1 - \beta^2) f_{3\gamma} m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) \\
& - \frac{1}{64\sqrt{3}\pi^2 m_b^2} (1 - \beta) \left[(1 + \beta) e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) + 4(2 + \beta) (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \\
& - \left. 12(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_1(h_\gamma) \right] \mathcal{I}_{-2} \\
& + \frac{1}{16\sqrt{3}\pi^2} (1 - \beta) m_b^2 \left[(7 + 3\beta) e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) - (3 + \beta) (e_s - e_u) f_{3\gamma} m_b \psi^v(u_0) \right] \mathcal{I}_{\ell n} \\
& + \frac{1}{32\sqrt{3}\pi^2} (1 - \beta) \left[(5 + 3\beta) e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) + 6(2 + \beta) (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \\
& - \left. 18(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_1(h_\gamma) - (3 + \beta) (e_s - e_u) f_{3\gamma} m_b \psi^v(u_0) \right] \mathcal{I}_{-1} \\
& + \frac{1}{384\sqrt{3}\pi^2} (1 - \beta) m_b \left\{ 6m_b \left[\beta e_u m_0^2 + 4(2 + \beta) (e_b + e_u) m_b^2 \right] \langle \bar{s}s \rangle \right. \\
& - \left. 6m_b \left[\beta e_s m_0^2 + 4(2 + \beta) (e_b + e_s) m_b^2 \right] \langle \bar{u}u \rangle - 72m_b^3 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_1(h_\gamma) \right. \\
& + \left. (3 + \beta) (e_s - e_u) f_{3\gamma} (\langle g_s^2 G^2 \rangle + 12m_b^4) \psi^v(u_0) \right\} \mathcal{I}_1
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{576\sqrt{3}\pi^2 m_b} (1 - \beta) \left\{ 9m_b \left[e_b \left((3 + 2\beta)m_0^2 + (17 + 9\beta)m_b^2 \right) (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \right. \right. \\
& + \left. \left. \left((4 + 3\beta)m_0^2 + 12(2 + \beta)m_b^2 \right) (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) - 36m_b^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_1(h_\gamma) \right] \right. \\
& + \left. f_{3\gamma} \left[(3 + \beta)(e_s - e_u) \langle g_s^2 G^2 \rangle - 48(1 + \beta)e_u m_b \pi^2 \langle \bar{s}s \rangle + 48(1 + \beta)e_s m_b \pi^2 \langle \bar{u}u \rangle \right] \psi^v(u_0) \right\} \mathcal{I}_0 \\
& + \frac{e^{-m_b^2/M^2}}{576\sqrt{3}\pi^2 m_b^2} (1 + \beta) \left\{ 9e^{m_b^2/M^2} \left[(1 + \beta)e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) + 4(2 + \beta)(e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \right. \\
& - \left. \left. 12(e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_1(h_\gamma) \right] (\mathcal{I}_{-2} - 2m_b^2 \mathcal{I}_{-1} + m_b^4 \mathcal{I}_0) + m_b^2 \left[3 \langle g_s^2 G^2 \rangle (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}_1(h_\gamma) \right. \right. \\
& - \left. \left. (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left((2 + \beta) \langle g_s^2 G^2 \rangle - 2(1 + \beta) f_{3\gamma} m_0^2 \pi^2 \psi^v(u_0) \right) \right] \right\} .
\end{aligned}$$

The functions i_n ($n = 1, 2$), and $\tilde{j}_1(f(u))$ are defined as:

$$\begin{aligned}
i_0(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) (k - u_0) \theta(k - u_0) , \\
i_1(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \theta(k - u_0) , \\
i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta(k - u_0) , \\
i_3(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(k - u_0) , \\
i_4(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta''(k - u_0) , \\
\tilde{j}_1(f(u)) &= \int_{u_0}^1 du f(u) , \\
\tilde{j}_2(f(u)) &= \int_{u_0}^1 du (u - u_0) f(u) , \\
\mathcal{I}_n &= \int_{m_b^2}^\infty ds \frac{e^{-s/M^2}}{s^n} , \\
\mathcal{I}_{\ell n} &= \int_{m_b^2}^\infty ds e^{-s/M^2} \ell n \frac{m_b^2}{s} ,
\end{aligned}$$

where

$$k = \alpha_q + \alpha_g \bar{v} , \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} .$$

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Figure captions

Fig. (1) The dependence of the of the form factor f_2^- on $\cos\theta$ at six fixed values of the Borel mass parameter M^2 , and at the fixed value $s_0 = 42 \text{ GeV}^2$ of the continuum threshold for the $\Sigma_b^0 \rightarrow \Lambda_b \gamma$ transition.

Fig. (2) The same as Fig. (1), but at the fixed value $s_0 = 44 \text{ GeV}^2$ of the continuum threshold.

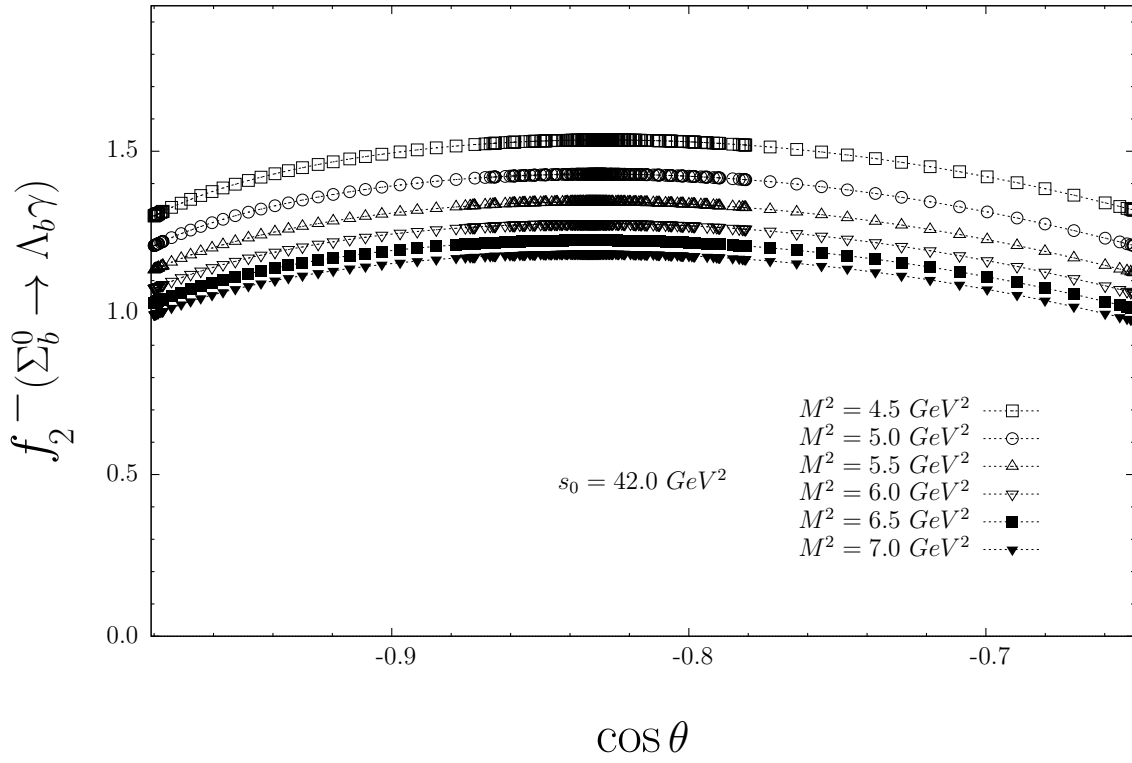


Figure 1:

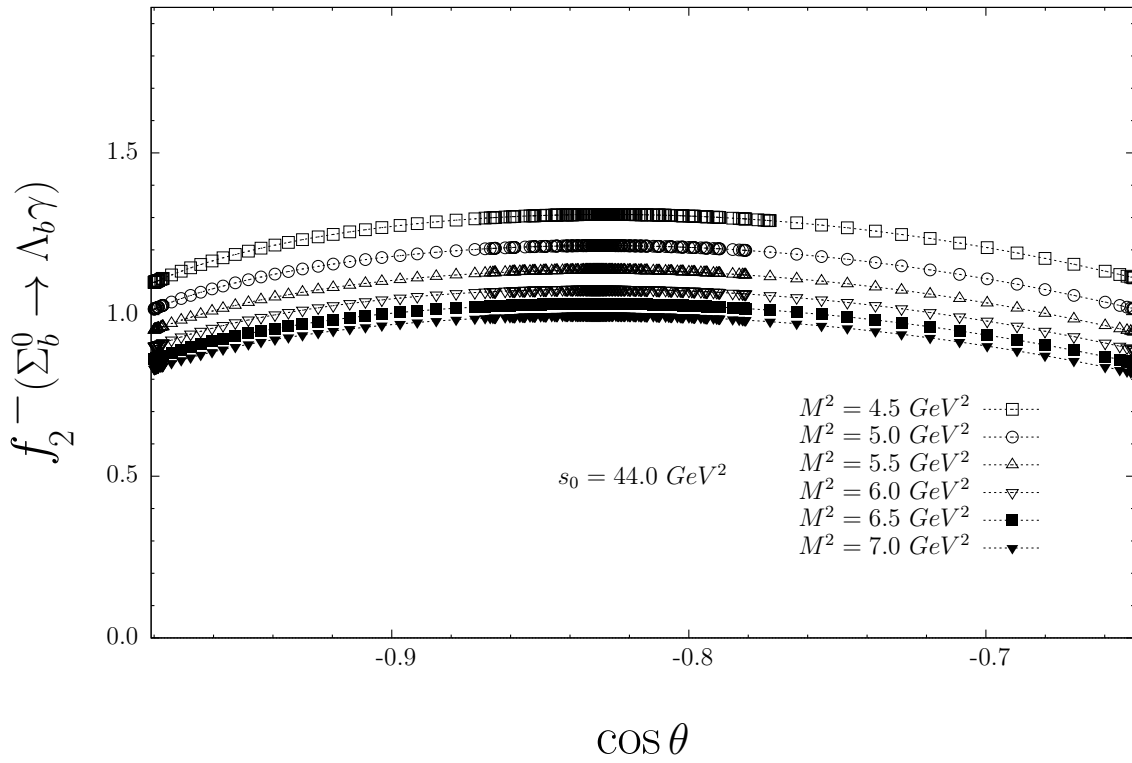


Figure 2: