Fluctuation sound absorption in quark matter

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We investigate the sound absorption in quark matter due to the interaction of the sound wave with the precritical fluctuations of the diquark-pair field above T_c. The soft collective mode of the pair field is derived using the time dependent Ginzburg-Landau functional with random Langevin forces. The strong absorption near the phase transition line may be viewed as a manifestation of the Mandelshtam-Leontovich slow relaxation time theory.

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I. INTRODUCTION

QCD under extreme conditions has been a subject of intense study for the last decade. While the properties of quark-gluon matter at high temperature and zero chemical potential are theoretically investigated in great detail, understanding of the quark matter physics in the regime of non-zero density and moderate or low temperature remains challenging. This is due to the fact that the high T and zero μ region of the QCD phase diagram is accessible to lattice simulations. In the non-zero density regime Monte Carlo simulations fail and one has to resort to models, like the NJL one. On the experimental side, information obtained at RHIC and LHC corresponds in bulk to the high temperature and low density region. Non-zero density and moderate or low temperature conditions may exist in neutron stars and will be possibly realized in future experiments at FAIR and NICA.

According to the present understanding of the QCD phase structure, attractive interaction between quarks in color antitriplet state leads to the formation of the color superconducting phase in the moderate and high density domain [1–3]. Some important features of this phase are, however, very different from that of the BCS superconductor [4]. In particular, instead of an almost sharp dividing line between the normal and superconducting phases in the BCS case, in color superconductor the transition is significantly smeared. Correspondingly, an exceedingly narrow precritical fluctuation region in the BCS superconductor is replaced by a rather wide and physically important one in color superconductor. The fluctuation contribution to the physical quantities is characterized by the Ginzburg-Levanyuk number Gi which for the quark matter may be estimated as [4]

$$Gi \simeq \frac{\delta T}{T_c} \simeq \left(\frac{T_c}{\mu}\right)^4 \simeq 10^{-4}, \tag{1}$$

where $T_c \cong 40$ MeV in the critical temperature for the $2SC$ phase (u and d quarks pairing), $\mu \cong 400$ MeV is the quark chemical potential. Note that for the BCS superconductor $Gi \sim 10^{-12} - 10^{-14}$. Fluctuation quark pairing which takes place when the temperature approaches T_c from above manifests itself in the characteristic temperature dependence of a number of physical quantities. In BCS superconductors such phenomena have been intensively studied for more than three decades [6]. In our previous paper [5] we have calculated the fluctuation electrical conductivity of quark matter. It has been shown it is large and greatly exceeds the Drude one.

In the present paper we investigate the fluctuation sound absorption and show that it has even more pronounced temperature dependence than the electrical conductivity. Let us point from the very beginning that we consider the hydrodynamical, or first, sound.

The reason for the strong energy dissipation of the sound wave in the precritical region has a general nature. The idea goes back to the seminal paper [7] in which Mandelshtam and Leontovich formulated the slow relaxation time theory (see also [8–12]). Suppose that the relaxation time corresponding to the equilibrium restoration is large. Then during the equilibration process strong energy dissipation occurs. Propagation of the sound wave changes the critical temperature in the compression – rarefraction regions. The fluctuation pairing is a slow process and the resulting nonequilibrium results in the sound wave energy loss [13, 14]. Based on the above ideas we shall calculate the fluctuation sound absorption in quark matter. Unlike other transport coefficients (shear viscosity, electrical conductivity, etc.), sound propagation in quark matter did not receive much attention in the literature. The quantity $(1/3 - c_s^2)$ reflects the breaking of

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the conformal symmetry and may serve as a measure of the interaction. Lattice calculation of this quantity may be found in [15]. The squared speed of the sound c_s^2 as a function of T at zero density was also calculated in this work and the minimum which corresponds to the softest point of the EoS was found. Different problems related to sound propagation in quark-gluon matter have been discussed in [16].

The paper is structured as follows. In Sec.II we introduce the time dependent Ginzdurg-Landau functional with random Langevin forces and derive the fluctuation propagator (FP) In Sec.III we present the Aslamazov-Larkin (AL) diagram for the polarization operator. Using the expression for the FP derived in Sec.II we evaluate the AL diagram and show that it gives rise to a strong sound absorption near the critical temperature. The final Sec.IV is devoted to a summary and concluding remarks.

II. COLLECTIVE MODE PROPAGATOR

Throughout this paper we use the natural system of units $\hbar = c = k_B = 1$. The Matsubara fermion propagator has the form

$$G(\mathbf{p}, \varepsilon_n) = \frac{1}{\gamma_0(i\varepsilon_n + \mu) - \boldsymbol{\gamma}\mathbf{p} - m}. \quad (2)$$

Here $\varepsilon_n = \pi T(2n+1), \mu$ is the quark chemical potential. Integration in the vicinity of the Fermi surface is performed making use of the variable ξ defined as

$$\xi = \sqrt{\mathbf{p}^2 + m^2} - \mu. \quad (3)$$

Then

$$\int \frac{d\mathbf{p}}{(2\pi)^3} \simeq \int d\xi \rho(\xi) \simeq \simeq \int d\xi \left[\rho(\mu) + \left(\frac{\partial \rho}{\partial \xi}\right)_\mu \xi\right] d\xi = = \frac{p_0 \mu}{2\pi^2} \int d\xi + \frac{\mu}{2\pi^2}\left(\frac{v_0^2+1}{v_0}\right) \int d\xi\, \xi. \quad (4)$$

Here p_0 is the Fermi momentum, $v_0 = p_0/\mu$ is the Fermi velocity. The second term in (4) takes into account the energy dependence of the density of states at the Fermi surface. As will be shown below only the contribution from this term enters into the final result for the fluctuation sound absorption.

Fluctuations of the pair field in the vicinity of T_c are described by the collective mode, or the fluctuation propagator (FP) [6]. In [6] and references therein the FP $L(\mathbf{q}, \omega_k)$ was introduced in the framework of the BCS theory making use of the nonrelativistic kinematics and Green's functions. In [5] it was derived for the relativistic quark system solving the Dyson equation with Matsubara propagators (2). Graphically the Dyson equation is represented in Fig.1.

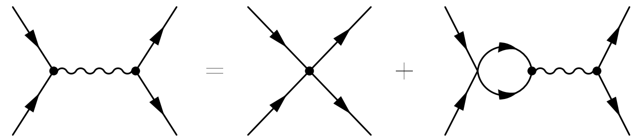

FIG. 1. Dyson equation for FP (wavy line).

Here the FP will be obtained using the time-dependent Ginzburg-Landau functional TDGL [6, 17, 18] and the stochastic Langevin forces. In absence of external electromagnetic field the TDGL for the fluctuating pair field ψ reads

$$-\gamma \frac{\partial}{\partial t}\psi(\mathbf{r}, t) = \frac{\delta F[\psi]}{\delta \psi *} + \eta(\mathbf{r}, t). \quad (5)$$

Here γ is the order parameter relaxation time constant. The GL functional with the quartic term dropped has the form

$$F[\psi] = F_0 + \int d\mathbf{r} \left\{a|\psi(\mathbf{r}, t)|^2 + b|\nabla\psi(\mathbf{r}, t)|^2\right\}, \quad (6)$$

where $a = \nu\varepsilon,\ \nu = \rho(\mu) = \frac{p_0\mu}{2\pi^2}, \varepsilon = \frac{T-T_c}{T_c}, b = \nu\varkappa^2, \varkappa^2 = \frac{\pi}{8T_c}D, D$ is the diffusion coefficient, $\gamma = \frac{\pi\nu}{8T_c}$ (for details see [4, 6, 17, 18]). With $F[\psi]$ given by (6) we return to (5) and write

$$-\left[\gamma\frac{\partial}{\partial t} + \nu\left(\varepsilon + \varkappa^2\mathbf{q}^2\right)\right]\Psi(\mathbf{r}, t) \equiv \hat{L}^{-1}\Psi(\mathbf{r}, t) = \eta(\mathbf{r}, t). \quad (7)$$

The solution of (7) may be formally written as

$$\Psi(\mathbf{r}, t) = \hat{L}\eta(\mathbf{r}, t). \quad (8)$$

Let us assume that the correlator of the Langevin forces has a gaussian form

$$\langle \eta^*(\mathbf{r}, t)\eta(\mathbf{r}', t')\rangle = \gamma\, \delta(\mathbf{r} - \mathbf{r}')\, \delta(t - t'). \quad (9)$$

According to the fluctuation-dissipation theorem [18] the retarded propagator (the FP in our case) is given by the equal time correlator $\langle \Psi^*(\mathbf{r}, t)\Psi(\mathbf{r}', t)\rangle$. From (8) one can write

$$\langle \Psi^*(\mathbf{r}, t)\Psi(\mathbf{r}', t)\rangle = \gamma \int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')} \times \times \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \hat{L}^*(\mathbf{r}, \omega)\hat{L}(\mathbf{r}, \omega) = = -\int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\omega^{-1}\, \mathrm{Im}\, \hat{L}(\mathbf{r}, \omega), \quad (10)$$

where

$$\hat{L}(\mathbf{r},\omega) = -\frac{1}{\nu}\frac{1}{\varepsilon + \frac{\pi}{8T_c}\big(-i\omega + D\mathbf{q}^2\big)}. \quad (11)$$

The FP (11) describes the slow diffusion mode near the critical temperature. At small ω and q close to T_c the quantity $L(\mathbf{q},\omega)$ can be arbitrary large and is rapidly varying. This will be an important point in the calculation of the Aslamazov-Larkin diagram for the sound absorption.

III. PRECRITICAL SOUND ABSORPTION

In this work we study the effects caused by the quark pair field fluctuations. The possible role of the gluon field fluctuations has been studied in detail in [4] and also in [20]. According to [4] the gluon field fluctuations lead to a shift in T_c and to possible replacement of the second-order phase transition to the first-order one. However, the increase of the quark density leads to a suppression of the gluon fluctuations [4]. The authors of Ref. [20] also came to the conclusion that the fluctuations of the pair field dominate those of the gauge field in the strong coupling regime.

With the FP at our disposal, we can evaluate the Aslamazov-Larkin (AL) [13, 14] contribution to the sound absorption in the fluctuation region. Based on the experience gained in condensed matter physics [6], we assume that it is of major importance among other quantum fluctuation effects. Previously it was shown that AL paraconductivity exceeds the Drude one [5]. In [21] preliminary results on the AL term in lepton-pair production were presented.

The Feynman diagram representing the AL sound absorption is shown in Fig.2. It contains two wavy lines

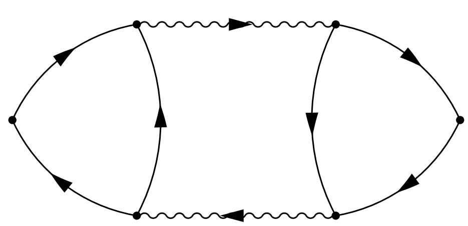

FIG. 2. Feynman diagram for the AL polarization operator for the sound absorption.

corresponding to the FP and this makes this contribution the most important in the vicinity of T_c. The sound absorption is determined by the imaginary part of the polarization operator given by the AL diagram. The in- and out- vertices in this diagram are equal to the constant g of the phonon-quark interaction. The solid lines are the quark propagators (2). The AL diagram corresponds to the following polarization operator

$$\Pi(\mathbf{k},\omega_\nu) = 4T\sum_{\Omega_j}\int\frac{d\mathbf{q}}{(2\pi)^3}B^2(\mathbf{k},\mathbf{q},\omega_\nu,\Omega_j)\times \\ \times L(\mathbf{q}+\mathbf{k},\Omega_j+\omega_\nu)L(\mathbf{q},\Omega_j). \quad (12)$$

Here $\mathbf{k}$ is the sound wave momentum, ω_ν is the Matsubara sound frequency, $B(\mathbf{k},\mathbf{q},\omega_\nu,\Omega_j)$ is the block of the three propagators

$$B(\mathbf{k},\mathbf{q},\omega_\nu,\Omega_j) = g\,T\sum_{\varepsilon_n}\int\frac{d\mathbf{p}}{(2\pi)^3}\times \\ \times\mathrm{tr}\big\{G(\mathbf{p},\varepsilon)G(\mathbf{p}+\mathbf{k},\varepsilon_n+\omega_\nu)G(\mathbf{q}-\mathbf{p},\Omega_j-\varepsilon_n)\big\}. \quad (13)$$

The dependence of the FP-s $L(\mathbf{k}+\mathbf{q},\Omega_j+\omega_\nu)$ and $L(\mathbf{q},\Omega_j)$ on Ω_j and ω_ν is much stronger than that of the Green's functions. The closeness to the transition point is enclosed in the FP-s at small values of frequencies and momenta. Therefore we shall keep in the propagators entering into B only the dependence on the fermionic frequencies and momenta. In this approximation one easily obtains

$$\mathrm{tr}\big\{G(\mathbf{p},\varepsilon_n)\,G(\mathbf{p},\varepsilon_n)\,G(-\mathbf{p},-\varepsilon_n)\big\} = \\ \frac{2m}{E}\frac{1}{(\xi-i\varepsilon_n)^2(\xi+i\varepsilon_n)} \simeq \\ \frac{2m}{\mu}\frac{1}{(\xi-i\varepsilon_n)^2(\xi+i\varepsilon_n)}. \quad (14)$$

Next we transform according to (4) the integration over $d\mathbf{p}$ in (14) into the integration over $d\xi$ and perform the summation over ε_n. Only the second term in (4) proportional to $\left(\frac{d\rho}{d\xi}\right)_\mu$ gives nonzero contribution in the integral over $d\xi$. The result for B yields

$$B = g\,T\,\frac{2m}{\mu}\sum_{\varepsilon_n}\int d\xi\frac{\xi}{(\xi-i\varepsilon_n)^2(\xi+i\varepsilon_n)}\left(\frac{\partial\rho}{\partial\xi}\right)_\mu = \\ g\,\frac{m}{2\pi^2}\left(\frac{v_0^2+1}{v_0}\right)ln\frac{\omega_D}{2\pi T_c}. \quad (15)$$

The critical temperature for 2SC superconducting phase is $T_c \simeq 40$ Mev, the ultraviolet cutoff $\omega_D \simeq 800$ MeV [2–4], therefore $ln\frac{\omega_D}{2\pi T_c} \simeq 1$.

Upon the substitution of (15) into (12) one gets

$$\Pi(\mathbf{k},\omega_\nu) = g^2\frac{m^2}{\pi^4}\left(\frac{v_0^2+1}{v_0}\right)^2\ln^2\frac{\omega_D}{2\pi T_c}\times \\ \times T\sum_{\Omega_j}\int\frac{d\mathbf{q}}{(2\pi)^3}L(\mathbf{q}+\mathbf{k},\Omega_j+\omega_\nu)L(\mathbf{q},\Omega_j). \quad (16)$$

To proceed further, we shall assume that the acoustic wavelength is much larger than the correlation radius of fluctuations, i.e.,

$$\varepsilon \gg \frac{\pi}{8T_c} D k^2. \tag{17}$$

For $T\tau \ll 1$ the diffusion coefficient is $D = \frac{1}{3} v_0^2 \tau$. As an order of magnitude estimate we take $\tau \simeq 0.3 fm$, $T_c = 40 MeV$, $v_0^2 = \frac{1}{3}$. Then

$$\frac{k^2}{MeV^2} \ll 10^6 \varepsilon. \tag{18}$$

Therefore (17) imposes a very weak restriction on the phonon momentum. The inequality (17) allows to neglect the $\mathbf{k}$-dependence of the FP $L(\mathbf{q}+\mathbf{k}, \Omega_j + \omega_\nu)$. To evaluate the sum over Ω_j in (16), we can use a technique of replacing the summation by contour integration [22, 23]. At the first step, this leads to the following result for the polarization operator

$$\Pi(\omega) = \frac{2B^2}{\pi} \int \frac{d\mathbf{q}}{(2\pi)^3} \int\limits_{-\infty}^{+\infty} dz \coth\frac{z}{2T} \Big[L^R(\mathbf{q}, -iz - i\omega) + \\ + L^A(\mathbf{q}, -iz + i\omega) \Big] \operatorname{Im} L^R(\mathbf{q}, -iz), \tag{19}$$

where $z = i\Omega_j$, $\omega = i\omega_\nu$, and L^R and L^A are the retarded and advanced FP-s. The next step is to expand the integral in powers of ω and to substract the zeroth order term which would lead to Meissner effect above T_c. Alternatively, this may be regarded as imposing the Word identity on the polarization operator. Keeping in (19) the term proportional to ω and integrating by parts, one has

$$\Pi(\omega) = -i\omega B^2 \frac{4T}{\pi} \int \frac{d\mathbf{q}}{(2\pi)^3} \int\limits_{-\infty}^{+\infty} dz \frac{(\operatorname{Im} L^R)^2}{z^2} = \\ -i\omega \frac{\pi B^2}{\nu^2} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{1}{\left(\varepsilon + \frac{\pi}{8T_c} D\mathbf{q}^2\right)^3}. \tag{20}$$

The final result for the fluctuation sound absorption coefficient reads

$$\operatorname{Im}\Pi = -\,\omega\, g^2 \,\frac{m^2}{2^5 p_0^4 \varkappa^3}\,(v_0^2+1)^2 \times \\ \times \ln^2 \frac{\omega_D}{2\pi T_c} \left(\frac{T_c}{T - T_c}\right)^{3/2}, \tag{21}$$

where $\varkappa^2 = \dfrac{\pi}{8\,T_c} D$ (see definition following Eq. (6))

IV. CONCLUDING REMARKS

In the paper we have examined the sound absorption in quark matter at moderate density and temperature due to the precursory fluctuations of the the quark pair field. The absorption is caused by the interaction of phonons with the soft collective mode of the quark field. The results are in line with the Mandelshtam-Leontovich slow relaxation time theory. The sound propagation changes the critical temperature in the compression-rarefaction regions. The fluctuation pairing is a slow process and the resulting inequilibrium leads to the intense sound wave energy loss. Assumptions which have been made in the course of the derivation were clearly exposed. The dependence of the sound absorption on the proximity to the critical temperature is even more pronounced than in case of the electrical conductivity [5], where it was possible to compare the fluctuation contribution with the Drude one. For the quark matter we are not aware of the "normal" sound absorption calculations. For the BCS superconductor it was shown that the fluctuation sound absorption exceeds the normal one in a wide range of parameter [13, 14].

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