

# Meson cloud effects on the pion quark distribution function in the chiral constituent quark model

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We investigate the valence quark distribution function of the pion  $v^\pi(x, Q^2)$  in the framework of the chiral constituent quark model and evaluate the meson cloud effects on  $v^\pi(x, Q^2)$ . We explicitly demonstrate how the meson cloud effects affect  $v^\pi(x, Q^2)$  in detail. We find that the meson cloud correction causes an overall 32% reduction of the valence quark distribution and an enhancement at the small Bjorken  $x$  regime. Besides, we also find that the dressing effect of the meson cloud will make the valence quark distribution to be softer in the large  $x$  region.

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## I. INTRODUCTION

The parton distribution functions (PDFs) of the hadrons delineate the partonic structure of the hadrons and have been intensively studied for several decades. However their non-perturbative nature prevents any direct application of the fundamental theory of the strong interaction, quantum chromo dynamics (QCD), since the usual perturbative technique is not applicable here. Lattice QCD can provide some moments of the PDFs, and recently new attempts to calculate the  $x$  dependencies of PDFs on the lattice have been done motivated by the X. Ji's proposal [1, 2], although the extraction of the light-cone PDFs from the quasi-PDFs is still under discussion. Therefore, it remains one of the greatest challenges in hadronic physics to understand the PDFs of the hadrons from the fundamental principles of QCD.

Fortunately one is still able to make connections between the hadronic phenomenology at low energy and QCD in indirect ways. For example, the  $SU(3)$  flavour symmetry of QCD provides us the clues to study the property of the hadronic bound states even though the strange quark masses explicitly break this symmetry. Furthermore we have also learned that the light pseudoscalar mesons are the Nambu-Goldstone bosons. It is the consequence of the spontaneously chiral symmetry breaking of the QCD vacuum. Those Nambu-Goldstone bosons play dominant roles in the low energy hadronic structures and interactions because they will couple to particles and develop the meson clouds around the core. The meson clouds have important impacts on the hadronic interactions and structures.

Actually it also explains why the constituent quark picture works so well in some aspect of low energy hadronic phenomenology because there are always meson clouds surrounding the light quarks such that the light quarks appear to be much heavier objects called the constituent quarks. Furthermore, the sea-quarks and the gluons are absorbed into constituent quarks as the components of the meson clouds. The hadrons are now treated as the bound states of these constituent quarks. One may expect to use such a picture to compute the PDFs of the quarks at low  $Q^2$  and the sea-quarks and gluons will reappear as the PDFs are evolved by the DGLAP equation to higher  $Q^2$  in which high energy scatterings occur. However, to apply the leading-order or even the next-to-leading-order (NLO) QCD evolution when the initial  $Q$  value is far below 1 GeV is hard to justify because the value of  $\alpha_s$  is usually too large. Hence it is necessary to take the meson cloud effects into account when one computes the PDFs in constituent quark models since the inner structure of the constituent quarks is not completely irrelevant [3, 4].

There are many applications of this meson cloud dressing scheme to the PDFs, but most of them are devoted to the PDFs of the nucleon. For example, it has been found that the Goldstone boson fluctuations generate a flavour

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asymmetry of the quark distribution functions inside the nucleon and significant depolarization effects reducing the fraction of the nucleon spin carried by the quarks [5]. However the study of the effects of Goldstone boson fluctuation on the quark distribution functions inside the pion was less active because it is much difficult to extract the distributions experimentally. Our knowledge of the pion quark distribution is only from the Drell-Yan di-muon production by charged pion incident on the fixed nuclear target  $\pi^\pm N \rightarrow \mu^+ \mu^- X$  [6–8].

Nevertheless, the pion is still one of the most important subjects in hadronic physics because of its Nambu-Goldstone character and its simplest quark content so that there are several theoretical efforts focusing on the PDFs of the pion [9–20] and some related lattice studies [21, 22]. In particular, the valence quark distribution functions of the pion have been extracted from the analysis including the next-to-leading-logarithmic threshold resummation effects in the calculation of the Drell-Yan cross section [23]. They found that at large  $x$  the distribution becomes much softer than those shown in well-known PDF sets of the pion, e.g., SMRS [24] and GRS [25]. Interestingly, their results are in very good agreement with those of the preceding study based on Dyson-Schwinger equations [13].

In this article we would like to investigate the meson cloud dressing effects of the quark distribution functions of the pion qualitatively. At first we look for the bare quark distribution functions which can reproduce the phenomenological satisfactory result in Ref. [23] after the dressing processes. Then we study how the meson cloud dressing effects modify the  $x$ -dependence of the bare quark distribution functions. We also figure out how the meson cloud effects in  $v^\pi(x, Q^2)$  change as  $Q^2$  changes.

This article is organized as follows: We briefly review how to calculate the meson cloud dressing effects in general in Sec. II. In Sec. III we explain the way we study the meson cloud effects on the pion valence quark distribution functions. We present and discuss our numerical results in Sec. IV and give our conclusion in Sec. V.

## II. DRESSING CORRECTIONS TO CONSTITUENT QUARKS

In this section we give a brief review of the method to compute the meson cloud dressing corrections on constituent quarks following Ref. [5]. The interactions between the Nambu-Goldstone bosons and the constituent quarks are described by the following effective Lagrangian,

$$\mathcal{L}_{int} = -\frac{g_A}{f} \bar{\psi} \gamma^\mu \gamma_5 (\partial_\mu \Pi) \psi, \quad (1)$$

with the constituent quark field  $\psi$  and the Nambu-Goldstone boson fields  $\Pi$  defined as

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (2)$$

Here  $f$  and  $g_A$  stands for the pseudoscalar decay constant and the quark axial-vector coupling constant, respectively. The dressed and bare states of constituent quarks are related by the following relations,

$$\begin{aligned} |U\rangle &= \sqrt{Z} |u_0\rangle + a_\pi |d\pi^+\rangle + \frac{a_\pi}{2} |u\pi^0\rangle + a_K |sK^+\rangle + \frac{a_\eta}{6} |u\eta\rangle, \\ |D\rangle &= \sqrt{Z} |d_0\rangle + a_\pi |u\pi^-\rangle + \frac{a_\pi}{2} |d\pi^0\rangle + a_K |sK^0\rangle + \frac{a_\eta}{6} |d\eta\rangle, \end{aligned} \quad (3)$$

where  $Z$  is the renormalization constant for a bare constituent quark.  $|a_\alpha|^2$  means the probability of discovering a Goldstone boson  $\alpha$  in the dressed state of a constituent quark. The meson cloud dressing effects are depicted as the diagrams in Fig. 1. In Fig. 1 (a) the Goldstone bosons around the constituent quarks are just spectators and the virtual photons directly couple to the constituent quarks. In Fig. 1 (b) the virtual photons couple to the sea-quark pair emitted by the Goldstone boson dressing the constituent quarks. Those diagrams are evaluated in the infinite momentum frame so that the calculation will be simplified a lot by neglecting the  $Z$ -graph terms. The quark distribution with the correction corresponding to Fig. 1 (a) is given by

$$q_j(x) = \int_x^1 \frac{dy}{y} P_{j\alpha/i}(y) q_i\left(\frac{x}{y}\right), \quad (4)$$

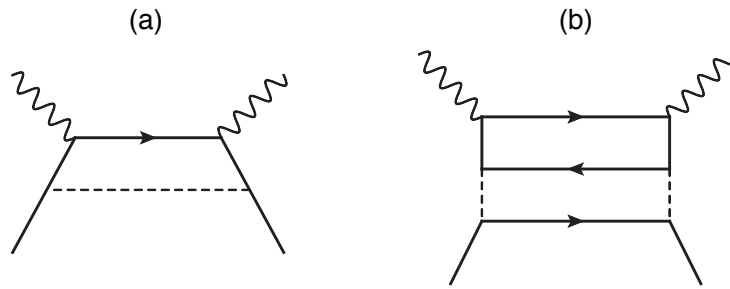


FIG. 1: Diagrams representing the Goldstone boson dressing corrections to a constituent quark. The wavy lines denote the virtual photons. The thick lines denote the constituent quarks and the dashed lines represent the Goldstone bosons. (a) describes the process in which the virtual photons couple directly to the constituent quarks and the Goldstone boson is emitted and absorbed by the constituent quark but does not couple to the photons. (b) describes the process where the virtual photons couple to the sea-quark pair emitted by the Goldstone boson dressing the constituent quarks.

where  $P_{j\alpha/i}$  is the splitting function which gives the probability to find a constituent quark  $j$  carrying the momentum fraction  $y$  together with a spectator Goldstone boson  $\alpha$ , whose parent constituent quark is denoted by  $i$ . With the masses of the constituent quarks and the pseudoscalar mesons,  $m_i$ ,  $m_j$ , and  $m_\alpha$ ,  $P_{j\alpha/i}$  is expressed as

$$P_{j\alpha/i}(y) = \frac{1}{8\pi^2} \left( \frac{g_A \bar{m}}{f} \right)^2 \int dk_T^2 \frac{(m_j - m_i y)^2 + k_T^2}{y^2 (1-y) (m_i^2 - M_{j\alpha}^2)^2}. \quad (5)$$

$M_{j\alpha}^2$  is the invariant mass squared of the final state expressed as

$$M_{j\alpha}^2 = \frac{m_j^2 + k_T^2}{y} + \frac{m_\alpha^2 + k_T^2}{1-y}, \quad (6)$$

and  $\bar{m} = (m_i + m_j)/2$  is the average of the constituent quark masses. To perform the integration of Eq. (5), a cutoff is required. Therefore we replace  $g_A$  by the following function [26],

$$g_A \exp\left(\frac{m_i^2 - M_{j\alpha}^2}{4\Lambda^2}\right), \quad (7)$$

where  $\Lambda$  is the cutoff parameter. The moments of the splitting functions are defined by

$$\langle x^{n-1} P_{j\alpha/i} \rangle \equiv \int_0^1 dx x^{n-1} P_{j\alpha/i}(x), \quad (8)$$

and the first moments with  $n = 1$ ,

$$\langle P_{j\alpha/i} \rangle = \langle P_{\alpha j/i} \rangle \equiv \langle P_\alpha \rangle, \quad (9)$$

are also defined. The renormalization constant  $Z$  is then expressed as

$$Z = 1 - \frac{3}{2} \langle P_\pi \rangle - \langle P_K \rangle. \quad (10)$$

Here only the pion and kaon contributions are considered. Since the contribution from  $\eta$  is relatively small compared to them due to the factor  $1/6$  which appears in Eqs. (3), we simply neglect it in this study.

The diagram Fig. 1 (b) represents the process in which the sea-quark pair emitted from Goldstone boson is probed. The corresponding contribution is given by

$$q_k(x) = \int \frac{dy_1}{y_1} \frac{dy_2}{y_2} V_{k/\alpha} \left( \frac{x}{y_1} \right) P_{\alpha j/i} \left( \frac{y_1}{y_2} \right) q_i(y_2). \quad (11)$$

Here  $P_{\alpha j/i}(x) = P_{j\alpha/i}(1-x)$  and  $V_{k/\alpha}$  is the quark distribution function with the flavor index  $k$  in the Goldstone boson  $\alpha$ , for which the normalization condition,

$$\int_0^1 dx V_{k/\alpha}(x) = 1, \quad (12)$$

is satisfied.

TABLE I: The set of parameters chosen in this article.

$g_A$	$m_u$	$m_d$	$m_s$	$m_\pi$	$m_K$	$f_\pi$	$\Lambda$
1.0	360 MeV	360 MeV	570 MeV	140 MeV	494 MeV	93 MeV	1.4 GeV

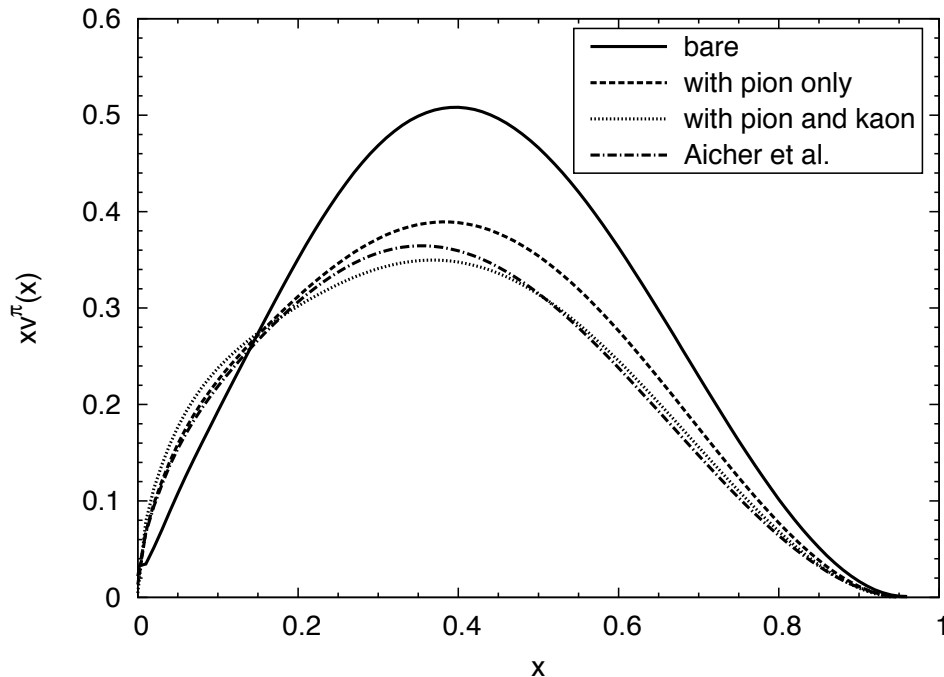


FIG. 2: The valence quark distribution functions of the pion at  $Q = 4$  GeV as a function of Bjorken  $x$ . The solid curve shows the bare distribution  $v_{\text{bare}}^\pi(x)$ . The dashed and dotted lines represent the dressed  $v^\pi(x)$  including only the pion dressing effect, and both the pion and kaon dressing effects, respectively. The dashed-dotted line represents the empirical curve provided in Ref. [23].

### III. DRESSING CORRECTIONS TO THE VALENCE QUARK DISTRIBUTION FUNCTIONS OF PION

In this section we will apply the procedure in the previous section to the pion valence quark distribution functions. To evaluate the meson cloud dressing corrections, we choose the values for some parameters shown in Table I. These are typical values guided by preceding studies based on the Nambu–Jona-Lasinio model [27–29]. It has been observed for the nucleon that the obtained value of the Gottfried sum rule with this parameter set in the present framework is consistent with the experimentally observed empirical one [5].

Our first task is to choose the appropriate bare quark distribution functions of  $\pi^+$ ,  $u_0(x)$  and  $\bar{d}_0(x)$  satisfying the normalization conditions,

$$\int_0^1 dx u_0(x) = \int_0^1 dx \bar{d}_0(x) = 1. \quad (13)$$

Through this article we adopt the following short hand notation,

$$P \otimes q \equiv \int_x^1 \frac{dy}{y} P(y) q\left(\frac{x}{y}\right). \quad (14)$$

According to Eq. (4) and Eq. (11), after the dressing the distribution functions of  $u$  and  $\bar{u}$  quarks in  $\pi^+$  become as

follows,

$$\begin{aligned}
u(x) = & Z u_0(x) + \frac{1}{2} P_{u\pi/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi d/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi \bar{u}/\bar{d}} \otimes \bar{d}_0 \\
& + \frac{1}{4} V_{u/\pi} \otimes P_{\pi u/u} \otimes u_0 + \frac{1}{4} V_{u/\pi} \otimes P_{\pi \bar{d}/\bar{d}} \otimes \bar{d}_0 + V_{u/K} \otimes P_{Ks/u} \otimes u_0,
\end{aligned} \tag{15}$$

$$\bar{u}(x) = P_{\bar{u}\pi/\bar{d}} \otimes \bar{d}_0 + \frac{1}{4} V_{\bar{u}/\pi} \otimes P_{\pi u/u} \otimes u_0 + \frac{1}{4} V_{\bar{u}/\pi} \otimes P_{\pi \bar{d}/\bar{d}} \otimes \bar{d}_0. \tag{16}$$

We then define the valence quark PDF of the pion as  $v_{\text{dressed}}^\pi(x) = u_v^{\pi^+}(x) = \bar{d}_v^{\pi^+}(x) = d_v^{\pi^-}(x) = \bar{u}_v^{\pi^-}(x)$ , and obtain the following expression,

$$\begin{aligned}
v_{\text{dressed}}^\pi(x) = & u(x) - \bar{u}(x) \\
= & Z u_0(x) + \frac{1}{2} P_{u\pi/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi d/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi \bar{u}/\bar{d}} \otimes \bar{d}_0 \\
& + V_{u/K} \otimes P_{Ks/u} \otimes u_0 - P_{\bar{u}\pi/\bar{d}} \otimes \bar{d}_0.
\end{aligned} \tag{17}$$

Integrating both sides of Eq. (17), we obtain

$$\int_0^1 dx v_{\text{dressed}}^\pi(x) = \int_0^1 dx \{u(x) - \bar{u}(x)\} = Z + \frac{3}{2} \langle P_\pi \rangle + \langle P_K \rangle = 1, \tag{18}$$

which shows the correct normalization, following Eq. (10).

To make the physics to be more transparent, we divide the dressed valence PDF into four terms according to their origins,

$$v^\pi(x, Q^2) = v^{\pi,0}(x, Q^2) + v^{\pi,a}(x, Q^2) + v^{\pi,b1}(x, Q^2) + v^{\pi,b2}(x, Q^2), \tag{19}$$

where each term is defined as follows,

$$\begin{aligned}
v^{\pi,0}(x, Q^2) &= Z u_0(x), \\
v^{\pi,a}(x, Q^2) &= \frac{1}{2} P_{u\pi/u} \otimes u_0 - P_{\bar{u}\pi/\bar{d}} \otimes \bar{d}_0, \\
v^{\pi,b1}(x, Q^2) &= V_{u/\pi} \otimes P_{\pi d/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi \bar{u}/\bar{d}} \otimes \bar{d}_0, \\
v^{\pi,b2}(x, Q^2) &= V_{u/K} \otimes P_{Ks/u} \otimes u_0.
\end{aligned} \tag{20}$$

$v^{\pi,a}(x, Q^2)$  is the contribution from Fig. 1 (a).  $v^{\pi,b1}(x, Q^2)$  and  $v^{\pi,b2}(x, Q^2)$  are the contributions from Fig. 1 (b) with the pion cloud and Fig. 1 (b) with the kaon cloud, respectively. For simplicity, we assume here the same initial  $u$  quark distribution of the involved  $K^+$  as pion's. To perform the  $Q^2$  evolution, we utilize the code produced by Kobayashi, Konuma, and Kumano [30], and the results presented in the next section are obtained at NLO accuracy.

#### IV. RESULTS AND DISCUSSION

In this section we present and discuss our numerical results. First of all, we shall fix the bare valence quark distribution function of the pion  $v_{\text{bare}}^\pi(x, Q_0^2)$ . We assume that only the valence quarks exist at the initial scale, and start with the symmetric distribution,  $v_{\text{bare}}^\pi(x, Q_0^2) \propto x^\alpha(1-x)^\alpha$ . We try to find the value of  $\alpha$  by fitting  $v_{\text{dressed}}^\pi(x, Q^2)$  to the distribution obtained from the fit 3 in Ref. [23] at  $Q = 4$  GeV, varying the initial scale  $Q_0$ . Since our purpose of this study is to discuss the meson cloud effects qualitatively, we simply choose an integer value for  $\alpha$ , and then fix the bare distribution as

$$v_{\text{bare}}^\pi(x, Q_0^2) = N x^{1.8} (1-x)^{1.8}, \tag{21}$$

where  $N$  is the normalization constant. The initial scale  $Q_0 = 0.50$  GeV is determined by fitting the resulting first moment to that of Aicher et al. at  $Q = 4$  GeV [23]. Our bare  $v^\pi(x, Q^2)$  at  $Q = 4$  GeV is depicted as the solid line in Fig. 2. We also present the dressed  $v^\pi(x, Q^2)$  with only pion cloud (in dashed line) and the one fully dressed with the pion and kaon clouds (in dotted line) at  $Q = 4$  GeV. One can compare our full result with the phenomenologically satisfactory one (in dashed-dotted line) given by Ref. [23]. Our full result agrees with the empirical one excellently

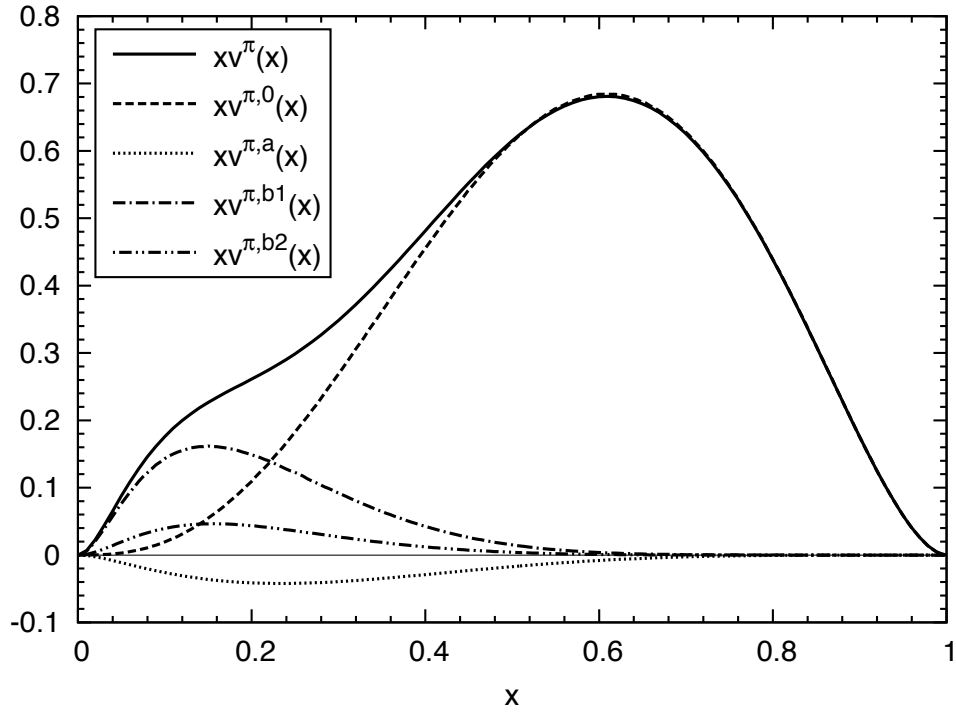


FIG. 3: The contributions of each term in Eq. (19) to  $xv_{\text{dressed}}^{\pi}(x)$  at  $Q = Q_0$ . The dashed, dotted, dashed-dotted and dashed-double dotted lines represent the contributions of the first term  $v^{\pi,0}(x)$ , second term  $v^{\pi,a}(x)$ , third term  $v^{\pi,b1}(x)$  and fourth term  $v^{\pi,b2}(x)$ , respectively. The solid line represents the sum of the each terms.

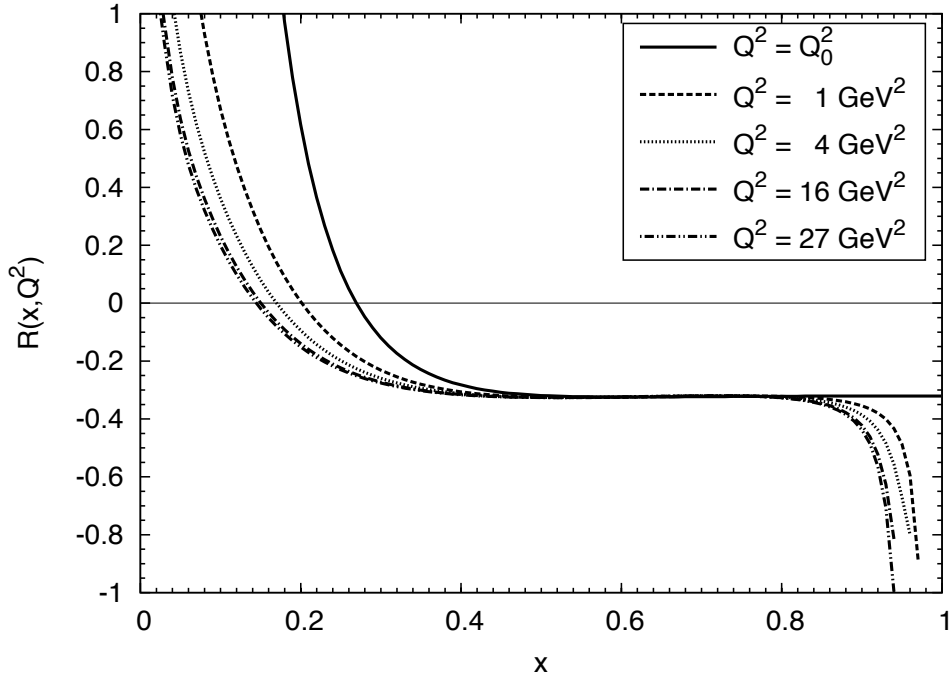


FIG. 4: The ratio of the valence quark distribution functions of the pion,  $R(x, Q^2)$  defined in Eq. (22), as a function of Bjorken  $x$  for various  $Q^2$ . The solid, dashed, dotted, dashed-dotted and dashed-double dotted lines stand for  $Q^2 = Q_0^2, 1, 4, 16, 27 \text{ GeV}^2$  cases, respectively.

when  $x \geq 0.5$  and becomes a slightly smaller between  $x = 0.4$  and  $0.8$ , and slightly larger when  $x \leq 0.2$ . However the difference between our result and the empirical curve is always less than few percents such that our study in this article is qualitatively reliable and not just a prototype study.

Fig. 3 provides a clear anatomy of the meson cloud dressing effects on the pion valence quark distribution function at  $Q = Q_0$  since the contribution of each term in Eq. (19) is demonstrated explicitly there. The solid curve corresponds to  $xv^{\pi,0}(x)$  which is just the bare valence quark PDF multiplying the renormalization factor  $Z$ . The value of  $Z$  is about  $2/3$ . Actually one finds that  $xv^{\pi,0}(x)$  and the total dressed  $xv^\pi(x)$  are almost identical when  $x \geq 0.5$ . Namely, at  $Q = Q_0$  the meson cloud dressing effect is exclusively the effect of the renormalization factor  $Z$  in the large  $x$  regime. The contribution from Fig. 1 (a) represented in the dotted line in Fig. 3 is negative and its magnitude is relatively small compared with  $xv^{\pi,0}(x)$  and  $xv^{\pi,b1}(x)$ . On the contrary, the contribution from Fig. 1 (b) involving the pion cloud is positive and much larger than  $xv^{\pi,a}(x)$  in the magnitude. It is represented by the dashed-dotted line in Fig. 3. But it is only nonzero within  $0 \leq x \leq 0.6$ . Moreover its effect is cancelled by the  $v^{\pi,a}$  in the regime  $0.4 \leq x \leq 0.6$ . Hence its effect is most significant in the regime  $0 \leq x \leq 0.4$ . The contribution of Fig. 1 (b) involving the kaon cloud represented in the dashed-double dotted line in Fig. 3 is small but positive. Its contribution is largely cancelled by the  $xv^{\pi,a}(x)$  between  $x = 0$  and  $0.2$ . Hence the curve corresponding to the total pion valence quark distribution function at  $Q = Q_0$  is just the sum of  $xv^{\pi,0}(x)$  and  $xv^{\pi,b1}(x)$  approximately.

Naively one may think that there is no dressing effect in the large  $x$  regime except the overall 32% reduction according to the above analysis. Nevertheless the further analysis shows otherwise. To observe the total dressing effect one is able to define the following quantity,

$$R(x, Q^2) = \frac{v_{\text{dressed}}^\pi(x, Q^2) - v_{\text{bare}}^\pi(x, Q^2)}{v_{\text{bare}}^\pi(x, Q^2)}. \quad (22)$$

The quantity characterizes the extent of the modification of the valence quark distribution function due to the Goldstone boson dressing effects. Our results are demonstrated in Fig. 4. At  $Q = Q_0$  indeed the dressing effect is simply overall 32% reduction due to the renormalization factor  $Z$  in the large  $x$  region. However, when one goes to a higher  $Q$  value, a suppression of  $R(x, Q^2)$  appears in the region of  $x \geq 0.8$ . Namely, the curves of  $R(x, Q^2)$  drop rather dramatically at the right end of the Fig. 4. This suppression becomes more pronounced as the  $Q^2$  increases. Since the large  $x$  behavior of the valence quark PDF usually can be characterized by some exponent  $\beta$  such that  $v^\pi(x) \sim (1-x)^\beta$  as  $x \sim 1$ . The dressing effect will increase the value of  $\beta$  substantially as  $Q^2$  increases. Furthermore, there is a plateau between  $x = 0.4$  and  $0.8$  where the dressing effect is just overall 32% reduction for all curves at different  $Q^2$  values. On the other hand, in the small  $x$  regime, the Goldstone boson dressing effect is to enhance the  $v^\pi(x)$  significantly. But the magnitude of this enhancement decreases very fast as  $Q^2$  increases. For example, one can find in Fig. 4, at  $x = 0.2$  and  $Q = 1$  GeV the enhancement of  $v^\pi$  is about 30%, but the enhancement is reduced to only 15% at  $Q = 2$  GeV, moreover, at  $Q = 4$  GeV the enhancement becomes only 10%. The enhancement is mainly due to the contribution of the Fig. 1 (b) involving the pion cloud according to Fig. 3. We conclude that the meson cloud dressing effect on the valence quark distribution function is an overall 32% reduction with an enhancement at the small  $x$  regime  $0 \leq x \leq 0.4$  and an additional suppression in the  $x \geq 0.8$  region. This generic feature is shared by the curves with different  $Q^2$  values.

## V. CONCLUSION

In this article, we have investigated the meson cloud dressing effects on the pion valence quark distribution function  $v^\pi(x, Q^2)$  in detail. We first choose the bare  $v^\pi(x, Q^2)$  judiciously such that the dressed  $v^\pi(x, Q^2)$  agrees with the phenomenologically satisfactory  $v^\pi(x, Q^2)$  given in Ref. [23], then we study the individual contribution of the meson cloud dressing effects. We find that the meson cloud effect is mainly an overall 32% reduction due to the renormalization factor  $Z$  with a suppression in the large  $x$  regime,  $x \geq 0.8$ , and an enhancement in the small  $x$  region,  $x \leq 0.4$ . The extent of the enhancement decreases fast as  $Q^2$  increases, on the contrary, the magnitude of the suppression increases as  $Q^2$  increases. We also find that the enhancement at the small  $x$  regime is mainly from the pion cloud contribution.

Currently, COMPASS Collaboration at CERN is taking data of the pion-induced Drell-Yan process. Absolute production cross sections will be measured there, and the preceding predictions for the valence quark distribution function of the pion will be tested with the data. Besides, the pion-induced Drell-Yan experiment is also planned at J-PARC in Japan. In this experimental facility, the beam energy is much lower compared to that of the COMPASS experiment, so the cross sections are sensitive to the larger  $x$  region. Hence, our understanding of the valence quark

distribution of the pion in the region will be improved so that our understanding of the bare quark distribution function also will be improved via the framework presented in this article.

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- [1] X. Ji, Phys. Rev. Lett. **110**, 262002 (2013).
  - [2] J. W. Chen, S. D. Cohen, X. Ji, H. W. Lin and J. H. Zhang, Nucl. Phys. B **911**, 246 (2016).
  - [3] W. Melnitchouk and W. Weise, Phys. Lett. B **334**, 275 (1994).
  - [4] S. A. Kulagin, W. Melnitchouk, T. Weigl and W. Weise, Nucl. Phys. A **597**, 515 (1996).
  - [5] K. Suzuki and W. Weise, Nucl. Phys. **A634**, 141(1998).
  - [6] J. S. Conway *et al.*, Phys. Rev. D **39**, 92 (1989).
  - [7] P. Bordalo *et al.* [NA10 Collaboration], Phys. Lett. B **193**, 373 (1987).
  - [8] B. Betev *et al.* [NA10 Collaboration], Z. Phys. C **28**, 9 (1985).
  - [9] T. Shigetani, K. Suzuki and H. Toki, Phys. Lett. B **308**, 383 (1993).
  - [10] R. M. Davidson and E. Ruiz Arriola, Phys. Lett. B **348**, 163 (1995).
  - [11] H. Weigel, E. Ruiz Arriola and L. P. Gamberg, Nucl. Phys. B **560**, 383 (1999).
  - [12] A. E. Dorokhov and L. Tomio, Phys. Rev. D **62**, 014016 (2000).
  - [13] M. B. Hecht, C. D. Roberts, and S. M. Schmidt, Phys. Rev. **C63**, 025213 (2001).
  - [14] T. Nguyen, A. Bashir, C. D. Roberts and P. C. Tandy, Phys. Rev. C **83**, 062201 (2011).
  - [15] K. Wijesooriya, P. E. Reimer and R. J. Holt, Phys. Rev. C **72**, 065203 (2005).
  - [16] A. V. Belitsky, Phys. Lett. B **386**, 359 (1996).
  - [17] S. i. Nam and C. W. Kao, Phys. Rev. D **85**, 034023 (2012).
  - [18] S. i. Nam and C. W. Kao, Phys. Rev. D **85**, 094023 (2012).
  - [19] S. i. Nam, Phys. Rev. D **86**, 074005 (2012).
  - [20] C. Chen, L. Chang, C. D. Roberts, S. Wan and H. S. Zong, Phys. Rev. D **93**, no. 7, 074021 (2016).
  - [21] D. Daniel, R. Gupta and D. G. Richards, Phys. Rev. D **43**, 3715 (1991).
  - [22] W. Detmold, W. Melnitchouk and A. W. Thomas, Phys. Rev. D **68**, 034025 (2003).
  - [23] M. Aicher and A. Schafer and W. Vogelsang, Phys. Rev. Lett. **105**, 252003 (2010).
  - [24] P. J. Sutton, A. D. Martin, R. G. Roberts, and W. J. Stirling, Phys. Rev. D **45**, 2349 (1992).
  - [25] M. Gluck, E. Reya, I. Schienbein, I. Eur. Phys. J. **C 10**, 313 (1999).
  - [26] J. Speth and A. W. Thomas, Adv. Nucl. Phys. **24**, 83 (1997).
  - [27] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
  - [28] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961).
  - [29] T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994).
  - [30] R. Kobayashi, M. Konuma and S. Kumano, Comput. Phys. Commun. **86**, 264 (1995).