

Higgs Potential from Derivative Interactions

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Abstract

A formulation of the linear σ model with derivative interactions is studied. The classical theory is on-shell equivalent to the σ model with the standard quartic Higgs potential. The mass of the scalar mode only appears in the quadratic part and not in the interaction vertices, unlike in the ordinary formulation of the theory. Renormalization of the model is discussed. A non power-counting renormalizable extension, obeying the defining functional identities of the theory, is presented. This extension is physically equivalent to the tree-level inclusion of a dimension six effective operator $\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$. The resulting UV divergences are arranged in a perturbation series around the power-counting renormalizable theory. The application of the formalism to the Standard Model in the presence of the dimension-six operator $\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$ is discussed.

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I. INTRODUCTION

The discovery of the Higgs boson at the LHC [1, 2] has by now firmly established the existence of a scalar particle as a fundamental ingredient of the spontaneous symmetry breaking (SSB) mechanism for electroweak theory [3–6].

On the other end, further investigation is required in order to understand the properties of the SSB potential. In addition to the Standard Model (SM) quartic Higgs potential, many other possibilities can be considered. The model-independent approach based on the effective field theory (EFT) technique allows to disentangle the phenomenological consequences of higher dimensional operators [7, 8]. The one-loop anomalous dimensions of dimension-six operators have been studied in [9–13]. Prospects of measuring anomalous Higgs couplings at the LHC and at future colliders have been considered e.g. in [14–16].

In this paper we discuss the field-theoretical properties of a formulation of the SSB potential based on higher derivatives interactions [17] that, at the classical level, is physically equivalent to the quartic Higgs potential.

The main advantage of this formulation is that the mass of the physical Higgs excitation only enters in the mass term of the physical field and not in the coupling constant (unlike in the ordinary quartic potential).

This has a number of consequences. The functional equations governing the theory include the equation of motion for the physical massive mode. Let us denote it by X_2 . In addition to the quadratic mass term, one can include in the action a kinetic term for X_2

$$\int d^4x \frac{z}{2} \partial_\mu X_2 \partial^\mu X_2.$$

Once such a term is introduced into the classical action, the equation of motion for X_2 is modified by a contribution linear in X_2 and still survives quantization (due to the fact that the breaking is linear in the quantized fields). On the other hand, power-counting renormalizability is lost, since new divergences arise, vanishing at $z = 0$.

This remark leads to a perturbative definition of the non power-counting renormalizable theory as a series expansion around $z = 0$, i.e. the coefficients of the expansion in z of the amplitudes are fixed in terms of amplitudes of the renormalizable theory at $z = 0$.

By going on-shell and eliminating X_2 via the relevant equation of motion we obtain the linear σ model with a quartic Higgs potential plus the dimension six operator $\sim z \partial_\mu \Phi^\dagger \Phi \partial^\mu \Phi^\dagger \Phi$.

In this model the mass of the physical scalar particle is given by $M_{phys}^2 = \frac{M^2}{1+z}$. The limit where the mass scale of the theory M goes to infinity while keeping M_{phys} fixed coincides with the strongly interacting regime $z \rightarrow \infty$.

The X_2 -equation allows one to control the UV divergences of the one-particle-irreducible (1-PI) Green's functions with X_2 -legs in terms of amplitudes with insertions of external sources with a better UV behaviour.

This allows one to disentangle some new relations between 1-PI amplitudes involving X_2 -external lines that are not apparent on the basis of the power-counting of the underlying effective field theory.

One might then conjecture that such relations will translate into consistency conditions between the Green's functions in the ordinary Higgs EFT once one eliminates X_2 through the equations of motion of the auxiliary fields. This is currently under investigation [18].

The paper is organized as follows. In Sect. II we introduce our notation and discuss the theory at $z = 0$. The BRST symmetry is given and the tree-level on-shell equivalence with the linear σ model in the presence of a quartic Higgs potential is discussed. In Sect. III the mechanism guaranteeing the on-shell equivalence of the derivative interactions with the usual quartic Higgs potential is elucidated on some sample tree-level computations. In Sect. IV the functional identities of the theory are presented and the renormalization of the model at $z = 0$ is carried out. The one-loop divergences are computed and the on-shell normalization conditions are presented. Sect. V describes the Standard Model (SM) action in the derivative representation of the Higgs potential. The BRST symmetry of the SM is provided. In Sect. VI the non power-counting renormalizable theory at $z \neq 0$ is considered and the differential equation defining the Green's functions of the theory as an expansion around $z = 0$ is introduced. In Sect. VII the UV subtraction of the model at $z \neq 0$ is studied. The ambiguities in the choice of the finite parts of the counterterms at $z \neq 0$ are related to the insertion at zero momentum of the quadratic operator $X_2 \square X_2$ in the amplitudes of the power-counting renormalizable theory at $z = 0$. In Sect. VIII we analyze how non-local symmetric deformations of the X_2 -propagator can induce a UV completion of the theory, restoring power-counting renormalizability also at $z \neq 0$, as well as the UV completion realized through the addition of further physical scalars, while preserving locality of the classical action. Conclusions are presented in Sect. IX.

II. FORMULATION OF THE THEORY

We start from the following action written in components

$$S_0 = \int d^4x \left[\frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a - \frac{M^2}{2} X_2^2 + \frac{1}{v} (X_1 + X_2) \square \left(\frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 - vX_2 \right) \right]. \quad (2.1)$$

The fields (σ, ϕ_a) belong to a SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\phi_1 + \phi_2 \\ \sigma + v - i\phi_3 \end{pmatrix}, \quad (2.2)$$

X_2 is a SU(2) singlet. v is the scale of the spontaneous symmetry breaking.

The equation of motion of X_1 is

$$\frac{\delta S_0}{\delta X_1} = \frac{1}{v} \square \left(\frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 - vX_2 \right). \quad (2.3)$$

Going on-shell with X_1 this yields (neglecting zero modes of the Laplacian¹)

$$X_2 = \frac{1}{2v} \sigma^2 + \sigma + \frac{1}{2v} \phi_a^2, \quad (2.4)$$

which, substituted into Eq.(2.1), gives

$$S_0|_{\text{on-shell}} = \int d^4x \left[\frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a - \frac{1}{2} \frac{M^2}{v^2} \left(\frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 \right)^2 \right], \quad (2.5)$$

i.e. at tree level one finds the ordinary SU(2) linear σ model with a quartic Higgs potential of coupling constant $\lambda = \frac{1}{2} \frac{M^2}{v^2}$.

It should be remarked that the right sign of the potential, triggering the spontaneous symmetry breaking in Eq.(2.5), is dictated by the sign of the mass term of the field X_2 , which in turn is fixed by the requirement of the absence of tachyons in the theory.

Notice that by going on-shell with X_1 , asymptotically σ coincides with X_2 , as can be seen by taking the linearization of the r.h.s. of Eq.(2.4). In particular, σ acquires a mass M , as in Eq.(2.5).

¹ A rigorous argument of why this is legitimate will be given in Subsection II A after Eq.(2.16).

A. Off-shell Formalism

The off-shell implementation of the constraint in Eq.(2.4) can be realized *à la* BRST [19–21]. The construction works as follows [17]. One introduces a pair of ghost c and antighost \bar{c} such that the BRST variation of the antighost is the constraint:

$$s\bar{c} = \Phi^\dagger\Phi - vX_2 - \frac{v^2}{2} = \frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2. \quad (2.6)$$

The Lagrange multiplier field X_1 , enforcing the constraint in the action S_0 , pairs with the ghost c into a BRST doublet [22–24]

$$sX_1 = vc, \quad sc = 0. \quad (2.7)$$

A set of variables u, v such that $su = v, sv = 0$ is known as a BRST doublet or a trivial pair [25] since it does not affect the cohomology $H(s)$ of the BRST differential s . We recall that $H(s)$ is the set of local polynomials in the fields and their derivatives such that two polynomials \mathcal{I}_1 and \mathcal{I}_2 are equivalent if and only if they differ by a s -exact term: $\mathcal{I}_1 = \mathcal{I}_2 + s\mathcal{K}$ for some \mathcal{K} . $H(s)$ identifies the local physical observables of the theory [22, 24]. If one considers local functionals (e.g. the action) and allows for integration by parts, one defines in a similar way the cohomology $H(s|d)$ of s modulo the exterior differential d [22, 24]. $H(s|d)$ controls the local deformations of the classical action (including counterterms) as well as (in the sector with ghost number one) the potential anomalies of the theory.

By Eq.(2.7) X_1 and c are not physical fields of the theory (as expected, since X_1 is a Lagrange multiplier and c is required in the algebraic BRST implementation of the off-shell constraint but should not affect the physics itself). All other fields are BRST invariant:

$$s\sigma = s\phi_a = sX_2 = 0. \quad (2.8)$$

s is nilpotent.

One recovers BRST invariance of the action by adding to S_0 the ghost-dependent term

$$S_{ghost} = - \int d^4x \bar{c} \square c \quad (2.9)$$

so that the full action of the theory is

$$S = S_0 + S_{ghost}. \quad (2.10)$$

Notice that the ghost is a free field. It should be stressed that the BRST symmetry s is not associated with a local gauge symmetry of the theory. It implements algebraically the (SU(2)-invariant) constraint in Eq.(2.4).

We remark that also the pair $\bar{c}, \mathcal{F} = \frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2$ forms a BRST doublet according to Eq.(2.6) and therefore drops out of the cohomology $H(s)$. The cohomology $H(s)$, respecting all the relevant symmetries of the theory, is thus given by Lorentz-invariant, global SU(2)-invariant polynomials constructed out of the doublet Φ and derivatives thereof, X_2 being cohomologically equivalent to $\Phi^\dagger\Phi$ according to Eq.(2.6). This is the cohomology of the linear σ model, as expected, since the introduction of the fields X_1, X_2, \bar{c}, c in order to enforce the constraint in Eq.(2.4) should not alter the physics of the theory.

Since the BRST transformation of the antighost \bar{c} is non-linear in the quantized fields, one needs one external source \bar{c}^* in order to control the renormalization of the BRST variation $s\bar{c}$. The latter need to be coupled to the source \bar{c}^* , known as an antifield [22, 25].

Thus the tree-level vertex functional of the theory is finally given by

$$\begin{aligned} \Gamma^{(0)} = \int d^4x & \left[\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}\partial^\mu\phi_a\partial_\mu\phi_a - \frac{M^2}{2}X_2^2 \right. \\ & - \bar{c}\square c + \frac{1}{v}(X_1 + X_2)\square\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\right) \\ & \left. + \bar{c}^*\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\right) \right]. \end{aligned} \quad (2.11)$$

Notice that the second line can be rewritten as a s -exact term as follows:

$$\begin{aligned} S_{constr} &= \int d^4x \left[-\bar{c}\square c + \frac{1}{v}(X_1 + X_2)\square\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\right) \right] \\ &= \int d^4x s\left(\frac{1}{v}\bar{c}\square(X_1 + X_2)\right). \end{aligned} \quad (2.12)$$

We see that the first line of Eq.(2.11) describes the (SU(2)-invariant) action of the linear σ -model (in the derivative representation of the potential), the second (BRST-exact) line the off-shell implementation of the constraint in Eq.(2.4), while the third line contains the antifield-dependent sector.

BRST invariance can be translated into the following Slavnov-Taylor (ST) identity

$$\int d^4x \left(vc \frac{\delta\Gamma}{\delta X_1} + \frac{\delta\Gamma}{\delta\bar{c}^*} \frac{\delta\Gamma}{\delta\bar{c}} \right) = 0 \quad (2.13)$$

which holds for the full vertex functional Γ (the generator of the 1-PI amplitudes, whose leading order in the loop expansion coincides with $\Gamma^{(0)}$).

The ghost field c has ghost number $+1$, the antighost field \bar{c} has ghost number -1 . All other fields and the external source \bar{c}^* have ghost number zero. Γ has ghost number zero.

Some comments are in order. According to Eq.(2.6) we have

$$\Phi^\dagger\Phi - \frac{v^2}{2} = vX_2 + s\bar{c} \quad (2.14)$$

and thus when $X_2 = 0$ the operator $\Phi^\dagger\Phi - \frac{v^2}{2}$ is physically equivalent to the null operator. In this case the theory reduces to the non-linear σ model [17], enforcing off-shell the non-linear constraint $\Phi^\dagger\Phi - \frac{v^2}{2} = 0$.

When X_2 is different than zero, the theory has the same degrees of freedom as the linear σ model. Notice in particular that the X_1 -equation of motion

$$\frac{\delta S_0}{\delta X_1} = \frac{1}{v}\square\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\right) = 0 \quad (2.15)$$

yields the most general solution

$$X_2 = \frac{1}{2v}\sigma^2 + \sigma + \frac{1}{2v}\phi_a^2 + \chi \quad (2.16)$$

with χ a massless degree of freedom satisfying the free Klein-Gordon equation $\square\chi = 0$. Compatibility between Eqs.(2.14) and (2.16) entails that χ must be cohomologically equivalent to the null operator and thus one can safely set $\chi = 0$ when going on-shell with the auxiliary field X_1 .

Since X_2 is a scalar singlet, one can add any polynomial in X_2 and ordinary derivatives thereof to $\Gamma^{(0)}$ without breaking BRST symmetry. With the conventions adopted, σ and X_2 have zero vacuum expectation value. This prevents to add the X_2 -tadpole contribution to the action. The simplest term is then a mass term for X_2 as in Eq.(2.1). This turns out to be compatible with power-counting renormalizability [17].

B. Propagators

Diagonalization of the quadratic part of $\Gamma^{(0)}$ is achieved by setting $\sigma = \sigma' + X_1 + X_2$. Then the propagators are

$$\begin{aligned} \Delta_{\sigma'\sigma'} &= \frac{i}{p^2}, & \Delta_{\phi_a\phi_b} &= \frac{i\delta_{ab}}{p^2}, & \Delta_{\bar{c}c} &= \frac{i}{p^2} \\ \Delta_{X_1X_1} &= -\frac{i}{p^2}, & \Delta_{X_2X_2} &= \frac{i}{p^2 - M^2}. \end{aligned} \quad (2.17)$$

Notice the minus sign in the propagator of X_1 . This entails that the combination $X = X_1 + X_2$ has a propagator which falls off as p^{-4} for large momentum:

$$\Delta_{XX} = \frac{iM^2}{p^2(p^2 - M^2)}. \quad (2.18)$$

The physical states of the theory are identified by standard cohomological methods [25, 26]. The asymptotic BRST charge Q acts on the fields as the linearization of the BRST differential s . X_1 is not invariant under Q and thus it does not belong to the physical space $\mathcal{H} = \text{Ker } Q / \text{Im } Q$, as well as σ' (since $[Q, \sigma'] = [Q, \sigma - X_1 - X_2] = -vc$). The combination $\sigma' + X_1$ is BRST invariant, however it is BRST exact, since it is generated by the variation of the antighost field \bar{c} :

$$[Q, \bar{c}]_+ = \sigma' + X_1 \quad (2.19)$$

and thus it does not belong to \mathcal{H} . The only physical modes are the scalar X_2 and the fields ϕ_a , namely the degrees of freedom of the linear σ model.

III. TREE-LEVEL

A. On-shell amplitudes

We check in this Section the on-shell equivalence between the theory and the linear σ model on the tree-level 3- and 4-point amplitudes.

1. 3-point amplitude

The off-shell 3-point amplitude is

$$\mathcal{A}_{X_2 X_2 X_2} = -\frac{i}{v} \sum_{i=1}^3 p_i^2 \quad (3.1)$$

where the sum is over the momenta of the particles. By going on shell $p_i^2 = M^2$ we get

$$\mathcal{A}_{X_2 X_2 X_2} |_{\text{on-shell}} = -\frac{3i}{v} M^2, \quad (3.2)$$

which coincides with the amplitude of the $\text{SU}(2)$ linear sigma model for the coupling constant $\lambda = \frac{1}{2} \frac{M^2}{v^2}$.

2. 4-point amplitude

The situation is more involved here. Diagrams contributing with an exchange of a σ' and X_1 of momentum q sum up to cancel out the unphysical pole at $q^2 = 0$, yielding a finite contribution

$$\mathcal{A}_{X_2 X_2 X_2 X_2}^{(1)} \Big|_{\text{on-shell}} = i \frac{16M^2}{v^2}. \quad (3.3)$$

Diagrams where a X_2 particle is exchanged give in turn

$$\mathcal{A}_{X_2 X_2 X_2 X_2}^{(2)} \Big|_{\text{on-shell}} = -i \frac{19M^2}{v^2} - i \frac{9M^4}{v^2} \left(\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2} \right), \quad (3.4)$$

as a function of the usual Mandelstam variables.

The sum is

$$\mathcal{A}_{X_2 X_2 X_2 X_2} \Big|_{\text{on-shell}} = -i \frac{3M^2}{v^2} - i \frac{9M^4}{v^2} \left(\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2} \right). \quad (3.5)$$

The first term is the one arising in the linear sigma model from the contact four-point vertex, the last three are those generated by diagrams with the exchange of a propagator and two trilinear couplings. Notice that the contact interaction contribution is controlled by the sum of two terms, originating both from $\mathcal{A}^{(1)}$ and $\mathcal{A}^{(2)}$.

IV. RENORMALIZATION

A. Functional identities

In addition to the ST identity in Eq.(2.13), the theory obeys a set of functional identities constraining the 1-PI Green's functions and their UV divergences:

- the ghost and the antighost equations

$$\frac{\delta\Gamma}{\delta\bar{c}} = -\square c, \quad \frac{\delta\Gamma}{\delta c} = \square\bar{c}. \quad (4.1)$$

These equations imply that the ghost and the antighost fields are free to all order in the loop expansion.

- since the ghost is free, one can take a derivative w.r.t. c of the ST identity and use the first of Eqs.(4.1) to get the X_1 equation for the full vertex functional

$$\frac{\delta\Gamma}{\delta X_1} = \frac{1}{v} \square \frac{\delta\Gamma}{\delta\bar{c}^*}. \quad (4.2)$$

- the shift symmetry

The shift symmetry

$$\delta X_1(x) = \alpha(x), \quad \delta X_2(x) = -\alpha(x), \quad (4.3)$$

gives

$$\frac{\delta\Gamma}{\delta X_1} - \frac{\delta\Gamma}{\delta X_2} = -\square(X_1 + X_2) + M^2 X_2 + v\bar{c}^*. \quad (4.4)$$

The r.h.s. is linear in the quantized fields and therefore the classical symmetry can be extended at the full quantum level. By using the X_1 equation (4.2) into Eq.(4.4) we obtain the X_2 -equation

$$\frac{\delta\Gamma}{\delta X_2} = \frac{1}{v}\square\frac{\delta\Gamma}{\delta\bar{c}^*} + \square(X_1 + X_2) - M^2 X_2 - v\bar{c}^*. \quad (4.5)$$

- global SU(2) invariance

$$\int d^4x \left[-\frac{1}{2}\alpha_a\phi_a\frac{\delta\Gamma}{\delta\sigma} + \left(\frac{1}{2}(\sigma+v)\alpha_a + \frac{1}{2}\epsilon_{abc}\phi_b\alpha_c\right)\frac{\delta\Gamma}{\delta\phi_a} \right] = 0. \quad (4.6)$$

In the above equation α_a are constant parameters and Γ denotes the full 1-PI vertex functional (the generator of 1-PI amplitudes).

B. Power-counting

The potentially dangerous interaction terms are the ones involving two derivatives arising from the fluctuation around the SU(2) constraint, namely

$$\int d^4x \frac{1}{v}(X_1 + X_2)\square\left(\frac{1}{2}\sigma^2 + \frac{1}{2}\phi_a^2\right). \quad (4.7)$$

1-PI Green's functions involving external X_1 and X_2 legs are not independent, since they can be obtained through the functional identities Eqs. (4.2) and (4.5) in terms of amplitudes only involving insertions of σ' , ϕ_a and \bar{c}^* . For these amplitudes the dangerous interaction vertices in Eq.(4.7) are always connected inside loops to the combination X . Since the propagator Δ_{XX} falls off as $1/p^4$ for large momenta, it turns out that the theory is still renormalizable by power counting.

The UV indices of the fields and the external source \bar{c}^* are as follows: σ' and ϕ_a have UV dimension 1, \bar{c}^* has UV dimension 2.

C. Structure of the counterterms

We consider the action-like sector independent of X_1 and X_2 , since amplitudes involving these latter fields are controlled by Eqs.(4.2) and (4.5).

Eq.(4.6) entails that the dependence on σ and ϕ_a can only happen through action-like functionals invariant under global SU(2) symmetry, namely

$$\begin{aligned} \mathcal{L}_{ct,1} = & -\mathcal{Z}\left(\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}\partial^\mu\phi_a\partial_\mu\phi_a\right) - \mathcal{M}\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2\right) \\ & - \mathcal{G}\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2\right)^2 - \mathcal{R}_1\bar{c}^*\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2\right). \end{aligned} \quad (4.8)$$

There also two invariants depending only on \bar{c}^* , i.e.

$$\mathcal{L}_{ct,2} = -\mathcal{R}_2\bar{c}^* - \frac{1}{2}\mathcal{R}_3(\bar{c}^*)^2. \quad (4.9)$$

The most general counterterm Lagrangian at $X_1 = X_2 = 0$ is thus given by

$$\mathcal{L}_{ct} = \mathcal{L}_{ct,1} + \mathcal{L}_{ct,2}. \quad (4.10)$$

Notice the appearance of a quartic potential term absent in the classical action (2.1). It can be introduced from the beginning into the classical action without violating power-counting renormalizability, as was done in [17]. Notice that if one adds the invariant $\int d^4x \mathcal{G}^{(0)}\left(\frac{1}{2}\sigma^2 + 2v\sigma + \frac{1}{2}\phi_a^2\right)^2$ at tree level, the physical content of the theory does not change ($\mathcal{G}^{(0)}$ is not an additional physical parameter). Indeed this term can be rewritten as

$$\int d^4x \mathcal{G}^{(0)}\left(\frac{1}{2}\sigma^2 + 2v\sigma + \frac{1}{2}\phi_a^2\right)^2 = \int d^4x s\left[\mathcal{G}^{(0)}\bar{c}\left(\frac{1}{2}\sigma^2 + 2v\sigma + \frac{1}{2}\phi_a^2 + vX_2\right)\right] + \mathcal{G}^{(0)}v^2X_2^2 \quad (4.11)$$

and this amounts to a redefinition of the mass parameter $M^2 \rightarrow M^2 - \frac{1}{2}\mathcal{G}^{(0)}v^2$ plus a s -exact term that does not affect the physics. The relevant deformations of the functional identities controlling the theory when $\mathcal{G}^{(0)}$ is non-zero has been given in [17].

D. One-loop divergences

The one-loop divergences are controlled by the six coefficients $\mathcal{Z}^{(1)}, \mathcal{M}^{(1)}, \mathcal{G}^{(1)}, \mathcal{R}^{(j)}$. Amplitudes are dimensionally regularized in D dimensions.

The amplitude² $\Gamma_{\bar{c}^*}^{(1)}$ fixes $\mathcal{R}^{(2)} = -\frac{1}{16\pi^2} \frac{M^2}{4-D}$. $\Gamma_{\bar{c}^* \bar{c}^*}^{(1)}$ fixes $\mathcal{R}^{(3)} = \frac{1}{4\pi^2} \frac{1}{4-D}$. $\Gamma_{\bar{c}^* \sigma'}^{(1)}$ fixes $\mathcal{R}^{(1)} = -\frac{1}{8\pi^2} \frac{M^2}{v^2} \frac{1}{4-D}$.

Moreover

$$\Gamma_{\phi_a \phi_b}^{(1)} \Big|_{UVdiv} = \frac{1}{8\pi^2} \frac{M^4}{v^2} \frac{\delta_{ab}}{4-D}. \quad (4.12)$$

The divergent part has no momentum dependence. This implies that $\mathcal{Z}^{(1)} = 0$ and $\mathcal{M}^{(1)} = \frac{1}{8\pi^2} \frac{M^4}{v^2} \frac{1}{4-D}$. Finally from the amplitude $\Gamma_{\sigma' \sigma'}^{(1)}$, one obtains the coefficient $\mathcal{G}^{(1)} = \frac{1}{8\pi^2} \frac{M^4}{v^4} \frac{1}{4-D}$.

Let us now compute the divergences of the one- and two-point functions of X_1 and X_2 . This requires to use Eqs.(4.2) and (4.5). One finds (we denote the external momenta as arguments of the fields)

$$\Gamma_{X_{1(0)}}^{(n)} = \Gamma_{X_{2(0)}}^{(n)} = 0, \quad n \geq 1. \quad (4.13)$$

Notice that Eqs.(4.2) and (4.5) holds in the $\sigma - X_1 - X_2$ (canonical) basis. If one performs explicit computations in the most convenient diagonal $\sigma' - X_1 - X_2$ basis one needs to take into account the contributions arising from the field redefinition from σ' to σ , namely on the example of the one-point functions (we denote by an underline the Green's functions in the diagonal basis whenever they differ from those in the canonical basis)

$$\int d^4x \left(\Gamma_{\sigma' \sigma'}^{(1)} \sigma' + \underline{\Gamma}_{X_1}^{(1)} X_1 + \underline{\Gamma}_{X_2}^{(1)} X_2 \right) = \int d^4x \left[\Gamma_{\sigma'}^{(1)} \sigma + (\underline{\Gamma}_{X_1}^{(1)} - \Gamma_{\sigma'}^{(1)}) X_1 + (\underline{\Gamma}_{X_2}^{(1)} - \Gamma_{\sigma'}^{(1)}) X_2 \right] \quad (4.14)$$

so that one finds by explicit computations

$$\Gamma_{\sigma'(0)}^{(1)} = \underline{\Gamma}_{X_1(0)}^{(1)} = \underline{\Gamma}_{X_2(0)}^{(1)} = \frac{1}{16\pi^2} \frac{M^2}{v} A_0(M^2) \quad (4.15)$$

in terms of the standard Passarino-Veltman scalar function $A_0(M^2)$ (we use the conventions of [27, 28]). Hence from Eq.(4.14)

$$\Gamma_{X_1(0)}^{(1)} = \underline{\Gamma}_{X_1(0)}^{(1)} - \Gamma_{\sigma'(0)}^{(1)} = 0, \quad \Gamma_{X_2(0)}^{(1)} = \underline{\Gamma}_{X_2(0)}^{(1)} - \Gamma_{\sigma'(0)}^{(1)} = 0, \quad (4.16)$$

consistent with Eq.(4.13).

² We denote by a subscript the fields and external sources w.r.t. which one differentiates, e.g. $\Gamma_{\bar{c}^*} = \frac{\delta \Gamma}{\delta \bar{c}^*}$. It is understood that we set all fields and external sources to zero after differentiation.

For the two-point functions in the canonical basis we get

$$\Gamma_{X_2(-p)X_2(p)}^{(n)} = \Gamma_{X_1(-p)X_1(p)}^{(n)} = \Gamma_{X_1(-p)X_2(p)}^{(n)} = \frac{1}{v^2} p^4 \Gamma_{\bar{c}^*(-p)\bar{c}^*(p)}^{(n)}, \quad n \geq 1, \quad (4.17)$$

so that the common UV divergent part is

$$\frac{1}{v^2} p^4 \Gamma_{\bar{c}^*(-p)\bar{c}^*(p)}^{(n)} \Big|_{UV\,div} = \frac{1}{4\pi^2} \frac{p^4}{v^2} \frac{1}{4-D}.$$

E. Normalization conditions

The two-point functions of X_1 and X_2 in the diagonal basis read

$$\underline{\Gamma}_{X_1(-p)X_1(p)}^{(n)} = \underline{\Gamma}_{X_2(-p)X_2(p)}^{(n)} = \underline{\Gamma}_{X_1(-p)X_2(p)}^{(n)} = \frac{p^4}{v^4} \Gamma_{\bar{c}^*(-p)\bar{c}^*(p)}^{(n)} - \Gamma_{\sigma'(-p)\sigma'(p)}^{(n)}. \quad (4.18)$$

The finite part of the coefficient $\mathcal{R}_3^{(n)}$ is chosen order by order in the loop expansion so that

$$\underline{\Gamma}_{X_1(-p)X_1(p)}^{(n)} \Big|_{p^2=M^2} = \underline{\Gamma}_{X_1(-p)X_2(p)}^{(n)} \Big|_{p^2=M^2} = 0. \quad (4.19)$$

This ensures that there is no mixing between X_1 and X_2 at the pole $p^2 = M^2$. By Eq.(4.18) this also implies that

$$\underline{\Gamma}_{X_2(-p)X_2(p)}^{(n)} \Big|_{p^2=M^2} = 0, \quad (4.20)$$

i.e. one is performing an on-shell renormalization, fixing the position of the physical pole of the X_2 mode at its tree level value M^2 .

The finite part of the coefficient $\mathcal{R}_2^{(n)}$ is chosen order by order in the loop expansion in such a way to ensure the absence of a tadpole contribution to X_2 :

$$\underline{\Gamma}_{X_2(0)}^{(n)} \Big|_{p^2=M^2} = -\frac{p^2}{v} \Gamma_{\bar{c}^*(0)}^{(n)} \Big|_{p^2=M^2} + \Gamma_{\sigma'(0)}^{(n)} \Big|_{p^2=M^2} = 0. \quad (4.21)$$

The finite part of the coefficient $\mathcal{R}_1^{(n)}$ is adjusted so that the mixing between σ' and X_2 vanishes on the pole $p^2 = M^2$:

$$\underline{\Gamma}_{X_2(-p)\sigma'(p)}^{(n)} \Big|_{p^2=M^2} = -\frac{p^2}{v} \Gamma_{\sigma'(-p)\bar{c}^*(p)}^{(n)} \Big|_{p^2=M^2} + \Gamma_{\sigma'(-p)\sigma'(p)}^{(n)} \Big|_{p^2=M^2} = 0. \quad (4.22)$$

Eqs.(4.19), (4.21) and (4.22) ensure that the massive physical scalar mode is described by the field X_2 to all orders in the loop expansion.

The finite parts of the remaining coefficients $\mathcal{M}^{(n)}$, $\mathcal{G}^{(n)}$ and $\mathcal{Z}^{(n)}$ are chosen in order to ensure the absence of a tadpole for σ' and the on-shell normalization conditions for σ' , namely

$$\Gamma_{\sigma'(0)}^{(n)} \Big|_{p^2=0} = 0, \quad \Gamma_{\sigma'(-p)\sigma'(p)}^{(n)} \Big|_{p^2=0} = 0, \quad \frac{d}{dp^2} \Gamma_{\sigma'(-p)\sigma'(p)}^{(n)} \Big|_{p^2=0} = 1. \quad (4.23)$$

V. INCLUSION IN THE STANDARD MODEL

We now discuss how the Higgs potential in the derivative representation is included into the Standard Model (SM).

X_2, X_1 and the ghosts c, \bar{c} are invariant under the electroweak gauge group $SU_L(2) \times U_Y(1)$ of weak isospin and hypercharge. The $SU_L(2)$ doublet Φ transforms as usual under an infinitesimal gauge transformation

$$\delta\Phi = \left(-\frac{i}{2}\alpha_Y + i\frac{\sigma_a}{2}\alpha_a \right)\Phi \quad (5.1)$$

where α_a and α_Y are the gauge parameters of $SU_L(2)$ and $U_Y(1)$ respectively and σ_a are the Pauli matrices. This implies that the couplings with the gauge fields and the fermions are the same as in the SM.

The SM action can be written as the sum of five terms:

$$S_{SM} = S_{YM} + S_H + S_F + S_{g.f.} + S_{ghost}. \quad (5.2)$$

$S_{YM}, S_H, S_F, S_{g.f.}, S_{ghost}$ are respectively the Yang-Mills, the Higgs, the fermion, the gauge-fixing and the ghost parts. S_{YM} and S_F are the same as in the ordinary formulation of the theory and are given for the sake of completeness in Appendix A, where we also collect our notations.

The Higgs part S_H is obtained from Eq.(2.1) upon replacement of ordinary derivatives with the covariant ones and by adding the Yukawa sector as in the ordinary formulation of the SM:

$$\begin{aligned} S_H = \int d^4x & \left[(D_\mu\Phi)^\dagger D^\mu\Phi - \frac{M^2}{2}X_2^2 \right. \\ & - \sum_{i,j} \left(g_{ij}\bar{\Psi}_i^L\Psi_{j,-}^R - \Phi + \tilde{g}_{ij}\bar{\Psi}_i^L\Psi_{j,+}^R + \Phi^C + h.c. \right) \\ & \left. + \frac{1}{v}(X_1 + X_2)\square\left(\Phi^\dagger\Phi - \frac{v^2}{2} - vX_2\right) \right]. \quad (5.3) \end{aligned}$$

Φ^C is the charge conjugated field $\Phi^C = i\sigma^2\Phi^*$.

Spontaneous symmetry breaking induces a mixing between ϕ_a and ∂A_a . With the choice of the doublet as in Eq.(2.2) the mixed bilinear terms read

$$\int d^4x \left(\frac{g_2 v}{2}\phi_a\partial A_a + \delta_{a3}\frac{g_1 v}{2}\partial B\phi_3 \right). \quad (5.4)$$

ϕ_3 is coupled to the divergence of the Z field, obtained from the Weinberg rotation as (A_μ is the photon)

$$A_\mu = c_W B_\mu - s_W A_{3\mu}, \quad Z_\mu = s_W B_\mu + c_W A_{3\mu}. \quad (5.5)$$

The sine and cosine of the Weinberg angle are given by

$$c_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \quad s_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}. \quad (5.6)$$

It is also convenient to introduce the charged combinations

$$W^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2), \quad \phi^\pm = \frac{1}{\sqrt{2}}(\phi^1 \mp i\phi^2) \quad (5.7)$$

The masses of the gauge bosons W^\pm, Z are $M_W = \frac{g_2 v}{2}$, $M_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}$. The mixings in Eq.(5.4) are cancelled in a renormalized ξ -gauge by choosing

$$S_{g.f.} = \int d^4x \left(b^+ \mathcal{F}^- + b^- \mathcal{F}^+ + b^Z \mathcal{F}^Z + b^A \mathcal{F}^A + \xi_W b^+ b^- + \frac{\xi_Z}{2} (b^Z)^2 + \frac{\xi_A}{2} (b^A)^2 \right) \quad (5.8)$$

with the gauge-fixing functions

$$\mathcal{F}^\pm = \partial W^\pm + \xi_W M_W \phi^\pm, \quad \mathcal{F}^Z = \partial Z + M_Z \xi_Z \phi_3, \quad \mathcal{F}^A = \partial A. \quad (5.9)$$

Finally one constructs the ghost dependent part by summing the SM ghost sector and Eq.(2.9)

$$S_{ghost} = \int d^4x \left(-\bar{c}^+ \mathfrak{s} \mathcal{F}^- - \bar{c}^- \mathfrak{s} \mathcal{F}^+ - \bar{c}^Z \mathfrak{s} \mathcal{F}^Z - \bar{c}^A \mathfrak{s} \mathcal{F}^A - \bar{c} \square c \right). \quad (5.10)$$

In the above equation \mathfrak{s} is the BRST differential associated with the $SU_L(2) \times U_Y(1)$ electroweak gauge group presented in Appendix A 1.

The full BRST symmetry of the theory is given by $\tilde{s} = s + \mathfrak{s}$. $\tilde{s}^2 = 0$ since both \mathfrak{s} and s are nilpotent and s and \mathfrak{s} anticommute, as a consequence of the fact that the constraint in Eq.(2.6) is invariant under \mathfrak{s} .

The physical states of the theory can then be identified as those belonging to the space

$\mathcal{H}_{phys} = \text{Ker } \tilde{Q}_0 / \text{Im } \tilde{Q}_0$, where \tilde{Q}_0 is the asymptotic charge associated with \tilde{s} :

$$\begin{aligned}
[\tilde{Q}_0, A_\mu] &= \partial_\mu c^A, [\tilde{Q}_0, Z_\mu] = \partial_\mu c^Z, [\tilde{Q}_0, W^\pm]_\mu = \partial_\mu c^\pm, \\
[\tilde{Q}_0, \sigma] &= 0, [\tilde{Q}_0, \phi^\pm] = M_W c^\pm, [\tilde{Q}_0, \phi_3] = M_Z c^Z, \\
[\tilde{Q}_0, X_2] &= 0, [\tilde{Q}_0, X_1] = v c, \\
[\tilde{Q}_0, \Psi_i^L]_+ &= 0, [\tilde{Q}_0, \Psi_{i,\sigma}^R]_+ = 0, \\
[\tilde{Q}_0, \bar{c}]_+ &= v(\sigma - X_2), [\tilde{Q}_0, \bar{c}^A]_+ = b^A, [\tilde{Q}_0, \bar{c}^Z]_+ = b^Z, [\tilde{Q}_0, \bar{c}^\pm]_+ = b^\pm, \\
[\tilde{Q}_0, b^A] &= [\tilde{Q}_0, b^Z] = [\tilde{Q}_0, b^\pm] = 0, \\
[\tilde{Q}_0, c]_+ &= [\tilde{Q}_0, c^A]_+ = [\tilde{Q}_0, c^Z]_+ = [\tilde{Q}_0, c^\pm]_+ = 0.
\end{aligned} \tag{5.11}$$

One sees that the physical states are the physical polarizations of the gauge fields W_μ^\pm, A_μ, Z_μ , the fermion fields and the scalar X_2 . Notice that σ is \tilde{Q}_0 -invariant, however it belongs to the same cohomology class of X_2 , since

$$X_2 = \sigma - \frac{1}{v} [\tilde{Q}_0, \bar{c}]_+. \tag{5.12}$$

The antighosts and the Nakanishi-Lautrup fields drop out of \mathcal{H}_{phys} being BRST doublets, the ghosts and the pseudo-Goldstone bosons do not belong to \mathcal{H}_{phys} , since they also form BRST doublets. The standard quartet mechanism [20, 29–32] is at work.

The equations (4.1), (4.2) and (4.5) controlling the dependence of the vertex functional Γ on X_1, X_2, \bar{c} and c do not change.

We remark that the sector spanned by X_1, X_2, \bar{c} and c respects custodial symmetry provided that X_1, X_2, \bar{c} and c do not transform under the global $SU_L(2) \times SU_R(2)$ group. This can be seen by introducing the matrix

$$\Omega = (\Phi^C, \Phi). \tag{5.13}$$

Since $\Phi^\dagger \Phi = \frac{1}{2} \text{Tr}(\Omega^\dagger \Omega)$, we see that the last line of Eq.(5.3) is invariant under the custodial transformation $\Omega' = V_L \Omega V_R^\dagger$, $V_L \in SU_L(2)$, $V_R \in SU_R(2)$.

VI. THE z -MODEL

There is a unique term that can be added to the classical action in order to preserve Eq.(4.5) at the quantum level by deforming its r.h.s. by a linear term in the quantized

fields, namely a kinetic term for X_2

$$\int d^4x \frac{z}{2} \partial^\mu X_2 \partial_\mu X_2. \quad (6.1)$$

By the same arguments leading to Eq.(2.5), upon eliminating X_2 by imposing the X_1 -equation of motion one obtains at tree-level the dimension-six operator

$$\int d^4x \frac{z}{v^2} \partial_\mu \Phi^\dagger \Phi \partial^\mu \Phi^\dagger \Phi. \quad (6.2)$$

The functional identities controlling the theory are unchanged with the exception of Eq.(4.5), which becomes

$$\frac{\delta\Gamma}{\delta X_2} = \frac{1}{v} \square \frac{\delta\Gamma}{\delta \bar{c}^*} + \square(X_1 + (1-z)X_2) - M^2 X_2 - v \bar{c}^*. \quad (6.3)$$

Notice that this equation is still valid for the SM with the inclusion of the kinetic term for the X_2 -field in Eq.(6.1). The propagator of X_2 is modified as follows

$$\Delta_{X_2 X_2} = \frac{i}{(1+z)p^2 - M^2}. \quad (6.4)$$

Power-counting renormalizability is lost since the cancellation mechanism between the propagator of X_1 and X_2 is no more at work at $z \neq 0$. Indeed the propagator for the combination $X = X_1 + X_2$ is now

$$\Delta_{XX} = \frac{i(-zp^2 + M^2)}{p^2[(1+z)p^2 - M^2]}, \quad (6.5)$$

so that at $z \neq 0$ the propagator falls off as p^{-2} for large momentum p and therefore cannot compensate the contributions from the derivative interaction vertices.

The dependence on z can be controlled by the following differential equation

$$\frac{\partial\Gamma}{\partial z} = \int d^4x \frac{\delta\Gamma}{\delta R(x)} \quad (6.6)$$

where $R(x)$ is the source coupled in the classical action to the composite operator $\mathcal{O}(x) = -\frac{1}{2}X_2 \square X_2$. Notice that the insertion of the operator $\mathcal{O}(x)$ happens at zero momentum (due to the integration over d^4x).

This in turn entails that the Green's functions at $z \neq 0$ can be defined as a perturbative expansion around the power-counting renormalizable model at $z = 0$.

A comment is in order here. Eq.(6.3) relates the Green's functions with the insertion of X_2 -lines with those with the insertion of the source \bar{c}^* , which has a better UV behaviour.

The simplest example is the two-point function for X_2 , which is obtained by differentiating Eq.(6.3) w.r.t. X_2 and then by replacing $\Gamma_{X_2\bar{c}^*}$ by using once more Eq.(6.3), this time after differentiation w.r.t. \bar{c}^* . In momentum space we find

$$\Gamma_{X_2(-p)X_2(p)}^{(n)} = \frac{1}{\nu^2} p^4 \Gamma_{\bar{c}^*(-p)\bar{c}^*(p)}^{(n)}, \quad n \geq 1. \quad (6.7)$$

The above equation states that the divergence of the two-point function $\Gamma_{X_2(-p)X_2(p)}^{(n)}$ in the canonical basis does not have a constant or a p^2 -term. Therefore there cannot be any mixing between the operator $X_2\Box^2 X_2$ and the operator X_2^2 or $X_2\Box X_2$ (off-shell and by forbidding field redefinitions). This goes along the same bulk of patterns as those observed in the one-loop anomalous dimensions studied in [9–13], although one cannot establish an immediate and straightforward correspondence, due to the fact that field redefinitions are used in [9–13] and moreover those results are valid for one-loop on-shell matrix elements (while Eq.(6.7) holds off-shell and to all orders in the loop expansion). We stress the fact that Eq.(6.7) is valid at any value for z .

A systematic study of the mixing constraints arising from Eq.(6.3) in the full SM with the dimension-six operator induced by the kinetic term in Eq.(6.1) is beyond the scope of the present paper and is currently under investigation [18].

VII. EXPANSION AROUND $z = 0$

Eq.(6.6) states that the contribution of the order z^n of a 1-PI Green's function γ is obtained by a repeated insertion of the integrated operator $\int d^4x \mathcal{O}(x)$ in the X_2 -lines of each diagram contributing to γ in the theory at $z = 0$.

This can be easily understood since the propagator $\Delta_{X_2X_2}$ can be Taylor-expanded according to

$$\Delta_{X_2X_2} = \frac{i}{(1+z)p^2 - M^2} = \frac{i}{(p^2 - M^2) \left[1 + \frac{zp^2}{p^2 - M^2} \right]} = \frac{i}{p^2 - M^2} \sum_{n=0}^{\infty} (-1)^n \left(\frac{zp^2}{p^2 - M^2} \right)^n. \quad (7.1)$$

The term of order n in the r.h.s. of the above equation is generated by the insertion of n vertices izp^2 on the X_2 -line, namely n insertions of the operators $\int d^4x \mathcal{O}(x)$ at zero external momenta (see Fig. 1).

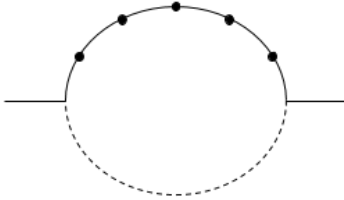


FIG. 1: Mass insertions (the dots) on a X_2 propagator (drawn as a solid line)



FIG. 2: Diagrams contributing to $\Gamma_{\sigma'\sigma}^{(1)}$ (thin solid lines denote the X_2 propagator, thick solid lines the σ' propagator. A dashed line denote a X_1 propagator)

We remark that the decomposition of the propagator $\Delta_{X_2 X_2}$ at $z \neq 0$ can be expressed as the result of repeated mass insertions according to the following identity

$$\Delta_{X_2 X_2} = \sum_{n=0}^{\infty} (-1)^n z^n \left[1 + (1 - \delta_{n,0}) \sum_{k=1}^n \binom{n}{k} \frac{M^{2k}}{k} \frac{\partial^k}{\partial (M^2)^k} \right] \frac{i}{p^2 - M^2}. \quad (7.2)$$

A. Two-point function

The z -dependence of the diagrams can be derived according to a simple prescription, namely one multiplies each diagram in the power-counting renormalizable theory at $z = 0$, with N internal X_2 -lines and contributing to γ , by a prefactor $(1 + z)^{-N}$ and replaces $M^2 \rightarrow M^2/(1 + z)$. Then the UV divergences are related directly to those of the amplitudes evaluated at $z = 0$.

We consider here as an example the two-point amplitude $\Gamma_{\sigma'\sigma}^{(1)}$ in the linear σ model. There are five diagrams contributing to this amplitude, depicted in Fig. 2.

The first two diagrams are UV convergent, the third and the fourth contain one X_2 -line, the fifth two internal X_2 -lines. This dictates the behaviour of the prefactors $1/(1 + z)$ in front of each amplitude.

The physical mass of the canonically normalized X_2 field is

$$M_{phys}^2 = \frac{M^2}{1+z}. \quad (7.3)$$

If one fixes M_{phys} (which, when the model is embedded as the scalar sector of the electroweak theory, corresponds to the measured Higgs mass), then for amplitudes expressed in terms of M_{phys} there are no further sources of z -dependence other than the prefactors $1/(1+z)$.

Considering M as the mass of new physics, the limit $M \rightarrow \infty$ is equivalent to $z \rightarrow \infty$. In this limit the diagrams involving the exchange of internal X_2 -lines go to zero and one gets back the non-linear σ model described in [17].

In terms of M_{phys} the one-loop divergence of $\Gamma_{\sigma'\sigma'}^{(1)}$ at finite z is

$$\Gamma_{\sigma'\sigma'}^{(1)UV} = \frac{M_{phys}^2}{8\pi^2 v^2 (4-D)} \left[3M_{phys}^2 \frac{1-z}{(1+z)^2} - p^2 \frac{z}{(1+z)^2} \right]. \quad (7.4)$$

For large z it is of order $1/z$, as expected (since the diagrams 1 and 2 are not UV divergent and the remaining three contain at least one X_2 -line). At $z = 0$ one recovers the divergence of the power-counting renormalizable theory.

B. Finite renormalizations

Consider now in the theory at $z = 0$ an amplitude γ with superficial degree of divergence $\delta(\gamma)$. Let us denote by $\gamma_{R(q_1)\dots R(q_n)}|_{q_k=0}$ the diagram with n insertions of the operator \mathcal{O} at zero momentum. The superficial degree of divergence $\delta(\gamma_{R(q_1)\dots R(q_n)}|_{q_k=0}) = \delta(\gamma)$, since each of the terms in the series of the r.h.s. of Eq.(7.1) tends to 1 for large p and thus does not alter the overall behaviour as $1/p^2$ of the propagator $\Delta_{X_2 X_2}$.

This implies that in the power-counting renormalizable theory at $z = 0$ the external source R has UV dimension zero.

Consequently when one allows in the theory at $z = 0$ for the insertion of the operator $\int d^4x \mathcal{O}(x)$, the most general form of the counterterms is no more given by Eqs.(4.8) and (4.9).

The problem of identifying the counterterms in the power-counting renormalizable theory at $z = 0$ in the presence of the source $R(x)$ amounts to find all possible Lorentz-covariant, global SU(2)-invariant local monomials in the fields, the external sources and their derivatives of dimension ≤ 4 .

We can safely disregard all monomials involving derivatives of $R(x)$, since we are only interested in zero momentum insertions. Then Eq.(4.10) is modified as follows:

$$\begin{aligned} \mathcal{L}_{ct,R} = & \mathcal{A} - \mathcal{Z}_R \left(\frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a \right) - \mathcal{M}_R \left(\frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 \right) \\ & - \mathcal{G}_R \left(\frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 \right)^2 - \mathcal{R}_{1,R\bar{C}^*} \left(\frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 \right) - \mathcal{R}_{2,R\bar{C}^*} - \frac{1}{2} \mathcal{R}_{3,R} (\bar{C}^*)^2, \end{aligned} \quad (7.5)$$

where $\mathcal{Z}_R, \mathcal{M}_R, \mathcal{G}_R$ and $\mathcal{R}_{j,R}$, $j = 1, 2, 3$ (which we collectively denote by $\mathcal{K}_{l,R}$, with \mathcal{K}_l standing for the corresponding quantities in Eqs.(4.8) and (4.9)) become analytic functionals of the integrated source $R(x)$:

$$\mathcal{K}_{l,R} = \mathcal{K}_l + \sum_{k=1}^{\infty} \frac{1}{k!} \int \prod_{i=1}^k d^4 x_i K_l^k R(x_1) \dots R(x_k) \quad (7.6)$$

while

$$\mathcal{A} = \sum_{k=1}^{\infty} \frac{1}{k!} \int \prod_{i=1}^k d^4 x_i A^k R(x_1) \dots R(x_k)$$

controls the renormalization of the \mathcal{O} -insertions into vacuum amplitudes.

The finite parts of the coefficients A^k, K_l^k are unconstrained by the symmetries of the theory and have to be chosen by an (infinite) set of normalization conditions. This is the counterpart at $z = 0$ of the ambiguities induced by the loss of power-counting renormalizability at $z \neq 0$.

VIII. UV COMPLETION

Under the assumption that all scalar fields should obey at the classical level asymptotically Klein-Gordon equations of motion, the most general X_2 -equation is given by Eq.(6.3) and includes the kinetic term controlling the violation of power-counting renormalizability. The fact that X_2 is a $SU(2)$ singlet entails however on symmetry grounds that more general quadratic terms in the tree-level action can be considered without violating the defining functional identities of the theory.

These terms might capture some features of the UV completion of the model from a more fundamental theory, valid at a much higher scale Λ . This possibility is a peculiar feature of the higher derivative formulation where the physical parameters of the theory are embodied in the two-point sector.

As an example, the bad UV behaviour of the X propagator at $z \neq 0$ can be regularized by the following choice of the X_2 -propagator

$$\Delta_{X_2 X_2} = \frac{i}{(1+z)p^2 - M^2} - \frac{i}{\Lambda^2} \left[\exp\left(-\frac{z}{z+1} \frac{\Lambda^2}{p^2}\right) - 1 \right]. \quad (8.1)$$

For large momenta $\Delta_{X_2 X_2}$ goes as

$$\Delta_{X_2 X_2} \sim_{p \rightarrow \infty} \frac{i}{p^2} + i \frac{M^2 - \frac{\Lambda^2}{2} z^2}{(1+z)^2 p^4} \quad (8.2)$$

so that Δ_{XX} goes as p^{-4} , making the derivative interaction terms harmless. Of course such a deformation is not unique (for instance the replacement $\exp\left(-\frac{z}{z+1} \frac{\Lambda^2}{p^2}\right) \rightarrow \exp\left(-z \frac{\Lambda^2}{(1+z)p^2 - M^2}\right)$ into Eq.(8.1) would also work).

This implies that one can regularize the propagator of the X_2 field in the UV in such a way to preserve power-counting renormalizability without destroying the symmetries of the model and without introducing new physical poles in the spectrum.

Power-counting renormalizability at $z \neq 0$ can also be restored by adding new physical SU(2)-invariant modes. For that purpose consider the following propagator for the field X_2 (now a field with a non-trivial Källén-Lehmann spectral density):

$$\Delta_{X_2 X_2} = \frac{i}{(1+z)p^2 - M^2} + \sum_{j=1}^N \frac{i}{z_j p^2 - M_j^2}, \quad (8.3)$$

where $z_j > 0$. The propagator in Eq.(8.3) describes a set of physical resonances at mass $M^2/(1+z)$ and M_j^2/z_j . A suitable choice of the z_j allows to cancel the UV behaviour in $1/p^2$ of Δ_{XX} , therefore re-establishing power-counting renormalizability. For simplicity we choose all the z_j to have the same common value z_c . Then the leading order in $1/p^2$ of the propagator Δ_{XX} is cancelled provided that one chooses

$$z_c = N \frac{1+z}{z} = N \frac{M^2}{M^2 - M_{phys}^2}. \quad (8.4)$$

The positivity condition on z_c yields $M > M_{phys}$.

Both with a UV-regularized propagator and with the addition of further resonances at higher masses, the X_2 -equation now takes the form (written for convenience in the Fourier space)

$$\frac{\delta\Gamma}{\delta X_2(p)} = -\frac{1}{v} p^2 \frac{\delta\Gamma}{\delta \bar{c}^*(p)} - p^2 X_1 + \mathcal{K}(p) X_2(p) - v \bar{c}^*(p), \quad (8.5)$$

where

$$\mathcal{K}^{-1}(p) = -i\Delta_{X_2 X_2}(p). \quad (8.6)$$

The r.h.s. of Eq.(8.5) is still linear in the quantized fields.

It is suggestive that such pieces of information about the UV completion can be embodied into the two-point function of the X_2 scalar, without the need to modify the interaction sector of the theory.

Linearity of Eq.(8.5) imposes strong constraints on the allowed potential. As an example, let us consider the theory with two physical massive scalars, i.e. $X_2 = \chi_1 + \chi_2$ where χ_1 has mass $M/\sqrt{1+z}$ and χ_2 has mass $M_1/\sqrt{z_1}$. Since both χ_1 and χ_2 are BRST-invariant, bilinear mixing terms involving these fields can be removed and the quadratic part of the action reads

$$S_2 = \int d^4x \left(-\frac{1}{2}\sigma'\square\sigma' + \frac{1}{2}X_1\square X_1 - \frac{z_1}{2}\chi_1\square\chi_1 - \frac{M_1^2}{2}\chi_1^2 - \frac{1+z}{2}\chi_2\square\chi_2 - \frac{M^2}{2}\chi_2^2 \right). \quad (8.7)$$

The propagator of X_2 is then given by Eq.(8.3). The interaction terms are obtained by replacing $X_2 = \chi_1 + \chi_2$. The inclusion of the field χ_1 allows to reproduce the Higgs singlet model (HSM), where in addition to the Higgs doublet the real scalar singlet χ_1 is introduced [33–40]. The most general tree-level potential compatible with SU(2) symmetry and of dimension ≤ 4 is

$$V = -v\mu_\Phi^2 X_2 + \frac{M_2^2}{2}X_2^2 + v\mu_{\Phi\chi}X_2\chi_1 + v^2\frac{\lambda_{\Phi\chi}}{2}X_2\chi_1^2 + \mu_\chi^3\chi_1 + \frac{M_1^2}{2}\chi_1^2 + \frac{\mu'_\chi}{3}\chi_1^3 + \frac{\lambda_\chi}{4}\chi_1^4 \quad (8.8)$$

Going on-shell with X_1 one again finds $X_2 = \frac{1}{v}\Phi^\dagger\Phi$ and upon substitution in Eq.(8.8) we obtain the standard formulation of the HSM potential. The coefficient of the quartic term $(\Phi^\dagger\Phi)^2$ is $\lambda_\Phi = \frac{M_2^2}{2v^2}$.

Imposing linearity of the X_2 -equation yields the constraint

$$\lambda_{\Phi\chi} = 0 \quad (8.9)$$

Provided that $\lambda_\chi > 0$, positivity of M_2 and Eq.(8.9) ensures the fulfillment of the vacuum stability condition, namely $\lambda_\Phi > 0, \lambda_\chi > 0, 4\lambda_\Phi\lambda_\chi > \lambda_{\Phi\chi}^2$ [40].

IX. CONCLUSIONS

The linear σ model can be formulated in such a way that the mass of the physical scalar particle only appears in the quadratic part of the action (and not also in the potential

coupling) by introducing suitable derivative interactions and additional unphysical fields pairing into BRST doublets.

In the present paper we have given all technical tools required to study such a model and its extension to the SM.

Power-counting renormalizability has been established and the BRST symmetry guaranteeing the cancellation of the unphysical degrees of freedom has been given.

The 1-PI amplitudes involving the physical massive scalar field X_2 are determined in terms of external sources with a better UV behaviour by a functional identity implementing at the quantum level the equation of motion for X_2 .

Remarkably, such an identity admits a unique deformation, given by a kinetic term for X_2 whose coefficient we have denoted by z .

Once such a kinetic term is introduced into the classical action, power-counting renormalizability is lost. Violation of power-counting renormalizability is controlled by the parameter z . At $z = 0$ we recover the original power-counting renormalizable model.

The amplitudes of the full theory can be expanded as a power series in z with coefficients given by diagrams of the original theory at $z = 0$. Each order in z is associated to an additional zero-momentum insertion of the kinetic operator into X_2 -lines.

This property can be used to express the structure of the counterterms of the non power-counting renormalizable theory at $z \neq 0$ in terms of analytic functionals of the integrated source $R(x)$ coupled to the kinetic term of the X_2 field.

The theory studied in this paper provides a novel example of a non power-counting renormalizable model, defined as a series expansion around the power-counting renormalizable theory at $z = 0$.

The proposed representation of the Higgs potential does not spoil any of the SM symmetries as well as the custodial symmetry, in the limit where the $U_Y(1)$ coupling constant vanishes.

The X_2 -functional equation at $z \neq 0$ holds in the SM deformed by the kinetic term for X_2 . This paves the way for a rigorous all-orders algebraic study of the renormalization properties of the effective operator $\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$ in the Higgs EFT.

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Appendix A: Electroweak SM Lagrangian

The generators of the weak isospin $SU_L(2)$ group are $I_a = \frac{\sigma_a}{2}$ where σ_a , $a = 1, 2, 3$ denote the Pauli matrices.

The Yang-Mills part S_{YM} of the SM Lagrangian is

$$S_{YM} = \int d^4x \left(-\frac{1}{4} G_{a\mu\nu} G_a^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \quad (\text{A1})$$

where the field strength $G_{a\mu\nu}$ is given in terms of the non-Abelian gauge fields $A_{a\mu}$ by (f_{abc} are the $SU_L(2)$ structure constants)

$$G_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + g_2 f_{abc} A_{b\mu} A_{c\nu} \quad (\text{A2})$$

and the Abelian $U_Y(1)$ field strength $F_{\mu\nu}$ is (B_μ is the hypercharge $U_Y(1)$ vector field)

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (\text{A3})$$

g_1, g_2 are the $U_Y(1)$ and $SU_L(2)$ coupling constants respectively.

The fermionic part S_F is

$$S_F = \int d^4x \left(i \sum_i \bar{\Psi}_i^L \not{D} \Psi_i^L + i \sum_{i,\sigma} \bar{\Psi}_{i,\sigma}^R \not{D} \Psi_{i,\sigma}^R \right) \quad (\text{A4})$$

where the sum is over all left fermionic doublets and the right singlets.

The index σ runs over the right fermion fields corresponding to the two components of the associated left doublet, namely if $\Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, then $\Psi_{R,+} = \nu_R, \Psi_{R,-} = e_R$.

The covariant derivative is defined by

$$D_\mu = \partial_\mu - ig_2 I_a A_{a\mu} + ig_1 \frac{Y}{2} B_\mu \quad (\text{A5})$$

where Y is the hypercharge. The electric charge is related to Y by the Gell-Mann-Nishijima formula $Q = I_3 + \frac{Y}{2}$.

1. BRST Symmetry

We collect here the BRST symmetry of the SM. c_a are the $SU_L(2)$ ghosts, c_0 is the $U_Y(1)$ ghost:

$$\begin{aligned}
\mathfrak{s}A_{a\mu} &= \partial_\mu c_a + g_2 f_{abc} A_{b\mu} c_c, \\
\mathfrak{s}B_\mu &= \partial_\mu c_0, \\
\mathfrak{s}\Phi &= \left(-ig_1 \frac{1}{2} c_0 + ig_2 \frac{\tau_a}{2} c_a \right) \Phi, \\
\mathfrak{s}\Psi_i^L &= \left(-ig_1 \frac{Y_i^L}{2} c_0 + ig_2 \frac{\tau_a}{2} c_a \right) \Psi_i^L, \quad \mathfrak{s}\Psi_{i,\sigma}^R = -ig_1 \frac{Y_{i,\sigma}^R}{2} c_0 \Psi_{i,\sigma}^R, \\
\mathfrak{s}X_1 &= \mathfrak{s}X_2 = \mathfrak{s}\bar{c} = \mathfrak{s}c = 0, \\
\mathfrak{s}c_a &= -\frac{g_2}{2} f_{abc} c_b c_c, \quad \mathfrak{s}c_0 = 0.
\end{aligned} \tag{A6}$$

The ghosts in the physical basis of the fields W_μ^\pm, A_μ, Z_μ are obtained by

$$c^\pm = \frac{1}{\sqrt{2}}(c^1 \mp ic^2), \quad c_A = c_W c_0 - s_W c_3, \quad c_Z = s_W c_0 + c_W c_3, \tag{A7}$$

where c_W, s_W are the cosine and sine of the Weinberg angle (see Eq.(5.6)). The Nakanishi-Lautrup fields in Eq.(5.8) form BRST doublets with the antighosts:

$$\mathfrak{s}\bar{c}^\pm = b^\pm, \quad \mathfrak{s}b^\pm = 0, \quad \mathfrak{s}\bar{c}^A = b^A, \quad \mathfrak{s}b^A = 0, \quad \mathfrak{s}\bar{c}^Z = b^Z, \quad \mathfrak{s}b^Z = 0. \tag{A8}$$

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