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Gauge-Higgs EW and Grand Unification[1]

Yutaka Hosotani

Department of Physics, Osaka University
Toyonaka, Osaka 560-0043, Japan
hosotani@phys.sci.osaka-u.ac.jp

Abstract

4D Higgs field is identified with the extra-dimensional component of gauge potentials in the gauge-Higgs unification scenario. $SO(5) \times U(1)$ gauge-Higgs EW unification in the Randall-Sundrum warped space is successful at low energies. The Higgs field appears as an Aharonov-Bohm phase θ_H in the fifth dimension. Its mass is generated at the quantum level and is finite. The model yields almost the same phenomenology as the standard model for $\theta_H < 0.1$, and predicts Z' bosons around 6 - 10 TeV with very broad widths. The scenario is genelarized to $SO(11)$ gauge-Higgs grand unification. Fermions are introduced in the spinor and vector representations of $SO(11)$. Proton decay is naturally forbidden.

1. Introduction

We are looking for a principle for the 125 GeV Higgs boson which regulates all Higgs couplings, explains electroweak (EW) symmetry breaking, and solves the gauge-hierarchy problem. One possible answer is the gauge-Higgs unification.[1, 2, 3] One considers gauge theory in higher dimensions, say, in five dimensions. The 4D gauge fields, photon, W, and Z, appear as zero modes of the four-dimensional components of gauge potentials, whereas the 4D Higgs field is identified with the zero mode of the extra-dimensional component of gauge potentials. When the fifth dimension is compact and is not simply connected, the Higgs field appears as an Aharonov-Bohm (AB) phase, θ_H, along the fifth dimension.

At the tree level the 4D Higgs field is massless. At the quantum level the effective potential for the AB phase, $V_{\rm eff}(\theta_H)$, becomes nontrivial, and a finite Higgs mass m_H is generated. At the same time dynamical breaking of the EW symmetry takes place. This is called the Hosotani mechanism.[1] The generated Higgs mass is finite, independent of a cutoff scale and regularization method. The gauge hierarchy problem is thus solved.

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Construction of a concrete model of gauge-Higgs EW unification is highly non-trivial. The standard model (SM) has $SU(2)_L \times U(1)_Y$ gauge symmetry, the Higgs field is an $SU(2)_L$ doublet, and quarks and leptons are chiral in interactions. All these features are naturally incorporated in the $SO(5) \times U(1)_X$ gauge-Higgs unification in the Randall-Sundrum warped space.[4]-[10]

2. $SO(5) \times U(1)_X$ gauge-Higgs EW unification

Zero modes of the extra-dimensional component of gauge potentials must appear as an $SU(2)_L$ doublet, and the custodial $SO(4)$ symmetry should result in the Higgs part in four dimensions. Further quark-lepton content must appear chiral. The minimal model is $SO(5) \times U(1)_X$ gauge theory defined on an orbifold. In five dimensions spacetime is either $M^4 \times (S^1/Z_2)$ or the Randall-Sundrum (RS) warped space. Only in the RS space consistent phenomenology is obtained.

The metric of the RS space is given by

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$, $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$, and $\sigma(y) = ky$ for $0 \leq y \leq L$. Topological structure of the RS space is the same as that of $M^4 \times (S_1/Z_2)$. The points y, $-y$, and $y + 2L$ are identified. There appear two fixed points $(y_0, y_1) = (0, L)$. The RS space is an AdS space sandwiched by the Planck brane (at $y = 0$) and the TeV brane (at $y = L$) with AdS curvature $\Lambda = -6k^2$.

Only physical quantities need to be single-valued on orbifolds. $SO(5)$ gauge fields $A_M(x, y)$ obey

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) P_j^{-1} \,,$$

$$A_M(x, y + 2L) = U A_M(x, y) U^{-1} \,, \quad U = P_1 P_0 \,. \tag{2}$$

We take

$$P_0 = P_1 = P_{\mathrm{vec}} = \mathrm{diag}\,(-1, -1, -1, -1, +1) \,. \tag{3}$$

$U(1)_X$ gauge fields $B_M(x, y)$ satisfy a condition similar to (2) where $P_j = 1$. At this stage $SO(5) \times U(1)_X$ symmetry breaks down to $SO(4) \times U(1)_X$. Parity even-even modes (zero modes) appear in the $SO(4)$ block of $SO(5)$ A_μ and in B_μ, and $SO(5)/SO(4)$ part of A_y ($A_y^{j5}, j = 1, \cdots, 4$). The latter is an $SO(4)$ vector, or an $SU(2)_L$ doublet of $SO(4) \simeq SU(2)_L \times SU(2)_R$, corresponding to the 4D Higgs field in the SM. In the gauge-Higgs unification the relevant quantity is the AB phase along the fifth dimension, and is given by

$$e^{i\hat{\theta}(x)/2} = P \exp \left\{ i g_A \int_0^L dy \, A_y(x, y) \right\}. \tag{4}$$

In the bulk ($0 < y < L$) we introduce quark and lepton multiplets Ψ_a ($a = 1, \cdots, 4$) in the vector representation of $SO(5)$ in each generation, and dark fermion multiplets Ψ_{F_i} ($i = 1, \cdots, n_F$) in the spinor representation. They satisfy

$$\Psi_a(x, y_j - y) = P_{\text{vec}} \Gamma^5 \Psi_a(x, y_j + y),$$

$$\Psi_{F_i}(x, y_j - y) = \eta_{F_i}(-1)^j P_{\text{sp}} \Gamma^5 \Psi_{F_i}(x, y_j + y), \quad \eta_{F_i} = \pm 1, \tag{5}$$

where $P_{\text{sp}} = \text{diag}(+1, +1, -1, -1)$. In addition, brane fermions $\hat{\chi}_\alpha$ in the $(\frac{1}{2}, 0)$ representation and brane scalar $\hat{\Phi}$ in the $(0, \frac{1}{2})$ representation of $SU(2)_L \times SU(2)_R$ are introduced on the Planck brane. The brane scalar $\hat{\Phi}$ spontaneously breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$. At the same time, couplings on the Planck brane among $\hat{\Phi}(x)$, $\hat{\chi}_\alpha$, and $\Psi_a(x, 0)$ generate additional mass terms.

The resultant symmetry is $SU(2)_L \times U(1)_Y$, which is subsequently broken to $U(1)_{\text{EM}}$ by the Hosotani mechanism when $e^{i\hat{\theta}(x)}$ is not proportional to I. Without loss of generality one may suppose that $\langle A_y^{45} \rangle \neq 0$, whereas $\langle A_y^{j5} \rangle = 0$ for $j = 1, 2, 3$. There appears one relevant AB phase θ_H;

$$\hat{\theta}_H(x) = \theta_H + \frac{H(x)}{f_H}, \quad f_H = \frac{2}{g_A} \sqrt{\frac{k}{z_L^2 - 1}} = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}}. \tag{6}$$

Here $H(x)$ is the canonically normalized 4D neutral Higgs field and the warp factor is $z_L = e^{kL} \gg 1$.

3. Why gauge-Higgs unification?

There are good reasons for pursuing the gauge-Higgs unification.

(a) Gauge principle governs the Higgs interactions.

The 4D Higgs field is a part of the gauge potentials so that all Higgs interactions emerges as gauge interactions in five dimensions. Various couplings in four dimensions appear as overlap integrals in the fifth dimension over three or more wave functions of participating 4D fields, or as loop effects. Yukawa couplings of quarks and leptons are in the former category, whereas the cubic and quartic couplings of Higgs self-interactions are in the latter category.

(b) The finite Higgs boson mass m_H is generated at the quantum level, free from a cutoff scale. The gauge-hierarchy problem is solved.

The Higgs field is a four-dimensional fluctuation mode of the AB phase θ_H in (4) and (6). The effective potential $V_{\text{eff}}(\theta_H)$ is flat at the tree level, but becomes nontrivial at the one loop level. θ_H-dependent part of $V_{\text{eff}}(\theta_H)$ turns out finite, irrespective of the regularization method and cutoff scale. This property is guaranteed by the gauge invariance in five dimensions. When the global minimum of $V_{\text{eff}}(\theta_H)$ is located at $\theta_H \neq 0$, the gauge symmetry is partially broken. The whole scheme is called the Hosotani mechanism.[1, 11, 12, 13]

The Higgs boson mass is given by

$$m_H^2 = \frac{1}{f_H^2} \frac{d^2 V_{\text{eff}}(\theta_H)}{d\theta_H^2}\bigg|_{\text{min}}, \tag{7}$$

and is finite. It is important to recognize that even though gauge theory in five dimensions is not renormalizable, the θ_H-dependent part of $V_{\text{eff}}(\theta_H)$ is found to be finite. In the evaluation one has to take into account contributions from all KK modes. The gauge hierarchy problem is solved. This should be contrasted to the situation in 4D gauge theory in which m_H^2 receives quantum corrections of $O(\Lambda^2)$ where Λ is typically the GUT scale.

(c) No vacuum instability.

Scalar field theory in four dimensions is plagued by the vacuum instability problem by quantum corrections.[14] In the gauge-Higgs unification the effective potential for the Higgs field is given by $V_{\text{eff}}[\hat{\theta}_H(x)]$ where $\hat{\theta}_H(x)$ is given in (6). The gauge invariance implies that $V_{\text{eff}}(\theta_H + 2\pi) = V_{\text{eff}}(\theta_H)$. It follows that the global minimum is located somewhere in $0 \leq \theta_H \leq 2\pi$, or $0 \leq H \leq 2\pi f_H$, up to the periodicity. There is no runaway instability.

(d) Almost SM phenomenology at low energies.

The $SO(5) \times U(1)_X$ gauge-HIggs unification in the RS space gives desired phenomenology at low energies and at the energy of 8 TeV LHC. The SM matter content is reproduced at low energies. Deviations of gauge couplings of quarks, leptons, W and Z from those in the SM turn out very tiny.[9, 15]

The Higgs couplings at the tree level receive corrections in an universal form. All HWW, HZZ, $Hq\bar{q}$, $H\ell\bar{\ell}$ couplings are suppressed by a common factor $\cos\theta_H$ compared to those in the SM. For $\theta_H < 0.2$ the correction is less that 2%, which is perfectly consistent with all data observed.[16]

One loop corrections also have been evaluated. As explained in the next section, the gauge-Higgs unification gives definitive predictions. It will be found that the Higgs decay rates are almost the same as in the SM, as far as $\theta_H < 0.2$, even when one loop corrections are included.[10, 17]

(e) Dynamical EW symmetry breaking takes place.

Once matter content is specified, $V_{\text{eff}}(\theta_H)$ is unambiguously evaluated. It is found that the minimal set of matter content described in Section 2 leads to the dynamical breaking of the EW symmetry to $U(1)_{\text{EM}}$. In the RS space contributions of light quarks and leptons become negligible. Contributions of gauge fields, top quark multiplet, and dark fermions are relevant. The existence of the top quark, whose mass is greater than m_W, is crucial.[10]

(f) Fermion mass hierarchy is explained by $O(1)$ parameters in RS.

There is a bonus in gauge-Higgs unification formulated in the RS space. Large hierarchy in the fermion mass spectrum is explained in terms of bulk mass parameters c of fermion multiplets in the RS space.[15] The third generation quark

multiplet has $0 < c < \frac{1}{2}$ so that the top quark acquires a mass, by the Hosotani mechanism, larger than m_W. On the other hand, quark multiplets in the first and second generations and all lepton multiplets have $c > \frac{1}{2}$. The mass hierarchy is easily generated with a factor z_L^{-c} for $c > \frac{1}{2}$.

4. Predictions

At low energies it is hard to distinguish the $SO(5) \times U(1)_X$ gauge-Higgs unification scenario from the SM. One needs to derive various predictions of the gauge-Higgs unification which can be tested by experiments and observations. The gauge-Higgs unification scenario is extremely restrictive, and therefore predictive.

(a) Universality

The model contains several parameters. The metric of the RS space is specified with two parameters k and $z_L = e^{kL}$, one of which is fixed by m_Z. Two gauge couplings of $SO(5)$ and $U(1)_X$ are related to the $SU(2)_L$ coupling g_w and $\sin^2 \theta_W$. The bulk mass parameters of quark/lepton multiplets are determined by the quark/lepton spectrum, in combination with brane interaction terms. There are a few parameters in the dark fermion sector, one of them is fixed by the Higgs boson mass m_H. In particular, the number of dark fermions, n_F, is arbitrary. In the minimal model, z_L and n_F may be treated as free parameters. Once (z_L, n_F) is given, $V_{\text{eff}}(\theta_H)$ is evaluated, from which the location of its global minimum, the value of θ_H, is determined.

One of the striking results in the $SO(5) \times U(1)_X$ gauge-Higgs unification is that many of the physical quantities depend, in a very good approximation, only on θ_H. They are independent of the details in the dark fermion sector, particularly n_F. This gives strong prediction power to the model.[10]

First of all gauge couplings of the SM particles are almost the same as in the SM. Deviations are typically less than 1%, except for the couplings of t and b quarks. The μ-e universality remains almost intact. The deviation in the WWZ coupling is less than 0.1%. Three point Higgs couplings are given by [6, 16, 18]

$$g_{HWW}, \ g_{HZZ}, \ g_{Hq\bar{q}}, \ g_{H\ell\bar{\ell}} \sim (\text{SM values}) \times \cos \theta_H \ . \tag{8}$$

The KK mass scale, the masses of the first KK modes of Z and γ, the mass of $SU(2)_R$ Z_R are given by

$$m_{\text{KK}} \sim \frac{1352 \, \text{GeV}}{(\sin \theta_H)^{0.786}} \ ,$$

$$m_{Z^{(1)}} \sim \frac{1044 \, \text{GeV}}{(\sin \theta_H)^{0.808}} \ ,$$

$$m_{\gamma^{(1)}} \sim \frac{1056 \, \text{GeV}}{(\sin \theta_H)^{0.804}} \ ,$$

$$m_{Z_R^{(1)}} \sim \frac{1038 \, \text{GeV}}{(\sin \theta_H)^{0.784}} \ . \tag{9}$$

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The Higgs self-couplings arise at the one loop level. The cubic and quartic couplings are found to be

$$\lambda_3^H/\text{GeV} \sim 26.7 \cos\theta_H + 2.84 \cos^2\theta_H \ ,$$

$$\lambda_4^H \sim 0.0214 + 0.0304(\cos 2\theta_H - 1) + 0.00159(\cos 4\theta_H - 1) \ . \tag{10}$$

These numbers should be compared with $\lambda_3^H = 31.5\,\text{GeV}$ and $\lambda_4^H = 0.0320$ in the SM. λ_3^H vanishes at $\theta_H = \frac{1}{2}\pi$ due to the H parity.[19, 20] The negative λ_4^H for large θ_H does not imply the instability, as $V_{\text{eff}}(\theta_H)$ is bounded from below.

Once the value θ_H is determined, say, from the mass of $Z^{(1)}$, then all other quantities are predicted.

(b) Loop corrections of KK modes in $H \to \gamma\gamma, gg, Z\gamma$ are finite and small.

In the SM, the decay $H \to \gamma\gamma$ takes place through one-loop processes in which W and top quark t run. In higher dimensional theory KK modes of W and t also run inside the loop. Their contributions may add up to large, even diverging, corrections. The same consern applies to $H \to gg$ and $H \to Z\gamma$.

In the gauge-Higgs unification miraculous cancellation takes place among contributions of KK modes.[10, 17] The decay rate for $H \to \gamma\gamma$ is given by

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 g_w^2}{1024\pi^3} \frac{m_H^3}{m_W^2} \left| \mathcal{F}_W + \frac{4}{3}\mathcal{F}_t + n_F \mathcal{F}_F \right|^2 , \tag{11}$$

where $\mathcal{F}_W$, $\mathcal{F}_t$, $\mathcal{F}_F$ represent contributions from W, t, and dark fermion loops. $\mathcal{F}_W$ is given by

$$\mathcal{F}_W = \sum_{n=0}^{\infty} \frac{g_{HW^{(n)}W^{(n)}}}{g_w m_W} \frac{m_W^2}{m_{W^{(n)}}^2} F_1(\tau_{W^{(n)}})$$

$$= \sum_{n=0}^{\infty} I_{W^{(n)}} \frac{m_W}{m_{W^{(n)}}} \cos\theta_H F_1(\tau_{W^{(n)}}) \tag{12}$$

where $\tau_i = 4m_i^2/m_H^2$ and $F_1(\tau) \sim 7$ for large τ. $I_{W^{(n)}}$ for large n is approximately given by

$$I_{W^{(n)}} \simeq (-1)^n \big\{ 0.0759 - 0.0065 \ln n + 0.0022(\ln n)^2 \big\} \ . \tag{13}$$

Similar behavior is found for $\mathcal{F}_t$ and $\mathcal{F}_F$ as well. In other words the sum in each $\mathcal{F}$ behaves as $\sum (-1)^n (\ln n)^\alpha / n$ ($\alpha = 0, 1, 2$) and rapidly converges. Moreover the contributions from $n \geq 1$ are suppressed by the ratio of the electroweak scale to the KK scale. The ratio of the amplitude to that with only zero modes is

$$\frac{\mathcal{F}_W + \frac{4}{3}\mathcal{F}_t + 4\mathcal{F}_F}{\mathcal{F}_{W^{(0)}\text{only}} + \frac{4}{3}\mathcal{F}_{t^{(0)}\text{only}}} = 1.0027 \tag{14}$$

at $\theta_H = 0.1153$. One finds that the contributions of the KK modes are less than 1% and negligible. For $H \to gg$ only the t tower loops contribute, and the behavior is similar.

For $H \to Z\gamma$ the cancellation mechanism is more intricate. In this case the KK number of particles running inside loops can change. Miraculous cancellation occurs only when all possible diagrams are summed. Numerically the correction due to KK modes amounts to only 0.07% at $\theta_H = 0.1153$.

(c) Signal strengths in the Higgs decay

As a consequence of (8) the decay widths of $H \to WW$, $H \to ZZ$, $H \to bb$ and $H \to \tau\tau$ are suppressed by $\cos^2 \theta_H$ at the tree level. The decay widths of the $H \to \gamma\gamma$, $H \to gg$ and $H \to Z\gamma$ are also suppressed by $\cos^2 \theta_H$ with the cancellation mechanism among KK contributions taken into accout. Consequently the branching ratios of the Higgs decay modes are almost the same as in the SM.

The Higgs boson production is dominated by $gg \to H$, and the production cross section is also suppressed by $\cos^2 \theta_H$. Therefore the signal strength of each decay mode $H \to j$, $\sigma(gg \to H)B(H \to j)/[\sigma(gg \to H)B(H \to j)]_{\rm SM}$, is approximately $\cos^2 \theta_H$. For $\theta_H \sim 0.1$, the deviation from the SM amounts to only 1%.

(d) Z' bosons

In the $SO(5) \times U(1)_X$ gauge-Higgs unification the first KK modes $Z_R^{(1)}$, $Z^{(1)}$, and $\gamma^{(1)}$, appear as Z' bosons in dilepton events at LHC. Here Z_R is the neutral gauge boson associated with $SU(2)_R$, which does not have a zero mode. At LHC they are produced and detected as

$$q \, \bar{q} \to Z_R^{(1)}, Z^{(1)}, \gamma^{(1)} \to e^+ e^-, \ \mu^+ \mu^- \ .$$

So far such events have not been observed, which put a constraint that their masses should be larger than $3 \, {\rm TeV}$.

In the gauge-Higgs unification left-handed quarks and leptons are localized near the Planck brane, whereas right-handed ones are localized near the TeV brane. The first KK modes of gauge fields are localized near the TeV brane so that right-handed quarks and leptons couple to $Z_R^{(1)}$, $Z^{(1)}$, and $\gamma^{(1)}$ more strongly than quarks and leptons couple to Z and γ. For instance, couplings of right-handed u, d, and e to $Z^{(1)}$ are about four times bigger than the corresponding couplings to Z. All of $Z_R^{(1)}$, $Z^{(1)}$, and $\gamma^{(1)}$ have large widths. Z' events are not SM-like. Masses and total decay widths of $Z_R^{(1)}$, $Z^{(1)}$, and $\gamma^{(1)}$ are summarized for $\theta_H = 0.114$ and 0.073 in Table 1.[21]

(e) Dark matter

In this model the lightest, neutral component of n_F $SO(5)$-spinor dark fermions becomes the dark matter of the universe.[22] The prediction concerning the dark matter, however, is not in the category of the universality explained above. The prediction depends on the details in the dark fermion sector.

The relic abundance of the dark matter determined by WMAP and Planck data is reproduced, below the bound placed by the direct detection experiment by LUX, by a model with one light and three heavier ($n_F = 4$) dark fermions with the lightest one of a mass from $2.3 \, {\rm TeV}$ to $3.1 \, {\rm TeV}$. The corresponding θ_H ranges from 0.097 to 0.074.

Table 1: Masses and total decay widths of Z' bosons

Z'	$\theta_H = 0.114$		$\theta_H = 0.073$	
	$m(\text{TeV})$	$\Gamma(\text{GeV})$	$m(\text{TeV})$	$\Gamma(\text{GeV})$
$Z_R^{(1)}$	5.73	482	8.00	553
$Z^{(1)}$	6.07	342	8.61	494
$\gamma^{(1)}$	6.08	886	8.61	1.04×10^3

5. $SO(11)$ gauge-Higgs grand unification

What is next? It is certainly necessary to incorporate strong interactions. The observed charge quantization in quarks and leptons, for instance, is most naturally explained in the framework of grand unification. It is desirable to have a unified theory of all gauge interactions. There have been many attempts for gauge-Higgs grand unification, most of which deals only with GUT symmetry breaking.[23]-[28]

We would like to have a gauge-Higgs grand unification scenario which carries over good features of $SO(5) \times U(1)$ gauge-Higgs EW unification. $SU(6)$ theory, for instance, does not meet this condition. It does not give consistent EW phenomenology at low energies. We propose $SO(11)$ gauge-Higgs grand unification in the RS space.[29]-[32]

(a) Model

$SO(11)$ gauge theory is defined in the RS space given by (1). $SO(11)$ orbifold boundary condition matrices P_0 and P_1 are given by

$$P_0^{\text{vec}} = \text{diag}(I_{10}, -I_1) , \quad P_1^{\text{vec}} = \text{diag}(I_4, -I_7) ,$$
$$P_0^{\text{sp}} = I_{16} \otimes \sigma^3 , \qquad P_1^{\text{sp}} = I_2 \otimes \sigma^3 \otimes I_8 \qquad (15)$$

in vectorial and spinorial representations. On the Planck brane P_0 breaks $SO(11)$ to $SO(10)$, whereas on the TeV brane P_1 breaks $SO(11)$ to $SO(4) \times SO(7)$. As a whole $SO(11)$ is broken to $SO(4) \times SO(6)$, which is isomorphic to $SU(2)_L \times SU(2)_R \times SU(4)$. $SO(11)$ gauge potentials satisfy (2). At this stage there appear parity even-even zero modes for A_μ in the $SO(4) \times SO(6)$ block. On the other hand zero modes of A_y appear only for $A_y^{a\,11}$ ($a = 1 \sim 4$) components, which are $SO(4)$ vector and $SO(6)$ singlet. The zero modes of A_y are identified with the four-dimensional $SU(2)_L$ doublet Higgs field in the SM.

We introduce a brane scalar $\Phi_{\mathbf{16}}$ on the Planck brane, in the spinor representation of $SO(10)$. We suppose that $\Phi_{\mathbf{16}}$ spontaneously develops $\langle \Phi_{\mathbf{16}} \rangle \neq 0$, which breaks $SO(10)$ to $SU(5)$ on the Planck brane. As a consequence $SO(4) \times SO(6)$ symmetry is broken to the SM symmetry, $G_{\text{SM}} = SU(2)_L \times SU(3)_C \times U(1)_Y$. G_{SM} is dynamically broken to $SU(3)_C \times U(1)_{\text{EM}}$ by the Hosotani mechanism.

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One immediate consequence is that all $SU(2)_L$, $U(1)_{\rm EM}$, $U(1)_Y$ charges, and the Weinberg angle at the GUT scale are determined to be

$$g_w = \frac{g}{\sqrt{L}} \ , \ \ e = \sqrt{\frac{3}{8}}\, g_w \ , \ \ g_Y = \sqrt{\frac{3}{5}}\, g_w \ , \ \ \sin^2\theta_W = \frac{3}{8} \ . \tag{16}$$

(b) Fermions

Fermions in the bulk are introduced in the spinor and vector representations of $SO(11)$. In each generation of quarks/leptons Ψ_{32}, Ψ_{11} and Ψ'_{11} are introduced. No additional brane fermions are necessary. In a sense bulk and brane fermions in the gauge-Higgs EW unification are unified in grand unification. The fermion fields obey

$$\Psi_{32}(x, y_j - y) = -\gamma^5 P_j^{\rm sp}\Psi_{32}(x, y_j + y) \ ,$$

$$\Psi_{11}(x, y_j - y) = (-1)^j \gamma^5 P_j^{\rm vec}\Psi_{11}(x, y_j + y) \ ,$$

$$\Psi'_{11}(x, y_j - y) = (-1)^{j+1} \gamma^5 P_j^{\rm vec}\Psi'_{11}(x, y_j + y) \ . \tag{17}$$

The content of these fermions is easily figured out. One finds that for Ψ_{32}

$$\Psi_{32} = \begin{pmatrix} \Psi_{16} \\ \Psi_{\overline{16}} \end{pmatrix}, \ \ \Psi_{16} = \begin{pmatrix} \nu \\ e \\ \hat{e} \\ \hat{\nu} \\ u_k \\ d_k \\ \hat{d}_k \\ \hat{u}_k \end{pmatrix}, \ \ \Psi_{\overline{16}} = \begin{pmatrix} \nu' \\ e' \\ \hat{e}' \\ \hat{\nu}' \\ u'_k \\ d'_k \\ \hat{d}'_k \\ \hat{u}'_k \end{pmatrix}, \ \ (k = 1 \sim 3),$$

$$\text{zero modes}: \ \ \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \ \begin{pmatrix} u_{kL} \\ d_{kL} \end{pmatrix}, \ \begin{pmatrix} \nu'_R \\ e'_R \end{pmatrix}, \ \begin{pmatrix} u'_{kR} \\ d'_{kR} \end{pmatrix}. \tag{18}$$

Here the notation is such that $\hat{e}$, $\hat{u}$, and $\hat{d}$ have charges $+1$, $-\frac{2}{3}$, and $+\frac{1}{3}$, respectively. For Ψ_{11} and Ψ'_{11}

$$\Psi_{11} = \begin{pmatrix} \hat{E} \ N \\ \hat{N} \ E \\ D_k \ \hat{D}_k \\ S \end{pmatrix}, \ \ \Psi'_{11} = \begin{pmatrix} \hat{E}' \ N' \\ \hat{N}' \ E' \\ D'_k \ \hat{D}'_k \\ S' \end{pmatrix},$$

$$\text{zero modes}: \ \ D_{kR}, \hat{D}_{kR}, \ \ D'_{kL}, \hat{D}'_{kL}. \tag{19}$$

All quarks and leptons fit in the zero modes of Ψ_{32}. Without the presence of Ψ_{11} and Ψ'_{11}, however, all quarks and leptons remain degenerate, acquiring the same mass by the Hosotani mechanism. To obtain the observed spectrum, one needs Ψ_{11} and Ψ'_{11}. An observed electron, for instance, is a linear combination of e, e', E and E' fields.

(c) Brane interactions and the fermion mass spectrum

On the Planck brane $SO(10)$ gauge invariance is strictly maintained. Bulk fermion fields which are parity even at $y = 0$ can form scalar interactions with $\Phi_{\mathbf{16}}$. $\Psi_{\mathbf{32}}$ decomposes into $\Psi_{\mathbf{16}}$ and $\Psi_{\overline{\mathbf{16}}}$, and $\Psi_{\mathbf{11}}$ into $\Psi_{\mathbf{10}}$ and $\Psi_{\mathbf{1}}$ under $SO(10)$. Participating fermion fields are $\Psi_{\mathbf{16}L}$, $\Psi_{\overline{\mathbf{16}}R}$, $\Psi_{\mathbf{10}R}$, $\Psi_{\mathbf{1}L}$, $\Psi'_{\mathbf{10}L}$ and $\Psi'_{\mathbf{1}R}$.

Six types of brane interactions are allowed from the symmetry.

$$
S_{\text{brane}} = \int d^5 x \sqrt{-\det G}\, \delta(y) \Big\{ - \kappa_1 \overline{\Psi}'_{\mathbf{1}R}\, \Phi^\dagger_{\mathbf{16}}\, \Psi_{\mathbf{16}L} - \kappa_2 \overline{\Psi}_{\mathbf{1}L}\, \tilde{\Phi}^\dagger_{\overline{\mathbf{16}}}\, \Psi_{\overline{\mathbf{16}}R}
$$

$$
- \kappa_3 (\overline{\Psi}_{\mathbf{10}R})_j\, \hat{\tilde{\Phi}}^\dagger_{\overline{\mathbf{16}}}\, \Gamma^j\, \hat{\Psi}_{\mathbf{16}L} - \kappa_4 (\overline{\Psi}'_{\mathbf{10}L})_j\, \hat{\Phi}^\dagger_{\mathbf{16}}\, \Gamma^j\, \hat{\Psi}_{\overline{\mathbf{16}}R}
$$

$$
- \mu_5 \overline{\Psi}'_{\mathbf{1}R}\, \Psi_{\mathbf{1}L} - \mu_6 \overline{\Psi}'_{\mathbf{10}L}\, \Psi_{\mathbf{10}R} - (\text{h.c.}) \Big\} . \tag{20}
$$

Here $\tilde{\Phi}_{\overline{\mathbf{16}}} = \hat{R}\,\Phi^*_{\mathbf{16}}$ transforms as $\overline{\mathbf{16}}$. 32 component notation has been adopted for $\hat{\Psi}_{\mathbf{16}L}$, $\hat{\Phi}_{\mathbf{16}}$ etc. In general κ_j and μ_j become 3-by-3 matrices in the generation space. With $\langle \Phi_{\mathbf{16}} \rangle \neq 0$, (20) generates six types of fermion mass terms, in which u and u' do not appear. The masses of up-type quarks are determined by the bulk mass parameters $c_{\Psi_{\mathbf{32}}}$ and θ_H. It turns out that the κ_2 term is responsible for m_ν/m_e, the κ_3 term for m_e/m_u, and the κ_4 and μ_6 terms for m_d/m_u. The observed fermion spectrum is reproduced by the Hosotani mechanism in combination with the brane interactions. Unfortunately there appear exotic light fermions associated with $\hat{u}$, $\hat{d}$ and $\hat{e}$ in this scheme.

6. Forbidden proton decay

Gauge-Higgs grand unification provides a new scheme of forbidding the proton decay.[29, 32] As seen in (18) and (19), all quarks and leptons reside in $\Psi_{\mathbf{32}}$, $\Psi_{\mathbf{11}}$ and $\Psi'_{\mathbf{11}}$ as particles, but not as anti-particles. In other words one can assign the Ψ-fermion number N_Ψ such that all quarks and leptons have $N_\Psi = 1$.

The gauge interactions as well as the brane interactions (20) preserve N_Ψ. N_Ψ is conserved. The proton has $N_\Psi = 3$ whereas the positron has $N_\Psi = -1$ so that the process $p \to \pi^0 e^+$, for instance, cannot take place.

This should be contrasted to 4D GUT. In the four-dimensional $SO(10)$ GUT, for instance, quarks and leptons are embedded in $\Psi_{\mathbf{16}L}$. In the notation in (18), $\hat{u}_L$ is identified with $(u^c)_L$ or $(u_R)^c$ so that gauge interactions do not conserve quark/lepton number, which induces the proton decay. In the gauge-Higgs grand unification u_L and u_R, for instance, are embedded as zero modes of 5D fields u and u' in $\Psi_{\mathbf{32}}$. Both u and u' have $N_\Psi = 1$.

7. Summary

Gauge-Higgs unification is promising. In the electroweak interactions we have $SO(5) \times U(1)_X$ gauge-Higgs EW unification. It is consistent with data and observations at low energies, including the data from 8 TeV LHC. It predicts Z' bosons in the energy range 6 to 10 TeV, which should be observed at 14 TeV LHC in a

few years. The deviation in the Higgs self-couplings from those in the SM is also predicted.

Grand unification is feasible in the gauge-Higgs unification scenario. We have proposed $SO(11)$ gauge-Higgs grand unification, incorporating strong interactions. Comparison of symmetry structure in the EW and grand unification is summarized in Table 2.

The concrete model of gauge-Higgs grand unification with fermions $\Psi_{\mathbf{32}}, \Psi_{\mathbf{11}}$ and $\Psi'_{\mathbf{11}}$ reproduces the observed quark-lepton mass spectrum. However, in the current minimal model there also appear light exotic fermions. We need further elaboration of the model.

We add that there have been many advances in the gauge-Higgs unification.[33]-[37] Dynamics of selecting orbifold boundary conditions has been explored.[38] The Hosotani mechanism has been examined not only in the continuum theory, but also on the lattice by nonperturbative simulations.[39, 40, 41]

Table 2: Symmetry structure in the gauge-Higgs EW and grand unification

EW unification	Grand unification
$SO(5) \times U(1)_X \times SU(3)_C$	$SO(11)$
$\downarrow$ BC	$\downarrow$ BC
$SO(4) \times U(1)_X \times SU(3)_C$	$SO(4) \times SO(6)$
$\downarrow \hat{\Phi}_{(0,\frac{1}{2})}$	$\downarrow \Phi_{\mathbf{16}}$
$SU(2)_L \times U(1)_Y \times SU(3)_C$	$SU(2)_L \times U(1)_Y \times SU(3)_C$
$\downarrow \theta_H$	$\downarrow \theta_H$
$U(1)_{\mathrm{EM}} \times SU(3)_C$	$U(1)_{\mathrm{EM}} \times SU(3)_C$

Acknowledgments

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