$SU(2N_F)$ symmetry of QCD at high temperature and its implications

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If above a critical temperature not only the $SU(N_F)_L \times SU(N_F)_R$ chiral symmetry of QCD but also the $U(1)_A$ symmetry is restored, then the actual symmetry of the QCD correlation functions and observables is $SU(2N_F)$. Such a symmetry prohibits existence of deconfined quarks and gluons. Hence QCD at high temperature is also in the confining regime and elementary objects are $SU(2N_F)$ symmetric "hadrons" with not yet known properties.

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INTRODUCTION

Nonperturbatively QCD is defined in terms of its fundamental degrees of freedom, quarks and gluons in Euclidean space-time. These fundamental degrees of freedom are never observed in Minkowski space, a property of QCD which is called confinement. Only hadrons are observed. It is believed, however, that at high temperature QCD is in a deconfinement regime and its fundamental degrees of freedom, quarks and gluons, are liberated. Is it true? Here we present results of our recent findings [1] that suggest that this is actually not true.

In Minkowski space-time the QCD Lagrangian in the chiral limit is invariant under the chiral transformations,

$$SU(N_F)_L \times SU(N_F)_R \times U(1)_A \times U(1)_V. \qquad (1)$$

The axial $U(1)_A$ symmetry is broken by anomaly [2]. The $SU(N_F)_L \times SU(N_F)_R$ symmetry is broken spontaneously by the quark condensate in the vacuum. According to the Bancs-Casher relation [3] the quark condensate in Minkowski space can be expressed through a density of the near-zero modes of the Euclidean Dirac operator,

$$\lim_{m\to 0} < 0|\bar{\Psi}(x)\Psi(x)|0 >= -\pi\rho(0)\ . \qquad (2)$$

Consequently, if we remove by hands the near-zero modes of the Dirac operator we can expect a restoration of the chiral $SU(N_F)_L \times SU(N_F)_R$ symmetry in correlation functions. If hadrons survive this "surgery", then the chiral partners should become degenerate. The chiral partners of the $J = 1$ mesons are shown in Fig. 1.

It was observed in $N_F = 2$ dynamical simulations with the overlap Dirac operator that indeed hadrons survive this truncation (except for the ground states of $J = 0$ mesons) and the chiral partners get degenerate [4–7]. Not only the $SU(2)_L \times SU(2)_R$ restoration was observed. Mesons that are connected by the $U(1)_A$ transformation get also degenerate. We conclude that the same low-lying modes of the Dirac operator are responsible for both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ breakings, which is consistent with the instanton-induced mechanism for both breakings [8].

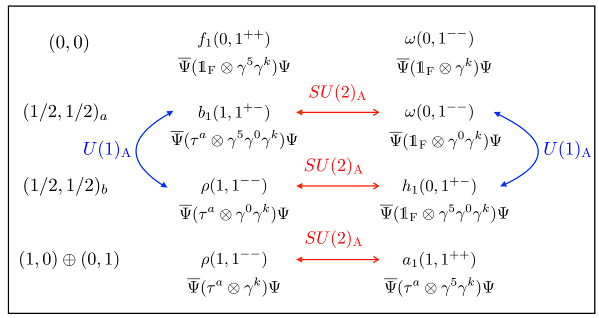

FIG. 1: $SU(2)_L \times SU(2)_R$ and $U(1)_A$ classification of the $J = 1$ meson operators.

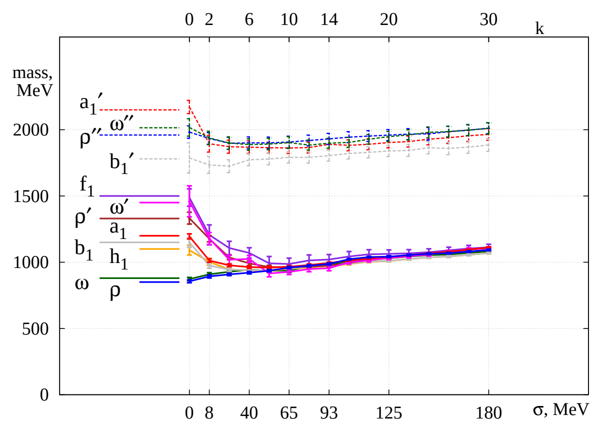

FIG. 2: $J = 1$ meson mass evolution as a function of the number k of truncated lowest-lying Dirac modes. σ shows energy gap in the Dirac spectrum.

Restoration of the full chiral symmetry $SU(2)_L \times SU(2)_R \times U(1)_A$ of the QCD Lagrangian assumes degeneracies marked by arrows in Fig. 1. However, a larger degeneracy that includes all possible chiral multiplets in Fig. 1 was detected, see Fig. 2.

This unexpected degeneracy implies a symmetry that is larger than the chiral symmetry of the QCD Lagrangian. This not yet known symmetry was reconstructed in refs. [9, 10] and turned out to be

$$SU(2N_F) \supset SU(N_F)_L \times SU(N_F)_R \times U(1)_A. \quad (3)$$

This group includes as a subgroup the $SU(2)_{CS}$ (chiralspin) invariance. The $SU(2)_{CS}$ chiralspin generators are

$$\mathbf{\Sigma} = \{\gamma^0, i\gamma^5\gamma^0, -\gamma^5\}\ , \qquad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk}\,\Sigma^k\ .$$

The Dirac spinor transforms under a global or local $SU(2)_{CS}$ transformation as

$$\Psi \to \Psi' = e^{i\varepsilon\cdot\mathbf{\Sigma}/2}\Psi\ . \quad (4)$$

The γ^5 generates a $U(1)_A$ subgroup of $SU(2)_{CS}$. The γ^0 and $i\gamma^5\gamma^0$ mix the left- and right-handed components of the Dirac spinors. When we combine the $SU(2)_{CS}$ generators with the $SU(N_F)$ generators we arrive at the $SU(2N_F)$ group.

$SU(2N_F)$ AS A HIDDEN CLASSICAL SYMMETRY OF QCD [11].

The $SU(4)$ symmetry of $N_F = 2$ Euclidean QCD was obtained in lattice simulations. This means that this symmetry must be encoded in the nonperturbative Euclidean formulation of QCD. Obviously the Euclidean Lagrangian for N_F degenerate quarks in a given gauge background $A_\mu(x)$,

$$\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x), \quad (5)$$

is not $SU(2)_{CS}$ and $SU(2N_F)$-symmetric, because the Dirac operator does not commute with the generators of $SU(2)_{CS}$. A fundamental dynamical reason for absence of these symmetries are zero modes of the Dirac operator, $\gamma_\mu D_\mu \Psi_0(x) = 0$. The zero modes are chiral, L or R. With a gauge configuration of a nonzero global topological charge the number of the left-handed and right-handed zero modes is according to the Atiyah-Singer theorem not equal. Consequently, there is no one-to-one correspondence of the left- and right-handed zero modes. The $SU(2)_{CS}$ chiralspin rotations mix the left- and right-handed Dirac spinors. Such a mixing can be defined only if there is a one-to-one mapping of the left- and right-handed spinors: The zero modes break the $SU(2)_{CS}$ invariance.

We can expand independent fields $\Psi(x)$ and $\Psi^\dagger(x)$ over a complete and orthonormal set $\Psi_n(x)$ of the eigenvalue problem

$$i\gamma_\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x), \quad (6)$$

$$\Psi(x) = \sum_n c_n \Psi_n(x), \qquad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi_k^\dagger(x), \quad (7)$$

where $\bar{c}_k, c_n$ are Grassmann numbers. The fermionic part of the QCD partition function takes the following form

$$Z = \int \prod_{k,n} d\bar{c}_k dc_n e^{\sum_{k,n}\int d^4x \bar{c}_k c_n (\lambda_n + im)\Psi_k^\dagger(x)\Psi_n(x)}. \quad (8)$$

In a finite volume the eigenmodes of the Dirac operator can be separated into two classes. The exact zero modes, $\lambda = 0$, and nonzero eigenmodes, $\lambda_n \neq 0$. It is well understood that the exact zero modes are irrelevant since their contributions to the Green functions and observables vanish in the thermodynamic limit $V \to \infty$ as $1/V$ [12–14]. Consequently, in the finite-volume calculations we can ignore the exact zero-modes.

Now we can read off the symmetry properties of the partition function (8). For any $SU(2)_{CS}$ and $SU(2N_F)$ rotation the Ψ_n and $\Psi_k^\dagger$ Dirac spinors transform as

$$\Psi_n \to U\Psi_n, \quad \Psi_k^\dagger \to (U\Psi_k)^\dagger, \quad (9)$$

where U is any transformation from the groups $SU(2)_{CS}$ and $SU(2N_F)$, $U^\dagger = U^{-1}$. It is then clear that the exponential part of the partition function is invariant under global and local $SU(2)_{CS}$ and $SU(2N_F)$ transformations, because

$$(U\Psi_k(x))^\dagger U\Psi_n(x) = \Psi_k^\dagger(x)\Psi_n(x). \quad (10)$$

The exact zero modes contributions

$$\Psi_0^\dagger(x)\Psi_n(x), \Psi_k^\dagger(x)\Psi_0(x), \Psi_0^\dagger(x)\Psi_0(x),$$

for which the equation (10) is not defined, are irrelevant in the thermodynamic limit and can be ignored. In other words, QCD classically without the irrelevant exact zero modes has in a finite volume V local $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. These are hidden classical symmetries of QCD.

The integration measure in the partition function is not invariant under a local $U(1)_A$ transformation [2], which is a source of the $U(1)_A$ anomaly. The $U(1)_A$ is a subgroup of $SU(2)_{CS}$. Hence the axial anomaly breaks $SU(2)_{CS}$ and $SU(2N_F) \to SU(N_F)_L \times SU(N_F)_R$.

In the limit $V \to \infty$ the otherwise finite lowest eigenvalues λ condense around zero and provide according to the Banks-Casher relation at $m \to 0$ a nonvanishing quark condensate in Minkowski space. The quark condensate in Minkowski space-time breaks all $U(1)_A$, $SU(N_F)_L \times SU(N_F)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ symmetries to $SU(N_F)_V$. In other words, the hidden classical $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are broken both by the anomaly and spontaneously.

RESTORATION OF $SU(2)_{CS}$ AND $SU(2N_F)$ AT HIGH TEMPERATURE [1]

Above the chiral restoration phase transition the quark condensate vanishes. If in addition the $U(1)_A$ symmetry is also restored [15–17] and a gap opens in the Dirac spectrum, then above the critical temperture the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are manifest. The precise meaning of this statement is that the correlation functions and observables are $SU(2)_{CS}$ and $SU(2N_F)$ symmetric.

These $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of QCD imply that there cannot be deconfined free quarks and gluons at any finite temperature in Minkowski space-time. Indeed the Green functions and observables calculated in terms of unconfined quarks and gluons in Minkowski space (i.e. within the perturbation theory) cannot be $SU(2)_{CS}$ and $SU(2N_F)$ symmetric, because the chromomagnetic interaction necessarily breaks both symmetries. Then it follows that above T_c QCD is in a confining regime. In contrast, color-singlet $SU(2N_F)$-symmetric "hadrons" (with not yet known properties) are not prohibited by the $SU(2N_F)$ symmetry and can freely propagate. "Hadrons" with such a symmetry in Minkowski space can be constructed [18].

PREDICTIONS

Restoration of the $SU(2)_{CS}$ and of $SU(2N_F)$ symmetries at high temperatures can be tested on the lattice.

Transformation properties of hadron operators under $SU(2)_{CS}$ and $SU(2N_F)$ groups are given in refs. [7, 10]. For example, the isovector $J = 1$ operators $\bar{\Psi}\vec{\tau}\gamma^i\Psi, (1^{--})$; $\bar{\Psi}\vec{\tau}\gamma^0\gamma^i\Psi, (1^{--})$; $\bar{\Psi}\vec{\tau}\gamma^0\gamma^5\gamma^i\Psi, (1^{+-})$ form an irreducible representation of $SU(2)_{CS}$. One expects that below T_c all three diagonal correlators will be different and the off-diagonal cross-correlator of (1^{--}) operators will not be zero. Above T_c a $SU(2)_{CS}$ restoration requires that all diagonal correlators should become identical and the off-diagonal correlator of (1^{--}) currents should vanish. A restoration of $SU(2)_{CS}$ and of $SU(N_F)_L \times SU(N_F)_R$ implies a restoration of $SU(2N_F)$.

A similar prediction can be made with the baryon operators.

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